

Utility Theory versus Labor Theory of Value

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I. Introduction

Among three different theories of value suggested by nineteenth century economists, namely, two versions of labor theory (labor-embodied theory and labor-commanded theory) and marginal utility theory, only the last one served as a basis for modern neo-classical explanation of price-determination. In Samuelson's *Economics*, price is explained in terms of demand-and-supply theory, with price (p) as a parameter, i.e. $D = f(p)$, $S = g(p)$, and $f(p) = g(p)$, where the quantity demanded (D) is derived from a utility-maximizing behavior of consumers, and an equilibrium price is determined at such a level that the quantity demanded is equal to the quantity supplied (S) at market equilibrium. In this book, labor theory of value, whatever the version is, is treated as an ancient theory that is proved to be erroneous by an advanced theory.

A historical re-examination, however, reveals that a neo-classical view of the labor theory of value is misleading. In our view, the marginal utility theory is compatible with the labor theory of value. In other words, the explanation of unequal exchange in labor-commanded terms is exactly equivalent to the marginal utility theory of value, and labor-embodied theory which holds true under equal exchange can be regarded as a special case of labor-commanded theory of value.

To prove this statement, we will define some basic concepts, including the new concept of economic surplus, in the next section. The third section is a digression on the distinction between real cost and alternative cost. The fourth section explains three principles of exchange, a law of

exchange in terms of both utility and labor-embodied, a determination of price by demand and supply, and a law of unequal exchange. The final section is a remark on the relationship between labor-embodied and labor-commanded theory of value.

II. Some preliminaries

A. Two Fundamental Necessities of an Economy

It goes without saying that the consumption and production of an economic good with a use value is indispensable for the survival of a human being. To produce use value, three factors of production — object of labor, means of labor, and labor — are necessary:

$$\text{object of labor} + \text{means of labor} + \text{labor} \rightarrow \text{use value} \quad (1)$$

The production process should be kept going to keep a human being alive. This belongs to the *laws of matter* in economics, as physical laws apply to a physical world. However, the difference between physical and economic processes is that a psychological factor plays a role in an economic process. In a sense, an economic process is also affected by the *laws of mind*, with which the Jevonian calculation of “pleasure and pain” or the Smithian notion of “toil and trouble” is always involved.¹

In the production process described in (1), an object of labor and means of labor can be represented as indirect labor — indirect in the sense that is subjectively remote from the current expenditure of human effects — and labor means direct labor which requires “toil and trouble”.² Both indirect and direct labor are considered to be concrete useful labor in Marx’s terms. Now the questions arise: How can we compare the amount of concrete useful labor which is heterogeneous and how can we find a commensurable measure? Marx’s concept of abstract human labor is a device to resolve this thorny question:

¹ The distinction between *laws of matter* and *laws of mind* are equivalent to Mill’s distinction between *laws of body* and *laws of mind* (see J. S. Mill, *A System of Logic*, 8th ed., 1930, p. 555).

² Malthus and Ricardo call direct labor as “immediate labor” and indirect labor as “accumulated labor” (see T. R. Malthus, *Definition in Political Economy*, 1827, p. 191).

It is apt to be forgotten that the magnitudes of different things can be compared quantitatively, only when those magnitudes are expressed in terms of the same unit. It is only as expressions of such a unit that they are of the same denomination, and therefore commensurable.³

Following Marx's method, we can express each concrete useful labor in labor-embodied terms measured by abstract human labor. On the other hand, use value or what Marx calls "a material thing," or "something useful" can be regarded as an utility or psychological pleasure from the point of the *laws of mind*.

In sum, we have the following relation:⁴

$$\begin{array}{rcc}
 \text{object of labor} + \text{means of labor} + \text{labor} & \rightarrow & \text{use value} \\
 \underbrace{\hspace{10em}} & & \downarrow \\
 \text{indirect labor} & & \text{direct labor} \\
 \underbrace{\hspace{10em}} & & \downarrow \\
 \text{labor embodied} & \longrightarrow & \text{utility}
 \end{array} \tag{2}$$

From this relation, we find that Marx's conception of commodity as a thing consisted of use value and value (or labor-embodied) is misleading. In our reformulation, what corresponds to use value is not value (or abstract human labor) but concrete useful labor, i.e. object of labor, means of labor and labor. In addition, what corresponds to value is not use value but utility based on a psychological factor.

