The Internal Structure of a Three-Dimensional City

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The internal structure of a closed monocentric three-dimensional city has been analyzed. It has been found that the rent function is quasi-convex and building height function is convex when the value of height is always negative. If the value of height is dependent upon the height amenity and disamenity, the direction of change in rent with respect to height will depend on the relative height. The population density function derived from the three-dimensional city model has been estimated. It has been found that the explicability of the three-dimensional city model is far greater than the standard two-dimensional one.

I. Introduction

Almost all existing models of urban internal structures are two-dimensional except the pioneering work by Wright(1971). Both the monocentric and multicentric city models do not explicitly allow for the existence of the vertical space(Alonso 1965; Mills 1972, 1984; Muth 1969; Fujita and Ogawa 1982; Papageorgiou and Casetti 1971).

But, cities are basically three-dimensional. There are high-rise apartments and office buildings and people are usually moving along both horizontal and vertical directions.

It is true that building heights are far shorter than horizontal distances of cities. But, for those living at the same distance from the Central Business District (CBD), the vertical distance can play significant roles in choosing the height of residence. Therefore, the ignorance of vertical space can yield the distorted understanding of the internal structures of cities. That is because the consideration of the vertical space is not a mere inclusion of one more factor with sustaining the dimension of the model but the extension of it. Therefore, new insights into the internal structures of real cities can be obtained through the analyses of three-dimensional cities.

In addition to this theoretical necessity, the analysis of three-dimensional cities is indispensable for obtaining empirically more relevant models. For example, the urban population density function derived from the two-dimensional model reveals that the population density of a certain location is dependent upon the horizontal distance between the CBD and that location. It is generally observed that the explicability of this population density function drastically decreases as the size of a city increases or the internal structure of a city becomes more complex.

This problem can be partially solved by the explicit consideration of the vertical space. Explanatory variables of three-dimensional population density function can be considered to be the horizontal distance and the vertical one, i.e., the building height. It is intuitively clear that the explicability of three-dimensional population density function may be much greater than the two-dimensional one.

Based upon these theoretical and empirical necessities, the closed monocentric three-dimensional model of residential land use will be analyzed. For this, the existence of vertical space will be considered in two different ways. The one is the case where the value of the vertical space is always negative due to the vertical moving cost. This will be the simple extension of standard closed monocentric two-dimensional model where the value of horizontal distance is always negative. The other is the case where the value of the vertical space is evaluated by considering both the height amenity and disamenity.

II. The Internal Structure of a Simple Three-Dimensional City

The internal structure of a closed monocentric three-dimensional city will be analyzed. This will be the simple extension of standard two-dimensional model because the value of height, i.e., the distance from the earth, is assumed to be always negative like the value of the horizontal distance from the CBD in a standard two-dimensional model.

Let z be the horizontal distance from the CBD and h be the height. Any location can be expressed by the two-tuple, (z, h). An agent living at any (z, h) consumes the composite commodity $x(z, h)$

and the space $s(z, h)$. p is the exogenously given price of composite commodity and $R(z, h)$ is the rent per unit space at (z, h). Also, y is the exogenously given income of an agent and r is the exogenously given opportunity cost of urban land use.

For each z, $H(z)$ is the maximum of h. Thus, $H(z)$ being determined endogenously will be called as the building height at z. $C(H)$ is defined as the per space construction cost necessary for constructing a building with height H.

The boundary of a city, b, is the boundary between the residential area of a city and the agricultural open space where there are assumed to be no buildings.

Assumption 1

At city boundary, the building height is zero, i.e., $H(b) = 0$

As for the construction industry, no scale economy is assumed (Mills and Hamilton 1984). Also, since it is necessary to buy lands for constructing bulidings, it is assumed that the per space construction cost with hegith zero is r.

Assumption 2

$$C'(H) > 0, \; C''(H) < 0 \text{ and } C(0) = r.$$

The possibility of constructing a building with a certain height will be dependent upon the willingness to pay per unit space and the per unit space construction cost. The construction industry faces with two types of decision making. The one is to determine whether or not to establish a building and the other is to decide the height of a building.

The construction industry is assumed to decide to establish a building when the willingness to pay per unit space of agents living on earth due to the non-existence of buildings is greater than the per unit space construction cost. Also, it is assumed that the height of a building is determined so that the willingness to pay per unit space of agents living at the top of a building is the same as the per unit space construction cost.

