Markov Property and Excess Sensitivity in Aggregate Consumption

Jisoon Lee
Seoul National University

I. Introduction

The hypothesis that dynamically optimizing economic agents try to maintain smooth consumption streams in the face of rather volatile income fluctuations has been extensively studied ever since Friedman (1957) put forward the permanent income hypothesis. Recently, with the advancement of time series econometrics and with the propagation of the rational expectations principle, this hypothesis has undergone major theoretical and empirical scrutiny. From these efforts several controversial issues have emerged that are conceptually important in understanding aggregate consumption behavior. Among these, we will study two closely related issues in this paper.

The first issue concerns stochastic properties of time series movements in aggregate consumption. This question was first raised by Hall (1978) who has argued that the most important implication of the permanent income hypothesis under rational expectations is that the best forecast of consumption in the next period, given current information, is a fixed function of current consumption only.[1] Hall's own empirical results partially reject the Markov property of aggregate consumption, while using different data Mankiw (1982) could not reject the Markov property. Muellbauer (1983) also rejects the Markov property using U.K. time series data. What are the implications of these empirical results for the intertemporal optimizing behavior of consumers? In particular, do they reject the consump-

[1] This implication is sometimes called the martingale property of consumption. However, it may better be called the Markov property.

[Seoul Journal of Economics, 1988, Vol 1, No. 2]

tion smoothing hypothesis?

The second issue concerns the apparent "excess sensitivity" of current consumption to current income. Studies by Bilson (1980), Flavin (1981, 1985), Hayashi (1982), and Muellbauer (1983) all report results showing that current consumption is sensitive to current income in regression equations that include some measure of lifetime wealth or permanent income. From these results the authors concluded that the permanent income hypothesis should be rejected. This conclusion raises two questions for us: First, what is the meaning of consumption being "too" sensitive to changes in current income? Second, if consumption is indeed excessively sensitive to changes in current income, does this observation refute in any way the view that consumers behave rationally?

The issues mentioned above can best be answered by studying general equilibrium models because the issues concern properties of the general equilibrium allocations. This means that most of the existing dynamic consumption studies are not well equipped to deal with these issues since they are typically partial equilibrium studies. In this paper, we will study a very simple general equilibrium model of aggregate consumption under uncertainty and clearly delineate the conditions for preferences and technology under which the Markov property of consumption can be true. For example, when consumers have time additive preferences and when production of output can be characterized by a simple neo-classical production function with i.i.d. productivity shocks, consumption has the Markov property. The model also demonstrates why general equilibrium responses in consumption to changes in income are expected to be stronger than its partial equilibrium responses. Here we extend Michener (1984) who emphasizes the intertemporal substitution effect of endogenous changes in real interest rates. Unlike Michener, however, we will emphasize the wealth effect of unexpected changes in income.

We will take a standard approach in studying dynamic rational expectations models. Thus in section II, we will first study a simple stochastic growth model to completely characterize the optimal decision rules. Then we will show that the optimal decision rules are equivalent to the solutions to decentralized competitive equilibrium problems. Using the properties of these general equilibrium allocations, we will investigate in section III whether consumption has the Markov property. In section IV, we will deal with the related issue of excess sensitivity of consumption to current income. Section V concludes the paper.

II. The Model

A. Optimal Growth Model

Consider the following one sector stochastic growth model of Brock and Mirman (1972):

$$Max\ E_t\ [\Sigma_{j=0}^{\infty}\ \beta^j\ u(c_{t+j})],\ 0 < \beta < 1 \tag{1}$$

subject to

$$c_{t+j} + k_{t+j+1} = f(k_{t+j},\ x_{t+j}) + (1-\delta)k_{t+j},\ j = 0,1,2,...$$

$$k_t > 0,\ \text{given},$$

where E_t, β, $u(\ )$, c_t, k_{t+1}, $f(\ )$, x_t, and δ denote a conditional expectation based on all available information at time t, a time discount factor, a utility function of consumption, the consumption at date t, the capital stock employed at date $t+1$, a production function, a random productivity shock, and the depreciation rate of the capital stock, respectively.