In this light, Marx's dialectic description of a commodity is incomplete. Marx should have been more careful in answering Hegel's question of "with which we should begin a science (Womit muß der Anfang der Wissenschaft gemacht werden)". In economics, the fundamental necessities can be found by starting with labor embodied and utility as *terminus a quo*. In my opinion, economic theories should stand on both *laws of matter*, in which Marx's materialism prevails, and *laws of mind* in which Smith's psychological monism holds true.⁵ Marx errs in characterizing an

³ K. Marx, *Capital*, Vol. I, p. 57.

⁴ In this reformulation, object of labor, means of labor, labor and use value belong to the abstract (das Abstrakte) or the unmediated (das Unmittelbare) in Hegel's terms, whereas labor-embodied and utility belong to the concrete (das Konkrete) or the mediated (das Vermittelte).

⁵ J. S. Mill expresses his view on psychological monism: 'All phenomena of society are phe-

economic good as consisting of labor-embodied (value) and use value because he adheres only to materialism, rejecting the psychological factor indispensable to economic considerations. He should have accepted labor-embodied and utility as two factors of goods.

B. Economic Surplus as the Objective of Economic Behavior

We believe that an economic agent maximizes the difference between pleasure and pain, as Jevons (1924, p. 23) argues:

The theory which follows is entirely based on a calculus of pleasure and pain: and the object of economics is to maximize happiness by purchasing pleasure, as it were, at the lowest cost of pain.

In terms of our reformulation in (2), “the lowest cost of pain” can be expressed as the lowest pain caused by labor-embodied. Setting aside the problem of measuring the amount of labor-embodied, we can say that the object of economic behavior is to maximize the economic surplus as the difference between utility (pleasure) and properly measured labor-embodied (pain).

C. Labor as a Measure of Utility

The impossibility to compare utility and labor-embodied directly has harassed economists for more than two centuries. Here we suggest that utility be measured by *the greatest quantity of labor* that a man is willing to expend. For instance, if someone is willing to spend ten hours of labor to get a deer, his utility of a deer will be equivalent to ten hours of labor, and he will give up a deer if he has to expend more than the hours of labor to get it.

We define *utility-labor* as the *greatest quantity* of labor that one will spend to achieve a certain degree of utility. This *quantity* will be different for different individuals. In addition, we assume that there exists the law

nomena of human nature... and it, therefore, the phenomena of human thought, feeling and action, are subject to fixed laws, the phenomena of society cannot but conform to fixed laws...”, Mill (1930, p. 572).

of diminishing marginal *utility-labor*.

Sensible readers may notice that our concept of *utility-labor* is similar to Marshall's notion of "demand price". However, it is regrettable that Marshall's derivation of demand price is incomplete in that he does not offer a causal explanation starting from *utility-labor*. Even more unfortunately, Marshall's notion of demand price, which is indispensable to the causal explanation of price determination, has been forgotten to modern textbooks.

As is often the case with important discoveries, the concept of *utility-labor* is not new. Malthus, one of the supporters of the labor-commanded theory of value, says:

The demand will be represented and measured by the sacrifice *in money* which the demanders are willing and able to make in order to satisfy their wants.⁶

If "*in money*" were replaced by "in labor", Malthus would have presented the same concept as ours. Böhm-Bawerk's (1966, p. 55) remark is also close to our concept:

That I have toiled over a thing is one fact, that the thing is worth the toil is another and a different fact, and that the two facts do not always go hand in hand is far too firmly established by experience to admit of any doubt.

Böhm-Bawerk's remark that "the thing is worth the toil" means none other than our *utility-labor*.

III. Real vs. Alternative Cost

Since real cost means labor-embodied and alternative cost is represented by sacrificed utility, a distinction between real and alternative cost corresponds to a distinction between labor-embodied and utility. Before explaining this relation, let us mention a brief history of the labor-embodied theory of value.

⁶ T. R. Malthus, *Principles of Political Economy*, 2nd ed., 1936, p. 62.

Ricardo(1917, p. 64), who is commonly regarded as the first developer of the labor-embodied theory of value, explains the price of a manufacturing good in terms of the *same* quantity of labor-embodied irrespective of the level of production:

But suppose corn to rise in price because more labor is necessary to produce it; that cause will not raise the price of manufactured goods in the production of which no additional quantity of labor is required.