Assumption 3

$$\text{for all } z < b, \; R(z,0) > C(H(z)) \text{ and } R(z, H(z)) = C(H(z))^{1}$$

[1]As will be clear after the proof of Proposition 2, this assumption actually implies that

t is the exogenously given marginal horizontal moving cost, i.e., the marginal moving cost necessary to move from the residential location on earth to the CBD. T is the exogenously given marginal vertical moving cost, i.e., the marginal moving cost necessary to move from a certain height to the earth. Then, the after moving cost income of an agent living at (z,h) will be $y - tz - Th$.[2]

Let u be the level of utility and $V(\cdot)$ be the indirect utility function. Then, the spatial equilibrium condition of a three-dimensional city can be expressed as follows.

Condition 1

$$V(P, R(z,h), y - tz - Th) = u. \text{ [3]}$$

Under Assumptions 1 through 3 and Condition 1, a three-dimensional city can be discribed by the following simultaneous equation system.

$$V(P, R(z, h), y - tz - Th) = u \tag{1}$$

$$-V_R / V_y = s(z,h) \tag{2}$$

$$-V_P / V_y = x(z,h) \tag{3}$$

$$R(b,0) = r \tag{4}$$

$$R(z,H(z)) = C(H(z)) \tag{5}$$

$$\int_0^b \int_0^{H(z)} 2\pi z / s(z,h) dh dz = N \tag{6}$$

where, N is the exogenously given population size. The endogenous variables are, $R(z,h)$, $s(z,h)$, $x(z,h)$, $H(z)$, b, and u. Among these endogenous variables, we are interested in variables which are new compared with the standard two-dimensional model; $R(z,h)$ and $H(z)$.

In the standard two-dimensional city model, rent function is found to be convex. But, in the three-dimensional city, it can be shown that the rent function, $R(z,h)$, is quasi-convex. Also, if z or h in-

the profit maximizing construction industry establishes a building with a certain height only when the willingness to pay per unit space at any height is greater than the per unit space construction cost.

[2]The general form of after moving cost income will be $y - tz + T(h)$, where $T'(h) \gtrless 0$. But in this section, the case of $T'(h) < 0$ will be considered. The general case will be considered in the next section.

[3]This spatial equilibrium condition implies that an individual can obtain at most u wherever he lives. Thus, this spatial equilibrium condition can be rewritten as $V(P, R(z,h), y - tz - Th) < u$.

creases, the after moving cost income will decrease. Therefore, $R(z,h)$ will also decrease.

Proposition 1

Under Condition 1, i) $R(z,h)$ is quasi-convex and ii) $R(z,h)$ is decreasing in z and h.

Proof:

i) Let $A = \{(z,h)\ R(z,h) \leq k\}$. Then, for all $(z, h) \in A$,

$$V(P, R(z,h), y - tz - Th) = u \geq V(P, k, y - tz - Th)$$

Define $\bar{y} \ni V(P,k,\bar{y}) = u$. Then, for all $(z,h) \in A$, $\bar{y} \geq y - tz - Th$.

For all (z',h'), $(z'',h'') \in A$ and for all $\lambda \ni 0 < \lambda < 1$,

define $(z,h) = \lambda (z',h') + (1 - \lambda)(z'',h'')$.

Since $y - tz' - Th' \leq \bar{y}$ and $y - tz'' - Th'' \leq \bar{y}$,

$$\begin{aligned} &\lambda (y - tz' - Th') + (1 - \lambda)(y - tz'' - Th'') \\ &= y - t(\lambda z' + (1 - \lambda)z'') - T(\lambda h' + (1 - \lambda)h'') \\ &= y - tz - Th \leq \bar{y}. \end{aligned}$$

Thus, $V(P, k, y - tz - Th) \leq u$. Therefore, $R(z,h) \leq k$ and $(z,h) \in A$.

ii) Suppose $z' > z$. Then, for all $\bar{h}$, $y - tz' - T\bar{h} < y - tz - T\bar{h}$. Also, Condition 1 implies that $V(P, R(z', \bar{h}), y - tz' - T\bar{h}) = u = V(P, R(z, \bar{h}), y - tz - T\bar{h})$. Therefore, $R(z', \bar{h}) < R(z, \bar{h})$.
The same logic can be applied in case of h.

