Entry Equilibrium under Asymmetric Information

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I. Introduction

Recent progress in the theory of entry-deterrence has cast strong doubts on the validity of the limit pricing theory for ad hoc nature of its underlying behavioral assumption-the Sylos postulate. The postulate states that a potential entrant behaves on an assumption that an established firm maintains its pre-entry output after entry has occurred. Many authors have criticized this assumption for its failure to describe convincingly the rules of the post-entry game. The established firm may find it best to reduce its output when entry actually occurs. Many of recent researches have abandoned the Sylos postulate and instead taken it as the rules of the post-entry game that a Nash equilibrium is established after entry occurs. In the words, the "fight" strategy is eliminated from the post-entry strategy set of the established firm. Instead, it has focused on pre-entry strategies of the establised firm that could realize an entry-deterring equilibrium. One example is a prior irrevocable commitment of the established firm such as sunk capacity (Spence 1977; Dixit 1979, 1980; Eaton and Lipsey 1981; Spulber 1981). In these models, depending on the underlying parameters such as discount factor, cost of capacity and profit structure, the established firm may choose an entry-deterring capacity.

There is another line of researches incorporating uncertainty. Milgrom and Roberts (1982a) show a possibility of an entry-deterring equilibrium when both the established firm and the potential entrant do not have complete information on each other's cost structure before an entry decision is made. In their model, the potential entrant tries to draw information from the pre-entry out-

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put level chosen by the established firm. Friedman (1983) introduces uncertainty on the post-entry demand condition and reaches a similar conclusion. In his model, the outcome depends on the subjective probability of the potential entrant attached to the post-entry demand function. In sum, in the aforementioned models, if entry is ever limited, it is because entry does not pay the potential entrant even in a duopoly market not because the former is feared of a threat by the latter.

The present model considers a different situation where the "fight" strategy is a justifiable, rational strategy of the established firm. Two basic conditions used for this situation are; (1) the established firm does not know financial condition of the entrant but the entrant knows that of established firm. That is, an asymmetric information situation is considered as contrasted with Milgrom and Roberts' work. A rationale for this may be that if a firm has been established as a monopolist in a market for a long while, it seems reasonable to imagine that more of characteristic of the firm is known to the potential entrant than the other way around. (2) A financial constraint is imposed on each firm that it must carry a certain amount of cash reserves to operate. Now, if we consider a two period model where entry occurs only in the first period if it ever does and the financial condition of the entrant is not revealed until the end of the first period, the above consideration changes the rules of the post-entry game in a nontrivial way. The established firm facing an entry threat may be interested in starting a war with a hope to identifying the type of an entrant rather than in sharing the market. If the firm is successful in making the entrant bankrupt by waging a war, it can regain its monopolistic position in the second period, although this strategy entails costs.

There are three related works to the present model, the models of Milgrom and Roberts (1982b), Kreps and Wilson (1982) and Benoit (1984). These models show that under asymmetric information the established firm may fight for its own benefits when entry occurs. This claim follows from a common factor shared by these models; one agent is uncertain of the "rationality" of the other agent. More specifically, in the models of Milgrom and Roberts, and Kreps and Wilson, potential entrants do not know whether they play a game with a rational incumbent or with an irrational one who always fights.[1] Benoit instead considers a reversed situation where

[1]In their study of the "chain-store paradox," Milgrom and Roberts (1982b) and Kreps

an established firm may face entry of a firm that is committed to enduring price wars until it goes bankrupt. Although the present model resembles the Benoit model because both consider financially constrained entry, it assumes no irrationality on either side of agents. Therefore, the current model can be classified as more direct extension of existing models of complete information. As an example, the Benoit model cannot predict a situation of entry deterring because of its fundamental assumption that there is some chance, however small, that a firm always enters. Furthermore, the present model deals with signaling incentive of a potential entrant that suffers negative externality created by the presence of another type of an entrant.

The present formulation generates a rich family of outcomes. In the conventional models, only pure strategy equilibria emerge. That is, either entry takes place and the market is shared or entry is limited depending on underlying parameters. By contrast, in this model where the "fight" strategy is an effective strategy, another interesting possibility of a mixed strategy equilibrium can arise in addition to the above traditional results. If entry is met with a harsh action by the established firm, an ex post price war can occur. Such a war occurs as an outcome of rational, deliberate choices of the established firm and potential entrants not as an outcome of irrational behavior of the firms as noted by Scherer (1980, pp. 246-7).

In Section II, the basic model is presented in the form of a game tree. In Section III, we examine the nature of equilibria emerging under different informational structure, focusing on the asymmetric information case. In Section IV, we discuss an incentive of a potential entrant to signal to avoid undesirable outcome under asymmetric information. The results obtained are summarized in Section V. Proofs to the propositions are relegated to Appendix.

II. The Basic Model

Suppose that an established firm (firm 1) has been a monopolist for a long while in the market, facing a threat of entry of a potential entrant in period 1. The potential entrant enters the industry only

and Wilson (1982) show that a predation could be a rational choice of the chain-store that wants to build up its reputation. The present model and also the Benoit model (1984) differ from those in that there is no room for the reputation effect.

in period 1 if it ever does by spending some fixed irrevocable entry cost T_0. Each firm is assumed to behave in a Nash way. The potential entrant may be either a financially strong firm (firm S) with cash reserves M_s or a financially weak firm (firm W) with cash reserves M_w where $M_s > M_w > T_0$. These two types of firms have the same cost conditon and discount factor $\beta_2\,(0< \beta_2 <1)$. Let $\beta_1\,(0< \beta_1 <1)$ be the discount factor of firm 1.