Then, Ricardo(1917, p. 40) argues that the price of an agricultural good is determined by the *greatest* quantity of embodied labor:

That corn which is produced by the greatest quantity of labor is the regulator of the price of corn.

Strictly speaking, Ricardo's theory that the price of a manufacturing good is determined by the greatest(marginal) quantity of labor-embodied in the *marginal* product is nothing but a theory of unequal exchange, because a theory of equal exchange cannot be applied to the intramarginal products, excluding marginal product.

J.S. Mill, who is often looked upon as one of the faithful followers of Ricardo, however, thinks that the price of a manufacturing good is also regulated by the *greatest* quantity of embodied labor. He explains "extra profit analogous to rent" in manufacturing by the greatest quantity of labor-embodied(Mill(1920, p. 476)). A more interesting change is that the expression of "the greatest quantity of labor" is replaced by "the greatest cost". "Permanent value," Mill says, "is determined by the greatest cost."⁷ For Mill, the amount of labor-embodied determines both price and cost. Mill's position on this point is a half-way house between Ricardo's explanation of price in terms of labor-embodied and Marshall's conception of labor-embodied as real cost.

This confusion is completely resolved by Marshall. He makes it clear that the quantity of embodied labor determines real cost. Marshall depicts

⁷ J. S. Mill, *Principles of Political Economy*, 1920, p. 569.

the greatest quantity of labor-embodied as the upward-sloping real-cost curve and the same quantity of labor-embodied as the horizontal real-cost line. It is to Marshall's credit to recognize that the difference between demand price and real cost determines an economic surplus which is the sum of consumers' and producers' surplus.

In passing, we may notice that Marx's labor theory of value is diametrically different from Ricardo-Mill's version in that Marx explains price in terms of the *average* quantity of labor-embodied. Marx's social or market value is nothing but the average of individual values:

If the ordinary demand is satisfied by the supply of commodities of average value, that is to say, of a value midway between the two extremes, then those commodities, whose individual value stands below the market-value, realize an extra surplus-value, or surplus profit, while those, whose individual value stands above the market-value cannot realize a portion of the surplus-value contained in them.⁸

I think that Marx's explanation of value in terms of the average quantity of embodied labor is erroneous because it is sensible only when the same quantity of labor is required in the production of goods.

Marx's explanation of a value in terms of the average quantity of embodied labor is not really a theory of equal exchange, since each good with a different individual value is not exchanged equally; if it is sold at the average value.

Now turning our attention to the theory of alternative cost, we find that Friedrich von Wieser is the first economist who explains this concept explicitly. Wieser argues:

The sacrifice consists in the exclusion or limitation of possibilities by which other products might have been turned out, had the material not been devoted to one particular product. Our definition in an earlier connection made it clear that cost-productive-means are productive agents which are widely scattered and have manifold uses. As such they promise a profitable yield in many directions. But the realization of one of these necessarily involves a loss

⁸ K. Marx, *Capital*, Vol. III, p. 210.

of all the others. It is this sacrifice that is predicated in the concept of costs; the costs of production or the quantities of cost-productive-means required for a given product and thus withheld from other uses.⁹

In Wieser's definition, alternative cost is measured by a value of means of production which could have been used in the production of products other than the product produced. This is, however, confused in that it has been already shown by Böhm-Bawerk that a value of means of production is determined by the marginal utility of marginal product. Schumpeter also argues that, "from this expression (cost phenomenon), the value of producers' goods must be distinguished."¹⁰

The relation between cost and the value of means of production is correctly explained by Kauder:

The total of all costs must be *lower*, and the sum of all values of the productive elements must be *equal* to the value of the finished consumer good.¹¹

Stigler defines alternative cost more concisely:

The cost of any productive service *X* in the production of any commodity *A* is the maximum amount that *X* would produce of any other commodity (*B,C,...*).¹²

A more general definition is also given by Stigler:

The theory of alternative costs is not vitiated by the fact that two or more productive services always cooperate to produce a commodity. ... We may accordingly restate the cost doctrine in more general terms: the cost of productive service *X* in the production of *A* is equal to the largest value of the marginal product of *X* in its other possible uses (*B,C,...*).¹³

We will adopt this definition in later discussions.

