Interpretation of Indexical Sentences

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Natural languages such as English abound in examples of indexical sentences, sentences that cannot be interpreted without knowing their contexts of utterance: the time and place of their utterance as well as their speakers and listeners. Consider:

(1) Ice floats on water.
(2) The snow is melting.
(3) I voted for you.

While sentence (1) can be understood as a general statement about the property of ice being lighter than water without any reference to its context of utterance, the interpretation of sentence (2) requires one’s knowledge of when and where it was spoken. What is being stated by (2) is not a general fact about the melting property of the snow. It states that the snow is melting at the time near the speaker’s vicinity. To understand sentence (3), one again has to have extralinguistic knowledge: he has to know who its speaker and listener are. Only then can he identify which individuals are referred to by the expression “I” and by “you” and determine whether or not sentence (3) is a true statement. Accordingly, one could suggest that each sentence be implemented with such contextual information. Sentences (2) and (3) would then be represented:

(4) [Aug. 31, 1973, Austin]: The snow is melting.
(5) [Pat Nixon said to Richard Nixon]: I voted for you.

These sentences may now be replaced by non-indexical sentences:

(6) The snow was melting on Aug. 31, 1973, in Austin.
(7) Pat Nixon voted for Richard Nixon.

Sentences (4) and (5) would, then, be interpreted as equivalent to sentences (6) and (7), respectively.

Although such an approach to the analysis of indexicals may seem obvious, it is not always the case that indexicals can be replaced by appropriate non-indexical expressions. It is immediately clear how difficult one would find it to replace the pronouns “I” and “you” in sentence (3) when he does not know the names of the speaker and listener. He might try to convert it into (8), but soon would recognize that he has not yet succeeded in getting rid of indexicals at all.

(8) The one who is saying “I voted for you” voted for the one who hears it.
By trying to replace indexical expressions with non-indexical expressions, we run into the problem of infinite regression. The same thing would be said about sentence (2) if we had no means of naming dates and places.

Indexicals cannot always be replaced by non-indexical expressions, and sometimes should not be so replaced. Consider:

(9) I am you.
(10) I am dead.
(11) Grass is green, but I don’t believe that it is.

Suppose we replace the pronouns “I” and “you” in these sentences by “John” and “Jack”, respectively:

(12) John is Jack.
(13) John is dead.
(14) Grass is green, but John doesn’t believe that it is.

These sentences no longer sound absurd or paradoxical, as the ones in (9-11) did. They are not equivalents having the same truth values in every interpretation, so the substitution of names for the pronouns was not legitimate.

These are classical puzzles that have plagued the minds of many logicians. From G. E. Moore to J. Hintikka (1962), these indexical sentences, particularly sentence (11), have been intensely scrutinized. Some attributed their absurdity to some peculiarities of the pronoun “I”, while others like Hintikka presented some pragmatic reasons independent of the peculiar use of “I”. For Hintikka, sentence (11) is not a simple contradiction of elementary logic, but has absurdity only due to the fact that it can easily be recognized as violating “the general presumption that the speaker believes or at least can conceivably believe what he says.” Sentence (14), Hintikka argues, would also be absurd if it is uttered by John. This shows that any sentence of the structure of (14) would be absurd if the subject of believe is the speaker and that sentence (11) is a particular instance of such a case. More recently, Prior (1968) argues that sentence (11) is absurd because of the difficulty of interpreting “I” as an observer unlike other pronouns such as “he”.

Whatever validity each of their arguments might have, their approach to the problems is essentially pragmatic-oriented. Pragmatics has now become a popular and yet serious topic of discussion among both logicians and linguists. This is borne out not only by an ever-increasing number of publications in this area, but also by such activities as the Texas Performadillo Conference (spring 1973), the Michigan Pragmatics Workshop (summer 1973), and the LSA Institute and the 2nd MSSB Workshop at Amherst, Mass., conducted in summer 1974.

