Exchange Rate Forecasting and the International Diversification of Liquid Asset Holdings

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I. Introduction

The gains from diversifying portfolios internationally rather than restricting investment choices solely to domestic assets are well established. (See, e.g., Grubel 1968; Levy-Sarnat 1971; and Lessard 1975 among others.) Although initial studies focused on long-term investments in stocks or bonds, the possibilities for diversifying portfolios of non-interest-bearing currencies or highly liquid interest-bearing assets have also been studied (Levy-Sarnat 1978, 1983 and Levy 1981).

The latter studies have been motivated by referring to the working capital positions of multinational corporations, although they ignore currency-specific transactions involving accounts receivable and accounts payable that are important in properly distinguishing between firms' perfectly hedged positions and open "speculative" positions in the foreign exchange markets.[1] An alternative interpretation might focus on the activities of private investors. In this context, however, the one-period utility maximizing framework where utility depends solely on end-of-period wealth and not on consumption decisions over time may be objectionable.[2]

[1] Makin (1978) discusses a portfolio approach to the hedging problem for international firms.

[2] In principle, investors should deflate nominal returns by subtracting the expected rate of change in their domestic consumer price index (unless they nave logarithmic utility functions, in

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In our view, a better interpretation for the one-period, liquid asset diversification studies is that they indicate the potential benefits of diversifying *money market mutual funds* internationally. Interestingly, at least one such money market fund, the Standard Chartered Offshore Money Market Fund Ltd., is currently available. The fund invests *solely* in cash deposits, CDs, and other money market instruments of "undoubted standing" and is prohibited from borrowing in either domestic or foreign currencies. Its liabilities consist solely of "shareholder" deposits.[3] Thus the international money market fund interpretation of earlier studies provides a possible institutional or legal justification for the often-employed assumption (retained in this study) that no short sales are allowed.

Even when interpreted this way, the conclusions of the above-mentioned portfolio diversification studies are all subject to an important qualification: they are based on *ex post* data rather than investors' *ex ante* expectations about asset returns. Consequently they provide little guidance as to how future investors should act so as to realize similar gains. As Levy (1971, p. 326) stresses, "the construction of the mean-variance efficient frontier with *ex post* data [involves] a sampling bias in favour of assets that are characterized by a low variance (or a high mean) in the sample."

In this paper, we make an initial attempt to base portfolio selection on *ex ante* forecasted returns rather than *ex post* returns. The assets considered are short-term, (non-Euro) money-market assets whose returns in terms of their respective currencies of denomination are known with certainty. Thus the assets are pure foreign exchange risks. (See the Appendix)

Our method of calculating expected returns differs in two ways from earlier studies. First, the interest rate for each liquid asset is the observable rate quoted in the market at the beginning of each period. Previous

which case only nominal returns matter). In the case where the home-currency inflation is assumed to be nonrandom, the investor's perfect-hedge portfolio obtained from an intertemporal consumption-investment problem a la Merton (1973) reduces to a single asset, namely the home-currency bank deposit or Treasury bill as we have assumed here. See Adler-Dumas (1983, pp 941-47, especially the Solnik-Sercu corollary on p. 947) for a detailed discussion.

[3] See the September 15, 1983 advertisement in the *Daily Telegraph.*

studies assume that the historical average interest rate over the entire sample period is received in each and every period even though investors clearly know the relevant interest rate when the investment is made in the short term asset.[4] Second, we use *forecasted* exchange rates for each upcoming period rather than *ex post* averages of actual realizations.[5]

Instead of using the variance-covariance matrix of *actual* returns (around their sample means) as a measure of risk as previous researchers have done, our measure of risk is based on the matrix of mean squared errors (MSE) of forecasted exchange rates. This is preferable for two reasons. First, it reflects the fact that the risks of various foreign-currency positions depend on the accuracy with which exchange rates can be forecasted as well as the correlation among forecast errors on various currencies, not exchange rate variability *per se.* Second, this measure of risk is consistent with our presumption that investors' *ex ante* return expectations may vary from period to period rather than remaining constant (at the sample mean) over time.[6]

Three simple exchange rate forecasting rules considered in the paper are discussed in Section II below. Section III lays out the portfolio selection problem that an internationally diversified money market fund faces when using forecasted returns and the MSE matrix. Section IV uses the

[4] Madura and Nosari (1982) have also pointed out that it would be preferable to use the (observable) interest rate at the beginning of each period rather than a historical average rate. Our study actually uses the average rate in the current period for some of the liquid assets, not the rate at the beginning of the current period; see fn. 12.

[5] Von Furstenberg (1981) uses *ex ante* real returns to calculate optimal portfolios for varying degrees of risk aversion. However, his forecastor for expected return in the next quarter is simply the average return over the previous quarters of the sample period. Testing this predictor on our own data, we found it to be consistently inferior to the martingale, and generally inferior to the forward rate, using MSE as the basis for comparison.