⁹ F. von Wieser, *Social Economics*, translated by A. F. Hinrichs, 1927, pp. 99-100.

¹⁰ J. Schumpeter, *The Theory of Economic Development*, Harvard University Press, 1934, p. 29; Schumpeter calls real cost as "a second concept of cost". (*ibid.*, p. 30).

¹¹ E. Kauder, *A History of Marginal Utility Theory*, 1965, p. 185.

¹² G. J. Stigler, *The Theory of Price*, revised ed., 1966, p. 96.

¹³ *ibid.*, p. 99.

IV. Three Laws of Exchange

In explaining laws of exchange, we will adopt three different stages of production *à la* Marx: The first is a direct barter of commodities ($C-C'$), the second a simple commodity production where money (M) is used as a medium of exchange ($C-M-C'$), and the third a capitalist production ($M-C < \overset{L}{P_m} \dots P \dots C'-M'$) in which labor (L) and means of production (P_m) are purchased with money capital (M) in the production (P) of a final commodity (C') which is again converted to money capital (M').

Throughout three different stages, the questions to be asked are:

- (1) What is the basis of exchange?
- (2) How is the rate of exchange determined?
- (3) Who gains and loses in this relations between men and men?

Short answers which we want to elaborate on are:

- (1) The basis of exchange has *two-fold* aspects, namely utility and labor-embodied (Law of exchange I: Exchange based on both utility and labor-embodied.)
- (2) The rate of exchange is determined by demand and supply (Law of exchange II: Determination of a price by demand and supply).
- (3) The exchange results in the gain or loss for one partner (Law of exchange III: Unequal exchange).

Law of Exchange I:

Suppose that, in a barter economy, x amount of a commodity C owned by a person A is exchanged for y amount of a commodity C' owned by a person B . The relative price of C in terms of C' is $P_c = y/x$. Let us denote the total amount of labor-embodied to produce x C as $L(xC)$ and that of labor-embodied to produce y C' as $L(yC')$, both of which are assumed to be given technically. The total utility which A can get by consuming x C and y C' will be written as $U_A(xC)$ and $U_A(yC')$, and B 's total utility will be $U_B(xC)$ and $U_B(yC')$. To compare utility and labor-embodied directly, we have to resort to the concept of *utility-labor* defined in the second section and transform utility into *utility-labor* written as $L_A(\cdot)$ and $L_B(\cdot)$.

When xC is exchanged against yC' , we have the following four relations:

$$\begin{aligned} L_A(yC') &> L(xC) \\ L_A(yC') &> L_A(xC) \\ L_B(xC) &> L(yC') \\ L_B(xC) &> L_B(yC'). \end{aligned} \tag{1-1}$$

(1-1) implies that (i) the total *utility-labor* of a commodity, which a person wants to get, is larger than the amount of labor-embodied of a commodity which he is about to dispense; (ii) the total *utility-labor* of a desired commodity is larger than the total *utility-labor* of a commodity he is about to supply; and (iii) the quantity supplied is equal to the quantity demanded.

Moreover, we know that $L_A(xC) > L(xC)$ and $L_B(yC') > L(yC')$, because, otherwise, A would not produce xC and B would not produce yC' . Therefore, we have two inequalities:

$$\begin{aligned} L_A(yC') &> L_A(xC) \\ L_B(xC) &> L_B(yC'). \end{aligned} \tag{1-2}$$

The difference between the two sides represents the *exchange surplus* of A and B .

(1-2) implies that a meaningful cost in a barter economy is not a real but an alternative cost and that an exchange is carried out on the basis of utility only. In other words, the marginal utility theory of value directly applies to a barter economy.

In a barter economy ($C-C'$) or a simple commodity production ($C-M-C'$), a producer is willing to supply his own good only when more satisfaction is achieved by exchanging it rather than by consuming it. In this sense, a commodity in these two stages is a *semi-commodity* or an *imperfect commodity*, in contrast with a *perfect commodity* which has to be sold instantly, completely, and at the highest price in a capitalist economy ($M-C < \frac{1}{p_m} \dots P \dots C' - M'$).

To a barter economy ($C-C'$) or a simple commodity production ($C-M-C'$) the price-determination theory with price(p) as a parameter, i.e.