1. Serious efforts had already been initiated by R. Montague to formalize pragmatics and to apply it to the analysis of natural language. In his “Pragmatics” (1968) and “Universal Grammar” (1970), he discussed how to treat indexicals such as tenses, personal
pronouns, and demonstratives and how to set up appropriate interpretation models. In his most recent work "The Proper Treatment of Quantification in Ordinary English" (1973), commonly called the PTQ, he added the tense as part of his logical system, but unfortunately no other indexicals have been introduced as part of his English fragment. Since the PTQ, however, seems to best illustrate Montague's efforts to apply formal logic to the analysis of natural language, it would seem very reasonable to analyze indexical expressions as an extension of Montague's PTQ.

In this paper, I shall simply restrict myself to the analysis of two indexicals "I" and "you" and to the delimitation of their possible denotation function. As a result, I hope that my extension of the PTQ would succeed in properly interpreting sentences like:

(15) a. I am you.
   b. I dreamt that I was B. Bardot and that I kissed me.

(16) a. I smoke.
   b. You smoke.
   c. John smokes.
   d. The dean smokes.

(17) a. I don't exist.
   b. I am dead. Don't look for me.

The interpretation of sentences (15a-b) raises the problem of identifying individuals across possible worlds, that of sentences (16a-b-c-d) illustrates how the extended PTQ would make them entail each other under certain contexts of utterance, and finally that of sentences (17a-b) involves the condition of actual existence that might be imposed on speakers. Moore's paradox will not be discussed in this paper, since it not only involves the interpretation of "I", but also the notions of belief and knowledge.

1.1 The PTQ grammar has three main parts: syntax, translation into intensional logic, and interpretation. The syntax of indexical pronouns is straightforward. The extended PTQ would contain the expressions "I" and "you" as belonging to the set of basic expressions of category T:

(18) I, you € B_T.

These indexicals would be treated like proper names and variables, which are also members of B_T. However, some syntactic rules like S4 Subject Formation and S17 Tense and Negation, which involve the subject-verb agreement, must be revised to allow sentences like (19), but to disallow sentences like (20).

(19) a. I smoke.
    b. I don't smoke.

(20) a.* I smokes.
    b.* You doesn't smoke.

Other morphological changes must also be made to accommodate the new lexical items.
In a fuller treatment of English, the performative analysis advocated by Ross, Lakoff, and Saddock may be adopted in writing syntactic rules like Imperative and Reflexivization rules. In the present treatment of English, however, it would not be assumed that every syntactic tree is headed by a performative expression like "I tell you that". Sentences (21) and (22), for instance, would be derived in our syntax, but each would have a different analysis tree.

(21) *I tell you that* I smoke pot.
(22) *I smoke pot.*

They would, nevertheless, be interpreted as equivalents by possibly setting up a meaning postulate to that effect.

Like proper names, the indexicals can be substituted for variables. Sentence (23), for instance, is derived by utilizing quantification rule S14.

(23) a. *I love every girl who loves me* $(F_{10-0})$
   b. *I $x_0$ love every girl who loves $x_0$*

Again like proper names, the indexicals do not display scope ambiguity so that the following two analyses are equivalent:

(24) a. *John was looking for you* $(F_4)$
   b. *John look for you* $(F_5)$

(25) a. *John was looking for you* $(F_{10-0})$
   b. *you John was looking for $x_0$* $(F_5)$

This will become obvious in the next section, where the translation of indexicals "I" and "you" is discussed.

1.2 The translation of "I" and "you" would also be like that of proper names. Both the pronouns and proper names are basic T-phrases, so they both are translated into the expressions of the same type in the language of intensional logic: $\langle\langle s, \langle s, e, t\rangle, t\rangle, t\rangle$, which is read as the set of properties of individual concepts.

(26) *John* $\Rightarrow [P[\langle i\rangle]]$
(27) *I* $\Rightarrow [P[\langle i\rangle]]$
(28) *you* $\Rightarrow [P[\langle y\rangle]]$

where $P$ is the variable $v_0$, $\langle s, \langle s, e, t\rangle, t\rangle$ which ranges over properties of individual concepts.

These translations would guarantee that sentences like (24) and (25) would be interpreted as synonymous, resulting in equivalent translations.

Our translation of the indexicals would also make the the translation of sentences like (29) parallel that of sentences like (30).
I am you.
John is Jack.
They are respectively translated into:

i = y.

j = j'.