The game-strategic interactions between firm 1 and the potential entrants are depicted by a game tree in Fig. 1 where the column vectors at the bottom of the tree represent two period discounted profits of firm 1, firm S and firm W, respectively. The game consists of three stage moves by Nature (a chance move), the potential entrant (either firm S or firm W) and firm 1. At the beginning of period 1, Nature picks one out of firms S and W according to known probabilities p for firm S and $1-p$ for firm $W (0 \leq p \leq 1)$. The probability p depends on some exogenous factors such as financial position, ability to gather information about profitability, past experience in the similar business and even a pure luck. Depending on the choice of Nature, either firm S or firm W plays. The potential entrant, if chosen, has two pure strategies. First, it may want to stay out of the industry. In this case, payoffs to each firm are immediately determined: Letting the subscripts 1 and 2 denote firm 1 and the potential entrant, firm 1 collects the monopolistic profits $\pi_1^m > 0$ in each period while the potential entrant gets zero profits ($\pi_2^m = 0$). If the potential entrant decides to enter, firm 1 is at either node 1_s or node 1_w depending on which type of firm plays at the second stage. In either case, it has two pure strategies. It either fights by maintaining its monopolistic strategy such as output level or shares the market by accommodating an entrant into the market. Let π_i^f and $\pi_i^d\,(i=1,2)$ denote one period profits of firm i when firm 1 chooses to fight and to share the market, respectively. The former situation is called a price war. A few assumptions are made on the profit and financial structure as follows:

Assumptions

(1) $\pi_1^m > \pi_1^d > \pi_1^f$ and $\pi_2^d > \pi_2^m = 0 > \pi_2^f$.

(2) $-T_0 + M_w + \pi_2^f < 0$ and $-T_0 + M_s + \pi_2^f > 0$.

(3) $-T_0 + \pi_2^d (1 + \beta_2) > 0$.

The first assumption is standard. For the second assumption, the left hand sides of the both inequalities are cash holdings of firms W and S, respectively at the beginning of period 2 if they enter and are met with a price war. Firm W ending up with negative cash holdings cannot go over to period 2 and is forced to go out of the market. In other words, firm W cannot survive a price war and goes bankrupt. In the case of bankruptcy, it is assumed that the firm takes only limited responsibilities up to cash holdings at the moment. This means that firm W may lose at most M_w. On the other hand, firm S has initial cash reserves large enough to survive a price war as the second inequality indicates and therefore can operate in period 2. Meanwhile, firm 1 is assumed to hold substantial amount of cash reserves reflecting its long-held monopolistic position so that no entrant even attempts to make firm 1 go bankrupt. The third assumption implies that it is profitable for any type of potential entrant to enter if firm 1 is willing to share the market in both periods.

Next, consider the behavior of firm 1 in period 2, which is the last period. If no entry occurs or the entrant goes bankrupt in period 1, firm 1 simply produces the monopolistic output in period 2. If the entrant does not go bankrupt in period 1 either because firm 1 does not fight a war or because the entrant happens to be the financially strong type, the only strategy available to firm 1 is to share the market. As a result, a duopoly market is established. This is because only one period is left and they behave in the Nash fashion. Therefore, in this case, each firm will receive π_1^d and π_2^d, respectively. In sum, in period 2, there are no real strategic interactions. This observation enables us to calculate the two period payoff vectors given at the bottom of the game tree in Fig. 1.

Finally, a remark on the informational structure is in order. There is informational asymmetry about the financial position between firm 1 and the entrants in the following manner: Firm 1 has no way of distinguishing, only knowing the chance of each type being chosen. This means that nodes 1_s and 1_w of the game tree are indistinguishable to firm 1 so that they are included in one information set depicted as a dotted circle in Fig. 1.

III. Equilibrium

Before presenting the analysis of the nature of equilibrium, we will examine it when no informational asymmetry exists as a ben-

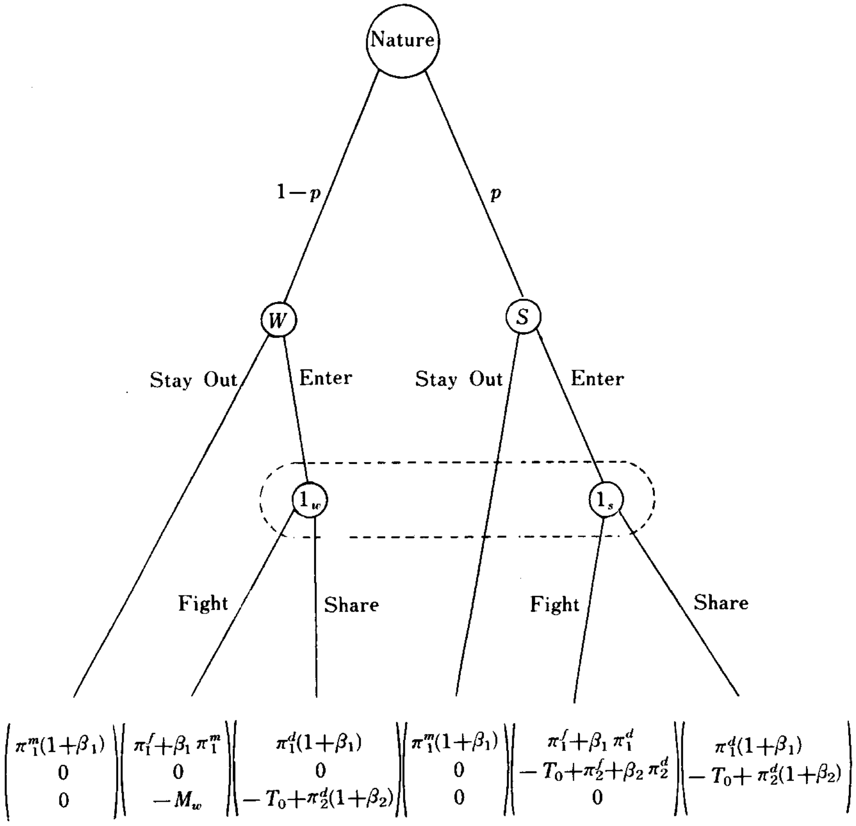

FIGURE 1

chmark. Let q_i $(i=s,w)$ be the probability that firm i enters and q_{1i} $(i=s,w)$ be the probability that firm 1 shares the market with firm i.

