A Brief Introduction to Neo-Austrian Capital Theory

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A concise introduction to Neo-Austrian approach to capital theory with particular emphasis on the accessibility of the representation is provided. A basic T-period model for a centrally planned economy is introduced. Then we show how certain conditions on technology (superiority, roundaboutness) and preferences (impatience) serve to guarantee a positive rate of interest. Finally, it is briefly indicated how the model has been reformulated for market economies and how it has been generalized in order to address questions of structural change, particularly those related to environmental pollution and the depletion of natural resources.

I. Introduction

Austrian capital theory goes back to Böhm-Bawerk (1889).[1] Until about 1930, it was just as widespread as its neoclassical counterpart, but then it fell into oblivion. In the 1970s, however, when it became increasingly clear that the neoclassical approach is of limited use for the treatment of questions of technical progress and the accompanying process of structural change, Austrian capital theory faced a renaissance.

The ideas of Böhm-Bawerk were taken up by three different "Neo-Austrian" schools, which are united by the emphasis they lay on the time structure of the production process.

One such approach was initiated by Hicks (1973) who modelled

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[1] A modern account of the development of Austrian capital theory is given by Negishi (1989, ch. 8).

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the production process as a continous flow of inputs and outputs at different dates,[2] whereas the second one, initiated by Weizsäcker (1971), centers around the Böhm-Bawerkian concept of the production-period.[3] For a detailed survey of these two approaches, we refer the reader to Faber (1986).[4]

In the present paper, we want to concentrate on the third approach that goes back to Bernholz (1971) and was developed further by Bernholz and Faber (1973), Bernholz, Faber and Reiß (1978) and Jaksch (1975). A comprehensive treatment of the issues raised in these papers can be found in Faber (1979).

A common feature of these papers is that they involve a considerable amount of formalism and apply unusual concepts. As a result, Neo-Austrian capital theory is not easily accessible. The purpose of this paper is to change this state of affairs by providing a quick introduction to the theory.

The paper is organized as follows. In Section II, the technology of the T-period model of Bernholz and Faber (1973) is introduced. In particular, we touch upon the specifically Austrian concepts of roundaboutness and superiority and discuss how the introduction of new technologies can be handled in the model. In Section III, the demand side of the model is discussed, with particular emphasis on the question of time-preference. In Section IV, the conditions for the positivity of the rate of interest are given. Finally, in Section V, we briefly mention generalizations and applications of Neo-Austrian capital theory.

II. The Technology of the T-period model.

A. Fundamental Concepts

Consider an economy with the following commodities: a non-producible factor (labor), a producible factor (capital), and a consumption good.

The technology consists of the following linear production processes:

[2]See also Burmeister (1974), Belloc (1981), Kim (1981).

[3]See also Orosel (1979), Reetz (1984).

[4]One should also mention the Austro-American school (see Kirzner 1973) in this context. However, it differs from the other approaches in many respects. The relation between Austro-Americans and Neo-Austrians was investigated by Pellengahr (1986).

R_1: l_1 units of labor$\rightarrow$1 unit of the consumption good
R_2: l_2 units of labor $\oplus$[5] k_2 units of capital$\rightarrow$1 unit of the consumption good $\oplus$ $(1-c)$ k_2 units of capital.
R_3: l_3 units of labor$\rightarrow$1 unit of capital.

In this introductory model, we assume that the production of the consumption good is "timeless" in the sense that the output of processes R_1 and R_2 is available for consumption in the period when the inputs are employed, while the production of the capital good takes one period of time. Furthermore, we assume that capital deteriorates at a constant rate c $(0 \leq c \leq 1)$. The consumption good can be used only in the period when it is produced, i.e. it is non-storable.

We define a *production technique* to be the minimal set of production processes with which the consumption good can be produced. A production technique therefore contains the process in which the consumption good itself is produced as well as all the processes that are necessary to produce intermediate goods.

Obivously, in our simple model there are two techniques. The first technique contains only the process R_1, because R_1 allows the direct production of the consumption good from the non-producible input labor; intermediate goods are not necessary:

$$T_1 = \{R_1\} .$$

Technique T_2 consists of processes R_2 and R_3. In R_3, the capital good, which is necessary for the production of the consumption good with process R_2, is produced. Hence,

$$T_2 = \{R_2, R_3\} .$$

Both techniques generate the consumption good from the non-producible input; the difference is that T_1 allows for immediate consumption, while for T_2 one period of time is needed, because the capital good has to be produced first.

Some additional notation is useful. Let $t = 1,\cdots, T$ be the periods considered, while $j = 1, 2, 3$ are indices for the three processes. Then we use the following abbreviations:

$x_j(t)$: the intensity of process R_j in period t, i.e. the output that is available through the application of the process in

[5]The notation $\oplus$ is used to indicate that l_2 units of labor are combined with k_2 units of capital.

period t (for R_1, R_2) and $t+1$ (for R_3), respectively.

$L_j(t)$: the amount of labor that is used in process R_j in period t.