Both of these translations are well-formed meaningful expressions of type \( \langle t \rangle \) in intensional logic, where \( i, y, j, \) and \( j' \) are particular distinct members of the set of constants of type \( \langle e \rangle \). Expression (29) would then be true under a certain interpretation if and only if both \( i \) and \( y \) refer to the same individual \( a_K \) under the same interpretation: that is,

\[
\text{Ext}_{t}^k(i = y) = 1 \text{ iff } \text{Ext}_{t}^k(i) = \text{Ext}_{t}^k(y).
\]

This means that, under a certain circumstance, I can be you just as both John and Jack can refer to the same individual. Although I do not know whether this is a fortunate situation for natural language, our extended PTQ does not rule out sentences like (29) as being a down-right contradiction or absurdity. Assuming, however, that it is a simple logical contradiction, try to interpret sentence (34):

[Archie Bunker says to B.B.]: I dreamt that I was you.

Then, we find our assumption being contradicted, since sentence (34) states that it was true at least in Archie’s dream that he was B.B. It seems again certain that a contradictory statement cannot be an argument of the predicate \( \text{dream} \), as attested by:

*I dreamt that I was dead and (that) I wasn’t dead at the same time.

This shows the plausibility of the 1st conjunct of sentence (36) in the extended PTQ.

I dreamt that I was B. Bardot and that I kissed me.

The extended PTQ, however, fails to interpret the 2nd conjunct of this sentence in any other ways than in an acrobatic sense or in a sense that one kisses(?) one’s own hand or other parts of his body. In other words, given a particular distinct constant \( j \) in \( \text{MB}_{\langle e \rangle} \), its extension with respect to a given interpretation, say \( \langle a, i, j \rangle \) cannot refer to two different individuals. Each possible denotation function of \( j \), \( F(j) \), defined in PTQ is a function that maps onto unique values, but not a relation that may give different values for a given argument. For example, (37) would be acceptable, while (38) would not be:

\[
\begin{align*}
\text{F}(j) : \{ & \langle i', j' \rangle \rightarrow a^9 \} \\
\text{F}(j) : \{ & \langle i', j' \rangle \not\rightarrow a^9 \}
\end{align*}
\]

So, it would not be possible in the PTQ to make any terms including “I” refer to two different individuals at the same time even in a dream world.

This has been pointed out by Stalnaker (1972) as a shortcoming of Montague’s pragmatics. In order to deal with cases of what he calls scope ambiguity, he suggests a scheme of separating the possible world coordinate from the coordinates of contextual features. These contextual coordinates first would disambiguate sentences into appropriate proposi-
tions, and then these propositions each map possible worlds onto truth values. Let us apply this scheme to our analysis of indexicals. Suppose that our interpretation model \( a \) is:

(39) \(<A, I, J, F'>\), where 

- \( A \) = the set of possible individuals \( \{a_0, a_1\} \),
- \( I \) = the set of possible worlds \( \{i_0, i_1\} \),
- \( J \) = the set of pairs of possible speakers and some other possible contextual features, \( \\{\langle c_0, c'_0\rangle, \langle c_0, c'_1\rangle\} \)

Now, instead of letting \( F(i) \) be a function from \( I \times J \) to \( A \), let it be:

(40) \( F(i): [J \to [I \to A]] \).

In particular,

(41) \[ \langle c_0, c'_0\rangle \to [i_0 \to a_0] \]

\[ \langle c_0, c'_1\rangle \to [i_1 \to a_1] \]

Then, by allowing \( F'(i) \) to be either \( F(i)\langle c_0, c'_0\rangle \) or \( F(i)\langle c_0, c'_1\rangle \), we would have:

(42) for any \( i' \) in \( I \),

\[ F'(i)(i') = \text{Ext}^i(i) = a_0 \]

or \[ F'(i)(i') = \text{Ext}^i(i) = a_1 \]

Then, the extension of (43a) can either be (43b) or (43c):

(43) a. \( \text{kiss}(i,i) \).

b. \( F'(\text{kiss})(i')(a_0, a_1) \).

c. \( F'(\text{kiss})(i')(a_0, a_0) \).

Hence, in this system, "I" can refer to different individuals in one world depending on a certain context and sentence (36) "I dreamt that I kissed me" is interpretable. For the present, however, I shall accept the PTQ as it is, and further pursue other problems involving the interpretation of indexicals "I" and "you".