[6] The MSE structure of the various exchange rate forecasts is assumed to remain constant over time. In principle, portfolio risk as well as returns ought to be calculated on an *ex ante* basis. In the absence of *a priori* knowledge about the second moments of investors' expectations regarding returns on various assets, some type of stationary assumption is required for empirical work. If the error covariance matrix changes dramatically over time, however, portfolio selection based on *ex post* data may be far from optimal. Maldonado and Saunders (1981) have recently provided evidence of considerable instability in the covariance matrix for returns on common stocks. Investors might justifiably expect similar instability in the international investment arena. No attempt is made to address this problem here.

alternative forecasting methods to construct *ex ante* efficiency frontiers and compares these to the *ex post* frontier that is typically employed. For a given risk-free domestic interest rate, the optimal portfolio of risky foreign-currency assets is calculated. We compare the *ex ante* return from the optimal (*ex ante*) portfolio to the returns that were actually realized *ex post.* This is done for both a buy-and-hold and a quarterly rebalancing investment strategy over the last eight quarters of our sample period (1980:II — 1982:I). Section V summarizes our major conclusions.

II. Exchange Rate Forecasting

The forecasted percentage return for time period t on a liquid asset in currency j equals:

$$\hat{R}_t^j = \frac{1}{e_t^j} (1 + r_t^j) \, {}_t\hat{e}_{t+1}^j - 1, \tag{1}$$

which, of course, depends on the forecast (denoted by hats) of the future spot rate ${}_t\hat{e}_{t+1}^j$ and the one-period rate of interest r_t^j on asset j (and paid in units of currency j).

To illustrate the implications of using exchange rate forecasts in the portfolio selection process, we consider simple exchange rate forecasting rules based on the martingale, open interest parity, and the forward rate models. The *martingale* model assumes that the exchange rate process is such that the best forecast at time t of the future exchange rate is just the current rate:

$${}_t\hat{e}_{t+1}^j = e_t^j. \tag{2}$$

According to the *open interest parity* or "Fisher open" relationship, on the other hand, the future exchange rate is expected to differ from the current rate by the amount of the current interest differential:

$${}_t\hat{e}_{t+1}^j = e_t^j\left(\frac{1+r_t}{1+r_t^j}\right) \tag{3}$$

where r_t^j is the risk-free interest rate in country j and r_t is the corresponding rate in the home country (i.e., the U.S.).

The third method uses the *forward exchange rate* as a forecast of the future spot rate on the grounds that it is a market consensus:

$$ {}_t\hat{e}^j_{t+1} = {}_tF^j_{t+1} \tag{4} $$

where ${}_tF^j_{t+1}$ is the forward rate prevailing at t for delivery of currency j at time $t+1$.

Several comments should be made about the alternative exchange rate forecasting methods. Some of these involve conceptual matters; others are empirical observations. First, Sims (1980) has shown that "under general conditions, which allow among other things for risk aversion among market participants, competitive asset prices ought to be locally — over small units of time — 'martingale-like' ". The question of how small the time units have to be to justify the empirical use of the martingale model remains open.

Second, it should also be emphasized that if *covered* interest parity (IRP) held *exactly,* that is: ${}_tF^j_{t+1} = {}_te^j_{t+1}\ (1 + r_t)/(1 + r^j_t)$, then the forecasts of the future spot rate based on open interest parity and those based on the forward rate would yield identical results. Frenkel and Levich (1977) present considerable evidence that although IRP holds when considering offshore or Eurocurrency markets, it does not hold continuously for onshore investments due to transactions costs, existing or potential capital controls, political risk, or borrowing and lending limits on arbitrage funds. We use *on shore* assets, for which international diversification is most likely to be profitable. Because IRP need not hold here, the open interest parity and forward rate models are both considered.

Third, from a conceptual standpoint the forward rate may or may not be an unbiased predictor of the future spot rate even if the forward exchange market is efficient. An *equilibrium* discrepancy between the expected future spot rate and the forward rate is possible, reflecting a risk premium on holdings of a particular currency, (see, e.g., Kouri 1977 and Hodrick 1981). Unless investors are risk neutral or exchange risk is completely diversifiable so that the risk premium is zero, this fact is problematic when using forecasts based on the forward rate in portfolio diversification exercises which presume that nondiversifiable risk is present. Ideally, one would like to forecast exchange rates, construct optimally diversified portfolios, and *simultaneously* adjust the forecasting method to reflect the equilibrium risk premium on each currency. To our knowledge, no one has yet discovered a way of doing this.[7]

Of the three predictors, the forward rate has been somewhat less accurate in practice (see Giddy and Dufey 1975; Levich 1979a; Meese and Rogoff 1983). The martingale and open interest parity models have performed about equally well in tests of forecast accuracy, though Levich (1979b) found the open interest parity relationship with external (offshore) interest rates to be slightly preferable. Although many authors compare the martingale and open interest parity models, it should be pointed out that it is extremely difficult to distinguish between them statistically, particularly in studies using daily, weekly, or even monthly data. After all, the difference between the two models is $(1 + r)/(1 + r^j)$ which is extremely close to unity over short time intervals even when differences in annual interest rates are very large by historical standards. The large variability in exchange rates typically overwhelms any influence due to this difference between the martingale and open interest parity forecasts.