A. Symmetric Information

In this simple case, the notion of the backward induction or the subgame perfection is directly applicable. Suppose that firm 1 is at node 1_w. Then, it can always make firm W go bankrupt by starting a price war whereby it can regain its monopolistic position in period 2. But at the same time this strategy entails costs of reduced profits in period 1. As a result, whether or not firm 1 fights a war is determined by the relative magnitudes of gains and costs of carrying out such a strategy. It will share the market ($q_{1w}=1$) if $\pi_1^f+\beta_1\pi_1^m < \pi_1^d(1+\beta_1)$ and fight ($q_{1w}=0$) otherwise. One immediate consequence from this observation is that the threat by firm 1 facing entry of firm W that it will maintain its monopolistic output may or may not be empty as contrasted with Dixit's argument in his one period model (1982). In such a model, it is always not wise for firm 1 to start a price war because it only hurts itself with no gains. Next, firm W that realizes this fully will enter in the former case but not in the latter case. If firm 1 is at node 1_s, it will definitely choose to share the market ($q_{1s}=1$) because $\pi_1^d(1+\beta_1)>\pi_1^f+\beta_1\pi_1^d$, recalling that firm 1 cannot make firm S go bankrupt. Going back to the second stage, firm S knowing the behavior of firm 1 will enter without any fear of being caught up in a price war. So in this case, a Nash equilibrium will be realized in both periods.

Finally, in this symmetric information case, there is no room for a price war; only pure strategy equilibrium is possible except for an uninteresting case of, for example, $\pi_1^f+\beta_1\pi_1^m=\pi_1^d(1+\beta_1)$. Every possibility is perfectly foreseen and taken into account by each firm before the game is played. Firm W enters only if firm 1 will not fight a war. On the other hand, firm S enters with rational belief that firm 1 will share the market.

B. Asymmetric Information

In this case, nodes 1_s and 1_w are in the same information set because firm 1 cannot distinguish at which nodes he arrives. Let q_1 be the probability that firm 1 shares the market when it reaches the information set. We will calculate conditional expected profits of firms on condition that each firm reaches its information set.[2] Let $U_i(i=1,s,w)$ be the conditional expected profits of firm 1, firm S and firm W, respectively. Then,[3]

$$\begin{aligned} U_1= &\{1/(pq_s+(1-p)q_w)\}\ \{pq_s(q_1\pi_1^d(1+\beta_1)+(1-q_1)(\pi_1^f+\beta_1\pi_1^d)) \\ &+(1-p)q_w(q_1\pi_1^d(1+\beta_1)+(1-q_1)(\pi_1^f+\beta_1\pi_1^m))\} \\ U_s= &q_s\{q_1(-T_0+\pi_2^d(1+\beta_2))+(1-q_1)(-T_0+\pi_2^f+\beta_2\pi_2^d)\} \\ U_w= &q_w\{q_1(-T_0+\pi_2^d(1+\beta_2))-(1-q_1)M_w\}. \end{aligned} \tag{1}$$

The first order derivatives of U_i with respect to q_i ($i=1,s,w$) are

[2]The reason for using conditional expected profits is to find local best replies(see Selten (1975) for this concept). This further implies that either $q_s>0$ or $q_w>0$ or both.

[3]For U_1, the fact is used that the conditional probability of an entrant being firm S is $pq_s/(pq_s+(1-p)q_w)$. For U_w recall that in case of bankruptcy, the firm is responsible only for the amount it has at the moment, that is, M_w-T_0.

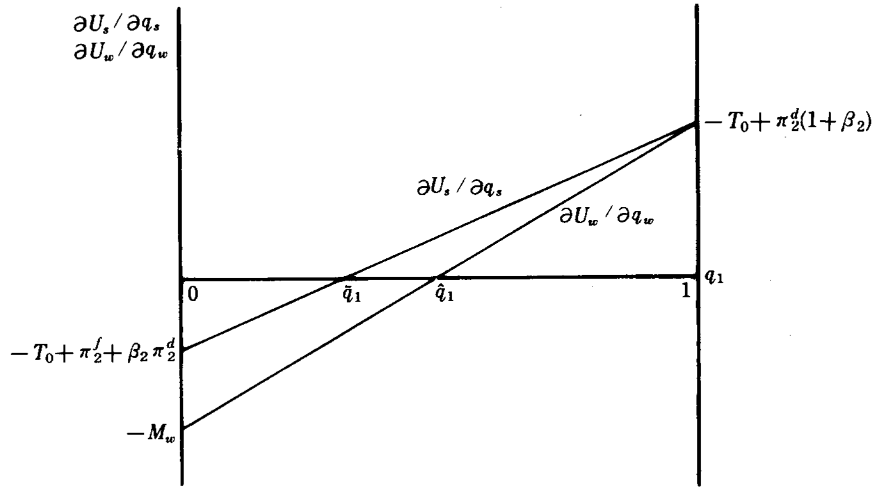

FIGURE 2

$$
\begin{aligned}
\partial U_1/\partial q_1 &= 1/(pq_s + (1-p)q_w)\{pq_s(\pi_1^d - \pi_1^f) \\
&\quad + (1-p)q_w(\pi_1^d(1+\beta_1) - (\pi_1^f + \beta_1 \pi_1^m))\} \\
\partial U_s/\partial q_s &= -T_0 + q_1 \pi_2^d(1+\beta_2) + (1-q_1)(\pi_2^f + \beta_2 \pi_2^d) \\
\partial U_w/\partial q_w &= q_1(-T_0 + \pi_2^d(1+\beta_2)) - (1-q_1)M_w
\end{aligned} \tag{2}
$$