1.3 The interpretation of indexical sentences can be carried out in the same manner as non-indexical sentences: the meaning of indexical sentences would be determined by the meaning of each of its parts including indexical expressions. Let me first illustrate how the following non-indexical sentences would be interpreted in the PTQ:

(44=16c) John smokes.

(45=16b) The dean smokes.

Let our interpretation model \( a \) be:

(46) \(<A, I, F'>\), where 

- \( A \) = the set of possible individuals \( \{a_0, a_1\} \),
- \( I \) = the set of possible worlds \( \{i_0, i_1\} \)

\[ F(j): [i_0 \to a_0] \]

\[ i_1 \to a_0 ] \]

Note that, by meaning postulate (1) in the PTQ, proper names are regarded as rigid designators invariant with respect to every \( i \) in \( I \), and thus that \( F(j) \) must be a constant function that maps every possible world into one designated individual. The following
function of \( j \) would then be regarded as logically impossible:

\[
(47)^* F(j) : \begin{cases} 
  i_0 \rightarrow a_0 \\
  i_1 \rightarrow a_1
\end{cases}
\]

We also have to define the denotation function of \( \text{smoke}' \). Let it be:

\[
(48) \quad F(\text{smoke}') : i_0 \rightarrow \begin{cases} 
  i_0 \rightarrow a_0 & \rightarrow 0 \\
  i_1 \rightarrow a_0 & \rightarrow 0 \\
  i_0 \rightarrow a_1 & \rightarrow 1 \\
  i_1 \rightarrow a_0 & \rightarrow 0 \\
  i_0 \rightarrow a_1 & \rightarrow 1 \\
  i_1 \rightarrow a_0 & \rightarrow 0
\end{cases}
\]

Since \( \text{smoke} \) is an IV-phrase, where \( IV = \langle t/e \rangle \), its translation \( \text{smoke}' \) is a meaningful expression of the type \( \langle\langle s, e \rangle, t \rangle \). Hence, the denotation function \( F(\text{smoke}') \) is:

\[
(2^{\langle A \rangle})^{1 \times J}
\]

as in (48).

Since the translation of (44) is:

\[
(49) \quad \text{smoke}'(\wedge i)
\]

we would have:

\[
(50) \quad \text{Ext}^i \left[ \text{smoke}'(\wedge i) \right] = \text{Ext}^i \left[ \text{smoke}' \right] (\text{Ext}^i[\wedge i]) = F(\text{smoke}') (\langle i \rangle) F(j)
\]

At \( i_0 \), \( \text{Ext}^i \left[ \text{smoke}'(\wedge i) \right] = 0 \),

At \( i_1 \), \( \text{Ext}^i \left[ \text{smoke}'(\wedge i) \right] = 1 \).

According to the interpretation (46, 48), sentence (44) is a false statement at world \( i_0 \), but a true statement at world \( i_1 \). Suppose that \( I \) is a set of utterance times such that \( i_0 \) is the present, and \( i_1 \) is the past, i.e., \( i_1 < i_0 \). Then, sentence (44) means that John does not smoke now, but smoked before, as is clearly shown by (51):

\[(51) \quad \text{a. [Mary says now:]} \quad \text{"John smokes."} \]

\[\quad \text{b. [Mary said before:]} \quad \text{"John smokes."}\]

We have just shown that sentences like (44) which do not contain any overt indexicals may have different truth-values according to the time of utterance.

In order to interpret sentence (45), we must define the function \( F \) which gives the meaning, or sense, of the expression "the dean". For simplicity’s sake, let \( F(\text{the dean}') \) be like \( F(j) \) except that it is allowed to designate different individuals according to any \( i \) in \( I \). Let the following be part of our interpretation model.
(52) \( F(\text{the dean'}) : \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \).

Then, we would have:

(53) \( \text{Ext}^t_{\text{smoke'}}(\text{the dean'}) = 0 \)

The extension of \( [\text{smoke'}(\text{the dean'})] \) at \( i_1 \) is also 0. The individual \( a_0 \), named John, is the dean now, and he does not smoke. But in the past, i.e. at the time of utterance \( i_1 \), the dean was someone else, and he didn't smoke.

Let us now consider indexical sentences:

(54) I smoke.