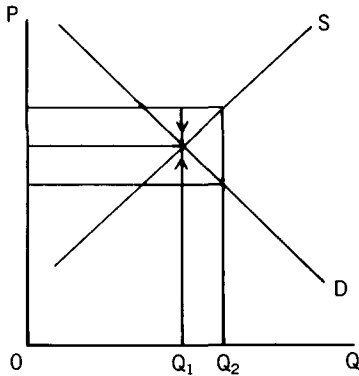


FIGURE 1

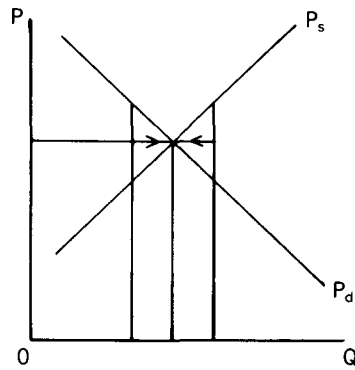


FIGURE 2

$D=f(p)$, $S=g(p)$, and $f(p)=g(p)$, is applicable. In this framework, the supply curve, $g(p)$, represents an alternative cost or what Wicksteed calls “reversible supply curve” (see Figure 1). On the other hand, a capitalist production ($M-C\cdots P\cdots C'-M'$) can be explained by Marshall's theory of price with quantity as a parameter, i.e. $P_d=F(Q)$, $P_s=G(Q)$, and $F(Q)=G(Q)$, where P_d is a demand price and P_s supply price (see Figure 2). In this case, the supply curve, $G(Q)$, represents the real cost curve, and $P_d=F(Q)$ implies that Q amount is to be sold instantly, completely and at the highest price.

In a barter economy, excess supply, Q_1Q_2 in Figure 1, does not cause a problem to a producer, because he himself as a consumer may consume it for his own satisfaction. In a capitalist economy in which producers are not identical with consumers, a glut does not lead to satisfaction but a loss of producers.

This historical perspective reveals the reason why only an alternative cost has meaning in a barter economy or a simple commodity production. M. Blaug aptly expresses the characteristics of alternative cost as “supply as reverse demand”.¹⁴ For example, A will supply x_C at the sacrifice of the utility of $U_A(x_C)$ because he wants $y_{C'}$. However, Blaug should have mentioned that this concept does not directly apply to a capitalist produc-

¹⁴ M. Blaug, *Economic Theory in Retrospect*, 4th ed., 1986, p. 489.

tion, in which the concept of real cost makes sense.

Law of Exchange II:

In explaining how the exact rate of exchange is determined in (I-2), we may take advantage of Walras' achievement.¹⁵ Suppose that m amount of the first commodity(A) is exchanged against n amount of the second commodity(B). The price of the first in terms of the second commodity is written as $P_a (=n/m)$ and the price of the second in terms of the first commodity as $P_b (=m/n)$, i.e. $P_a = 1/P_b$.

If demand for the first commodity, D_a , is matched by supply of the second commodity, O_b , when the price is P_a , we have:

$$O_b = D_a P_a \quad \text{or} \quad D_a = O_b P_b. \quad (\text{II-1})$$

Similarly, if supply of the first commodity, O_a , is matched by demand for the second commodity, D_b , we obtain:

$$D_b = O_a P_a \quad \text{or} \quad O_a = D_b P_b. \quad (\text{II-1}')$$

Next, let us define Walras' concept of *rareté* or marginal utility. The *rareté* of the owner of the second commodity towards the first commodity is written as r_a , and r_b is defined as the *rareté* given by the consumption of the second commodity. The *rareté* is a decreasing function of the quantity consumed, i.e. $r_a = Q_a(\cdot)$ and $r_b = Q_b(\cdot)$. At equilibrium, the exchange ratio is equal to the reciprocal of the ratio between r_a and r_b :

$$r_a = P_a r_b. \quad (\text{II-2})$$

If the owner of the second commodity supplies O_b out of his initial endowment q_b , and demands d_a amount of the first commodity, we have:

$$Q_a(d_a) = P_a Q_b(q_b - O_b) \quad \text{or} \quad Q_a(d_a) = P_a Q_b(q_b - D_a P_a).$$

Solving this equation with respect to d_a , we can obtain the individual demand function as a function of price:

$$d_a = f_a(P_a).$$

¹⁵ L. Walras, *Elements of Pure Economics*, translated by W. Jaffé, 1954, Part 2.

