A Partial Adjustment Model of Interrelated Prices and Wages with its Applications to Korea and Japan

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I. Introduction

One of the most controversial issues in the relations between wages and prices is whether inflation is caused by an increase in wages or an increase in wages is caused by an increase in the living costs due to inflation.

A series of statements by government authorities strongly supports the idea that the level of prices can be stabilized by suppressing an increase in wage rates. This idea is not novel. Standard economics admits that the wage is an important item in the production costs and that an increase in the wage rates causes an increase in costs which, in turn, causes an increase in the general price level. This process is known as the cost-push inflation.

If inflation of our real world is due to the cost-push process, then the suppression of an increase in the wage rates is certainly an effective device to cure inflation. But the relation between wages and inflation may be the other way round. According to the demand-pull theory of inflation, inflation is caused by an excess demand in general relative to the aggregate supply. If an excess demand causes inflation, inflation, in turn, increases the cost of living, which leads to an increase in the wage rates.

During the 1960s and 1970s, the Korean economy experienced high levels of inflation and high increses in the nominal wage rates. But the direction of causality has not been fully clarified. Those who want to

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suppress the wage increase tend to have relied on the theory of cost-push inflation, while those who want the higher wage rates tend to have relied on the theory of demand-pull theory of inflation. But neither side has a strong empirical ground for its arguments.

If we want to know whether wages lead prices or prices lead wages, then we need to clarify the mechanism of the wage-price interrelations. Intuitively, as well as theoretically, we should admit that both the theory of cost-push inflation and the theory of demand-pull inflation contain a bit of truth. We cannot deny any one of them *a priori*. Therefore our argument on the causality of wages and prices should be focused not on the direction but on the magnitude of its effect. In other words, we need to measure the magnitude empirically.

For the empirical magnitude of the effect, we need an econometric model that can explain the interrelations of wages and prices as well as the external effects of major exogenous variables. It seems that existing models of inflation are not sufficiently adequate for this purpose. Therefore, our first task is to develop a model that fits this purpose. The model is a partial adjustment model which is specifically familiar in the theory of investments.

II. Theoretical Analysis

A. The Partial Adjustment Model of Prices and Wages

According to the acceleration theory of investment, the level of investment is determined by the changes in the level of the aggregate demand and not by the level of the demand itself. For example, to maintain the level of the aggregate demand of 100 billion won in a specific year, we only need to maintain a constant level of production capacity, and we do not need any new investment. But to support an increase in aggregate demand of 10 billion won a year, we need a new investment of, say, 30 billion won to increase the level of the production capacity.

The flexible accelerator model of investment is a modification of the acceleration theory. According to this model, the investment requirement of 30 billion won in the specific year may not be realized fully within that year. It may be realized only partially, say, 20 billion won in that year. The reason is that investment behavior accompanies the adjustment cost which increases as the rate of investment increases. For example, as the net investment increases from 10 billion won a year to 20 billion won, the adjustment cost increases more than twice.

One of the earliest and rigorous developments of the flexible accelerator model of investment was provided by Eisner and Strotz(1963). They showed that, if a firm is assumed to maximize its net present value by choosing the appropriate time path of investment, the path would be the same as that of the flexible accelerator model. Lucas(1967) generalized the model to the case of an arbitrary number of production factors and derived the interrelated factor demand functions. One of the empirical applications of the model is Nadiri and Rosen(1974). The flexible accelerator model is also called the partial adjustment model. Since the latter terminology is more appropriate in connection with our problem, hereafter, we will use it more frequently.

Consider the problem of changes in the price level. The general price level is a reflection of the economic conditions as a whole. If we denote the expected level of prices at period t by p_t^*, we may expect that it changes when external variables such as the foreign exchange rate, the imported prices of raw materials, and/or the domestic money supply changes. If we denote the actual price level at t by p_t, then the required change in the level may be expressed as $p_t - p_{t-1}$. Then, will it be desirable to realize the required change within one period?

In the process of changing the actual price level p_{t-1} to the desired level p_t^*, we may think of various frictions which entail a social cost of adjustment. As the rate of change in the price level changes, the adjustment cost may increase more than proportionately. Therefore, it may be desirable that the actual change in the price level in period t, p_t-p_{t-1}, is less than the desired change, p_t^*-p_{t-1}, or,

$$p_t - p_{t-1} = b(p_t^* - p_{t-1}), \ 0 < b < 1. \tag{1}$$

This relation can be rewritten as:

$$p_t = bp_t^* + (1 - b)p_{t-1}. \tag{2}$$

This is the partial adjustment model of the price level. It shows that the price level at period t depends on the previous level p_{t-1} and the expected level p_t^*. Therefore, if the expected level p_t^* can be expressed in terms of observable variables, then we can estimate the parameter b.

The expected level of prices p_t^* may be affected by various exogenous factors. For simplicity, if we assume that the number of exogenous vairables is two, x_{1t} and x_{2t}, then we can express p_t^* as:

$$p_t^* = a_0^* + a_1^*x_{1t} + a_2^*x_{2t} + u_t^*, \tag{3}$$

where u_t^* is the disturbance term representing the aggregate effects of the minor exogenous factors which affect p_t^*.

If we combine the partial adjustment model (2) with the expression of p_t^* (3), we get:

$$p_t = a_0 + a_1x_{1t} + a_2x_{2t} + cp_{t-1} + u_t, \tag{4}$$

where $a_i = ba_i^*$, i = 1, 2, and $c = 1 - b$. This is the econometrically estimable partial adjustment model.

We have developed above the partial adjustment model of the price level. Likewise, we can develop the partial adjustment model of wage rates under a set of similar assumptions. But if the level of general prices and the level of wage rates are closely interrelated, then the above separate model may not be appropriate. In this case, we may need a combined model. The discrepancy between the expected price level and the realized level of the previous period, p_t^*-p_{t-1}, may affect not only the price level p_t, but also the wage level w_t. Likewise, the discrepancy between the expected wage level and the realized level of the previous period, w_t^*-w_{t-1}, may also affect both p_t and w_t. These considerations lead to the following relations:

$$\begin{aligned} p_t - p_{t-1} &= b_{11}(p_t^*-p_{t-1}) + b_{12}(w_t^*-w_{t-1}) \\ w_t - w_{t-1} &= b_{21}(p_t^*-p_{t-1}) + b_{22}(w_t^*-w_{t-1}). \end{aligned} \tag{5}$$

If we assume that the expected values, p_t^* and w_t^*, are affected by two exogenous variables, x_{1t} and x_{2t}, then we have:

$$\begin{aligned} p_t^* &= a_{10}^* + a_{11}^* x_{1t} + a_{12}^* x_{2t} + u_{1t}^* \\ w_t^* &= a_{20}^* + a_{21}^* x_{1t} + a_{22}^* x_{2t} + u_{2t}^*, \end{aligned} \tag{6}$$

where u_{1t}^* and u_{2t}^* are disturbance terms. Combining the partial adjustment relation (5) and the relation (6), we get the following:

$$\begin{aligned} p_t &= a_{10} + a_{11}x_{1t} + a_{12}x_{2t} + c_{11}p_{t-1} + c_{12}w_{t-1} + u_{1t} \\ w_t &= a_{20} + a_{21}x_{1t} + a_{22}x_{2t} + c_{21}p_{t-1} + c_{22}w_{t-1} + u_{2t}, \end{aligned} \tag{7}$$

where u_{1t} and u_{2t} are disturbances, and the coefficients *a's* and *c's* are obvious from the derivation. For example, $c_{11}=1-b_{11}$ and $c_{12}=1-b_{12}$. The relations in (7) are an econometrically estimable partial adjustment model of prices and wages. In this model, *x's*, *p's* and *w's* are observable variables. From the properties of *b's*, we see that c_{ii} are between 0 and 1.