(55) You smoke.

Just as the truth-values of sentences (44) and (45) depended on the time of utterance, the interpretation of these indexical sentences depends on who their speakers and listeners are. Let \( J \) be the set of ordered triplets, where the first coordinate refers to the time of utterance, the second to the speaker, and the third to the listener:

(56) \( J = \{ (i_0, c_0, c_1), (i_0, c_1, c_0), (i_1, c_0, c_1), (i_1, c_1, c_0) \} \)

abbreviated as, \( \{ i_{001}, i_{010}, i_{101}, i_{110} \} \)

Note that I could have included in \( J \) the cases in which one speaks to himself, or one speaks to a group of individuals, but that I have excluded them merely for simplicity’s sake. Let this \( J \) be part of our interpretation model \( a \) (46, 48). Then, we would have:

(57) \( I \times J = \{ (i_0, j_{001}), (i_0, j_{010}), \ldots, (i_1, j_{101}), (i_1, j_{110}) \} \) (\( I \times J \) contains 8 pairs.)

The meaning functions of \( i \) and \( y \) may now be assigned:

(58) \( F(i) : \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \)

(59) \( F(y) : \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \)

where \( k, m, n \) range over \( \{0, 1\} \).

Note that \( F(i) \) is invariant with respect to the 1st and 3rd as well as the possible world coordinate, while \( F(y) \) is invariant with respect to the 1st and 2nd coordinate of \( J \) and to the \( I \) coordinate. \( F(j) \), \( F(\text{smoke'}) \), and \( F(\text{the dean'}) \) must also be revised to incorporate the contexts of utterance as part of their arguments. But since they are invariant with respect to these contextual coordinates, the previously defined functions may be used for the interpretation of sentences (54-55), replacing \( (i_0) \) and \( (i_1) \) by \( (i_{00}) \) and \( (i_{11}) \), respectively.

According to our extended model \( a' \), which is \( \langle A, I, J, F \rangle \), the interpretation of sentence (54) is as follows:

(60) \( \text{Ext}_{t,s}^t[\text{smoke'}('i')]=F(\text{smoke'})(i)F(i) \)

At \( (i_0, j_{000}) \), \( \text{Ext}_{t,s}^t[\text{smoke'}('i')] = 0 \)

At \( (i_0, j_{100}) \), \( \text{Ext}_{t,s}^t[\text{smoke'}('i')] = 1 \)

Let \( a_1 \) be an individual named Bill. Then, according to this analysis, sentence (54) would be either in the context (61a) or in (61b):
Interpretation of Indexical Sentences

(61) a. [John says]: I smoke.
     b. [Bill says]: I smoke.

Under these contexts of utterance, sentence (61a) is equivalent to sentence (44) "John smokes", whereas (61b) is not. We can also easily recognize that sentences (44) and (45) would entail each other when they are augmented with the contexts, \( \langle i_k, j_{m0} \rangle \) and \( \langle i_k, j_{m10} \rangle \), respectively. To spell out:

(62) a. [John says to Bill]: I smoke.
     b. [Bill says to John]: You smoke.

Note that for indexical sentences entailment would be a relation between sentence tokens, say \( \text{\text{smoke}('t_i', \langle i_k, j_{m0} \rangle}) \) and \( \text{\text{smoke}('t_y', \langle i_k, j_{m10} \rangle}) \). And the extended PTQ seems adequate for dealing with such entailment.

1.4 In defining the denotation function of \( i \) or \( y \), one can easily recognize that not all possible functions of \( i \) or \( y \) would be reasonable candidates for the interpretation of indexical expressions in English. Let us consider a very simple model \( \langle A, I, J, F \rangle \), where \( A \) has as its members two possible individuals \( a_0 \) and \( a_1 \), I two possible worlds \( i_0 \) and \( i_1 \), and \( J \) two possible speakers \( c_0 \) and \( c_1 \). Then, there are 16 possible functions of \( i \), \([F(i): I \times J \rightarrow A]\), since the Cartesian product \( I \times J \) contains 4 members and \( A \), 2 members. Some of the possible functions are, however, unreasonable for the interpretation of natural language. The following is such an example:

(63) \[ F(i) : \]
\[ \begin{cases} \langle i_0, c_0 \rangle \rightarrow a_0 \\
\langle i_0, c_1 \rangle \rightarrow a_1 \\
\langle i_1, c_0 \rangle \rightarrow a_1 \\
\langle i_1, c_1 \rangle \rightarrow a_0 \end{cases}, \] abbreviated as \( f_{0110} \).