The notation makes the working of the model almost self-explanatory. The representative consumer begins a period with an initial capital stock k_t which he puts into the production process. Then the nature reveals the value of the productivity shock x_t, and the level of current period output is determined by $f(k_t,\ x_t)$. During the course of production, δk_t of the initial capital stock wears out and the remaining gross output $y_t=f(k_t,\ x_t) + (1-\delta)k_t$ is divided between current consumption c_t and next period's capital stock k_{t+1}. Then the process repeats itself with k_{t+1} as the new initial capital stock. It is assumed that preferences are completely deterministic and time additive. Furthermore, the utility function $u:R_+ \rightarrow R_+$ is assumed to be strictly concave, increasing, twice differentiable, continuous and bounded, and $u'(0) = -\infty$. It is also assumed that conversion of capital goods into consumption goods or vice versa is costless, and the production technology is a simple neo-classical one without any adjustment costs or gestation periods. For a given value of x, the production function $f: R_+ \rightarrow R_+$ is assumed to be strictly concave, increasing, twice differentiable, and continuous with $f_1(0, x) = \infty$. Further more, it is assumed that for any given k, $f(k, x') > f(k, x)$ if and only if $x' > x$.

The productivity shock x_t is given broader interpretations. It may represent pure technological disturbances, such as weather conditions or technological breakthroughs, or it can be thought of as capturing disturbances to the *marginal productivity of capital* caused by various government policies. For example, fluctuations in real government spending in Barro's (1981) model of government behavior, or stochastic income tax, can be represented by x_t. We assume that the productivity shock $\{x_t\}$ follows a Markov process with a stationary transition probability function $F(x', x) = Prob\ [x_{t+1} < x' \mid x_t = x]$.

Under these assumptions it is straightforward to demonstrate that there exists a unique value function $v(\ \ )$ that solves the following functional equation:

$$v(k, x) = Max\ [u(c) + \beta \int v(k', x')\, dF(x', x)]$$
$$s.t.\ c + k' = f(k, x) + (1-\delta)k,\ k > 0,\ \text{given}$$

and there exists associated with the value function a pair of unique (policy) functions that solves the maximum problem given as the right hand side of the above functional equation. Denote these as,

$$c_t = c(k_t, x_t) \text{ and } k_{t+1} = g(k_t, x_t), \tag{2}$$

where the functions c( ) and g( ) are related through the budget constraint as,

$$c(k_t, x_t) = f(k_t, x_t) + (1-\delta)k_t - g(k_t, x_t),$$

and the consumption function $c(k_t, x_t)$ is given as the solution to the following equation:

$$\beta \int v_k[f(k_t, x_t) + (1-\delta)k_t - c_t, x_{t+1}]\, dF(x_{t+1}, x_t) = u'(c_t),$$

where $v_k(\ ) = dv(\ )/dk$. Under the above assumption, it can also be shown that both c and g are monotone increasing in k and x. That is, an increase in the endowment of the initial capital stock k or a realization of a larger value of x induces people to consume more. The same is true for capital accumulation.

The following example is used to illustrate the results stated above. Assume that $u(c) = ln\ c$ and $f(k, x) = k^{\alpha}x$, $0 < \alpha < 1$, where we have normalized all the inessential constants to be identically one. Then the optimal policy functions are given as (with $\delta = 1$),

$$c_t = (1-\alpha\beta)k_t^{\alpha}x_t \quad and \quad k_{t+1} = \alpha\beta k_t^{\alpha}x_t,$$

and the value function is given as $v(k, x) = A + B\ ln\ k + C\ ln\ x$, where A, B, and C are constants determined by α, β, and parameters describing the distribution function $F(\ )$.

B. The Decentralized Solution

The optimal policy functions given as the solution to the planning problem (1) are also competitive equilibrium allocation rules of the following decentralized economy. In the economy, there are a large number of identical consumers, each of whom owns a unit of labor services and capital stock k_t. In each period, the consumer inelastically supplies his labor services at the real wage rate w_t and rents out his capital stock k_t at the gross rental rate R_t. There are also a large number of identical firms, each of which hires one unit of labor services and rents capital stock k_t to produce a single product according to the production technology $f(k_t, x_t)$. The consumer's problem is then to choose c_t and k_{t+1} in each period to solve the problem:

$$\underset{(c_t,\ k_{t+1})}{Max}\ E_0\ [\Sigma_{t=0}^{\infty}\ \beta^t\ u(c_t)],\quad s.t.\ c_t + k_{t+1} = w_t + R_t k_t \tag{3}$$

taking the prices $\{w_t\}$ and $\{R_t\}$ as given. On the other hand the firm's problem is to choose k_t^* to maximize