Following Selten, the limiting behavior of the equilibrium in a sequence of perturbed games are studied where there is a slight chance of making mistakes in choosing the optimal strategies in each information set. That is, in a perturbed game, $q_i (i=1,s,w)$ is restricted by ε_i and $1-\varepsilon_i$ where ε_i is a sufficiently small positive number.

There are a few immediate things to be noted from eqs. (1) and (2). First, the behavior of each firm depends on the profit structure, entry costs and initial cash holdings of firm W. Second, the strategic behavior of firms S and W depends only on the strategy choice of firm 1 not on that of each other. A possible situation about $\partial U_s/\partial q_s$ and $\partial U_w/\partial q_w$, both of which are functions of q_1, are drawn in Fig. 2. In drawing the figure, we assume that $M_w > T_0 - (\pi_2^f + \beta_2 \pi_2^d)$. We confine the analysis only to this case.[4] Note

[4]In the other case of $M_w < T_0 - (\pi_2^f + \beta_2 \pi_2^d)$, there is no basic change in the analysis although slightly different outcomes obtain(see also footnote 6).

that depending on whether $-T_o+\pi_2^f+\beta_2\pi_2^d>0$ or not, $\partial U_s/\partial q_s$ may lie entirely above q_1 axis or intersect the axis while $\partial U_w/\partial q_w$ always intersects it as shown in the figure. In the case of $\partial U_s/\partial q_s$ intersecting q_1 axis, let $\tilde{q}_1$ and $\hat{q}_1$ denote the intersection points of $\partial U_s/\partial q_s$ and $\partial U_w/\partial q_w$ with the q_1 axis whose values are given by

$$\tilde{q}_1=(T_0-\pi_2^f-\beta_2\pi_2^d)/(\pi_2^d-\pi_2^f) \text{ if } T_0-\pi_2^f-\beta_2\pi_2^f>0, \\ \hat{q}_1=M_w/(\pi_2^d(1+\beta_2)+M_w-T_0). \tag{3}$$

It is obvious from Fig. 2 that $\tilde{q}_1$ and $\hat{q}_1$ are between 0 and 1.

As in the symmetric information case, we distinguish two cases depending on whether $\pi_1^d(1+\beta_1)>\pi_1^f+\beta_1\pi_1^m$ or not. The following result is immediate.

Proposition 1[5]

If $\pi_1^d(1+\beta_1)>\pi_1^f+\beta_1\pi_1^m$, there is a unique equilibrium point in a perturbed game: $q_1=1-\varepsilon_1$, $q_s=1-\varepsilon_s$, $q_w=1-\varepsilon_w$.

Taking the limit as ε_i $(i=1,s,w)$ goes to 0, the same type of Nash equilibrium is obtained as that in the symmetric information case. The equilibrium is perfect by its construction. If $-T_0+\pi_2^f+\beta_2\pi_2^d\leq 0$, no entrant can make positive profits by entering if firm 1 fights a war. This condition is a basis for the limit pricing theory combined with the Sylos postulate. In this case, the entry-deterring equilibrium $(q_1=q_s=q_w=0)$ is also a Nash equilibrium, though it is not perfect as indicated by the proposition. That is, a slight chance of entering forces firm 1 to choose to share the market.

In the following, the analysis is confined to the other and more interesting case of $\pi_1^d(1+\beta_1)<\pi_1^f+\beta_1\pi_1^m$. That is, firm 1 would fight a war if the entrant were known to be firm W. Now, for future reference, consider that if firms S and W have the same level of incentive to enter $(q_s=q_w<0)$, the posterior probabilities about the identity of the entrant are the same as the initial probabilities, p and $1-p$. Next, define z as $\partial U_1/\partial q_1$ evaluated at $q_s=q_w>0$. Then it is easy to show that

$$z=\pi_1^d(1+\beta_1)-(\pi_1^f+\beta_1(p\pi_1^d+(1-p)\pi_1^m)). \tag{4}$$

z is the net benefit of sharing the market without fighting. Notice that $\partial U_1/\partial q_1$ does not depend on the common value of q_s and q_w. For analytic simplicity, we assume that $z\neq 0$.

[5]The proposition also holds when $M_w<T_0-(\pi_2^f+\beta_2\pi_2^d)$.

Proposition 2[6]

Suppose that $\pi_1^f + \beta_1 \pi_1^m > \pi_1^d (1 + \beta_1)$. Then there are at most three types of equilibrium points of the following form in a perturbed game.

(A) $\varepsilon_1 \leq q_1 \leq \tilde{q}_1$, $q_s = \varepsilon_s$, $q_w = \varepsilon_w$ iff $-T_0 + \pi_2^f + \beta_2 \pi_2^d < -\varepsilon_1 (\pi_2^d - \pi_2^f)$.