Forecast errors tend to be large regardless of which of the three forecasting methods is used, reflecting the fact that it is extremely difficult to forecast exchange rate movements. Yet Meese and Rogoff (1983) show that the martingale and forward rate consistently outperform simple structural models of exchange rate determination for the dollar/mark, pound/dollar and dollar/yen rates (December 1976 — November 1980),

[7] Empirically, the possible bias of the forward rate forecast may or may not be particularly important when studying gains from portfolio diversification. Recent research (e.g., Geweke-Feige 1979; Hansen-Hodrick 1981, and Obstfeld-Cumby 1981) indicates that the bias is typically very small and often statistically insignificant. While the forecast errors may have zero mean, they seem to exhibit significant serial correlation over time. In light of these empirical findings, one might conclude either that exchange markets are inefficient that the risk premium varies over time. This suggests that it is advisable to be fairly open-minded about what forecasting methods are "reasonable" from a purely theoretical standpoint.

at least for forecasting horizons of less than one year. Furthermore, in their comparison based on out-of-sample forecasting ability, the martingale forecast consistently outperformed other forecasting methods including ARIMA and VAR models, the forward rate, and structural exchange rate models. (The open interest parity model was not considered.)

Since all three forecast described above are used extensively in theoretical as well as empirical work, we calculate average efficiency frontiers derived from each. In our later discussion of quarterly rebalancing and buy-and-hold strategies, we restrict our attention to the martingale on the belief that it is both theoretically and empirically most appealing. No consideration is given here to forecasts from structural models of exchange rate determination or professional exchange rate forecasting services, which at least in some cases appear to have produced profitable trading rules (see Levich 1980).

III. Optimum Portfolios under Alternative Forecasting Rules

First, consider the forecast method based on the martingale model (2). With this model, the expected foreign asset return (1) reduces to:

$$\hat{R}_t^j = r_t^j. \tag{5}$$

The return of foreign assets relative to the safe domestic asset is simply the uncovered interest differential $r_t^j - r_t$. The risk associated with asset j is measured by:

$$MSE^j = \left(\frac{1 + r_t^j}{e_t^j}\right)^2 E(e_{t+1}^j - e_t^j)^2. \tag{6}$$

Assets with forecasted returns above the domestic rate ($r_t^j > r_t$) may well enter efficient portfolios if the exchange risk in (6) is not too large. Even assets with expected returns below the domestic rate ($r_t^j < r_t$) may enter, depending on the correlations among forecast errors for the individual exchange rates (provided that $r_t^j > r_t$ for at least one asset j).

If the open interest parity relationship is used to forecast future exchange rates, the *ex ante* forecasted return on asset j in (1) reduces to:

$$\hat{R}_t^j = r_t \,. \tag{7}$$

That is, the foreign investment is always expected to yield a return just equal to the risk-free domestic rate. The risk of investing in the foreign asset, on the other hand, is positive unless investors expect the Fisher-open condition to be satisfied exactly at each and every moment in time. Consequently any forecaster who (with or without empirical justification) employs the open interest parity relationship always calculates the efficiency frontier (for period t) to be a single point at the risk-free domestic rate r_t. The optimal speculative portfolio for such risk-averse investors would *never* contain foreign assets. This is an analytical result and hence does not require computations of the sort in Section IV below. It differs sharply from results of previous empirical studies using *ex post* data, which indicate high returns to international diversification.

If the forward rate forecast is used in (4), the forecasted rate of return on liquid assets in currency j equals:

$$\hat{R}_t^j = \frac{1}{e_t^j} (1 + r_t^j)\, {}_tF_{t+1} - 1 \,. \tag{8}$$

The associated risk from taking a position in country/currency j depends on the mean square error of the forecast method. Only if the forecasting method is perfectly accurate, i.e., ${}_tF_{t+1}^j = e_{t+1}^j$ in *all* time periods, would the exchange risk associated with j be zero. Investors will compare the forecasted return on each risky foreign asset j as calculated in (8) with the riskless domestic rate r. The anticipated premium for accepting the foreign exchange risk equals:

$$\hat{R}_t^j - r_t = \frac{1}{e_t^j} (1 + r_t^j)\, {}_tF_{t+1}^j - (1 + r_t). \tag{9}$$

It might be noted from (9) that any risk-averse investor who rightly or wrongly believes that the IRP condition *always* holds for these particular assets would never take a speculative foreign currency position. Its excess return (relative to the risk-free rate) would equal zero, yet the investment is risky unless the forward rate is a *perfect* forecaster.

The foregoing discussion shows *analytically* that when the open interest parity relationship is used to predict future exchange rates, no international portfolio diversification is ever warranted regardless of the MSE structure of the forecasted rate-of-return data. Given this forecasting method, all foreign asset positions are expected to yield returns exactly equal to the risk-free domestic rate, yet foreign assets involve exchange rate risk. The efficiency frontier is, therefore, a single point at the domestic risk-free rate. The optimum portfolio for any risk-averse investor will include no open foreign-currency positions.