$$profits = f(k_t^*, x_t) + (1-\delta)k_t^* - w_t - R_t k_t$$

taking again the prices $\{w_t, R_t\}$ as given. The competitive equilibrium allocations rule is then a sequence of $\{c_t, k_{t+1}, w_t, R_t\}$ at which consumers maximize utility, producers maximize profits, and markets clear. In equilibrium, the firm's demand for the capital stock k_t^* is equal to its supply k_t by the household. Therefore, equilibrium prices $\{w_t, R_t\}$ in this economy are completely characterized by the following pair of equations:

$$\begin{aligned} w_t &= f(k_t, x_t) - k_t f_1(k_t, x_t) \text{ and} \\ R_t &= f_1(k_t, x_t) + 1 - \delta, \end{aligned} \tag{4}$$

together with the law of motion for the capital stock given in (2). Define real interest rate r_t so that $R_t = 1 + r_t - \delta$. Then, $r_t = f_1(k_t, x_t)$.

The observation that equilibrium prices are fixed functions of the state variables k_t and x_t has several important implications. First, they clearly indicate that equilibrium prices are not statistically independent of each other. In our case the distribution function of w_t is closely related to the distribution function of r_t. Therefore, when stochastic processes are specified to model time series movements of w_t and r_t, we cannot use any arbitrary specifications. Rather we must take into consideration the statistical relationship between w_t and r_t implied by the equilibrium theory. To illustrate this point, consider the example economy where $f(k_t, x_t) = k_t^\alpha x_t$. In this case the equilibrium price functions are given by $w_t = (1-\alpha) k_t^\alpha x_t$ and $r_t = \alpha k_t^{\alpha-1} x_t$. Taking the logarithms of these, we get:

$$\begin{aligned} \ln w_t &= \ln(1-\alpha) + \alpha \ln k_t + \ln x_t \\ \ln r_t &= \ln \alpha - (1-\alpha) \ln k_t + \ln x_t, \end{aligned}$$

which show that movements in w_t and r_t are governed by movements in k_t and x_t. Since the evolution of k_t is completely characterized by k_0, $[x_0, x_1, \ldots, x_t]$ and the function $k_{t+1} = g(k_t, x_t)$, distribution functions of w_t and r_t also depend on k_0, $\{x_0, x_1, \ldots, x_t\}$ and the function $k_{t+1} = g(k_t, x_t)$. Therefore, once consumer preferences, the production technology, and the stochastic property of $\{x_t\}$ are specified, we no longer have any degree of freedom in specifying the time series movements in w_t and x_t. Therefore, the usual practice of using arbitrary stochastic processes to model given prices is in most cases unwarranted.

Second, the equilibrium price functions are also closely related to each other in their functional forms. As a result there are tight cross-equational restrictions among equilibrium price functions. In most cases these take the form of nonlinear relationships among parameters of the model. In the case of the example economy, however, there are linear cross-equational restrictions. For example, the coefficient of $ln\ k_t$ is α in the equation for $ln\ w_t$ and $-(1 - \alpha)$ in the equation for $ln\ r_t$. In empirical studies, these cross-equational restrictions can be used to aid identification and estimation of the model. Note that the cross-equational restrictions mentioned here are distinct from those that we usually get in (linear) rational expectations models. Here they are restrictions among equilibrium price functions, whereas usual rational expectations cross-equational restrictions refer to relationships between equilibrium price functions and decision rules of the agents.[2]

Third, equilibrium prices in our model $\{w_t, r_t\}$, given the knowledge of k_t, jointly have Markov property, and distributions of future prices can be completely characterized with current prices $\{w_t, r_t\}$. Therefore, competitive allocation rules obtained by consumers can be characterized by

$$c_t = C(w_t, r_t, k_t),\ k_{t+1} = G(w_t, r_t, x_t), \tag{5}$$

which is the solution to the consumer's decentralized problem (3) treating $\{w_t, r_t, x_t\}$ as state variables.