According to this model, the impact multipliers of exogenous variables are denoted by *a's*, and the cross effects of interrelations between prices and wages are denoted by *c's*. If all of the endogenous and exogenous variables are expressed in natural logarithms, then the coefficients denote the elasticities.

We derived the above model (7) from some intuitive considerations. it can be shown that a rigorous derivation of the model is possible under the assumption of partial adjustment behavior. This derivation can also show the properties of the coefficients, at least qualitatively. For this derivation, we need not restrict the number of endogenous and exogenous variables. In the next section, we will derive the model.

B. The Derivation of the Partial Adjustment Model of Price Level: A Preliminary Model

Eisner and Strotz(1963) derived their flexible accelerator model of investment under the assumption that the firm would maximize its present value by choosing the optimal path of investment. To derive the partial adjustment model of price level, we may start from a maximizing model. We may assume that an economy will choose the time path of the price level to optimize something.

From the stand point of an economy, there may be an optimal level of prices at a point in time. Both higher and lower levels are not desirable, and the disadvantage will increase as the discrepancy from this optimal level increases. The relation between this disadvantage and the level of prices p may be denoted as:

$$F(p) = \frac{1}{2}ap^2 - bp + a_0;\ a,\ b,\ > 0, \tag{8}$$

where a's and b are constants. The sign restriction of a and b is due to the consideration that the function $F(p)$ would have a minimum at the optimal point of p which is positive.

From the point of view of the economy, changing the level of the price level also entails some cost, the adjustment cost. This cost is a function of the rate of change of the price level. As the rate of change increases, the cost of adjustment increases more than proportionately. If we denote the rate of change of the level by p°, then the adjustment cost may be denoted by the function $G(p^\circ)$:

$$G(p^\circ) = \frac{1}{2}cp^{\circ 2};\ c > 0, \tag{9}$$

where c is a constant, and p° is the time derivative of p.

If we define the disadvantage function of p and the adjustment cost function of p° by $F(p)$ and $G(p^\circ)$, respectively, then we may assume that the economy chooses the time path of p so that the present value of the sum, $F(p) + G(p^\circ)$, is minimized, i.e.,

$$min\ V[p(t)] = \int_0^\infty e^{-rt}[F(p(t)) + G(p^\circ(t))]dt, \tag{10}$$

where $p(t)$ is the time path of the price level, and r is the discount rate of the economy.

Now, finding the optimal time path of the price level amounts to solving the calculus-of-variations problem, the Euler-Lagrange equation given by:

$$cp^{\circ\circ} - rcp^\circ - ap - b = 0, \tag{11}$$

where $p^{\circ\circ}$ is the second time derivative of p. This is a second order differential equation, whole solution is composed of the particular solution and the complementary solution.

The particular solution of the differential equation can be obtained by putting $p^{\circ\circ}=p^{\circ}=0$. If we denote the solution by p^*, then we have:

$$p^* = b/a. \tag{12}$$

This solution may be called the equilibrium solution, or the desired level of p, in the sense that, over time, the economy pursues p^*.

To obtain the complementary solution of the differential equation, consider its characteristic equation:

$$cm^2 - rcm - a = 0, \tag{13}$$

whose roots are:

$$m_1,\ m_2 = \tfrac{1}{2}\left[rc \pm \sqrt{(rc)^2 + 4ac}\right]/c. \tag{14}$$

By assumption, a and c are positive, so that:

$$\sqrt{(rc)^2 + 4ac} > rc > 0.$$

Therefore, the two roots, m_1 and m_2, are all real: the smaller root, say, m_1 is negative and the larger root, m_2, is positive. If we denote the arbitrary constants by k_1 and k_2, then the solution of the differential equation (11) may be written:

$$p(t) = p^* + k_1 exp(m_1 t) + k_2 exp(m_2 t).$$

But for this solution to be stable, k_2 should be zero identically. Under this condition, if we denote $p(0) = p_0$, the solution becomes:

$$p(t) = p^* + (p_0 - p^*)exp(m_1 t). \tag{15}$$

To obtain our main result we differentiate this equation with respect to t:

$$p^{\circ}(t) = m_1(p_0 - p^*)exp(m_1 t) = m_1[p(t) - p^*],$$

or

$$p^{\circ}(t) = B[p^{*} - p(t)], \qquad (16)$$

where $B = -m_1$, by definition, so that it is positive. This relation shows that if the desired price level exceeds the prevailing level, the price level will rise. This is a model of partial adjustment.

C. The Derivation of the Partial Adjustment Model of Prices and Wages: The Main Model

Equation (16) is a preliminary partial adjustment model of price level. But that model does not allow us to deal with the interrelation between the price and wage levels. For the main purpose of our analysis, it is also desirable to separate the groups of price items, such as raw materials, capital goods, and consumer goods, because they may have different effects on the wage rates. Furthermore, for our empirical analysis, the continuous time model, such as equation (16), is not convenient. A discrete time model with difference equations is more appropriate. In this section we will derive the partial adjustment model that satisfies these considerations.

Consider the vector of price-wage items, p:

$$p = [p_1, p_2, \ldots, p_n],$$

where p_i, $i = 1, .., n$, denotes each of the price-wage items such as the price level of the raw materials, the price level of the consumer goods, and the level of wage rates. Therefore, this vector is a positive vector. Consider the generalization of the disadvantage function (8) using vector p:

$$F(p) = \tfrac{1}{2} p'Ap - b'p + a_0. \qquad (17)$$

At some optimal level of p this function should have a minimum. Mathematically, for this function to have a minimum at positive level of p, the coefficient matrix A should be positive definite, and the coefficient vector b should be a positive vector. We will assume that these conditions are satisfied.

If there is no problem of adjustment cost, the economy will always try to maintain the price-wage vector that minimizes the disadvantage function $F(p)$. But we cannot ignore the adjustment cost when the prevailing vector p is not identically minimized in the function. Consider the generalization of the adjustment cost function (9), using the price-wage vector p:

$$G(\triangle p) = \tfrac{1}{2}(\triangle p)'C(\triangle p), \tag{18}$$

where $\triangle p = p_{t+1} - p_t$, i.e., the first difference of p between time periods $t+1$ and t, and C is the diagonal matrix with positive diagonal elements. Note that the concept of the first difference has replaced the concept of the first derivative in (9). This adustment cost functions shows that the cost increases as the speed of adjustment of every price-wage item increases.

If we define the disadvantage function as a function of p, and the adjustment cost function as a function of $\triangle p$, then we may think of the objective function of the economy as the accumulated disadvantage of the combined cost evaluated in terms of the present value. In other words, we will assume that the economy minimizes the following by appropriately choosing the time path of the vector p:

$$min\ V[p_t] = \sum_{t=0}^{T} e^{-rt}[F(p_t) + G(\triangle p_t)], \tag{19}$$

where T is the time horizon and r is the discount rate of the economy assumed to be constant.