(B) $q_1 = \hat{q}_1$, $q_s = 1 - \varepsilon_s$, $q_w = (1 - \varepsilon_w) q_w'$ iff $z < 0$

$$\text{where } 1 > q_w' = \frac{p}{1-p} \frac{\pi_1^d - \pi_1^f}{\pi_1^f + \beta_1 \pi_1^m - (1 + \beta_1) \pi_1^d} > 0.$$

(C) $q_1 = 1 - \varepsilon_1$, $q_s = 1 - \varepsilon_s$, $q_w = 1 - \varepsilon_w$ iff $z > 0$.

Moreover, in each type of equilibrium, given the value of q_1, there is no more equilibrium values of q_s and q_w. That is, there exists a unique equilibrium in each type.

Now letting ε_i go to zero, the perfect equilibria of different types are obtained depending on the underlying parameter.[7]

Case (A): Entry-Deterring Equilibrium ($q_s = q_w = 0$)

In this case, no potential entrant (even firm S) enters and firm 1 keeps its monopolistic position, which is very similar to the limit pricing equilibrium. Firm 1's threat that it will fight with a high chance against the entrant is credible and the potential entrant is kept out of the industry. It is worth noting that for this type of equilibrium an extra condition that it would be in firm 1's interest to fight a war if it were known that firm W entered is needed in addition to the condition mentioned above after Proposition 1 that no potential entrant can make positive profits in a price war.

[6]In case of $M_w < T_0 - (\pi_2^f + \beta_2 \pi_2^d)$, there are also three possible equilibrium points of the following form: (A) $\varepsilon_1 \leq q_1 \leq \hat{q}_1$, $q_s = \varepsilon_s$, $q_w = \varepsilon_w$. (B) $q_1 = \tilde{q}_1$, $q_s = (1 - \varepsilon_w) q_s$, $q_w = 1 - \varepsilon_w$ iff $z > 0$ where $q_s' = 1/q_w'$ (C) $q_1 = 1 - \varepsilon_1$, $q_s = 1 - \varepsilon_s$, $q_w = 1 - \varepsilon_w$ iff $z > 0$. Notice that if $z < 0$, (A) is the only possible equilibrium this case.

[7]There arises a problem frequently seen in the game theoretic models that there may be multiple equilibria. For example, if $z < 0$ and $-T_0 + \pi_2^f + \beta_2 \pi_2^d$, then both (A) and (B) are the perfect equilibria. The present paper as it stands does not attempt to pick a more plausible equilibrium in such a situation. In this regard, it is worth noticing Shubik's remarks (1981) that "If we take the attitude that much of the normative component to our theorizing should take place at level of the design of institution, i.e., the rules of the game; then existence of the two types of equilibrium… should not bother us."

Case $(B)'$: Mixed Strategy Equilibrium $(q_1=\hat{q}_1,\ q_s=1,\ q_w=q'_w)$

In this equilibrium where $z<0$, firm S always enters if chosen while firm W does with probability $q'_w<1$. In response to this, firm 1 fights with probability $1-\hat{q}_1$. As a result, depending on strategies actually taken by each party, different ex post outcomes obtain. For example, if firm W is chosen and decides not to enter, firm 1 will keep its monopolistic position in two periods. If firm W decides to enter and firm 1 decides to fight, there will be a price war in period 1 and firm 1 regains the monopolistic position in period 2 while if firm 1 decides not to fight, a Nash equilibrium will be realized in both periods. The case of firm S being chosen can be similarly argued.

In passing, we make some comparative static analysis of parameters affecting $\hat{q}_1$ and q'_w. First, suppose that M_w increases slightly. Then from the equilibrium condition, it is immediate that new equilibrium includes a higher $\hat{q}_1$ and the same q'_w. Although the model is static in nature, the following heuristic argument about an adjustment process appears to be helpful in explaining the transition to a new equilibrium point: An increase in M_w leads to an increase in expected losses to firm W in the case of a price war. This will discourage the firm to enter the industry, reducing q_w. Firm 1 will be then less inclined toward fighting and thereby raises $\hat{q}_1$. Obviously, this reaction of firm 1 will increase the incentive of firm W to enter. And this process will continue until a new equilibrium is reached where q'_w returns to the original value while $\hat{q}_1$ increases enough. A similar argument can be made for other variables affecting $\hat{q}_1$ such as T_0, β_2 and π_2^d. Second, suppose that p increases. Then, it is immediate that in a new equilibrium $\hat{q}_1$ is the same as before while q'_w goes up. Similar arguments can be made as before. As p goes up so that the chance of the entrant being firm S increases, firm 1 will have less incentive to fight a war. Firm W will take advantage of this, increasing its chance to enter. Then, this will in turn pull up the decreasing incentive of firm 1 to fight. Again, this process continues until a new equilibrium is established. The same kind of reasoning can be applied to other variables affecting q'_w such as π_1^d, π_1^f and π_1^m. Table 1 shows the changes in the relevant variables that increase $\hat{q}_1$ and q'_w. On one hand, an increase in the parameters such as M_w and T_0 raises the cost of entry, reducing the willingness of firm W to enter. As a result, firm 1 switches to a softer policy (*i.e.*, a higher q_1). On the other hand, an

TABLE 1

	$\hat{q}_1$	q'_w
An increase in	M_w, T_0, $\hat{q}_1$	p, π_1^d
A decrease in	β_2, π_2^d	π_1^f, π_1^m, β_1

increase in the parameters such as π_1^f, π_1^m and β_1 increases the benefit of fighting. Expecting a harsher policy by firm 1, firm W will reduce its chance of entering.