Q. E. D.

Remark 1: If $R(z,h)$ is assumed to be continuously differentiable as usual, it can be shown that $\partial R(z,h) / \partial z = R_z = -t / s(z,h) < 0$ and $\partial R(z,h) / \partial h = R_h = -T / s(z,h) < 0$.

For clarifyng the characteristics of the building height function, $H(z)$, the following two lemmas will be necessary.

Lemma 1

Under Condition 1, $\partial s(z,h) / \partial z = s_z > 0$ and $\partial s(z,h) / \partial h = s_h > 0$.

Proof: From Condition 1, $S(P, R(z,h), y - tz - Th, u) = s(z,h)$.

Then, $s_z = \bar{S}_R\ R_z - tS_y$

$$= [\ \bar{S}_R - s(z,h)S_y\ ]R_z - tS_y$$

where, $\bar{S}_R$ implies the substitution effect and S_R and S_y have obvious meanings.

Since, $R_z = -t\ /\ s(z,\ h)$, s_z can be expressed as,

$$s_z = \bar{S}_R + s(z,h)S_y(t\ /\ s(z,h)) - tS_y$$

$$= \bar{S}_R\ R_z > 0.$$

By the same reasoning, it can be derived that

$$s_h = \bar{S}_R\ R_h > 0.$$

Q. E. D.

Lemma 2

Under Condition 1, $\partial^2 R(z,h)/\partial z^2 = R_{zz} > 0$, $\partial^2 R(z,h)/\partial h^2 = R_{hh} > 0$ and $\partial^2 R(z,h)/\partial z \partial h = R_{zh} > 0$.

Proof: Since $R_z = -t\ /\ s(z,h)$, it can be derived that

$$R_{zz} = s_z(t\ /\ s^2) > 0.$$

By the same reasoning,

$$R_{hh} = s_h(t\ /\ s^2) > 0.$$

From $R_z = -t\ /\ s(z,h)$, it can be derived that

$$R_{zh} = (t\ /\ s^2)s_h > 0.$$

Q. E. D.

Remark 2: Since $R_{zh} = R_{hz}$, it can be noticed that $ts_h = Ts_z$.

As z increases, after horizontal moving cost income will increase. This implies that the bid rent for height will decrease. Therefore, this will cause the decrease in building height.

Proposition 2

Under Assumptions 2 and 3 and Condition 1, the building height function is decreasing and convex with respect to the distance from CBD, i.e., $dH(z)/dz = H' < 0$ and $dH'/dz = H'' > 0$.

Proof:

i) $R(z,\ H(z)) = C(H(z))$ from Assumption 3.

Thus, $R_z + R'_h H' = C'H'$.

Therefore, $H' = R_z / (C' - R_h) > 0$.

ii) From i), H'' can be obtained as follows

$$H'' = [\,1 / (C' - R_h)^2\,]\,[\,(R_{zz} + R_{zh}\ H')(C' - R_h) - R_z(C''H' - (R_{hz} + R_{hh}H'))]$$
$$= [\,1 / (C' - R_h)^2\,]\,[\,(R_{zz} + R_{zh}\ H')(C' - R_h) + R_z(R_{hz} + R_{hh}\ H') - R_z\ C''H'\,]$$

From Lemma 2 and Remark 2,

$$R_{zz} + R_{zh}\ H' = (s_z / s^2)(t + TH')$$
$$R_{hz} + R_{hh}\ H' = (s_h / s^2)(t + TH').$$

By substituting these, H'' can be rewritten as,

$$H'' = [\,1 / (C' - R_h)^2\,]\,[\,(1 / s^2)(t + TH')(C' - R_h) \cdot (s_z + H's_h) - R_z\ C''H'\,].$$

From Remark 1, $t + TH' = t[\,C's / (C's + T)\,] > 0$.

Also, from Lemma 1, Remark 1 and (i),

$$s_z + H's_h = \bar{S}_R\ R_z + \bar{S}_R\ R_h\ H'$$
$$= \bar{S}_R(R_z + R_h\ H') = \bar{S}_R\ R_z(C' / (C' - R_h)) > 0.$$

Therefore, $H'' > 0$.

Q. E. D.

In summary, if the money value of height is alwalys negative, the internal structure of a closed monocentric three-dimensional city can be described as follows. The rent function is quasi-convex and building height decreases as the distance from CBD increases.