The optimal time path of p_t should satisfy the following first order conditions:

$$\partial V/\partial p_t = [Ap_t - b - C(p_{t+1} - p_t) + C(p_t - p_{t-1})(1+r)](1+r)^{-t} = 0,\ t=1,\dots,T;$$
$$\partial V/\partial p_{T+1} = (1+r)^{-T}C(p_{T+1} - p_T) = 0.$$

We also assume that the second-order sufficient conditions are satisfied. We can easily show that this assumption is closely related to the previous assumptions of positive definiteness of both A and C.

The last of the first order conditions is satisfied when the time horizon T goes to infinity. From the remaining first order conditions we get the following:

$$Cp_{t+2}-[A+(2+r)C]p_{t+1}+(1+r)Cp_t+b = 0, \ t=0,1,...,T-1. \quad (20)$$

This is a second-order difference equation whose solution can be divided into two parts, the particular solution and the complementary solution. This solution determines the time path of vector p_t.

To find the particular solution, p^*, we put

$$p_{t+2} = p_{t+1} = p_t = p^*$$

in the above difference equation, so that we have

$$p^* = A^{-1}b. \quad (21)$$

If we define x_t and y_t such that

$$x_t = p_t - p^* \text{ and } y_t = x_{t+1},$$

then equation (20) can be rewritten as;

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} C^{-1}A+(2+r)I & -(1+r)I \\ I & 0 \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix}.$$

To find the solution of the equation, we need the characteristic roots and vectors of the coefficient matrix. By inspection, we see that finding a diagonal matrix which is similar to $C^{-1}A$ is a very useful intermediate step. If we define the positive diagonal matrix D by $D = C^{\frac{1}{2}}$, then $D^{-1}AD^{-1}$ is similar to $C^{-1}A$, since $D^{-1}AD^{-1} = D(C^{-1}A)D^{-1}$. Note that $D^{-1}AD^{-1}$ is a positive definite matrix. If M is the positive diagonal matrix with diagonal elements m_i which are the roots of this positive definite matrix, and Q is the orthonormal matrix whose columns are the characteristic vectors corresponding to m_i, then we always have the relation,

$$QD^{-1}AD^{-1}Q'=M,$$

which implies that matrices M, $D^{-1}AD^{-1}$, and $C^{-1}A$ are similar to each other. In particular,

$$M = T^{-1}(C^{-1}A)T,$$

where T is defined as $T = D^{-1}Q^{-1} = D^{-1}Q'$.

Consider the following operation:

$$\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}^{-1} \begin{bmatrix} C^{-1}A+(2+r)I & -(1+r)I \\ -I & 0 \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} = \begin{bmatrix} M+(2+r)I & -(1+r)I \\ I & 0 \end{bmatrix}.$$

This operation implies that the right-hand-side matrix is similar to the coefficient matrix of our difference equation system. Since the set of the characteristic roots of similar matrices are identical to each other, we can obtain the set of desired roots by solving the characteristic equation of the left-hand-side matrix. The equation is

$$\begin{vmatrix} (h-2-r)I - m & (1+r)I \\ -I & hI \end{vmatrix} = \prod_{i=1}^{n} [h(h-2-r-m_i) + (1+r)]$$

$$= \prod_{i=1}^{n} [h^2 - (2+r+m_i)h + (1+r)] = 0.$$

This is a 2nd degree polynomial equation. By inspection we see that for each m_i, two characteristic roots h_{i1} and h_{i2} correspond as follows:

$$h_{i1},\ h_{i2} = \tfrac{1}{2}\left[(2+r+m_i) \pm \sqrt{(2+r+m_i)^2 - 4(1+r)}\right]$$

$$= 1 + \tfrac{1}{2}\left[(r+m_i) \pm \sqrt{(r+m_i)^2 + 4m_i}\right],\ i=1,2,\ldots,n.$$

We can easily see that the roots are all positive and real. Furthermore, for each pair of the roots, the one, say h_{i1}, is less than 1 and the other, say h_{i2}, is greater than 1, so that we get the inequalities,

$$0 < h_{i1} < 1, \text{ and } h_{i2} > 1, \text{ for } i=1,2,\ldots,n.$$

From the definition of the characteristic equation, we get the following equation system:

$$\begin{aligned} &h_{i1}{}^2 - (2+r+m_i)h_{i1} + (1+r) = 0 \\ &h_{i2}{}^2 - (2+r+m_i)h_{i2} + (1+r) = 0 \end{aligned} \qquad \textit{for all } i=1,2,\ldots,n.$$

In matrix notation this equation system becomes:

$$
\begin{aligned}
&H_1{}^2-(2I+rI+M)H_1+(1+r)I=0\\
&H_2{}^2-(2I+rI+M)H_2+(1+r)I=0,
\end{aligned}
\tag{22}
$$

where H_1 and H_2 are $n \times n$ diagonal matrices with diagonal elements h_{i1}'s and h_{i2}'s, respectively.

The next intermediate step is to find out the characteristic vectors which correspond to the above characteristic roots. If we denote the 2nd characteristic vectors corresponding to H_1 and H_2 by

$$
V=\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}
$$

then, from the general relation between the characteristic roots and vectors, we get the matrix equation system,

$$
\begin{bmatrix} (2+r)I+M & -(1+r)I \\ I & 0 \end{bmatrix}
\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}=
\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}
\begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}.
$$

This equation system can be expanded as

$$
\begin{aligned}
(2I+rI+M)V_{11}-(1+r)V_{21} &= V_{11}H_1\\
V_{11} &= V_{21}H_1\\
(2I+rI+M)V_{12}-(1+r)V_{22} &= V_{12}H_2\\
V_{12} &= V_{22}H_2.
\end{aligned}
\tag{23}
$$

This system can, in turn, be reduced as follows:

$$
\begin{aligned}
(2I+rI+M)V_{21}H_1 - (1+r)V_{21} &= V_{21}H_1{}^2\\
(2I+rI+M)V_{22}H_2 - (1+r)V_{22} &= V_{22}H_2{}^2,
\end{aligned}
$$

or

$$
\begin{aligned}
(2I+rI+M)V_{21}H_1 &= V_{21}[H_1{}^2+(1+r)I]\\
(2I+rI+M)V_{22}H_2 &= V_{22}[H_2{}^2+(1+r)I].
\end{aligned}
\tag{24}
$$

But from (22), we see that

$$
\begin{aligned}
H_1{}^2+(1+r)I &= (2I+rI+M)H_1\\
H_2{}^2+(1+r)I &= (2I+rI+M)H_2,
\end{aligned}
$$

so that (24) becomes

$$\begin{aligned} M^*V_{21} &= V_{21}M^* \\ M^*V_{22} &= V_{22}M^*, \end{aligned} \tag{25}$$

where $M^*=2I+rI+M$. Since M^* is symmetric, the equation system (25) holds for any symmetric matrices V_{21} and V_{22}. The simplest case is when V_{21} and V_{22} are both an nth order identity matrix. So we will assume that

$$V_{21} = I, \text{ and } V_{22} = I.$$

If we substitute them in the second and the last equations in (25), we get

$$V_{11} = H_1, \text{ and } V_{12} = H_2.$$

Thus, we get the desired matrix V with characteristic vectors of the modified coefficient matrix. By inverse modification of V we get the matrix of the characteristic vectors of the original coefficient matrix as:

$$\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} H_1 & H_2 \\ I & I \end{bmatrix} = \begin{bmatrix} TH_1 & TH_2 \\ T & T \end{bmatrix}$$

Using the above results, we are able to get an appropriate expression of the solution of the difference equation system as

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} TH_1 & TH_2 \\ T & T \end{bmatrix} \begin{bmatrix} H_1^t & 0 \\ 0 & H_2^t \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix},$$

where k_1 and k_2 are arbitrary constant n-vectors. But for the stability of the system we need the condition, $k_2=0$. Otherwise, the system contains the term H_2^t which is divergent as t goes to infinity. Therefore we will assume that $k_2=0$.