The decentralized problem considered here is identical to the planning problem we have considered in the previous section. To see this, substitute the equilibrium prices into the budget constraint to get:

$$c_t + k_{t+1} = w_t + (1 + r_t)k_t$$

[2] These cross-equational restrictions are the dynamic analogue of the usual restrictions among elasticities in demand theory.

$$= f(k_t, x_t) - k_t f_1(k_t, x_t) + (1 + f_1(k_t, x_t) - \delta)k_t$$
$$= f(k_t, x_t) + (1-\delta)k_t,$$

which is exactly the same constraint for the planning problem. Since both problems deal with the same homogeneous consumers, the observation that they have essentially the same budget constraints is both necessary and sufficient to establish the equivalence result. Finally, the competitive equilibrium allocation rules are related (identical) to the optimal policy functions in the following way:

$$c_t = C(w_t, r_t, k_t) = C[w(k_t, x_t), r(k_t, x_t), k_t] = c(k_t, x_t)$$
$$k_{t+1} = G(w_t, r_t, k_t) = G[w(k_t, x_t), r(k_t, x_t), k_t] = g(k_t, x_t),$$

where $w(k_t, x_t)$ and $r(k_t, x_t)$ stand for the equilibrium prices given in (4).

Since it is shown that the solution to the planning problem is identical to the solution to the decentalized problem, we will use in the following section the solution to the planning problem to answer questions posed in the introduction.

III. Markov Property of Consumption

The first question we now ask is whether consumption has the Markov property[3] $E[c_{t+1} \mid I_t] = E[c_{t+1} \mid c_t]$, where I_t is an information set at t which contains, among others, c_t. Using the policy functions given in (2), we can easily show that the above question is equivalent to asking whether

$$E[c(g(k_{t+1}, x_{t+1}), x_{t+1}) \mid I_t] = E[c(g(k_{t+1}, x_{t+1}), x_{t+1}) \mid c_t]. \quad (6)$$

Since I_t contains k_t and x_t and since x_{t+1} depends only on x_t, we can denote the left-hand side of (6) as $h(k_t, x_t)$. Similarly we can write the right-hand side of (6) as $\Psi(c_t)$. Then the question is whether $h(k_t, x_t) = \Psi(c_t)$ for all realized values of x_t when c_t and k_t follows (2).

[3] Let $\{x_t\}$ be a sequence of well-defined random variables and let I_t denote an information set Then the random variable x_t is said to have a Markov property if and only if $E[x_t \mid I_{t+1}] = E[x_t \mid x_{t-1}]$.

It is obvious that this equality is not true in general. Even though knowing k_t and x_t separately is equivalent to knowing c_t, the converse is not always true since we cannot unscramble c_t to get values of k_t and x_t separately. To illustrate this point, let us take a look at the example economy once more. Here we have,

$$c_{t+1} = (1-\alpha\beta)k_{t+1}^{\alpha}x_{t+1} = (1-\alpha\beta)^{1-\alpha} \ (\alpha\beta)^{\alpha} \ c_t^{\alpha}x_{t+1}.$$

Taking conditional expectations of the above expression first on $I_t = [c_t, c_{t-1},\ldots, x_t, x_{t-1},\ldots, k_t, k_{t+1},\cdots]$ and then on c_t, we see that the question is whether $E[x_{t+1}|x_t] = E[x_{t+1}|c_t]$. Obviously the equality does not hold and therefore consumption does not have the Markov property. Under what conditions then does consumption have the Markov property? Given our model, consumption has the Markov property if and only if the productivity shock is such that $E[x_{t+1} \mid x_t] = E[x_{t+1}]$ for all t. Thus we conclude that even with time separable preferences and simple neo-classical technologies, consumption does not have the Markov property unless the productivity shock is independently distributed over time. As a corollary to this observation, it is easy to show that consumption does not have the Markov property if: i) preferences are dynamically nonseparable, ii) adjustment costs or multiple gestation periods are needed to reach the optimum capital stock, or iii) we have to model the productivity shock with non-Markov processes.

When consumption does not have the Markov property, it cannot be a martingale either, since a martingale is more restrictive than the Markov.[4] It has often been argued in the literature that the marginal utility of consumption should be a martingale when preferences are time separable. Here we show that this is not the case in general. The first order condition to the decentralized problem is $E_t[\beta u'(c_{t+1}) \ (1 + r_{t+1})] = u'(c_t)$, which is a relationship between marginal utility of consumption in different periods given real interest rate r_{t+1}. Now if r_{t+1} is independent of c_{t+1} and if $E_t \, r_{t+1} = Er_{t+1}$, then marginal utility of consumption is a

[4] See Muellbauer (1983) and MaCurdy (1985) both of whom argue that consumption is not a martingale.

martingale. But $c_{t+1} = c(k_{t+1}, x_{t+1})$, and consumption and real interest rates as random variables will never be contemporaneously independent. Furthermore, r_t, which in equilibrium is the marginal productivity of capital $f_1(k_t, x_t)$, would never be constant in a dynamic setting. Therefore, marginal utility of consumption would never be a martingale and lagged variables other than lagged consumption should help to predict consumption.