The total demand function is the sum of individual demand functions:

$$D_a = \sum_{i=1}^k f_{ai}(P_a) = F_a(P_a),$$

where k is the number of owners of the second commodity.

Similarly, we can derive the demand function for the second commodity:

$$D_b = F_b(P_a).$$

From (II-1) and (II-1'), we know that:

$$O_a = D_b P_b = F_b(P_b) P_b = F_b(1/P_a)(1/P_a)$$

$$O_b = D_a P_a = F_a(P_a) P_a = F_a(1/P_b)(1/P_b).$$

Finally, at market equilibrium, supply is equal to demand:

$$D_a = O_a \quad P_a P_b = 1 \quad \text{(II-3)}$$

$$D_b = O_b \quad P_a P_b = 1. \quad \text{(II-3')}$$

We can solve either (II-3) or (II-3') to get P_a or P_b . This derivation constitutes the essence of Walras' theory of relative price with price as a parameter.

However, Walras' approach is lacking in the causal explanation in that he does not derive his utility theory of value in (II-2) from the theory of exchange based on utility and labor-embodied as we did in (I-1) and (I-2). In addition, it is noteworthy that Walras' theory is basically applicable to a barter economy, in which buyers and sellers are searching for an exchange surplus expressed in utility, taking the price as a signal. On the other hand, in a capitalist society, what matters to a producer is the quantity produced which has to be sold instantly, completely, and at the maximum price. A similar criticism will apply to Jevons' equation of exchange and Wicksell's price theory.

In *Value, Capital and Rent*, Wicksell presents the theory of price determination in a barter economy:

$$\begin{aligned} F_1(a_1 - x_1)/f_1(y_1) = y_1/x_1 = p & \quad F_1(x_1 \hat{)/}f_1(b_1 - y_1 \hat{)} = y_1 \hat{}/x_1 \hat{'} = p \\ F_2(a_2 - x_2)/f_2(y_2) = y_2/x_2 = p & \quad F_2(x_2 \hat{)/}f_2(b_2 - y_2 \hat{')} = y_2 \hat{}/x_2 \hat{'} = p \end{aligned} \quad \text{(A)}$$

$$\begin{aligned}
 & \dots\dots\dots & \dots\dots\dots \\
 & F_m(a_m - x_m)/f_m(y_m) = y_m/x_m = p & F_n(x_n')/f_n(b_n - y_n') = y_n'/x_n' = p \\
 & x_1 + x_2 + \dots + x_m = x_1' + x_2' + \dots + x_n' & \text{(B)}
 \end{aligned}$$

$$y_1 + y_2 + \dots + y_m = y_1' + y_2' + \dots + y_n' \quad \text{(C)}$$

where a_1, a_2, \dots, a_m = the endowment of each owner of commodity C, b_1, b_2, \dots, b_n = the endowment of each owner of commodity C', x_1, x_2, \dots, x_m (or y_1, y_2, \dots, y_m) = the quantity of C (or C') which each owner of C can give (or get) in the exchange process,

x_1', x_2', \dots, x_n' (or y_1', y_2', \dots, y_n') = the quantity of C (or C') which each owner of C' can get (or give) in the exchange process ($x_1 \neq x_1', x_2 \neq x_2', \dots$),

$F_1(a_1 - x_1), F_2(a_2 - x_2), \dots, F_m(a_m - x_m)$ (or $f_1(y_1), f_2(y_2), \dots, f_m(y_m)$) = the marginal utility of each owner of commodity C with respect to C (or C'),

$F_1(x_1'), F_2(x_2'), \dots, F_n(x_n')$ (or $f_1(b_1 - y_1'), f_2(b_2 - y_2'), \dots, f_n(b_n - y_n')$) = the marginal utility of each owner of commodity C' with respect to C (or C'). The number of equations, (A) and either (B) or (C), is $2(m+n)+1$, which is equal to the number of unknowns ($x_1, x_2, \dots, x_m; y_1, y_2, \dots, y_m; x_1', x_2', \dots, x_n'; y_1', y_2', \dots, y_n'; p$), so that we may determine the relative prices.¹⁶