Under this condition, x_t becomes $x_t=TH_1^t k_1$. If the initial value of x_t at $t=0$ is known as x_0, then we can express k_1 as $k_1=T^{-1}x_0$, so that the specific solution of x_t becomes

$$x_t = TH_1^t T^{-1}x_0.$$

If we consider the definitional relation $p_t = p^*+x_t$, then we have

$$p_t = p^* + TH_1T^{-1}(p_0 - p^*),$$

where p_0 is p_t when $t=0$. We can express this using the first difference, $\triangle p_t = p_{t+1} - p_t$, as

$$\triangle p_t = T(H_1^{t+1} - H_1^t)T^{-1}(p_0 - p^*) = T(I - H_1)T^{-1}(p^* - p_t),$$

or

$$\triangle p_t = B(p^* - p_t), \tag{26}$$

where, by definition,

$$B = T(I - H_1)T^{-1}. \tag{27}$$

Equation (26) is the genenalized partial adjustment model of prices and wages at which we aimed. The properties of the model is concentrated at the n x n matrix B. Therefore, we will analyze this matrix.

D. The Analysis of Matrix B

From the definition of matrix B given by (27), we have

$$BT = T(I - H_1). \tag{28}$$

This implies that the diagonal elements, $1 - h_{i1}$, of the diagonal matrix $I - H_1$ are the characteristic roots of the matrix B. We already know that h_{i1}'s are positive fractions, so that

$$0 < 1 - h_{i1} < 1.$$

In the generalized partial adjustment model (26), this implies that the discrepancy between p^* and p_t diminishes as t increases.

Recall that the matrix T was defined as

$$T = D^{-1}Q'.$$

If we utilize this relation, we can express B as follows:

$$B = T(I - H_1)T^{-1} = D^{-1}Q'(I - H_1)QD,$$

or

$$B = D^{-1} B^* D, \tag{29}$$

where, by definition,

$$B^* = Q'(I - H_1)Q$$

whose (i,j)th element is b^*_{ij}. From this definition we see that B^* is symmetric and positive definite, so that

$$b^*_{ii} > 0, \text{ and } b^*_{ij} = b^*_{ji}, \quad \text{for all } i,j = 1,2,\ldots,n.$$

Considering that D is diagonal with diagonal elements d_i, the relation (29) implies

$$b_{ij} = d_i^{-1} b^*_{ij} \, d_j, \quad \text{for all } i,j = 1,2,\ldots,n, \tag{30}$$

so that

$$b_{ij} b_{ji} = b^{*\,2}_{ij} > 0.$$

This implies that B is sign-symmetric, though it is not symmetric in general. Relation (31) also implies that

$$b_{ii} = b^*_{ii} > 0, \text{ for all } i = 1,2,\ldots,n,$$

so that the diagonal elements of B are always positive. Furthermore,

$$b_{ij}/b_{ji} = (d_j/d_i)^2 = c_j/c_i, \tag{31}$$

where c_i, $i = 1,2,\ldots,n$, are the diagonal elements of the coefficient matrix C of the adjustment cost function defined earlier as

$$G(\triangle p) = (\triangle p)' C (\triangle p).$$

Relation (31) implies that the ratio of symmetric elements of B is equal to the ratio of the corresponding marginal adjustment costs. The marginal adjustment cost is variant with respect to the measurement cost of the corresponding element of p. Define q as the linear transformation of p such that

$$q = Dp.$$

Then, $\triangle q = D(\triangle p)$ and $\triangle p = D^{-1}(\triangle q)$, and we can express the adjustment cost G as the function of q as follows:

$$G = (\triangle p)'C(\triangle p) = (\triangle q)'D^{-1}CD^{-1}(\triangle q) = (\triangle q)'(\triangle q).$$

This implies that, if we express the adjustment cost as the function of $\triangle q$ instead of $\triangle p$, then the coefficient matrix becomes the $n \times n$ identity matrix. In turn, the partial adjustment matrix becomes B^* which is symmetric and positive definite.

E. The Econometric Partial Adjustment Model

We derived above the partial adjustment model of the form

$$\triangle p_t = B(p^* - p_{t-1}).$$

To modify this to an econometrically estimable model, we need to specify p^*. For this purpose we will assume that p^* itself is a function of a set of exogenous variables and disturbance terms, so that it becomes a function of time indirectly, i.e.,

$$p_t^* = G^* z_t + u_t^*.$$

Then the above model takes the following form:

$$p_t - p_{t-1} = B(G^* z_t + u_t^* - p_{t-1}),$$

or

$$p_t = BG^* z_t + (I - B)p_{t-1} + Bu_t^*.$$

We will redefine notations as

$$A = BG^*,$$
$$C = I - B,$$
$$u_t = Bu_t^*,$$

and

$$x_t = z_t,$$

then this model becomes

$$p_t = Ax_t + Cp_{t-1} + u_t. \tag{32}$$

This is the fundamental econometric model of our study. Given appropriate data sets on p_t and x_t we can estimate this model according to the conventional econometric procedure.

F. Some Dynamic Properties of the Model

According to the fundamental model (32), the coefficient matrix A shows the multipliers of exogenous variables x_t on the endogenous variables p_t. But, because this model is a dynamic one, the effects of the exogenous variables on the endogenous variables are not confined to the contemporary period. The multiplier matrix shows only the impact multiplier. The lagged or interim multipliers can be found according to the following procedure.

Let us introduce the lag operator L such that

$$Lp_t = p_{t-1}, \text{ and } L^s p_t = p_{t-s},\ s=1,2,\dots .$$

Then model (32) can be rewritten as

$$p_t = Ax_t + CLp_t + u_t,$$

or

$$(I-CL)p_t = Ax_t + u_t,$$

or

$$p_t = (I-CL)^{-1}Ax_t + (I-CL)^{-1}u_t. \tag{33}$$

Since the characteristic roots of $C=I-B$ are between 0 and 1, the non-stochastic part of (33) can be expanded as

$$(I-CL)^{-1}Ax_t = Ax_t + CAx_{t-1} + C^2Ax_{t-2} + \dots . \tag{34}$$

Therefore, we may say that C^sA, $s=0,1,2,\dots$, denotes the effect of x_t on p_{t+s}. Therefore, it is called the sth interim multiplier. If we sum the interim multipliers over s, then we get the total multiplier as:

$$(I - C)^{-1}A = B^{-1}BG^* = G^*. \tag{35}$$

(Note that the impact multiplier is the 0th interim multiplier.) Because we can estimate matrices A and C, we can also estimate the impact, interim, and total multipliers according to the above procedure.