One interesting observation is that $U_1=(1+\beta_1)\pi_1^d$, which is easily verified by substituting the equilibrium values of q_i. That is, in this equilibrium, if entry occurs, firm 1 gets profits that would be obtained if it shared the market from the beginning.

Next consider the chance of a price war occurring. Evidently, it occurs if (i) firm 1 decides to fight and (ii) either firm S is chosen or firm W is chosen and decides to enter. Thus, the probability of a price war occurring, L, is given by

$$L=(1-\hat{q}_1)(p+(1-p)q'_w) \tag{5}$$

As is obvious from the above equation, L depends positively on p and negatively on $\hat{q}_1$ and q'_w. Using the definition of $\hat{q}_1$ and q'_w, and the comparative static results in Table 1, we see that an increase in p, π_1^d, π_2^d and β_2 and a decrease in M_w, T_0, π_1^f, π_1^m and β_1 increase L. The case of an increase in p is worth mentioning. If p goes up, the chance of firm S entering directly rises. Meanwhile, it has two offsetting effects on the chance of firm W entering. It reduces the chance of firm W being chosen while raises q'_w by the reason discussed above. But in view of the definition of q'_w, it is easy to check that the net effect is positive. As a result, the chance of both firms entering and thereby that of a price war will increase. For other parameters, it is straightforward to see the effects of a change on L.

Case (C): Entry-Accommodating Equilibrium ($q_1=q_s=q_w=1$)

If $z>0$, an entry-accommodating equilibrium is realized. This is because if both firms have the same level of incentive to enter, firm 1 can do no better than sharing the market. (Refer to eq. (4) and its related argument.) Knowing this, both firms will enter without any fear of a price war, irrespective of whether they could survive a possible price war or not. Under asymmetric information, then, there are possibly two sets of conditions that realize this type of equilibrium, one in Proposition 1 and the other in Proposition 2. Also, it is noted that the information asymmetry even provides firm W with a chance of entering in contrast with the symmetric information case where firm W has no chance to enter. Finally, for high p (for example $p=1$), $z>0$. This means that if the chance of firm S being chosen is high enough, firm 1 will not attempt to identify the entrant because it is too costly.

IV. Signaling

In general, firm S is made worse-off under asymmetric information because of the presence of firm W: The firm may be blocked to enter (Case (A)) or met with a price war (Case (B)), which would not arise if its identity were known to firm 1. To see this point more closely, we calculate (unconditional) expected profits of each firm under different informational structure, confining ourselves only to Case (B) for the asymmetric information case. Let V_1^s and V_1^a be those of firm 1 under symmetric and asymmetric informaton, respectively. For firms S and W, they are analogously defined. Then,

$$
\begin{aligned}
V_1^s &= (1-p)\pi_1^m(1+\beta_1)+p\pi_1^d(1+\beta_1)\\
V_s^s &= p(-T_0+\pi_2^d(1+\beta_2))\\
V_w^s &= 0\\
V_1^a &= (1-p-(1-p)q_w')\pi_1^m(1+\beta_1)+(p+(1-p)q_w')\pi_1^d(1+\beta_1)\\
V_s^a &= p(\hat{q}_1(-T_0+\pi_2^d(1+\beta_2))+(1-\hat{q}_1)(-T_0+\pi_2^f+\beta_2\pi_2^d))\\
V_w^a &= 0.
\end{aligned}
\tag{6}
$$

(The fourth equality of eq. (6) follows from the fact that $U_1=\pi_1^d(1+\beta_1)$ as was shown in the previous section.) Now, comparing V_1^s and V_1^a, it is immediate that firm 1 loses in the asymmetric information case. For firm S, although $V_s^a>0$ from Fig. 2, it is clear that $V_s^a<V_s^s$. Therefore, firm S also loses because of asymmetric information even though it is still making positive profits. Firm W is indifferent, making zero profits.[8]

Firm S may want to make efforts to differentiate itself from firm

[8]Incidentally, similar phenomenon is observed in Rothschild and Stiglitz (1976) about the working of the insurance market, where "there are losses to the low-risk individuals, but the high-risk individuals are no better than they would be in isolation."

W to eliminate the informational asymmetry. In this section, we will consider a simple strategy of signaling on the part of firm S. Note that from Fig. 2 and the previous assumptions the following conditions must hold in addition to $z<0$ in such a regime:

$$max \{T_0,\ T_0 - \pi_2^f - \beta_2\pi_2^d\} < M_w < T_0 - \pi_2^f\ . \tag{7}$$

From now on, we will regard the entry cost as a choice variable of firm S denoted by T and T_0 as the minimal entry cost. Firm S faces two options when choosing the level of T. It can choose T to the extent (i) that the mixed strategy equilibrium is maintained or (ii) that the equilibrium is no more sustained. In the first case, it is not possible for firm S to differentiate itself from firm W. The case is nevertheless worth considering because an increase in the entry cost lowers the incentive of firm 1 fighting back ($\hat{q}_1$), which has positive effects on its expected profits, though it is a direct cost to firm S at the same time. Referring to Fig. 2, choosing option (i) requires the value of T to be within a certain range. First, T cannot exceed $\pi_2^d(1+\beta_2)$ and $M_w + \pi_2^f + \beta_2\pi_2^d$. Second, T also cannot be greater than M_w because otherwise firm W cannot afford it and therefore will not attempt to enter in any case. In fact, this point plays a key role in the signaling argument and will be discussed below. From the above observation,

$$T_0 \le T \le T_1 = min\{\pi_2^d(1+\beta_2),\ M_w + \pi_2^f + \beta_2\pi_2^d\ ,\ M_w\}\ . \tag{8}$$

Note that $\partial^2 V_s^u / \partial T^2 > 0$ for all T. Since $V_s^u > 0$ at $T = T_0$ and $=0$ at $T = \pi_2^d(1+\beta_2)$, it is immediate that V_s^u takes on its maximum at T_0 over the interval given in eq. (8). This means that profits of firm S are maximized by spending the minimal amount of entry cost T_0 so that it will not increase the entry cost above T_0 under the constraint that the mixed strategy equilibrium should be maintained.