In our notation, Wicksell's simultaneous equations can be translated into the following:

$$\begin{aligned}
 & L_{A1}(y_1 C') > L_{A1}(x_1 C) & L_{B1}(x_1' C) > L_{B1}(y_1' C') \\
 & L_{A2}(y_2 C') > L_{A2}(x_2 C) & L_{B2}(x_2' C) > L_{B2}(y_2' C') & \text{(D)} \\
 & \dots\dots\dots & \dots\dots\dots \\
 & L_{Am}(y_m C') > L_{Am}(x_m C) & L_{Bn}(x_n' C) > L_{Bn}(y_n' C').
 \end{aligned}$$

This *ex-ante* relation may lead to the following *ex-post* results:

$$\begin{aligned}
 & L(y_1 C') > \text{or} < L(x_1 C) & L(x_1' C) > \text{or} < L(y_1' C') \\
 & L(y_2 C') > \text{or} < L(x_2 C) & L(x_2' C) > \text{or} < L(y_2' C') & \text{(E)} \\
 & \dots\dots\dots & \dots\dots\dots \\
 & L(y_m C') > \text{or} < L(x_m C) & L(x_n' C) > \text{or} < L(y_n' C').
 \end{aligned}$$

¹⁶ K. Wicksell, *Value, Capital and Rent*, translated by S. H. frowein, 1954, pp. 71-2.

(E) shows the possibility of unequal exchange, with equal exchange as a special case.

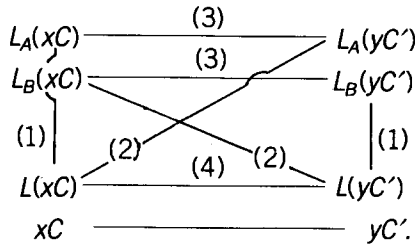
$$\text{In (E), it is shown that } \frac{y_1}{x_1} = \frac{y_2}{x_2} = \dots = \frac{y_m}{x_m} = p = \frac{y'_1}{x'_1} = \frac{y'_2}{x'_2} = \dots = \frac{y'_n}{x'_n}.$$

But, in general, the ratio between the quantities of labor-embodied of

$$\begin{aligned} \text{individual commodities is not equal, i.e., } & \frac{L(y_1 C')}{L(x_1 C)} \neq \frac{L(y_2 C')}{L(x_2 C)} \neq \dots \\ & \neq \frac{L(y_m C')}{L(x_m C)} \neq \frac{L(y'_1 C')}{L(x'_1 C)} \neq \frac{L(y'_2 C')}{L(x'_2 C)} \neq \dots \neq \frac{L(y'_n C')}{L(x'_n C)} \end{aligned}$$

Law of Exchange III:

Now coming back to the relation in (1-2), we may describe this as follows:



These relations can be rewritten as follows:

$$L_A(xC) > L(xC) \text{ and } L_B(yC') > L(yC') \tag{III-1}$$

$$L_A(yC') > L(xC) \text{ and } L_B(xC) > L(yC') \tag{III-2}$$

$$L_A(yC') > L_A(xC) \text{ and } L_B(xC) > L_B(yC'). \tag{III-3}$$

These three inequalities represent *ex-ante* motive of exchange; we know that exchange based on utility and labor-embodied can be reduced to the third relation (III-3) in a barter economy. The exact value of x and y will be determined by demand and supply, and the final result is the exchange of xC for yC'. This implies that the ratio of the amounts of labor-embodied which are exchanged in the actual process depends on demand and supply. This ratio is passively determined in the market, and

thus we cannot guarantee that equal exchange prevails in the *ex-post* relation of $L(xC)-L(yC')$. In other words, the direction of inequality goes either way, i.e. $L(xC) > L(yC')$ or $L(xC) < L(yC')$. Generally, labor-commanded theory of value holds true in a barter economy, as Malthus maintains:

In the very earliest periods, when not only land was in common, but scarcely any capital was used to assist manual exertions, exchanges would be constantly made with but little reference to the quantity of labour which each commodity might have cost. The greatest part of the objects exchanged would be raw products of various kinds, such as game, fish, fruits, &c. with regard to which the effects of labour are always uncertain. One man might have employed five days' labour in procuring an object which he would subsequently be very happy to exchange for some other object that might have cost a more fortunate labourer only two, or perhaps one day's exertion. And this disproportion between the exchangeable value of objects and the labour which they had cost in production would be of perpetual recurrence.¹⁷

Now we can conclude that an unequal exchange occurs, even though both agents benefit from the exchange, as is shown in (III-3). Gains or losses always exist in a free exchange. The more one agent is willing to buy a commodity the other has, the less favorable become the terms of trade against the other.