The dynamic properties of our model can be derived from the shape of the sth interim multiplier C^sA. Considering the definition of C, C^s can be written as:

$$C^s = (I-B)^s = TH_1^s T^{-1},\ s=0,1,2,\dots .$$

Since H_1 is the diagonal matrix with diagonal elements between 0 and 1, H_1^s steadily approaches 0 matrix as s goes to infinity. So does C^s. Therefore, the lag structure of our model resembles that of the geometric lag structure, which steadily approaches zero as the length of the lag increases.

III. Empirical Analysis

A. Some Concrete Procedures for Empirical Analaysis

We have derived the partial adjustment model of prices and wages in the previous part. The model itself is neutral with respect to the relative importance of prices and wages in the mechanism of the interrelationships. The quantitative magnitude of the effects of prices on wages and of wages on prices can be identified only through an empirical analysis of the model.

For our empirical analysis, we have partitioned the price variables into three groups: raw materials, capital goods, and consumer goods. The price indices of these groups, together with the wage rate in the manufacturing sector, constitute the endogenous variables in our empirical model. The exogenous variables of the model are the import price index, the money supply, and the gross national product. Besides, the quarterly seasonality variables and the constant term variable are included as the pure exogenous variables. We take the natural logarithms for all variables except the pure exogenous ones.

Our specific model is as follows:

$$P_{it}=a_{i1}^{*}+a_{i2}^{*}d_{2t}+a_{i3}^{*}d_{3t}+a_{i4}^{*}d_{4t}+a_{i1}p_{mt}+a_{i2}m1_t+a_{i3}Y_t+c_{i1}p_{rt-1}+ c_{i2}p_{kt-1}+c_{i3}p_{ct-1}+c_{i4}p_{wt-1}+u_{it},$$

for $i = r, k, c$, and w, where

p_r: the natural log of the price index for raw materials,

p_k: the natural log of the price index for capital goods,

p_c: the natural log of the price index for consumer goods,

p_w: the natural log of the wage rate of the manufacturing sector,

p_m: the natural log of the price index for imported goods,

d_j: the seasonal dummy variable for the jth quarter,

$m1$: the natural log of the money supply, $M1$,

Y: the gross national product in current prices.

For notational simplicity, we will express the four-equation model as:

$$p_t = A^*d_t + Ax_t + Cp_{t-1}, \tag{36}$$

where A^* is the 4x4 coefficient matrix of the pure exogenous variables, A is the 4x3 coefficient matrix of the other exogenous variables, and C is the 4x4 coeffcient matrix of the lagged endogenous variables. Note that model (36) is exactly the same model as (32) with minor notational modifications. Note, also, that each element of A and C denotes the elasticity of p_{it} with respect to the corresponding variables.

For our empirical analysis, we use the quarterly data sets of Korea and Japan for the period, 1966-1980. Because the data sets are under the conditions of strong multicollinearity, the ordinary least squares estimation, or the generalized least squares estimation does not give us econometrically satisfactory results. Therefore, we have tried the ridge regression technique proposed by Hoerl and Kennard(1970)(see Appendix 1). The following sections contain the summary of this empirical analysis.

B. Interrelationships between Prices and Wages

For the Korean economy, the empirically estimated matrix C, is as follows:

$$C_k = \begin{bmatrix} 0.302 & 0.267 & 0.188 & 0.061 \\ 0.091 & 0.187 & 0.089 & 0.052 \\ 0.175 & 0.227 & 0.221 & 0.120 \\ 0.147 & 0.315 & 0.313 & 0.236 \end{bmatrix} . \tag{37}$$

As explained above, each element of this matrix denotes an elasticity measure. For example, the first column denotes the elasticities of the prices and wages with respect to the price of raw materials one quarter ago. Similarly, the last column denotes the elasticities with respect to the wage rate one quarter ago. If we read this matrix by rows, each row denotes the elasticities of the price or wage group with respect to the prices and wage rate one quarter ago. More concretely, element c_{ij} means that we can expect that the price increase of the ith price group in the present quarter is c_{ij} percent due to a one percent price increase in the jth group. Therefore, we may think of c_{ij} as the measure of response received by i from j. The symmetric element c_{ji} measures the influence of i given to j.

From the above matrix, we perceive some conspicuous asymmetry. To understand the asymmetry, let us measure the net responsiveness of i from j as c_{ij}-c_{ji}. Then, the net responsiveness matrix is given by C-C'. If the influence and response are symmetric between each pair of price and wage groups, then this matrix becomes a zero matrix. From the above matrix we can get the responsiveness matrix for Korea as follows:

$$C_K - C_K' = \begin{bmatrix} 0 & 0.176 & 0.013 & -0.086 \\ -0.176 & 0 & -0.138 & -0.263 \\ -0.013 & 0.038 & 0 & -0.193 \\ 0.086 & 0.263 & 0.193 & 0 \end{bmatrix} \quad (38)$$

This matrix is skew-symmetric, or, the (i,j)th element is the negative of the (j,i)th element. From this matrix we see that the net responsiveness of wage rate is always positive (see column 4). This means that the effects of a wage increase on the other price groups are always smaller than the influences received by the wage rate from the corresponding price groups. On the other hand, the price of capital goods shows negative net responsiveness. Capital goods price gives net influence to the prices of other groups of goods and wage rate.

We may emphasize here the net positive responsiveness of wage rate because we may derive an important economic implication from this fact.

The positiveness of the fourth column elements c_{i4} of C implies that the wage rate does a positive role in the wage-price spiral of an inflationary process. But the net positive responsiveness of wage rate implies that the role of the wage rate in this spiral is smaller than the role of the prices. In fact, the ratios c_{i4}/c_{4i} are less than half for all $i=1,2$, and 3.

For the Japanese economy, the estimated C-matrix is as follows:

$$C_J = \begin{bmatrix} 0.256 & 0.371 & 0.250 & 0.027 \\ 0.118 & 0.243 & 0.159 & 0.008 \\ 0.098 & 0.228 & 0.183 & 0.041 \\ 0.061 & 0.207 & 0.291 & 0.188 \end{bmatrix} \tag{39}$$

From this matrix, we can derive the net responsiveness matrix as

$$C_J - C_J' = \begin{bmatrix} 0 & 0.253 & 0.152 & -0.034 \\ -0.253 & 0 & -0.069 & -0.199 \\ -0.152 & 0.069 & 0 & -0.250 \\ 0.034 & 0.199 & 0.250 & 0 \end{bmatrix}. \tag{40}$$

If we compare C_K and C_J, we may be surprised by the similar configuration of elements of the two matrices. This may be a reflection of the basic similarity of some relevant aspects of the Korean and Japanese economies. Comparison of the net responsiveness matrices, $C_K - C_K'$ and $C_J - C_J'$, gives a similar qualitative feelings.