$K_2(t)$: the amount of capital that is used in process R_2 in period t.

$L = L(\mathrm{t})$: the amount of available labor that is given exogenously in period t (and is assumed to be constant for simplicity).

$K(t)$: the amount of capital that is available in period t.

$Q(t)$: the amount of the consumption good that is consumed in period t.

The technology can now be summarized as follows:

$$x_1(t) = L_1(t)/l_1 \tag{1}$$

$$x_2(t) = \min[L_2(t)/l_2, K_2(t)/k_2] \tag{2}$$

$$x_3(t) = L_3(t)/l_3 \tag{3}$$

The evolution of the capital stock $K(t)$ is given by $K(t) = K(t-1) + x_3(t-1) - cK_2(t-1)$; the repeated application of this equation gives:

$$K(t) = K(1) + \sum_{s=1}^{t-1} x_3(s) - c\sum_{s=1}^{t-1} K_2(s). \tag{4}$$

We have the following restrictions for consumption $Q(t)$.

1) Consumption in period t is bounded by the production of the consumption good in that period (because of the non-storability assumption), i.e.

$$H_{tQ}: x_1(t) + x_2(t) - Q(t) \geq 0. \tag{5}$$

2) Labor input is bounded by L in each period $t \in \{1,\cdots, T\}$, i.e.

$$H_{tL}: L - l_1x_1(t) - l_2x_2(t) - l_3x_3(t) \geq 0. \tag{6}$$

3) Capital input in period t is bounded by the available capital stock, i.e.

$$K_2(t) \leq K(t).$$

The last restriction can be reformulated by assuming that $L_2(t)/l_2 \geq K_2(t)/k_2$ during the periods under consideration. Then (2) gives

$$x_2(t) = K_2(t)/k_2 \leftrightarrow K_2(t) = k_2x_2(t)$$

and hence, by (4),

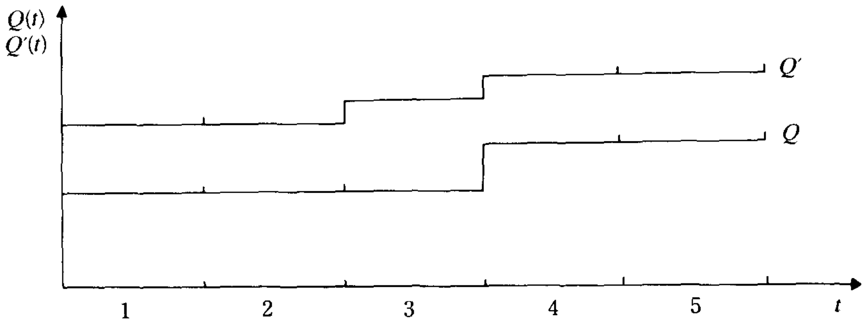

FIGURE 1
EFFICIENCY
Note: If Q' in Figure 1 is feasible, Q cannot be efficient.

$$K(t) = K(1) + \sum_{s=1}^{t-1} x_3(s) - c \sum_{s=1}^{t-1} k_2 x_2(s).$$

$K_2(t) \leq K(t)$ can therefore be reformulated as

$$H_{tk}: K(1) + \sum_{s=1}^{t-1} x_3(s) - ck_2 \sum_{s=1}^{t-1} x_2(s) - k_2 x_2(t) \geq 0. \qquad (7)$$

Let $x_j \equiv (x_j(1),\cdots, x_j(T))$ for $j = 1, 2, 3$, $Q \equiv (Q(1),\cdots, Q(T))$. A vector $[x, Q] = (x_1, x_2, x_3, Q) \in R^{4T}$ satisfying (5), (6) and (7) is called a *production program*. The corresponding vector $Q \equiv (Q(1), \cdots, Q(T))$ is called a *feasible consumption path*. We now set out to investigate the feasible consumption paths of an economy. We first define efficiency. (see Figure 1)

Efficiency

A feasible consumption path $Q \equiv (Q(1),\cdots, Q(T))$ (a production program $[x, Q]$) is called efficient, when there is no other feasible consumption path $Q' \equiv (Q'(1),\cdots, Q'(T))$ such that $Q' > Q$, i.e.

$$Q'(t) \geq Q(t) \text{ for each } t \in \{1,\cdots, T\}$$

and

$$Q'(s) > Q(s) \text{ for at least one } s \in \{1,\cdots, T\}.$$

B. *An Example: the Transformation Curve for $T = 2$*

Before we present the general T-period model, the concepts that have been introduced in sub-section A will be explained for the special case $T = 2$. Specifically, the set of efficient feasible consumption paths is investigated, which, in this case, is of course the transformation curve of the economy.

For simplicity, we assume that $K(1) = 0$. We then have $Q(1) \leq L/l_1$ for every feasible production path $(Q(1), Q(2))$; this is so because the output L/l_1 is obtained if the available labor is completely used in process R_1 in period 1. For $Q(1) = L/l_1$ it follows that $x_3(1) = 0$; hence there is no capital good available at the beginning of period 2, so that, as in period 1, R_1 is the only available production process for the consumption good in period 2. We therefore have $Q(2) = L/l_1$, if $Q(1) = L/l_1$ and $(Q(1), Q(2))$ is to be efficient. Hence, $A \equiv (L/l_1, L/l_1)$ is the point on the transformation curve with maximal $Q(1)$.