The marginal utility theory of value which is applicable to the explanation of an *ex-ante* behavior is equivalent to the labor-commanded theory of value or theory of unequal exchange, which is an *ex-post* result. This means that the Walras-Jevons-Wicksellian theory of price can be translated into the Malthusian theory of unequal exchange in a barter economy.

V. On Marx's Theory of Value

As one of the remaining problems, we will consider the relation between Marx's (average) embodied labor theory of value or theory of equal

¹⁷ T. R. Malthus, *Principles of Political Economy*, 1st ed., 1820, p. 87.

exchange and Malthus' commanded labor theory of value or theory of unequal exchange. Our simple conclusion is that the former is the special case of the latter.

The reason is easily found by asking a simple question: Does the owner of xC stop exchanging when he finds that the unequal amount of labor-embodied is exchanged? The answer is negative because he will continue to enjoy the exchange surplus defined in (III-3). The only case in which he will stop purchasing C' is the time when he is able to transfer his labor freely into the production of C' . This is possible only at the earliest stage of society in which (i) production is carried out by using natural materials only, without means of labor and produced material, and in which (ii) only simple labor exists. In other words, only at an early stage of a barter economy will an agent stop exchanging unequally, even though he might get an exchange surplus by continuing to barter. This stage presupposes the condition that all men can produce all goods. As Smith's famous example indicates, equal exchange between one beaver and two deer is carried out, "in that early and rude state of society which precedes both the accumulation of stock and the appropriation of land".¹⁸

It is at this stage that Marx's law of value holds true. Moreover, if everyone's ability to produce goods is equal, Marx's average embodied labor theory of value makes sense without any modification because individual labor time is equal to social or average labor time. On the other hand, if (i) a technical and a social division of labor is made to such a degree as to make it hard to obtain skills, and if (ii) means of labor and an object of labor, which are available after spending a long time, become necessary in the production, an agent may not have an option to break up an exchange as long as an exchange surplus exists.

Therefore, it is obvious that Marx's theory of equal exchange is applicable to a barter economy and that Malthus' labor-commanded theory of value as a theory of unequal exchange in a pre-capitalist economy has a more general applicability.

It is one of the most tragic events in the history of economics that the

¹⁸ A. Smith, *The Wealth of Nations*, Vol. 1, edited by E. Canrjan, p. 49.

clash of two theories of value ended up with a complete disregard of value theory in modern economics, because they are actually compatible. As Hegel once noted, "the truth is the whole."

References

- Blaug, M. *Economic theory in Retrospect*, 4th ed., 1986.
- Böhm-Bawerk, E. von. *Karl Marx and the Close of his System*, edited with an introduction by P. M. Sweezy, 1966.
- Jevons, W. S. *The Theory of Political Economy*, 1924.
- Kauder, E. *A History of Marginal Utility Theory*, 1965.
- Malthus, T. R. *Principles of Political Economy*, 1st ed., 1820.
- _____. *Principles of Political Economy*, 2nd ed., 1836.
- _____. *Definition in Political Economy*, 1827.
- Marx, K. *Capital*, Vols. I-III.
- Mill, J. S. *A System of Logic*, 1930.
- _____. *Principles of Political Economy*, 1920.
- Ricardo, D. *The Principles of Political Economy and Taxation*, Everyman's Library, 1917.
- Schumpeter, J. *The Theory of Economic Development*, Harvard University Press, 1934.
- Smith, A. *The Wealth of Nations*, Vol. 1, edited by E. Cannan.
- Stigler, G. J. *The Theory of Price*, revised ed., 1966.
- Walras, L. *Elements of Pure Economics*, translated by W. Jaffé, 1954.
- Wicksell, K. *Value, Capital and Rent*, translated by S. H. Frowein, 1954.
- Wieser, F. von. *Social Economics*, translated by A. F. Hinrichs, 1927.