Determining the efficiency frontier and optimum risky portfolio under the other two forecasting rules is more involved. The result can not be determined *a priori* in these cases. Rather, actual data must be used to calculate the mean square error matrix for each forecasting procedure. The matrix capturing risk among various investments can then be used in the standard quadratic programming problem to determine the efficiency frontier.[8] Because the forecasted returns obtained from inserting the exchange rate forecasts into (1) can change every period, however, the efficiency frontier can also change over time. Hence the quadratic program must be resolved each investment period.

Define $\underline{\hat{R}}_t = (\hat{R}_t^1, \hat{R}_t^2, \ldots, \hat{R}_t^n)$ as the vector of forecasted returns based on a particular exchange rate forecasting procedure and calculated according to (1). The relevant risk matrix Ω — analagous to the variance-covariance matrix of actual returns $\underline{R}_t$ in the existing literature — is based on mean square forecast errors for returns on the various foreign asset positions:

$$\Omega = \frac{1}{T} (\underline{R}_t - \underline{\hat{R}}_t - \bar{\varepsilon})(\underline{R}_t - \underline{\hat{R}}_t - \bar{\varepsilon})^T. \qquad (10)$$

The diagonal elements of Ω are essentially the MSE^j values defined by (6), which in turn depend on the forecast method. To allow for the possibility that the forecast may be biased, we substract off the mean forecast

[8] We have used the QPSOL FORTRAN subroutine, written in the Stanford Operations Research Department. We are indebted to Dr. Phillip Gill for providing QPSOL.

error $\bar{\varepsilon}$ for each asset prior to squaring.[9] The off-diagonal elements, which capture the correlations among the forecast errors on different currencies, are calculated analogously.

Solving the quadratic programming problem:

$$\min_{X_t} V = \underline{X}_t^T \; \underline{\Omega} \; \underline{X}_t \tag{11}$$

subject to:

$$\underline{X}_t^T \; \underline{R}_t = \mu \tag{12}$$

$$\underline{X}_t^T \; \underline{1} = 1 \tag{13}$$

$$\underline{X}_t \geqslant 0 \tag{14}$$

minimizes the portfolio risk V for a prespecified total portfolio return μ. Repeating the analysis for a range of μ values yields various points on the efficiency frontier for investment period t. $\underline{X}_t$ is the vector of portfolio proportions whose elements must, according to (13), sum to unity. Equation (14) assumes that each asset position must be non-negative. That is, no short positions are allowed. Although it is much simpler to calculate optimum portfolio proportions when unlimited short sales are allowed, because the optimization problem can then be solved analytically instead of numerically, we adopt this assumption for two reasons. First, we are concerned with the lack of institutional motivation in earlier studies. As discussed in the Introduction, these studies make good sense when interpreted as the optimizing behavior of internationally diversified money market funds. These funds are generally prohibited from levering up.

Second, Levy (1981) and Adler-Dumas (1983) show that allowing short sales yields optimal weights that are implausibly large. For example, Levy's speculative portfolio for a six-month holding period has weights as low as -1415.23 per cent for Belgium and as high as $+1486.89$ per cent for the Netherlands. The *smallest* weight in absolute value for the sixteen national assets considered is -41.28 per cent (for Canada). As

[9] The difference in MSE values due to subtraction of the mean error is very small.

Adler and Dumas (1983, p. 945) note, portfolio weights of this magnitude "suggest improbably that investors individually and in the aggregate should sell short some securities and hold more than 100% of the available supply of others. Beyond the possibility that the calculated weights violate typical short selling constraints, such a result, if sustained, would imply that international capital markets are not in equilibrium."

Due to difficulties interpreting results when short sales were permitted, we report only the "no short sales" case. Our computational procedure can easily be extended to incorporate less restrictive limits on short sales or additional constraints on portfolio proportions such as those specified by top-level foreign exchange management.

IV. Empirical Results

A. Data

Insofar as possible, we have tried to duplicate Levy's selection of highly liquid assets (see Appendix). It was, however, necessary to eliminate Brazilian and Israeli assets from consideration due to a lack of interest rate data.[10] Hence the investor may choose from 14 countries' liquid assets. When using the forward exchange rate as a predictor of the future spot exchange rate, three more assets (Australia, Italy, and South Africa) had to be excluded because forward markets did not exist for part or all of the sample period.

Our sample period extends from 1974:II to 1982:I. We deliberately excluded from consideration the period immediately following the breakdown of the Bretton Woods fixed-exchange-rate commitment. During this period of structural change, the MSE matrix of exchange rate forecasting schemes may not remain constant as our approach requires. The raw data came from the IMF's *International Financial Statistics* tape.

Table 1 presents the means and variances of the actual returns for the fourteen countries/assets over our 32-quarter sample period. Also reported for each asset is the mean *forecasted* return as well as its associ-

[10] The Brazilian interest rate data were not available for the last three quarters of the sample period, the Israeli data for the entire sample period.

