Three-dimensional Flow Physics Analyses Using Multi-dimensional Limiting Process

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ABSTRACT

In this paper, we apply three-dimensional limiting process for three-dimensional flow physics analyses. The basic idea of multi-dimensional limiting condition is that the multi-dimensionally interpolated values at a vertex point should be within the maximum and minimum cell-average values of neighboring cells for the monotonic distribution. By applying the MLP (Multi-dimensional Limiting Process) to the three dimensional Euler and Navier-Stokes equations, we can achieve monotonic characteristics, which results in the enhancement of solution accuracy, convergence behavior.

INTRODUCTION

Accurate monotonic schemes for hyperbolic conservation laws are developed based on one-dimensional flow physics through the analysis of TVD limiters [1], [2]. It shows the complete monotonic and accurate distribution in a one-dimensional discontinuity. However, if they are applied to a multi-dimensional problem, the interpolated property, without considering the effect of other flow directions, certainly leads to a non-monotonic distribution. In order to find out the monotonicity condition for multi-dimension, Kim et al. [3] extended the one-dimensional monotonic condition to two-dimensional problem and presented the two-dimensional limiting condition successfully. With the limiting condition, a multi-dimensional limiting process (MLP) is proposed which gives more accurate results for two-dimensional Euler and Navier-Stokes equations. It was the approach which prompted the work of the present paper. Basically, it extends the idea of MLP to three-dimensional problem. Thus, in this paper, we introduce a three-dimensional limiting condition and present the numerical investigation of test cases which include complex physical phenomena.

MLP FOR THREE-DIMENSIONAL FLOWS

After the three-dimensional limiting condition is applied, the general form of MLP is written as follows

$$\mathbf{\Phi}_{i+\frac{1}{2},L} = \mathbf{\Phi}_i + 0.5\max\left(0, \min\left(\alpha_L, \alpha_L r_L, \beta_L\right)\right)\Delta\mathbf{\Phi}_{i-\frac{1}{2}} \qquad (1)$$

$$\mathbf{\Phi}_{i+\frac{1}{2},R} = \mathbf{\Phi}_{i+1} - 0.5\max\left(0, \min\left(\alpha_R, \alpha_R r_R, \beta_R\right)\right)\Delta\mathbf{\Phi}_{i+\frac{3}{2}} \qquad (2)$$

where β determines the type of limiting and α is the multi-dimensional restriction coefficient as follows.

$$\alpha^{+} = g\left(2\max(1, r^{-x}_{i,j,k})\left[\frac{\left(1+\frac{\tan\bar{\theta}_{i+1,j,k}}{r^{+x}_{i+1,j,k}}+\frac{1}{r^{+x}_{i+1,j+1,k}\,r^{+y}_{i,j+1,k}}\frac{\tan\bar{\phi}_{i+1,j+1,k}}{\cos\bar{\theta}_{i+1,j+1,k}}\frac{\tan\bar{\theta}_{i,j,k}}{\tan\bar{\theta}_{i,j+1,k}}\right)}{(1+\tan\theta+\frac{\tan\phi}{\cos\theta})}\right]\right) \quad (3)$$

$$\alpha^{-} = g\left(2\max(1, r^{+x}_{i+1,j,k})\left[\frac{\left(1+\frac{\tan\bar{\theta}_{i+1,j,k}}{r^{-x}_{i,j,k}}+\frac{1}{r^{-x}_{i,j+1,k}\,r^{-y}_{i,j,k}}\frac{\tan\bar{\phi}_{i+1,j+1,k}}{\cos\bar{\theta}_{i+1,j+1,k}}\frac{\tan\bar{\theta}_{i,j,k}}{\tan\bar{\theta}_{i,j+1,k}}\right)}{(1+\tan\theta+\frac{\tan\phi}{\cos\theta})}\right]\right) \quad (4)$$

It is noted that α is a function of multi-dimensional flow parameters such as flow angle, cell-aspect ratio and local slopes.

NUMERICAL RESULTS

Here, we consider the three-dimensional normal shock discontinuity in order to investigate the shock-capturing characteristics of TVD MUSCL limiters and MLP. This test shows the advantages of MLP clearly in terms of monotonicity and convergence.

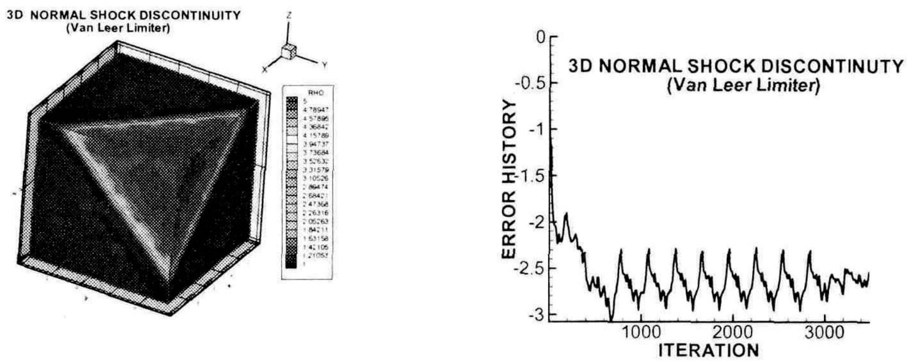

Figure 1. Density contour and error history : van leer limiter

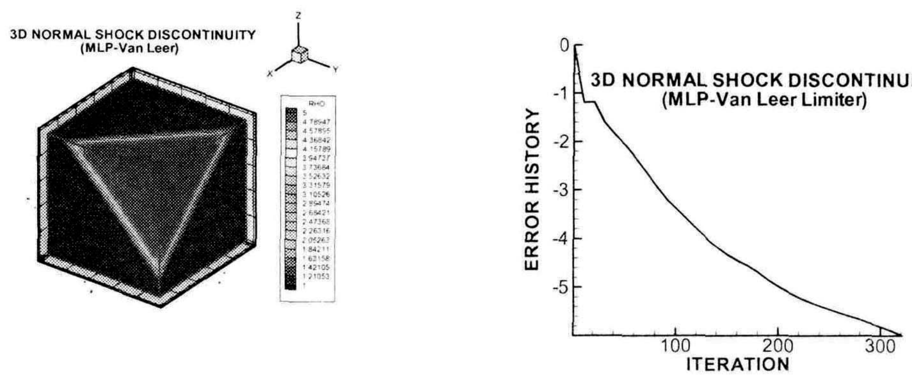

Figure 2. Density contour and error history : MLP-van leer limiter

REFERENCES

1. P. K. Sweby : "High resolution schemes using flux limiters for hyperbolic conservation laws". SIAM J. Numer. Anal. Vol.21, No.5, Oct. 1984
2. A Harten, B.Engquist, S.Osher, and Chakravarthy," Uniformly high order accurate essentially non-oscillatory schemes," J. of Computational Physics, 71, 231-303 (1987)
3. K. H. Kim, and C. Kim : "Accurate, Efficient and Monotonic Numerical Methods for Multi-dimensional Compressible Flows" accepted to J. of Computational Physics, 2005