To find out some quantitative difference between Korea and Japan, consider the matrices

$$C_K - C_J = \begin{bmatrix} 0.046 & -0.014 & -0.062 & 0.034 \\ -0.207 & -0.056 & -0.070 & 0.044 \\ 0.077 & -0.001 & 0.038 & 0.079 \\ 0.086 & 0.108 & 0.022 & 0.048 \end{bmatrix} \tag{41}$$

and

$$(C_K - C_K') - (C_J - C_J') = \begin{bmatrix} 0 & -0.077 & -0.139 & -0.052 \\ 0.077 & 0 & -0.069 & -0.064 \\ 0.139 & 0.069 & 0 & 0.057 \\ 0.052 & 0.064 & -0.057 & 0 \end{bmatrix} \tag{42}$$

In matrix (41), all elements of the fourth row are positive. This implies that the wage rate is more sensitive to the other price variables in Korea than in Japan. This may reflect the difference in the absorption capacity in the labor side of the two economies. We also notice that all elements of the fourth column are also positive. This implies that the impact of a wage rate change on the other price variables is relatively large in Korea compared to Japan. This may reflect the lack of an absorption capacity in the production side in the Korean economy.

Matrix (42) measures the difference in the net responsiveness of the two economies. The positive elements of the third row show that the price of the consumption goods is more net responsive to the changes in the other price-wage variables in Korea than in Japan.

C. The Analysis of the Multiplier Matrices

We have analyzed above the interactions of the price-wage variables that have two-way relations with each other. Now we will analyze the one-way impacts of exogenous variables on the endogenous price variables.

We know that matrix A in model (36) is the impact multiplier matrix of the price-wage variables with respect to the exogenous variables, i.e., the import-good prices, the money supply, and the gross national product. The empirically estimated matrix A of the Korean economy is as follows:

$$A_K = \begin{bmatrix} 0.382 & 0.009 & 0.004 \\ 0.125 & 0.059 & 0.073 \\ 0.123 & 0.047 & 0.075 \\ 0.122 & 0.135 & 0.222 \end{bmatrix} . \tag{43}$$

The (i,j)th element of A, a_{ij}, shows the impact of the jth exogenous variable on the ith endogenous variable. The magnitude of the elements of the first column is conspicuously large. This implies that the price of imported goods in the Korean economy is the most important exogenous determinant of the price-wage variables. Especially, the price of raw materials receives the largest impact from the import-good price.

According to matrix (43), the money supply and the gross national product have relatively small direct impacts on the price-wage variables. But, it may be interesting to observe that the main entry point of the impacts is wage rate. They give the direct impacts mainly to the wage rate.

Observing the impact multiplier matrix is not enough for us to understand the whole effects of exogenous variables to the endogenous variables. Because our model is a dynamic one, we should consider the interim and total multipliers. At the end of Part One, we have shown that the sth interim multiplier matrix can be expressed as C^sA.

Table 1 shows the interim multiplier matrices. According to this table, the series of the interim multiplier matrices approaches zero matrix relatively rapidly. Within twelve quarters, or three years, it becomes almost a zero matrix. This fact confirms our theory developed in the previous part.

The total multiplier is the sum of the impact multiplier and the whole of the interim multipliers. The total multiplier matrix in our case can be expressed as:

$$(I-C)^{-1}A=A+CA+C^2A+\dots$$

This shows the totality of the total multipliers of our system. The estimated total multiplier matrix for Korea is as follows:

$$(I-C)^{-1}A_K=\begin{bmatrix} 0.906 & 0.140 & 0.188 \\ 0.365 & 0.129 & 0.172 \\ 0.579 & 0.180 & 0.267 \\ 0.721 & 0.331 & 0.507 \end{bmatrix} \tag{44}$$

The impression of this matrix may be somewhat different from that of matrix A. The total elasticity of import price on the price of the raw materials is as high as 0.906. If the price of imported goods increases 10 percent, then we should expect the domestic price of the raw materials to increase about 9 percent in the long run. The total effect of imported price on the wage rate is also very high. The total effects of the money supply and the gross national product on the price-wage variables are

TABLE 1
THE SERIES OF THE INTERIM MULTIPLIER MATRICES (KOREA)

S	Pm	M1	GNP	S	Pm	M1	GNP
1				7			
	0.179	0.036	0.048		0.016	0.005	0.007
	0.075	0.022	0.032		0.008	0.002	0.003
	0.137	0.042	0.060		0.015	0.004	0.006
	0.163	0.066	0.099		0.022	0.006	0.009
2				8			
	0.110	0.029	0.041		0.011	0.003	0.005
	0.051	0.015	0.021		0.005	0.002	0.002
	0.098	0.029	0.041		0.011	0.003	0.004
	0.131	0.041	0.060		0.015	0.004	0.006
3				9			
	0.073	0.021	0.029		0.008	0.002	0.003
	0.035	0.010	0.014		0.004	0.001	0.002
	0.068	0.020	0.028		0.007	0.002	0.003
	0.094	0.028	0.040		0.010	0.002	0.004
4				10			
	0.050	0.014	0.020		0.005	0.002	0.002
	0.024	0.007	0.010		0.003	0.001	0.001
	0.047	0.014	0.019		0.005	0.001	0.002
	0.065	0.019	0.027		0.007	0.002	0.003
5				11			
	0.034	0.010	0.014		0.004	0.001	0.002
	0.017	0.005	0.007		0.002	0.001	0.001
	0.033	0.009	0.013		0.003	0.001	0.001
	0.045	0.013	0.019		0.005	0.001	0.002
6							
	0.024	0.007	0.010				
	0.012	0.003	0.005				
	0.022	0.006	0.009				
	0.031	0.009	0.013				

also high relative to the direct and impact effects.

For the Japanese economy, we also estimated the impact multiplier matrix as follows:

$$A_J = \begin{bmatrix} 0.384 & -0.002 & 0.008 \\ 0.147 & -0.013 & -0.011 \\ 0.098 & 0.036 & 0.041 \\ 0.023 & 0.274 & 0.278 \end{bmatrix} \tag{45}$$

The configuration of matrix A_J is very similar to that for Korea, or A_K. In

Japan, too, the price of imported goods is the most important external determinant for the price-wage variables. The price of raw materials is most easily affected by the imported good price. As for the effects of the money supply and the gross national product, the main entry point of the effects is the wage rate. Therefore, the structure of the mechanism of the two economies are very similar qualitatively. There appears some negative figures in the matrix, but they are not significant statistically.

To understand the quantitative difference between the two economies, we will examine the following matrix:

$$A_K - A_J = \begin{bmatrix} -0.002 & 0.011 & -0.004 \\ -0.022 & 0.072 & 0.084 \\ 0.025 & 0.011 & 0.034 \\ 0.099 & -0.139 & -0.056 \end{bmatrix} \tag{46}$$

One of the conspicuous facts of this matrix is that the elements of the third row are all positive. This may imply that the price of consumer goods is more vulnerable to external shocks in Korea than in Japan. The positive (4,3)th element reflects the relatively deep dependence of the wage goods on imports in Korea.

Next, consider the interim multiplier matrices. As we have explained above, we can get the estimated version of the matrices from C_J and A_J as $C_J^s A_J$. Table 2 shows the interim multiplier matrices for Japan.

In Table 2, the series of the interim multiplier matrices approaches zero matrix in the same fashion as for Korea. Within twelve quarters, or three years, the series becomes almost a zero matrix.