TABLE 1

1974:II — 1982:I

	Ex Post		*Martingale*			*Forward Rate*		
	Mean Actual Return ($\Sigma_{t=1}^{T} R^i/T$)	Variance of Actual Return ($\sigma^2(R^i)$)	Mean of Forecast Returns ($\Sigma_{t=1}^{T} \hat{R}_t^i/T$)	MSE (Ω_{ii})	Mean Forecast Error ($\Sigma_{t=1}^{T} (R_t^i - \hat{R}_t^i/T)$)	Mean of Forecast returns	MSE	Mean Forecast Error
Australia	0.0142	0.00145	0.0245	0.00142	−0.0104	—	—	—
Belgium	0.0165	0.00431	0.0195	0.00428	−0.00291	0.0156	0.00425	0.000927
Canada	0.0180	0.000560	0.0253	0.000428	−0.00728	0.0235	0.000452	−0.00553
Denmark	0.0281	0.00354	0.0363	0.00359	−0.00823	0.0238	0.00409	0.00432
France	0.0178	0.00302	0.0250	0.00302	−0.00718	0.0241	0.00322	−0.00027
Germany	0.0185	0.00350	0.0154	0.00359	0.00307	0.0244	0.00356	−0.00592
Italy	0.00105	0.00313	0.0329	0.00309	−0.0225	—	—	—
Japan	0.0243	0.00341	0.0191	0.00330	0.00518	0.0235	0.00365	0.000778
Netherlands	0.0202	0.00384	0.0183	0.00357	0.00192	0.0236	0.00380	−0.00333
Norway	0.0188	0.00220	0.0213	0.00223	−0.00245	0.0129	0.00250	0.00590
South Africa	0.00416	0.00205	0.0173	0.00214	−0.0132	—	—	—
Sweden	0.0165	0.00259	0.0249	0.00266	−0.00843	0.0197	0.00275	−0.00309
Switzerland	0.0289	0.00616	0.0121	0.00599	0.0168	0.0262	0.00618	0.00268
United Kingdom	0.0190	0.00292	0.0270	0.00277	−0.00806	0.0201	0.00290	−0.00111
Arithmetic Mean of Column	0.0182	0.00305	0.0228	0.00305	−0.00455	0.0216	0.00340	−0.000968

ated mean forecast error and mean square error using both the martingale and forward rate forecasting techniques. The martingale method seems to be slightly more reliable than the forward rate predictor, if the MSE criterion is used. It yields a lower MSE for nine of the eleven asset returns (where there was a forward market so that both forecasters could be calculated). Nonetheless, the MSE terms for both predictors are strikingly similar to the *ex post* return variance terms (see Appendix).

B. Efficiency Frontiers and Optimal Portfolios Based on Average Returns

In practice, investors would not base current-period portfolio diversification decisions on the *average* forecasted returns reported in Table 1. Each investment period the investor can use only currently available information to forecast returns for the period ahead. Every period, in principle, the investor could recalculate the efficiency frontier and optimum portfolio based on the most recent exchange rate forecast. The latter, of course, depends on the forecasting method employed.

Given our 32-quarter sample period, it would be expensive to make such calculations period-by-period for both the martingale and forward rate forecasting methods discussed above. In Section C below, we undertake a more limited experiment using the martingale model to calculate the optimum *ex ante* portfolios (but not the entire efficiency frontiers) for each of the last eight quarters of the sample period. In this context, period-by-period rebalancing as well as buy-and-hold investment strategies are considered.

It is useful to have some idea of the shape of the efficiency frontiers (not just the optimum portfolios) when investment decisions are based on *ex ante* forecasts rather than *ex post* returns. Hence in this section forecasted returns were averaged over the entire sample. Using these averages and the associated risk matrices defined by (10), "average" efficiency frontiers for both the martingale and forward rate forecasting methods were calculated.

The resulting frontiers are shown, along with the frontier based on *ex post* data, in figure 1. Both *ex ante* frontiers (based on the average