III. The Extension of the Model

The money value of height will be generalized in this section. In section II, the money value of height is assumed to be alwalys negative. But, in this section, it is assumed that the money value of height is the sum of the money value of height amenity and that of height disamenity.

An example of height amenity may be the view of sight. Therefore, it will be assumed that the height amenity is dependent upon the height itself. The height disamenity is assumed to be dependent upon the relative height, $h / H(z)$.

Assumption 4

The money value of height, $T(h, H(z))$, is the sum of the money value of hieght amenity being dependent upon the height, $A(h)$, and the money value of height disamenity being dependent of the relative height, $D(h / H(z))$, i.e., $T(h, H(z)) = A(h) - D(h / H(z))$, for all $H(z) > 0$.

Under Assumption 4, the spatial equilibrium condition can be re-written as follows.

Condition 2

$$V(P, R(z,h), y - tz - T(h,H(Z))) = u, \text{ for all } z.$$

It is assumed that the height amenity increases in decreasing rate and the height disamenity increases in increasing rate as the height increases. The height amenity and disamenity are obviously related to the height which is greater than zero. Thus, it is assumed that the height amenity and disamenity occur only when an agent is above the earth, i.e., $h > 0$.

Assumption 5

$$dA(h) / dh = A' > 0 \text{ and } dA'(h) / dh = A'' < 0 \quad (7)$$

$$dD(h) / dh = D' > 0 \text{ and } dD'(h) / dh = D'' > 0 \quad (8)$$

$$A(0) = D(0) = 0 \quad (9)$$

$$A'(0) - D'(0) > 0, \text{ for all } z. \quad (10)$$

Notice that, if equation(10) is not satisfied, i.e., $A'(0) - D'(0) < 0$, then equation (7) through equation (9) will imply that the money value of height is alwalys negative. Therefore, the internal structure of a city will be exactly the same as is analyzed in Section II.

There will be no buildings at the boundary of a city. Therefore, there will be no height amenity and disamenity.

Assumption 6

The money value of height is 0 if $H(z) = 0$.[4]

[4]Since $h < H$, h equals to 0 if $H(z) = 0$. Also, since $h / H(z)$ is not defined when both h and $H(z)$ are 0, this assumption for the boundary of a city is necessary.

From Assumptions 4 and 5, it can be shown that $T(0, H(z)) = 0$ and $\partial T(h,H(z)) / \partial H(z) = T_H > 0$.

Define $F(z)$ such that $\max_h T(h,H(z)) = T(F(z),H(z))$.
Also, define $G(z)$ such that $G(z) > 0$ and $T(G(z), H(z)) = 0$.
The relationship between $F(z)$, $G(z)$ and $H(z)$ can be explained by the following proposition.

Proposition 3

Under Assumptions 2 and 3, Assumptions 4 through 6 and Condition 2, $\exists$ $F(z)$, $G(z)$ and $H(z)$ such that $F(z) \leq G(z) \leq H(z)$, for all $z < b$.

Proof:
i) By definition of $F(z)$, $F(z)$ satisfies that

$$A'(F(z)) - D'(F(z) / H(z))(1 / H(z)) = 0$$

Bu Assumption 5, $\exists$ unique $F(z) > 0$.
Also, equcation (10) implies that $F(z) \leq G(z)$.

ii) By definition of $F(z)$, $\partial T(\cdot) / \partial h = T_h > 0$ if $h \leq F(z)$ and $T_h < 0$ if $h > F(z)$. Since $A(G(z)) - D(F(z) / H(z)) = 0$, the following relation can be obtained under Condition 2.

$$\text{for all } z,\ R(z,0) = R(z,G(z)) > C(H(z)) = R(z,H(z))$$

Since $G(z) > F(z)$, $R_h(z,G(z)) < 0$.

Therefore $G(z) \leq H(z)$.

Q. E. D.

Remark 3: At city boundary, $H(b) = 0$. Therefore, $F(b) = G(b) = H(b) = 0$ by definitions of $F(z)$, $G(z)$ and $H(z)$ and Assumptions 5 and 6.

In analyzing the internal structure of a three-dimensional city, it is natural to consider a city with the finite building height. The sufficient condition for the finite building height will be that the marginal money value of per unit space height amenity is less than the marginal per unit space construction cost.