IV. Excess Sensitivity of Consumption to Current Income

Recent empirical studies of dynamic consumption behavior indicate that consumption is too sensitive to changes in income to be compatible with the permanent income hypothesis. Hall (1978), Flavin (1980, 1985), Hayashi (1982), Hall and Mishkin (1982), and Muellbauer (1983) all report this excess sensitivity result and conclude that aggregate consumption data cannot adequately be explained by the permanent income hypothesis. As alternative explanations they cite the possible existence of liquidity constrained consumers, possibly nonoptimal income forecasting procedures, and the possibility of Bayesian learning of the model. In this section we first define when consumption is said to be "excessively" sensitive to change in income. We then show that the excess sensitivity issue is closely related to the question whether consumption is a Markov process. Finally, we demonstrate why in general equilibrium models consumption is expected to be more sensitive to changes in income than in partial equilibrium models.

Hall, Flavin, and Muellbauer all asked whether in the following consumption function the coefficient ϕ is empirically equal to zero:

$$c_{t+1} = \lambda c_t + \gamma [y_t - E_{t-1} y_t] + \phi y_t,$$

where λ, γ, and ϕ are coefficients to be estimated and $y_t - E_{t-1} y_t$ is new information revealed by current income (surprises in income movements). They regard consumption to be excessively sensitive to current income when $\phi = 0$. Their conclusion is based on the following chain of reasoning. In a dynamic setting, consumption is geared to the permanent income (or lifetime wealth) which does not change very much in a short period of time. In this case, the lagged consumption c_{t-1} contains all the information (about the permanent income) available up to the last period. Therefore, current consumption should not be too different from c_{t-1}. Any difference should be due to unexpected changes in income. Once the lagged consumption which contains all the information about wealth available up to the last period and the forecast error in income which contains all the new information about the permanent income are included in a consumption function, current income y_t should not play any additional role in explaining c_t. If, on the other hand, y_t turns out to be important ($\phi \neq 0$), consumption is regarded to be excessively sensitive to income. In fact, Hall and Flavin using U.S. data, and Muellbauer using U.K. data have found that they could reject the null hypothesis that the coefficient $\phi = 0$. From this they conclude that the permanent income hypothesis is not supported by data.

We now will show that the empirical result that $\phi = 0$ in the above consumption function does not constitute a rejection of the permanent income hypothesis. Indeed we will argue that the coefficient ϕ should be nonzero in most of the dynamic consumption models. We will do this by showing that the excess sensitivity question formulated by Hall and others is equivalent to asking whether consumption has a Markov property. The question asked by Hall and others is whether $E[c_t \mid c_{t-1}, y_t - E_{t-1}y_t, y_t] = E[c_t \mid c_{t-1}, y_t - E_{t-1}y_t]$. When this equality holds, $\phi = 0$ and $E[c_t \mid I_{t-1}] = E[c_t \mid c_{t-1}] = \lambda c_{t-1}$ since $E[y_t - E_{t-1}y_t] = 0$. But of course $E[c_t \mid I_{t-1}] = E[c_t \mid c_{t-1}]$ is nothing but the Markov properety of consumption. We have already shown in the previous section that consumption almost certainly does not have the Markov property. Therefore, there is no reason why the coefficient ϕ should be zero and the observation that ϕ is empirically nonzero in no way contradicts the permanent income hypothesis.

The above result can also be seen in the following way. When Hall's or Flavin's claim is correct, we should be able to get the exact value of c_t from the knowledge c_{t-1} and $y_t - E_{t-1}y_t$ alone. Recall $c_t = c(k_t, x_t)$. Therefore, if we know the value of k_t and x_t separately, we can get the value of c_t. Now knowing c_{t-1} and $y_t - E_{t-1}y_t$ which is equivalent to knowing $c(k_{t-1}, x_{t-1})$ and $f(k_t, x_t) - E_{t-1}f(k_t, x_t)$, is not sufficient to give the values of k_t and x_t separately. The most we get from that information is a value of k_t and possibly a value of the forecast error $x_t - E_{t+1}x_t$. On the other hand, when y_t is added to the information set, the values of k_t and x_t can be separately obtained to find the exact value of c_t. Therefore, adding y_t to the information set unambiguously helps to predict current consumption c_t.