As explained above, the total multiplier matrix is defined by the sum of the impact multiplier matrix and all of the interim multiplier matrices. The estimated version of the total multiplier matrix is given by the following:

$$(I-C)^{-1} A_J = \begin{bmatrix} 0.846 & 0.037 & 0.060 \\ 0.403 & 0.007 & 0.016 \\ 0.350 & 0.069 & 0.081 \\ 0.320 & 0.367 & 0.380 \end{bmatrix} \tag{47}$$

TABLE 2
THE SERIES OF THE INTERIM MULTIPLIER MATRICES (JAPAN)

S	Pm	M1	GNP	S	Pm	M1	GNP
1				7			
	0.178	0.011	0.016		0.010	0.001	0.001
	0.097	0.005	0.007		0.006	0.001	0.001
	0.090	0.015	0.017		0.006	0.001	0.001
	0.087	0.059	0.062		0.008	0.001	0.001
2				8			
	0.106	0.010	0.013		0.006	0.001	0.001
	0.060	0.005	0.007		0.004	0.000	0.000
	0.060	0.007	0.009		0.004	0.000	0.000
	0.073	0.017	0.019		0.005	0.001	0.001
3				9			
	0.066	0.007	0.008		0.004	0.000	0.001
	0.037	0.004	0.005		0.002	0.000	0.000
	0.038	0.004	0.005		0.002	0.000	0.000
	0.050	0.007	0.008		0.003	0.000	0.000
4				10			
	0.042	0.004	0.005		0.003	0.000	0.000
	0.023	0.002	0.003		0.001	0.000	0.000
	0.024	0.003	0.003		0.001	0.000	0.000
	0.032	0.004	0.005		0.002	0.000	0.000
5				11			
	0.026	0.003	0.003		0.002	0.000	0.000
	0.015	0.002	0.002		0.001	0.000	0.000
	0.015	0.002	0.002		0.001	0.000	0.000
	0.020	0.002	0.003		0.001	0.000	0.000
6							
	0.016	0.002	0.002				
	0.009	0.001	0.001				
	0.009	0.001	0.001				
	0.013	0.001	0.002				

The shape of this matrix is very similar to the corresponding matrix for Korea. But this matrix emphasizes the conspicuous importance of the price of imported goods as the external factor, and the importance of the wage rate as the entry point of the effects from the money supply and the gross national product. Note, also, that some negative figures in A_j has disappeared in this total multiplier matrix.

To understand the difference in the total effects of exogenous variables on the endogenous variables between the two economies, let us consider the difference matrix.

$$(I-C)^{-1}A_K-(I-C)^{-1}A_J=\begin{bmatrix} 0.060 & 0.103 & 0.128 \\ -0.038 & 0.122 & 0.156 \\ 0.229 & 0.111 & 0.186 \\ 0.401 & -0.036 & 0.127 \end{bmatrix}. \quad (48)$$

From this matrix, we may get an explanation similar to that of A_K-A_J. Because the Korean economy is not well structured compared to the Japanese economy, the Korean economy seems to be more vulnerable to external shocks.

D. The Two Group Model

Up to now, we have dealt with the four-group model in the sense that the number of the endogenous variables of prices and wages is four. Now, we want to reduce the model to a two-group one, so that the interactions between prices and wages come out more vividly. The new model is simply a modification of the previous one, i.e.,

$$P_{it}=a^*_{i1}+a^*_{i2}\,d_{2t}+a^*_{i3}d_{3t}+a^*_{i4}\,d_{4t}+a_{i1}p_{mt}+a_{i2}m1_t+a_{i3}Y_t+c_{i1}p_{gt-1}$$
$$+c_{i2}p_{wt-1}+u_{it},$$

for i = g and w, where

p_g: the natural log of the wholesale price index,

p_w: the natural log of the wage rate of the manufacturing sector, and the other notations are the same as before. The model can also be written as:

$$p_t = A^*d_t + Ax_t + Cp_{t-1}, \quad (49)$$

where the coefficient matrices A^*, A, and C are of orders 2x4, 2x3, and 2x2, respectively.

For our empirical analysis of the two-group model, we used similar data sets and similar estimation procedure as for the four-group model. The following two matrices are the estimated versions of matrix C for the Korean and Japanese economies, respectively:

$$C_K=\begin{bmatrix} 0.364 & 0.145 \\ 0.364 & 0.321 \end{bmatrix} \quad C_J=\begin{bmatrix} 0.363 & 0.039 \\ 0.302 & 0.208 \end{bmatrix}. \quad (50)$$

From these matrices, it is apparent that we can derive the same implications about the interactions of prices and wages, as we did with the four-group model. The general price level is more active than the wage rate in the price-wage spiral.

Consider the difference matrix,

$$C_K - C_J = \begin{bmatrix} 0.001 & 0.106 \\ 0.062 & 0.113 \end{bmatrix} \tag{51}$$

This matrix also shows that the Korean economy is more active in the price-wage interactions than the Japanese economy.

Now, consider the matrices,

$$C_K - C_K' = \begin{bmatrix} 0 & -0.219 \\ 0.219 & 0 \end{bmatrix}, \quad C_J - C_J' = \begin{bmatrix} 0 & -0.263 \\ 0.263 & 0 \end{bmatrix}$$

and

$$(C_K - C_K') - (C_J - C_J') = \begin{bmatrix} 0 & 0.044 \\ -0.044 & 0 \end{bmatrix}.$$

These matrices show that, though the general price level is more active than the wage rate in the price-wage spiral in both economies, absolutely, the wage rate is more active in Korea than in Japan, relatively.

The following matrices are the estimated versions of the impact multiplier matrices for Korea and Japan, respectively.

$$\begin{aligned} A_K &= \begin{bmatrix} 0.339 & 0.053 & 0.080 \\ 0.175 & 0.175 & 0.294 \end{bmatrix} \\ A_J &= \begin{bmatrix} 0.266 & 0.019 & 0.031 \\ 0.094 & 0.277 & 0.291 \end{bmatrix} \end{aligned} \tag{52}$$

These matrices also imply that the price of imported goods is the most important external determinant of the price-wage variables and that the entry point of the effect of the money supply and the gross national product is the wage rate, in both economies.

Consider the difference matrix,

TABLE 3
THE SERIES OF THE INTERIM MULTIPLIER MATRICES (KOREA)

S	Pm	M1	GNP	S	Pm	M1	GNP
0				6			
	0.339	0.053	0.080		0.008	0.003	0.005
	0.175	0.175	0.294		0.012	0.004	0.007
1				7			
	0.149	0.045	0.071		0.005	0.002	0.003
	0.180	0.076	0.123		0.007	0.002	0.004
2				8			
	0.080	0.027	0.044		0.003	0.001	0.002
	0.112	0.041	0.066		0.004	0.001	0.002
3				9			
	0.045	0.016	0.025		0.002	0.001	0.001
	0.065	0.023	0.037		0.001	0.001	0.001
4				10			
	0.026	0.009	0.015		0.001	0.000	0.001
	0.037	0.013	0.021		0.001	0.000	0.001
5				11			
	0.015	0.005	0.008		0.001	0.000	0.000
	0.021	0.008	0.012		0.001	0.000	0.000

TABLE 4
THE SERIES OF THE INTERIM MULTIPLIER MATRICES (JAPAN)

S	Pm	M1	GNP	S	Pm	M1	GNP
1				7			
	0.100	0.018	0.023		0.001	0.000	0.000
	0.100	0.063	0.070		0.001	0.000	0.000
2				8			
	0.040	0.009	0.011		0.000	0.000	0.000
	0.051	0.019	0.021		0.000	0.000	0.000
3				9			
	0.017	0.004	0.005		0.000	0.000	0.000
	0.023	0.007	0.008		0.000	0.000	0.000
4				10			
	0.007	0.002	0.002		0.000	0.000	0.000
	0.010	0.003	0.003		0.000	0.000	0.000
5				11			
	0.003	0.001	0.001		0.000	0.000	0.000
	0.004	0.001	0.001		0.000	0.000	0.000
6							
	0.001	0.000	0.000				
	0.002	0.000	0.001				

$$A_K - A_J = \begin{bmatrix} 0.073 & 0.034 & 0.049 \\ 0.081 & -0.104 & 0.003 \end{bmatrix} .$$

This matrix shows that the price-wage variables in Korea are more vulnerable than in Japan with respect to external shocks and that the wage rate in Japan is more sensitive to the money supply than in Korea.