Next, we examine the second option available to firm S, which is to raise T to break the mixed strategy equilibrium. A natural candidate for this purpose is $T = M_w$.[9] For this level of entry cost, firm W will be indifferent to whether to stay out or enter. But since a slightly higher amount will keep firm W out, M_w can be regarded as

[9]Firm S may want to move into another regime described in footnote 6 by raising t above $M_w + \pi_2^f + \beta_2\pi_2^d$ if it is less than M_w. But since $z<0$ in this case, the only possible equilibrium is an entry-deterring equilibrium. Therefore, the firm would not make such a choice.

the limit entry costs to firm W. Suppose that if firm S is chosen by Nature, it enters by spending M_w. Then, if entry occurs, the entrant will be certainly firm S because firm W never enters with that high entry cost. As a result, in case of firm S being chosen, a Nash equilibrium will be realized. Therefore, the expected profits of firm S will be $p(\pi_2^d(1+\beta_2)-M_w)$, which must be at least as great as pU_s for the strategy to be profitable. It is shown that the equation $\pi_2^d(1+\beta_2)-M_w-U_s=0$ has one positive root and one negative root for M_w. Let M_w^+ be the positive root. Then, we have

Proposition 3

For the signaling to be profitable, $M_w < M_w^+$.

The proposition says that cash reserves of firm W should be small enough for a signaling equilibrium to be realized. From eq. (7), if $M_w^+ > T_0 - \pi_2^f$, it always pays firm S to signal by spending entry cost of M_w while if $M_w^+ < \max\{T_0 - \pi_2^f - \beta_2\pi_2^d, T_0\}$, it does not pay. In between case, the result depends on the actual amount of M_w. Below, a numerical example is presented. Suppose that $\pi_2^f = -1$, $\pi_2^s = 1$, $\beta_2 = 1$ and $T_0 = 1$. Then $M_w^+ = 3^{1/2}$ while $1 < M_w < 2$. Therefore, if $M_w < 3^{1/2}$, firm S will signal and will not otherwise. Finally, if T_0 is interpreted as the expenditure on capacity for producing duopolistic output, the proposition implies that excess capacity may be observed to be carried by the entrant not by the established firm as contrasted with the traditional results (Spence 1977; Spulber 1981).

V. Summary

We have discussed the nature of equilibrium under both symmetric and asymmetric information about the financial position of the potential entrant. The argument can be easily extended to other types of heterogeneity such as cost conditions. The symmetric information case is simpler to analyze and yields similar results to those of traditional works except for a possible entry-deterrence when a financially weak firm is chosen by Nature. In the event of a financially strong firm entering, the usual Nash equilibrium is always realized.

Under asymmetric information, we examine a pooling equilibrium where the identity of an entrant is not revealed. Various possibilities obtain. If it does not pay the established firm to fight even when the financially weak firm is known to enter, only a Nash equilibrium is possible. On the other hand, if it pays the established firm to fight with the financially weak firm, there are three types of equilibria. First, an entry-deterring equilibrium without a prior commitment is possible if (i) the financially strong firm finds no incentive to enter if a price war is expected and (ii) it would be optimal to fight a price war if firm 1 knew the entrant to be firm *W*.

If it did not pay the established firm to fight when both types of entrants and the same level of incentive to enter, the second type of equilibrium can arise, a duopoly equilibrium. Otherwise, another type of equilibrium, a mixed strategy equilibrium can be obtained, which generates an interesting possibility of a price war in transition. In relation to this, a remark by Dixit (1980) is reminded that, "In reality, there may be no agreement about the rules of the post-entry game, and there may be periods of disequilibrium before any order is established. Financial position of the firms may then acquire an important role." In the present model, with complete understanding of the rules of the post-entry game, such chaos may occur as a result of equilibrium behavior due to the informational asymmetry.

As in other literature involving asymmetric information, a question of signaling arises because a financially strong firm generally suffers reduced profits due to the presence of a financially weak firm. The former will have an incentive to differentiate itself from the latter or to establish a separating equilibrium. A conclusion from this discussion is that an excessive spending by an entrant at the time of entry, possibly through a form of excess capacity, may be observed if the entrant is of a financially strong type. This is contrasted with the arguments found in the previous works that an established firm may carry excess capacity to deter entry.

Appendix

Proof of Proposition 1

From eq. (2), it is obvious that from any combination of q_s and q_w, $\partial U_1 / \partial q_1 > 0$ so that $q_1^* = 1 - \varepsilon_1$ where $*$ denotes a best reply. Then, Fig. 2 says that $\partial U_s / \partial q_s$ and $\partial U_w / \partial q_w$ are positive for small $\varepsilon_1 > 0$. The result then follows.