Condition 3

$$C'(H) > A'(H) / s.$$

If this condition is not satisfied, $[\, d(A(H) - D(h / H)) / dH \,] / s = A'(H) / s > C'(H)$. In other words, agents' per unit space money gain from increasing H, which determines the willingness to pay, will be greater than the unit space marginal cost of construction. Therefore, the building height will be infinite.

Propostion 4

Under Conditions 2 and 3, $dF(z) / dz = F' < 0$, $dG(z) / dz = G' < 0$ and $dH(z) / dz = H' < 0$, for all $z < b$.

Proof:

i) At $(z, H(z))$, the following conditions must be satisfied

$$V(P, R(z,H(z)), y - tz + T(H(z), H(z))) = u \tag{11}$$

$$R(z, H(z)) = C(H(z)). \tag{12}$$

From, the differentiation of (11) and (12) with respect to z, it can be derived that

$$H' = tV_y / [V_R C' - V_y A']$$

$$= tV_y / V_R(C' - (A' / s)).$$

Therefore, $H' < 0$ by Condition 3.

ii) By definition of $G(z)$, $T(G(z), H(z)) = 0$. Thus, $T_h G' + T_H H' = 0$. Since $H' < 0$ from i), $T_h < 0$ from Proposition 3 and $T_H > 0$, $G' < 0$.

iii) By the definition of $F(z)$, it must be satisfied that

$$A'(F(z)) - D'(F(z) / H(z))(1 / H(z)) = 0. \tag{13}$$

From the differentiation of (13) with respect to z, F' can be derived as

$$F' = -(D'HH' + D''FH') / (A''H^3 - D''H).$$

From Assumption 5 and i), $F' < 0$.

Q. E. D.

The sign of R_h is dependent upon whether h is greater than $F(z)$. In other words, $R_h(z,h) \geq 0$ if $h \leq F(z)$ and $R_h(z,h) < 0$ if $h > F(z)$. But the sign of R_z is uniquely determined, i.e., $R_z(z,h) = [tV_y - V_y T_H H'] / V_R < 0$.

These results also imply that rent functions for different horizontal distances do not intercept each other, i.e., if $z' \neq z''$, there does not exist h such that $R(z',h) = R(z'',h)$.

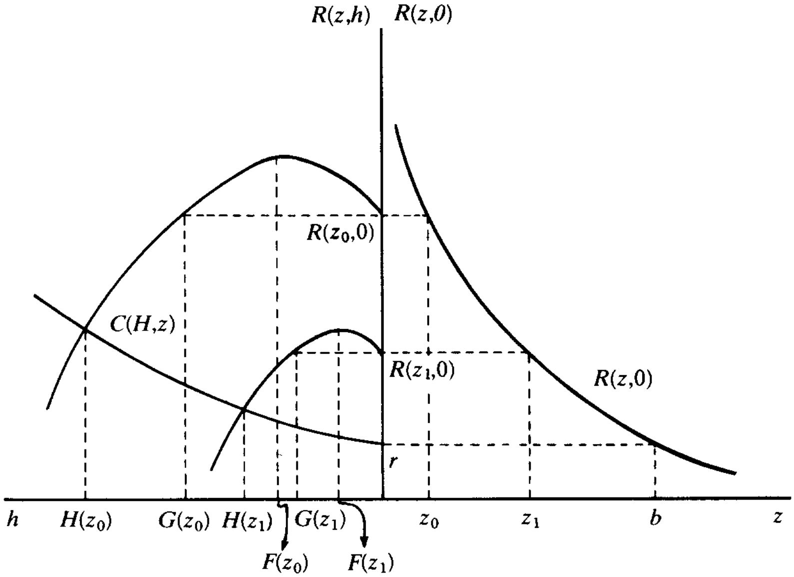

FIGURE 1
THE INTERNAL STRUCTURE OF A THREE-DIMENSIONAL CITY

The internal structure of a three-dimensional city analyzed in this section can be geometrically explained by the Figure 1.

IV. The Empirical Relevance of the Model

The empirical relevance of three dimensional city model developed in Sections II and III will be investigated in this section. For this, the population density function of the three-dimensional city model will be estimated and compared with the one based upon the standard two-dimensional city model.

The number of population at z is $\int_0^H 2\pi z / s(z,h)\ dh = N(z,H(z))$. Therefore, the population density at z, $PD(z)$, will be $PD(z) = N(z,H(z)) / 2\pi z$. That is, the population density at a certain location is dependent upon the horizontal distance from the CBD and the building height. Notice that the population density is only dependent upon the horizontal distance from the CBD in two-dimensional model.