This function is unacceptable because what it says is that individuals designated as speakers at world \( i_0 \) become different individuals at another world \( i_1 \). But the personal pronoun "I" is used in English to refer to the speaker independently of other contextual coordinates. In other words, the value of \( F(i) \) only depends on the speaker coordinate. Suppose the following sentence is uttered by John at a party, and again by John at another party.

(64) I dreamt that I was dead.

In either case, "I" must refer to John. Out of the 16 possible functions, only the following four are reasonable candidates:

(65) \[ F(i) : f_{0000}, f_{1010}, f_{0101}, f_{1111}. \]

In the case of \( f_{1010} \), the speaker \( c_0 \) at any \( i' \) refers to the individual \( a_1 \), and the speaker \( c_1 \) to the individual \( a_0 \). In the case of \( f_{0000} \) and \( f_{1111} \), however, two distinct speakers refer to the same individual \( a_0 \) and \( a_1 \), respectively at any \( i' \). This situation seems to me absurd. If so, the number of acceptable \( F(i) \) would be reduced to two: \( f_{0101} \) and \( f_{1010} \). This restriction would be stated in the PTQ by characterizing \( F(i) \) or \( F(y) \) as a one-one mapping from distinct speakers to distinct individuals.

This characterization of \( F(i) \) is not sufficient. It allows any possible object in \( A \) to be
a speaker. Suppose a₀ is a brick in the actual world i₀. We now have to ask whether it might be all right to allow a₀ to be a speaker in that world, say in our unimaginative world. Suppose a₁ is a dead person. Shall we make him a speaker? Its answer would depend on how one interprets sentences like:

(66) I don't exist.
(67) I am a brick.

Let a brick be a speaker, but let's agree that (66) is absurd. For this, one might suggest that we should have:

(68) I ⇒ P[E('i') & P('i')], where E is the predicate exist.

This then would make (66) a contradictory statement:

(69) [E('i') & ¬E('i')]

But this does not work in the PTQ, since (67) also allows (69), which is not a logical contradiction, but an unnecessary repetition:

(70) ¬[E(i) & E(i)]

Hence, translating "I" as in (68) would not guarantee the actual existence of speakers.

There is a simpler way of handling sentences like (66). Without making them logical contradictions, we show them to be semantically unacceptable, neither being true nor false. We now characterize F(i) as a partial function that would not be defined unless F(i)(<iₚ, cₚ>) is an individual actually existing at iₚ, i.e., a member of [F(E)(<iₚ>)](<iₚ>). Suppose that a₀, a₁ actually exist at i₀, while only a₁ exists at i₁. Then, F(i) would be:

(71) F(i): [<i₀, c₀> → a₀
         <i₁, c₁> → a₁
         undefined]

Or,

F(i₀): [[i₁, c₁] → a₁
        undefined]

According to this analysis, sentences like (66) cannot be understood as sincere statements. If so, their truth-values cannot be determined because the denotation function of i corresponding to the pronoun "I" is undefined in such a case. As a further illustration, consider:

(72) a. [John says now]: I smoke.
    b. [John says before he is born]: I smoke.

The context (b) violates our common presumption that one cannot speak before he is born, and thus the reference of i would be undefined. So would the truth-value of the sentence be undefined, and we have a semantically unacceptable sentence.

2. The problems dealing with the interpretation of indexical expressions are still numerous. The extended PTQ cannot properly interpret sentences like:

(73) I am dead. Do not look for me.

Before one kills himself, he can conceivably write such a note. At present, I have no formal mechanism explaining it except for an informal statement that the pronoun "I"
here is used like a proper name to designate rigidly the person who wrote the note. Along with the problem noted in section 1.2 this problem needs further study of pragmatics. However, I have shown that the extended PTQ is theoretically rich enough to deal with seemingly trivial but crucial problems arising from an attempt to fully interpret indexical sentences abundant in natural language.

REFERENCES


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