Proof of Proposition 2

1. (A) (Sufficiency) let $\varepsilon_w = \varepsilon_s q'_w$ where q'_w is defined in eq. (2). For q_1 such that $\varepsilon_1 \leq q_1 < \tilde{q}_1$, $\partial U_s / \partial q_s < 0$ and $\partial U_w / \partial q_w < 0$ so that $q_s^* = \varepsilon_s$ and $q_w^* = \varepsilon_s q'_w$. It is easy to see that for $q_s = \varepsilon_s$ and $q_w = \varepsilon_s q'_w$ $\partial U_w / \partial q_w = 0$. (Necessity) Obvious. (B) (Sufficiency) Let $\varepsilon_s = \varepsilon_w$. Then, for $q_1 = \hat{q}_1$, $\partial U_s / \partial q_s > 0$ and $\partial U_w / \partial q_w = 0$. Therefore, $q_s^* = 1 - \varepsilon_s$ and any q_w is a best reply, inparticular, $q_w^* = (1 - \varepsilon_s) q'_w$ as long as $\varepsilon_s \leq (1 - \varepsilon_s) q'_w \leq 1 - \varepsilon_s$. The left hand side inequality is automatically satisfied for small ε_s. The right hand side inequality is also satisfied because $z < 0$, which implies that $q'_w < 1$. Finally, for $q_s = 1 - \varepsilon_s$ and $q_w = (1 - \varepsilon_s) q'_w$ $\partial U_1 / \partial q_1 = 0$ so that $\hat{q}_1$ is a best reply. (Necessity) Suppose that $z > 0$ so that $q'_w > 1$. Then, for the equilibrium, there must be some $\varepsilon_w > 0$ such that $\varepsilon_w \leq (1 - \varepsilon_s) q'_w \leq 1 - \varepsilon_w$. From the right hand inequality, $0 < \varepsilon_w \leq 1 - (1 - \varepsilon_s) q'_w$. Taking the limit as ε_s goes to 0, $0 < \lim \varepsilon_w \leq 1 - q'_w < 0$, which is a contradiction. (C) (Sufficiency) Let $\varepsilon_s = \varepsilon_w$. For $q_1 = 1 - \varepsilon_1$, $\partial U_s / \partial q_s > 0$, $\partial U_w / \partial q_w > 0$ so that $q_s^* = q_w^* = 1 - \varepsilon_s$. Also for $q_s = q_w = 1 - \varepsilon_s$, $\partial U_1 / \partial q_1 = z > 0$, which proves the result. (Necessity) Suppose that $z < 0$. For $q_s = 1 - \varepsilon_s$ and $q_w = 1 - \varepsilon_w = (1 - \varepsilon_s) + (\varepsilon_s - \varepsilon_w)$, $\partial U_1 / \partial q_1 = (1 - \varepsilon_s) z + (\varepsilon_s - \varepsilon_w)(1 - p)(\pi_1^d (1 + \beta_1) - (\pi_1^f + \beta_1 \pi_1^m))$, which must be nonnegative. This implies that $\varepsilon_w \geq \varepsilon_s (1 + z/(1 - p)(\pi_1^f + \beta_1 \pi_1^m - \pi_1^d (1 + \beta_1))) - z/(1 - p)(\pi_1^f + \beta_1 \pi_1^m - \pi_1^d (1 + \beta_1))$. Taking the limit as ε_s goes to 0, $\lim \varepsilon_w > 0$, which establishes a contradiction.
2. Uniqueness can be easily shown by following the same argument in 1.
3. Finally, to show that there does not exist any more equilibrium, suppose $\tilde{q}_1 < q_1 < \hat{q}_1$. Then, $q_s^* = 1 - \varepsilon_s$ and $q_w^* = \varepsilon_w$. For such q_1 to be a best reply to $q_s = 1 - \varepsilon_s$ and $q_w = \varepsilon_w$, $\partial U_1 / \partial q_1 = 0$ or $\varepsilon_w = (1 - \varepsilon_s) q'_w$. Taking the limit as ε_s goes to 0, $\lim \varepsilon_w = q'_w$ which is a contradiction. For q_1, $\hat{q}_1 < q_1 < 1 - \varepsilon_1$, to be an equilibrium strategy, it is easy to show that $z = 0$. But this possibility was assumed to be ruled out.

References

Benoit, J. P. "Financially Constrained Entry in a Game with Incomplete Information." *The Rand Journal of Economics* 15 (Winter 1984): 491–9.
Dixit, A. "A Model of Duopoly Suggesting a Theory of Entry Barriers." *Bell*

Journal of Economics 10 (Spring 1979): 20-32.

________. "The Role of Investment in Entry-Deterrence." *Economic Journal* 90 (March 1980): 95-106.

________. "Recent Development in Oligopoly Theory." Papers and Proceedings, *American Economic Review* (May 1982): 12-7.

Eaton, B. C., and Lipsey, R. G. "Capital, Commitment, and Entry Equilibrium." *Bell Journal of Economics* 12 (1981): 593-604.

Kreps, D. M., and Wilson, R. "Reputation and Imperfect Information." *Journal of Economic Theory* 27 (1982): 253-79.

Milgrom, P., and Roberts, J. "Predation, Reputation, and Entry Deterrence." *Journal of Economic Theory* 27 (1982): 280-312.

Rothschild, M., and Stiglitz, J. E. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Insurance." *Quarterly Journal of Economics* 90 (1976): 629-50.

Scherer, F. M. *Industrial Market Structure and Economic Performance.* Chicago, 1970.

Selten, R. "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games." *International Journal of Game Theory* 4 (1975): 25-55.

Shubik, M. "Perfect Robust Noncooperative Equilibrium: A Search for the Philosopher's Stone?" in Mannheim, *Essays in Game Theory and Mathematical Economics,* 1981.

Spence, A. M. "Entry, Investment and Oligopolistic Pricing." *Bell Journal of Economics* 8 (Autumn 1977): 534-44.

Spulber, D. F. "Capacity, Output, and Sequential Entry." *American Economic Review* 71 (1982): 503-14.