The most widely estimated population density function of two-dimensional city model has the form of semi-log linear; $\log PD(z) = \alpha_0 + \alpha_1 z + \varepsilon$. Here, ε is the random error. In accord-

TABLE 1
THE COMPARISION OF THREE-DIMENSIONAL AND TWO-DIMENSIONAL POPULATION DENSITY FUNCTION

	Constant	DIST	HGT	SHGT	BD	R^2
(1)	10.9970	−0.1383				0.2225
	(57.55)	(−7.04)				
(2)	10.5269	−0.0558			−1.2068	0.3505
	(54.58)	(−2.44)			(−5.82)	
(3)	9.1282	−0.1067	0.2911			0.4862
	(36.13)	(−6.51)	(9.39)			
(4)	8.7822	−0.0966	0.4902	−0.0247		0.5108
	(32.05)	(−5.89)	(6.59)	(−2.93)		
(5)	9.1348	−0.0663	0.2503		−0.6551	0.5188
	(37.25)	(−3.35)	(7.73)		(−3.40)	
(6)	8.8061	−0.0583	0.4408	−0.0241	−0.6304	0.5408
	(33.06)	(−2.97)	(5.97)	(−2.86)	(−3.34)	

ance with this, semi-log linear $PD(z)$ of three-dimensional city model will be considered; $log\ PD(z) = \beta_0 + \beta_1 z + \beta_2 H(z) + \varepsilon$.[5] Analyses of Sections II and III imply that β_1 is negative and β_2 is positive. Also, it is expected that α_1 is negative.

Regressions have been run as for the city of Seoul, which is the prime city of Korea. Administrative areas and population sizes of precincts, the smallest administrative unit, have been obtained from the "1980 Municipal Yearbook of Seoul." Because some precincts include Han-River, areas of Han-River are extracted from the administrative area for those precincts. Distances between CBD and precincts have been measured by 0.5km unit from the map.

The data for building heights cannot be obtained directly. But, the data for the apartments units for each precinct can be obtained from the "1980 Census of Population and Housing." Therefore, as a proxy variable for the building height, log of apartment units per one square kilometer has been used.

Also, most precincts containing the boundary of a city were on developing in 1980. Areas of those precincts are about five to seven times greater than the average area of other precints and the migra-

[5]The building height, $H(z)$, is an endogenous variable in the theoretical model. Therefore, in estimating the population density function of a three-dimensional city, the equation $H(z)$ will be also necessary. But, because the data for $H(z)$ is not available, the proxy varlable for $H(z)$ will be used. Thus, the equation for $H(z)$ is enforced not to be considered.

tion to those precincts has not been finished at that time. Therefore, boundary dummy variable is used. Because of reasons just mentioned, the coefficient of boundary dummy variable is expected to have the negative sign.

Results of regressions are summarized by Table 1. In Table 1, DIST is the distance from the CBD, HGT is the building height, SHGT is the squared value of HGT and BD is the boundary dummy variable. Also, numbers in parentheses are t-values. Regression (1) and (2) in Table 1 are based upon the two-dimensional city model and (3) through (6) are based upon the three-dimensional one.

From regression results of Table 1, it may be concluded that the three-dimensional city model has far greater explicability as for the urban population density than the two-dimensional one.

V. Conclusion

Three-demensional city models have been analyzed with explicit considerations of the third dimension, i.e., the building height. Major findings of this paper can be summarized as follows.

The internal structure of a closed monocentric three-dimensional city with the negative value of height can be explained as the simple extension of standard two-dimensional one. The rent function is quasi-convex and the building height funtion is convex such that the building height decreases as the distance from the CBD increases and the building height at the boundary of a city is zero.

If height amenities and disamenities are considered together, the shape of rent function can be summarized as follows. For any given height, the level of bid rent curves for different horizontal distances do not intercept each other. Levels of bid rents with respect to heights are dependent upon the relative heights. In other words, as height increases the bid rent increases at first and then decreases.

Even though the shape of rent function is complex, the change in building height is the same as before. The building height decreases as the horizontal distance increases and the building height at the boundary of a city is zero.

Regression results of Section IV may imply that the explicability of three-dimensional one is greater than the standard two-dimensional one. Therefore, it may be concluded that the three-dimensional city model is theoretically and empirically more viable than the two-dimensional one.

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