These discussions can also be stated in the following manner. Note first that for our problem the optimum choice of consumption and capital accumulation can be written as functions of current output y_t only. This results from the observation that once y_t is known there is no need to know k_t and x_t separately to determine current consumption and capital accumulation. Therefore, $c_t = c^*(y_t)$ and $k_{t+1} = g^*(y_t)$. Of course, in determining the optimum decision rules $c^*(\ )$ and $g^*(\ )$, the consumer has to know how income y_t evolves over time and how his choice of k_{t+1} affects future evolution of income. In terms of this alternative formulation, the question of whether consumption has Markov property is to ask whether

$$E[c^*(y_{t+1}) \mid I_t] = E[c^*(y_{t+1}) \mid c^*(y_t)].$$

When the consumption function $c^*(\ )$ is single valued, then the question is equivalent to asking whether $E[y_{t+1} \mid I_t] = E[y_{t+1} \mid y_t]$. Since $y_{t+1} = f(g^*(y_t), x_{t+1}) + (1 - \delta)g^*(y_t)$,

$$E[y_{t+1} \mid I_t] = E[f(g^*(y_t), x_{t+1}) \mid I_t] + (1 - \delta)g^*(y_t),$$

because I_t contains y_t. Clearly,

$$E[f(g^*(y_t)), x_{t+1} \mid I_t] \neq E[f(g^*(y_t)), x_{t+1} \mid y_t],$$

unless $\{x_t\}$ is independently distributed over time. Therefore, consumption does not have the Markov property whenever the productivity shock x_t is serially correlated.

Our model demonstrates why general equilibrium responses in consumption to changes in income must be stronger than partial equilibrium responses. Here we extend earlier results suggested by Michener (1984). Michener argues that an individual consumer's consumption smoothing activity is mitigated to a certain extent as a result of an endogenous change in the interest rate. When consumers face an unexpected increase in income, they try to maintain smooth consumption patterns by putting most of the income increase into savings. But the increase in savings in aggregate exerts a downward pressure on interest rates which in turn induce consumers to increase current consumption. Thus the general equilibrium response in consumption to an increase in current income is stronger than its counterpart in the partial equilibrium setting.

Whereas Michener's explanation emphasizes the intertemporal substitutional effect of endogenous changes in real interest rates, we provide explanations based on the wealth effect of changes in income. In our model a temporarily large income means that firms experience a larger realization of the productivity shock x_t. This in turn translates into increases in both current labor income w_t and current rental rate R_t. These increases then induce consumers to increase both consumption and savings (capital accumulation). In particular, if the change in income is perceived as only transitory, then each consumer in our economy would put most of their income increases into savings (more capital accumulation). Hence the initial response is a small increase in consumption. However, the increase in the capital stock means that from next period on income will be larger than they originally would be. That is, the transitory increase in current income is translated to a certain extent into a persistent increase in income through a larger accumulation of capital stock. This then induces the consumer to increase current consumption. Hence there is a stronger response in consumption to an increase in current income than the initial response.

The reasoning involved in the last point can be explained using a simple two period model. Consider the following problem:

$$Max \; U(c_1) + \beta U(c_2)$$

subject to

$$c_1 + s = y \text{ and } c_2 = (1 + r)s + \pi,$$

where c_1, c_2, s, y, r, and π are respectively the consumption today, the consumption tomorrow, savings, income, real interest rate, and profits. This is a general equilibrium formulation in that both r and π are endogenously determined depending on the magnitude of the savings. Of course, each individual would regard r and π as given. The first order condition of the above problem is:

$$U'(c_1) - \beta U'[(1 + r)(y - c_1) + \pi](1 + r) = 0. \qquad (7)$$

From the first order condition, the partial equilibrium response in c_1 to changes in y is given as:

$$\partial c_1/\partial y = [\beta U''(1 + r)^2]/[U'' + \beta U''(1 + r)^2], \qquad (8)$$

whereas the general equilibrium response of the same change is given as:

$$dc_1/dy = \frac{\beta U''(1 + r)^2}{\beta U''(1 + r)^2 + U''} + \frac{r'[U' + (y - c_1)U'']}{\beta U''(1 + r)^2 + U''} + \frac{(1 + r)U''\pi'}{\beta U''(1 + r)^2 + U''} \qquad (9)$$