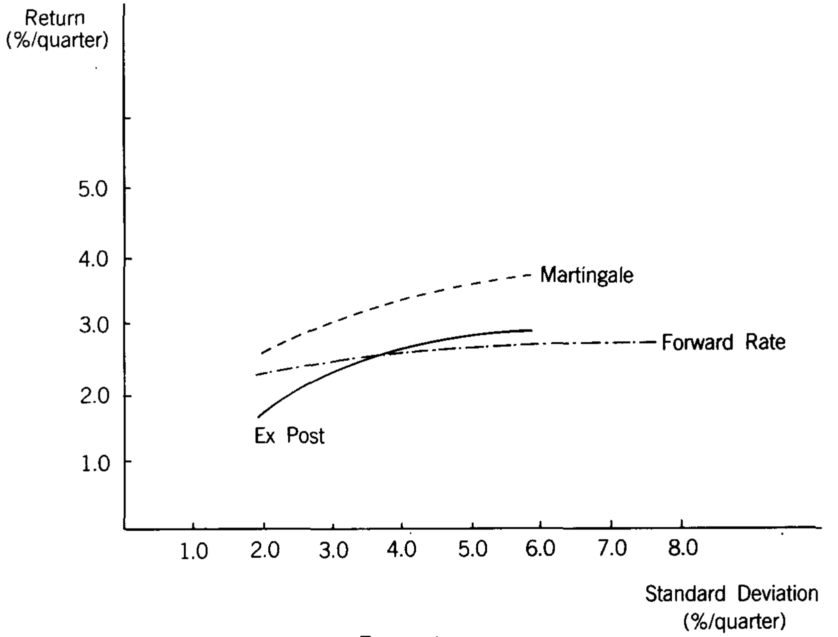

FIGURE 1

martingale and forward rate forecasts respectively) suggest greater gains from diversification than does the *ex post* frontier. Although this isn't apparent by casual inspection in the case of the forward rate frontier, the slope of the capital market line (using the average risk-free U.S. interest rate of 2.06 percent) is 0.16. This exceeds the 0.12 slope of the *ex post* market line. The martingale frontier, which yields a market line with a slope of 0.34, suggests substantially greater benefits from international diversification than does the forward rate.

The data presented in Table 1 provide a possible explanation for the more favorable appearance of the *ex ante* frontiers, and in particular, why the martingale frontier lies well outside the other two. The key appears to be the overestimation of actual returns on some of the available assets by the two forecasting techniques. Even in the absence of statistically significant forecast error (which may not be uniformly true here), *any* forecast method inevitably overestimates some returns and underestimates others. Since the optimal portfolio contains assets that bear high returns but add relatively little portfolio risk, assets whose returns are overestimated will

TABLE 2

Frontier	Slope of[1] Market Line	Forecast[2] Return	Actual[3] Return	Optimal Portfolio Shares
Ex Post	0.12	0.028	0.028	Denmark = 0.82
				Switzerland = 0.18
Martingale	0.34	0.028	0.018	Canada = 0.69
				Denmark = 0.13
				Italy = 0.18
Forward Rate	0.16	0.024	0.017	Canada = 0.84
				France = 0.09
				Japan = 0.01
				Switzerland = 0.05

[1] Assuming a risk-free rate of 0.0206.
[2] Percentage return per quarter (arithmetic average).
[3] Quarterly return implied by holding optimal portfolio throughout sample period. Reinvesting of interest is not assumed.

tend to be over-represented when the optimal portfolio is calculated using forecasted returns. (Levy (1981) also notes this; see our introductory quotation.)

The optimal portfolios associated with the three frontiers (*ex post,* martingale, and forward rate), given the risk-free U.S. rate of 2.06 percent, are summarized in Table 2. As in Levy's study (which used 1970-78 data), the Danish krone receives a high weight in the *ex post* portfolio. The optimal portfolios based on the martingale and forward rate forecast averages, however, have the Danish krone entering with a low or zero weight. This implies that the high returns earned by the Danish asset could not be anticipated *a priori.* On the other hand, the Canadian asset is quite prominent in the *ex ante* portfolios. The forecast data reveal that the return on the Canadian asset is moderately large, relatively easy to predict, and negatively correlated with some of the other assets returns. The mean forecast error using either method is also seen to be large relative to the corresponding MSE for the Canadian asset.

Suppose that an investor were to hold the best portfolio implied by the average forecasted returns using either the martingale or forward rate model. What would his actual return have been? As Table 2 reveals, the forecasted returns on the optimum portfolio substantially overstate the returns that were ultimately realized. (Further evidence on this in the case of period-by-period portfolio rebalancing is reported in Table 3 below.) As noted above, a selection bias problem is present; i.e., there is a tendency for assets whose returns were overestimated to find their way into the optimal portfolio, and for those whose returns were underpredicted to be excluded. Thus the *ex ante* expected return on the optimum portfolio can be a biased predictor of realized returns even if the forecasted returns on the individual assets are unbiased.

C. Buy-and-Hold Strategy vs. Portfolio Rebalancing

While the efficient frontiers derived from *average* forecasted returns provide an interesting comparison between the use of *ex post* data and forecasted returns, a real-world investor must make periodic decisions about the future on the basis of existing information. In our next empirical exercise, we calculate the aggregate portfolio return that would be earned by an investor using the martingale to predict exchange rates over the last eight quarters of the sample period.[11]

The particular decision considered is whether the investor should rebalance his portfolio for each of the eight quarters from 1980:II to 1982:I or hold a single portfolio for the entire two years. We make no attempt here to measure the costs of recalculating a new optimal portfolio every period or the transactions cost of buying and selling assets. The difference in returns from the two strategies will, however, give us an idea of how high these costs must be in order to make an investor indifferent between the "buy and hold" and portfolio rebalancing strategies.

Using the martingale forecast method, the forecasted return on each foreign asset is just its interest rate over the appropriate holding period.[12]

[11] We choose the martingale because the exchange rate forecasting literature has almost universally found it to be superior to the forward rate (as noted in Section II). Our own results based on the MSE of the martingale and forward rate predictors in Table 1 also suggest that the martingale is a more accurate predictor.