Next, consider the interim multiplier matrices. Table 3 and 4 are the series of the interim multiplier matrices for Korea and Japan, respectively. As a whole the behavior of the series is very similar to that of the four-group model, though the series for Japan approaches the zero matrix more rapidly.

To understand the total effects of the exogenous variables on the endogenous variables, consider the total multiplier matrix. The following are the estimated versions of this matrix for Korea and Japan, respectively:

$$(I-C)^{-1}A_K = \begin{bmatrix} 0.674 & 0.162 & 0.255 \\ 0.619 & 0.345 & 0.569 \end{bmatrix} \tag{53}$$

$$(I-C)^{-1}A_J = \begin{bmatrix} 0.436 & 0.053 & 0.072 \\ 0.285 & 0.370 & 0.394 \end{bmatrix} . \tag{54}$$

These two matrices give the same implications as the four-group model except the price variables are integrated into one.

Consider the difference matrix,

$$(I-C)^{-1}A_K - (I-C)^{-1}A_J = \begin{bmatrix} 0.238 & 0.109 & 0.183 \\ 0.334 & -0.025 & 0.175 \end{bmatrix} .$$

This matrix may also be interpreted as reflecting the vulnerability of the Korean economy to external shocks, due to its structural weakness relative to the Japanese economy.

E. Concluding Remarks

In the theoretical part of this study, we have developed the partial adjustment model of prices and wages, which enables us to analyze the interrelations of prices and wages in one integrated process. In the applications part, we have estimated this model using statistical data sets for the Korean and Japanese economies.

According to the theoretical development of the model, our model gives an appropriate device for the analysis of the interplay of prices and wages in the price-wage spiral process. According to the empirical analysis of the model, the interactions of the prices and wage rate are highly asymmetric. Based on these empirical results, we may say that in the course of the price-wage spiral of the Korean and Japanese economies, the prices led the process and the wage rates followed. The fact is that an increase in the wage rate does not entail a corresponding increase in the price level, in both of the Korean and Japanese economies. This may reflect the absorption capacity of the economies, which, of course, includes the increase in the productivity. Our results reflect, even, the difference in the absorption power of the Korean and Japanese economies.

Incidentally, our empirical analysis shows the effects of the external variables on the endogenous price-wage variables. Our analysis also shows the detailed process through the impact, interim, and total multiplier matrices. These results should give us some useful policy implications of prices and wages.

Appendices

A. Some Remarks on the Estimation Procedures

For the estimation of an econometric model, the most popular procedure is the least squares method. If the model is under the serial correlation, the better alternative is the generalized least squares(GLS) method.

The following matrix is the C_K-matrix estimated by the GLS:

$$
\begin{bmatrix}
0.669 & 0.052 & 0.067 & 0.063 \\
(6.18) & (0.29) & (0.40) & (0.84) \\
0.314 & 0.740 & -0.047 & 0.035 \\
(0.41) & (5.67) & (0.42) & (0.72) \\
0.090 & 0.014 & 0.765 & 0.082 \\
(1.24) & (0.11) & (6.96) & (1.64)
\end{bmatrix}
$$

$$\begin{bmatrix} -0.101 & -0.114 & 0.526 & 0.632 \\ (0.64) & (0.43) & (2.17) & (5.63) \end{bmatrix}.$$

(The figures in the parentheses are the t-values.) For each regression equation, the value of the coefficient of determination is 0.9999 or over.

But from this matrix we feel some dissatisfaction. First, the matrix does not fit the implications of our theory. According to the theory, this matrix should be sign-symmetirc. But this matrix violates this at three pairs. Second, some elements are absolutely too big to be economically meaningful. All of these dissatisfactory results may be due to some undesirable conditions of the explanatory variables, say, the phenomenon of multicollinearity.

Consider the GLS-estimated C_J-matrix for the Japanese economy:

$$\begin{bmatrix} -0.016 & 0.337 & -0.105 & -0.026 \\ (0.27) & (1.97) & (0.52) & (0.65) \\ -0.125 & 0.900 & -0.080 & 0.062 \\ (2.07) & (5.66) & (0.42) & (1.47) \\ -0.074 & 0.173 & 0.605 & 0.054 \\ (1.56) & (1.65) & (4.70) & (1.38) \\ 0.245 & 0.225 & -0.038 & 0.053 \\ (2.24) & (1.13) & (0.15) & (0.44) \end{bmatrix}.$$

This matrix also has the same kind of problems. Four pairs of the elements violate the condition of sign-symmetry, and one diagonal element violates the condition of positivity.

As explained in the text, and shown in Appendix B, our data sets contain economic time series data which have strong trends. Therefore, our X-matrix has the problem of multicollinearity. To solve this problem, we have relied on the ridge estimation technique introduced by Hoerl and Kennard(1970). According to this technique, we add some positive constant a to the diagonal elements of the standardized $X'X$-matrix. By this procedure, we introduce an intentional bias to the parameter estimation.

But with a suitable choice of the value of the constant a, we can reduce the mean-squared error of the estimates.

For our empirical study, we tried all values of a from 0 to 0.3 with interval of 0.05. It turned out that, for every regression equation, the optimal value of a is 0.1 by the criterion of minimum ISRM value. Our empirical analyses have been based on this estimation procedure.

B. The Data Sets Used

Every economic time series data used by us is a standard one in the sense that they are compiled by the central authorities of both Korea and Japan. Especially, the Japanese data sets are the same as stored in the data base of the Economic Planning Agency.

The ensuing tables are concrete *X*-matrices and *Y*-vectors actually used for this study. Note that they are in the form before logarithmic transformation.

References

Eisner, R., and Strotz, R. "Determinants of Business Investments." in CMC, *Impacts of Monetary Policy.* 1963.

Hoerl, A. E., and Kennard, R. W. "Ridge Regression: Biased Estimation for Non-Orthogonal Problems." *Technometrics,* Vol. 12 (1970): 55-67.

Jeong, Ki-Jun. "A Flexible Accelerator Model of Prices." *The Korean Economic Journal,* Vol. 20 (1981): 245-61. (In Korean)

Lucas, R. E. "Optimal Investment Policy and the Flexible Accelerator." *International Economic Review,* Vol. 8 (1967): 78-85.

Nadiri, M. I., and Rosen, S. *A Disequilibrium Model of Demand for Factors of Production.* Columbia University Press for NBER, 1974.