There are two additional terms in the general equilibrium case. It is clear from (8) and (9) that when $r' = \pi' = 0$, the general equilibrium response coincides with the partial equilibrium response. But in general the real interest rate r is a decreasing function, while the profit π is an increasing function of the capital stock. Therefore, the general equilibrium response is larger than the partial equilibrium response. Note finally that what Michener argues is that r' is negative in general due to the diminishing marginal productivity of capital, whereas what we emphasize here is that the profit π is an increasing function of the capital stock. Since π' is positive, an increase in saving is equivalent to an increase in future income. That is, a temporary increase in current income gets transformed to some extent into a permanent increase via the additional capital accumulation. Hence a stronger response in current consumption to transitory changes in income exists than predicted by the partial equilibrium version of the permanent income hypothesis.

Empirically, the intertemporal substitutional effect suggested by Michener is too weak to explain why consumption is "excessively" sensitive to changes in income (see Hall 1982; Muellbauer 1983). The importance of the wealth effect suggested in this paper, on the other hand, is yet to be examined. Nevertheless, it may provide a more satisfactory explanation as to why aggregate consumption appears to be more sensitive to changes in income than that predicted by the partial equilibrium version of the permanent income hypothesis.

V. Concluding Remarks

Using a simple stochastic growth model we have shown that consumption does not possess the Markov property and that the excess sensitivity of current consumption to current income may not in any way contradict the view that consumers rationally choose consumption and savings so that they can maintain smooth consumption streams in the face of rather volatile income fluctuations. Since this conclusion is derived from studying a very simple dynamic model, when we study more complicated models that allow for an endogenous choice of labor supply, intertemporally non-separable preferences, production technologies with adjustment costs, or production technologies that require time to build capital stock, we will get even stronger conclusions. That is, in these more general models, consumption will almost never be a Markov process.

We have also shown why the general equilibrium responses in consumption to unexpected changes in income are expected to be stronger than the partial equilibrium responses. This is so because in a general equilibrium setting unexpected changes in income, which mainly affect consumers' savings behavior eventually get translated into changes in wealth via changes in capital stock. Our explanation, which emphasizes the wealth effect, is then different from Michener (1984) who derived the same conclusion by emphasizing the intertemporal substitutional effect of endogenous changes in real interest rates caused by unexpected changes in income.

These conclusions then cast serious doubts on existing empirical aggregate consumption studies which tend to reject the consumption-smoothing hypothesis based on observations that consumption does not have the Markov property and that consumption is too sensitive to changes in current income.

Finally, we must note that the conclusions we have arrived at in this paper are conceptually the same as those suggested in general equilibrium asset pricing models, which show that asset prices do not have the Markov property.

References

Barro, R. "Output Effects of Government Purchases." *Journal of Political Economy* 89 (1981): 1086-121.

Brock, W., and Mirman, L. "Optimal Economic Growth and Uncertainty: The Discounted Case." *Journal of Economic Theory* 4 (1972): 479-513.

Flavin, M. A. "The Adjustment Cost in Consumption to Changing Expectations about Future Income." *Journal of Political Economy* 89 (1981): 974-1009.

____________. "Excess sensitivity of Consumption to Current income: Liquidity Constraints or Myopia?". *Canadian Journal of Economics* 18 (1985): 117-36.

Friedman, M. *A Theory of Consumption Function*. Princeton University Press, 1957.

Hall, R. E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86 (1978): 971-87.

Hall, R., and Mishkin, F. "The Sensitivity of Consumption to Transitory Income; Estimates from Panel Data on Households." *Econometrica* 50 (1982): 461-81.

Hayashi, F. "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables." *Journal of Political Economy* 90 (1982): 895-916.

MaCurdy, T. "Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty." in J. Heckman and B. Singer (eds.), *Longitudinal Analysis of Labor Market Data,* 1985.

Michener, R. "Permanent Income in General Equilibrium." *Journal of Monetary Economics* 13 (1984): 297-305.

Muellbauer, J. "Surprises in the Consumption Function." *Economic Journal* 93 (supplement, 1983): 34-49.