[12] Ideally, he would use beginning-of-period quarterly interest rates in the rebalancing case and a two-year interest rate on investment beginning in 1980:II for the buy-and-hold strategy. As noted earlier, most of the interest rates taken from the IFS tape are period averages. While this is probably not a severe problem for a period as short as one quarter, it cannot be assumed that the

TABLE 3

	Portfolio Value			Portfolio Proportions (Rebalancing)				
	Buy and Hold	Rebalancing (Actual value)	Rebalancing (Expected value)	Canada	Denmark	Italy	Japan	U.K.
Initial value at end of:	$1.00	$1.00	$1.00					
1980:II	1.09	1.09	1.03	0.60	0.08	0.10	0.06	0.1
1980:III	1.11	1.11	1.06	0.53	0.17	0.14	0.04	0.1
1980:IV	1.13	1.10	1.10	0.52	0.33	0.15	—	—
1981:I	1.14	1.12	1.15	0.75	0.07	0.18	—	—
1981:II	1.12	1.12	1.20	0.76	0.03	0.21	—	—
1981:III	1.15	1.18	1.25	0.76	0.03	0.20	—	0.0
1981:IV	1.22	1.23	1.31	0.68	0.09	0.23	—	—
1982:I	1.19	1.22	1.36	0.61	0.13	0.26	—	—
Geometric Mean of Quarterly Return	2.21%	2.53%	3.91%					

Recall equation (5) above. The risk matrix is calculated from the prediction errors in the 24 quarters prior to 1980:II.[13]

Imagine that at the beginning of 1980:II, an investor places $1.00 in an optimal portfolio (on the basis of expected returns in 1980:II) to be held for two years. At the same time, he places another dollar in the same portfolio, but rebalances the second portfolio every quarter in response to changing quarterly return forecasts. Table 3 traces the cumulative returns from both strategies on the assumption that both principal and interest are reinvested each quarter. Also shown is the forecasted end-of-quarter value of the investment in the case of quarterly rebalancing.

period average is a good measure of the beginning-of-period rate for an interval as long as one or two years. We therefore assume that the investor considers the expected return on the two-year portfolio to be determined by quarterly interest rates at the beginning of our experiments period in 1980:II. This somewhat understates the expected two-year return since long-term rates have generally exceeded short-term interest rates in recent years. Consequently our calculations will make the buy-and-hold strategy appear a bit less attractive than it truly is.

[13] In light of the added computational cost, we do not update the risk matrix every quarter as the portfolio is rebalanced. If the error process is indeed stationary, it should make little difference which period is used to calculate the risk matrix, so long as it is a subset of the post-1973 period.

As might be expected, the investor does better by rebalancing his portfolio every period than by holding a single portfolio for several periods. Still, the difference is not dramatic. The (geometric) average quarterly returns are 2.53% and 2.21% respectively for the rebalancing and buy-and-hold strategies. So if transactions costs of rebalancing every period exceeded 0.32% of the investment, the investor would have earned more using the buy-and-hold approach. Two important qualifications must be made, however. First of all, this result depends on the sample period. The return for 1980:II is an enormous 9.4%. Since this return is common to both cases, it tends to make the two strategies look equally good. Indeed, if the first quarter were omitted, transactions costs could be twice as high and still the rebalancing strategy would have yielded the greater net-of-cost return. Second, because the buy-and-hold portfolio does not remain optimal in subsequent quarters (1980:III-1982:I), the investor is not compensated for risk as well as he would have been in the rebalancing case.

As was the case in Section B, we again observe that the forecasted returns on optimal portfolios exceed their actual returns *ex post* in virtually every period. The overstatement is substantial (despite the huge first-period return when, in a sense, the investor "got lucky").

Table 3 also reports the shares of various assets in each period's optimal portfolio for the rebalancing case. The risk-free U.S. interest rate as well as the forecasted returns on foreign assets are allowed to change each quarter. The Canadian asset accounts for over half of the portfolio in each of the either periods, just as the martingale-optimal portfolio derived from *average* forecast returns had suggested above. Still, there is a fair bit of rebalancing to be done from period to period. Over the seven quarters in which rebalancing of the $1.00 portfolio can occur, an average of $0.22 worth of assets must be bought or sold. Presumably, transactions costs are comprised of a fixed component (e.g., the cost of contacting one's broker) plus a component that varies with the amount of rebalancing to be done. The changes in the composition of the optimal portfolio from period to period indicate that this cost, which further reduces the potential gains from international diversification, is not trivial.

V. Conclusions

This paper re-examines the potential gains from diversifying a money market fund internationally. It recognizes the fact that international investors must *forecast* the expected returns and evaluate the exchange risks associated with foreign assets before the optimally diversified portfolio can be selected. Using three possible techniques (the martingale, open interest parity and forward rate models), we conclude that the warranted degree of international diversification, in terms of the number of assets that should be held, is quite limited when only liquid onshore assets are considered (as would typically be required by money market funds).

We find that the expected *ex ante* return from undertaking such diversification is modest. It is in the range of two to four percent per quarter depending on the associated risk, according to the *ex ante* efficiency frontiers shown in Figure 1. The difference in returns between buy-and-hold and quarterly-rebalancing portfolio management strategies appears to be slight. More importantly, the *realized* returns received by funds that diversified optimally *ex ante* were invariably less than anticipated. This may indicate that investors are not being adequately compensated (in terms of realized returns) for the risks they incur in diversifying their liquid asset portfolios internationally, or that our simple forecasting rules result in biased estimates of portfolio returns.

One thing is clear, it has proved to be extremely difficult to make accurate exchange rate forecasts in the post-Bretton Woods floating-exchange-rate environment. Any realistic examination of the potential gains from international portfolio diversification should certainly include some treatment of this problem.

References

Adler, M., and Duma, B. "International Portfolio Choice and Corporation Finance: A Synthesis." *Journal of Finance* 38, 3 (June 1983): 925-84.

Cuddington, J. T., and Gluck, J. A. "A Note on International Portfolio Diversification with Short

Sales." Mimeo., 1983.

Frenkel, Jacob A., and Levich, Richard A. "Transactions Costs and Interest Arbitrage: Tranquil versus Turbulent Periods." *Journal of Political Economy* 85, 6 (December 1977): 1209-26.

Geweke, John, and Feige, Edgar. "Some Joint Tests of the Efficiency of Markets for Forward Foreign Exchange." *Review of Economics and Statistics* LXI, 3 (August 1979).

Giddy, I., and Dufey, G. "The Random Behavior of Flexible Exchange Rates: Implications for Forecasting." *Journal of International Business Studies* (Spring 1975): 1-32.

Grubel, Herbert G. "Internationally Diversified Portfolios: Welfare Gains and Capital Flows." *American Economic Review* 58 (1968): 1299-314.

Hansen, Lars P., and Hodrick, Robert J. "Forward Exchange Rates as Optimal Predictors and Future Spot Rates," *Journal of Political Economy* 88, 5 (October 1980): 829-53.

Hodrick, Robert J. "International Asset Pricing with Time-Varying Risk Premia." *Journal of International Economics* 11, 4 (November 1981): 573-88.

Kouri, Pentti J. K. "International Investment and Interest Rate Linkages Under Flexible Exchange Rates." in R. Z. Aliber (ed.), *The Political Economy of Monetary Reform.* Montclair, N. S.: Osmun and Co. 1977.

Lessard, Donald. "International Portfolio Diversification: A Multinational Analysis for a Group of Latin American Countries." *Journal of Finance* (June 1973): 619-33.

Levi, M. D. "Taxation and 'Abnormal' International Capital Flows." *Journal of Political Economy* 85 (1977): 635-46.

Levich, Richard M. "On the Efficiency of Markets for Foreign Exchange." in R. Dornbusch and J. A. Frenkel (eds.), *International economic Policy: Theory and Evidence.* Baltimore: Johns Hopkins University Press, 1979.

———. "Analyzing the Accuracy of Foreign Exchange Advisory Services: Theory and Evidence," in R. Levich and C. Wihlborg (eds.), *Exchange Risk and Exposure: Current Developments in International Financial Management.* Lexington, Mass. D. C. Heath, 1980.

Levy, Haim. "Optimal Portfolio of Foreign Currencies with Borrowing and Lending." *Journal of Money, Credit and Banking* 13, 3 (August 1981): 325-41.

Levy, H., and Sarnat, M. "Exchange Rate Risk and the Optimal Diversification of Foreign Holdings." *Journal of Money, Credit and Banking* 10, 4 (November 1978): 453-63.

———. "International Portfolio Diversification." in R. J. Herring (ed.), *Managing Foreign Exchange Risk.* Cambridge University Press, 1983.

Madura, J., and Nosari, E. J. "Optimal Portfolios of Foreign Currency with Borrowing and Lending." Comment in *Journal of Money, Credit and Banking* 14 (November 1982): 531.

Makin, J. "Portfolio Theory and the Problems of Foreign Exchange Risk." *Journal of Finance* 33, 2 (May 1978): 517-34.

Maldonado, R., and Saunders, A. "International Portfolio Diversification and The Intertemporal Stability of International Stock Market Relationships, 1957-78." *Financial Management* (Autumn 1981): 54-63.

Meese, Richard, and Rogoff, Kenneth. "Empirical Exchange Rate Models of the Seventies: Do They Fit Out-of-Sample?" *Journal of International Economics* (forthcoming), 1981.

Merton, R. "An Intertemporal Capital Asset Pricing Model." *Econometrica* 41 (1973): 867-87.

Obsfeld, M., and Cumby, R. T. "A Note on Exchange-Rate Expectations and Nominal Interest Differentials: A Test of the Fisher Hypothesis." *Journal of Finance* 36, 3 (June 1981): 697-703.

Sims, C. A. "Martingale-Like Behavior of Prices," NBER Working Paper no. 489, 1980.

Solnki, Bruno H. "Why Not Diversify Internationally Rather Than Domestically?" *Financial Analysts Journal* (July-Aug. 1974): 48-54.

Von Furstenberg, George M. "Incentives for International Currency Diversification by U.S. Financial Investors." *IMF Staff Papers* (September, 1981): 477-94.