Furthermore, every feasible consumption path has to satisfy $Q(1) \geq 0$. For efficient paths it is also necessary that first-period investment is not too high.

We distinguish between two cases:

i) $l_1 \leq l_2$: In this case, it is never efficient to invest. The "transformation curve" consists only of the point $A \equiv (L/l_1, L/l_1)$. Only technique T_1 is applied.

ii) $l_1 > l_2$: It is by no means possible to produce more than L/l_2 in period 2 because, by (2), $x_2(t) = \min[L_2(t)/l_2, K_2(t)/k_2]$ and therefore $x_2(t) = L/l_2$ for every capital stock $K(t) \geq Lk_2/l_2$. It is therefore inefficient to produce more than $x_3(1) = Lk_2/l_2$ units of the capital good in period 1: a greater $x_3(1)$ would not improve production possibilities in period 2, but would use up labor for the production of the capital good that could alternatively be used for the production of the consumption good in sector 1.

Hence, for efficient production programs

$$L_3(1) = l_3x_3(1) \leq l_3Lk_2/l_2.$$

As the second process cannot be used in period 1 because $K(1) = 0$, we have $L_2(1) = 0$ and hence, by the last equation,

$$L_1(1) = L - L_3(1) \geq L - L\frac{l_3k_2}{l_2} = L(\frac{l_2 - l_3k_2}{l_2}). \tag{8}$$

Of course, this inequality is only meaningful if the last expression is positive. Otherwise, i.e. if $l_2 < l_3k_2$, $Q(1) \geq 0$ is the only *lower* bound that $Q(1)$ has to obey. If, however, $l_2 > l_3k_2$, (8) implies $Q(1) \geq L(\frac{l_2 - l_3k_2}{l_1l_2}) > 0$ for efficient production programs, so that this restriction is economically meaningful.

The slope of the transformation curve is $-(l_1 - l_2)/l_3k_2$: If $Q(1)$

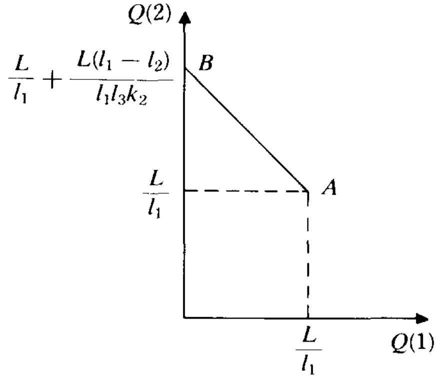

FIGURE 2-1
THE TRANSFORMATION CURVE
FOR $l_2 < l_3k_2$

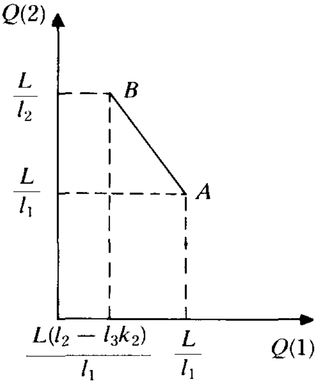

FIGURE 2-2
THE TRANSFORMATION CURVE
FOR $l_2 > l_3k_2$

$> L[(l_2 - l_3k_2)/l_1l_2]$, the reduction of first-period consumption by 1 unit allows to use an additional l_1 units of labor for the production of l_1/l_3 units of capital in process R_3. With this additional capital, production of the consumption good in R_2 can be increased by $dx_2 = l_1/l_3k_2$ units if an additional $l_2dx_2 = l_2l_1/l_3k_2$ units of labor are supplied. The only way to obtain this additional labor is by a sufficient reduction of the labor input in process R_1. More precisely, one has to reduce the output of process R_2 by $-dx_1 = l_2/l_3k_2$ units. The reduction of consumption by 1 unit therefore allows the consumption of an additional $dx_1 + dx_2 = (l_1 - l_2)/l_3k_2$ units in period 2, which shows that the slope of the transformation curve is in fact $-(l_1 - l_2)/l_3k_2$.

Using the knowledge that $A \equiv (L/l_1, L/l_1)$ is on the transformation curve, it follows from the formula for the straight line that the transformation curve is given by

$$Q(2) = (1 + \frac{l_1 - l_2}{l_3k_2})\frac{L}{l_1} - (\frac{l_1 - l_2}{l_3k_2})Q(1). \tag{9}$$

In the case $l_2 \leq l_3k_2$, one therefore obtains the curve depicted in Figure 2-1. For $l_2 > l_3k_2$, the value of the point B on the transformation curve with minimal $Q(1)$ is therefore $B \equiv [L(l_2 - l_3k_2)/l_1l_2, L/l_2]$ by (8), so that the transformation curve is given as in Figure 2-2.

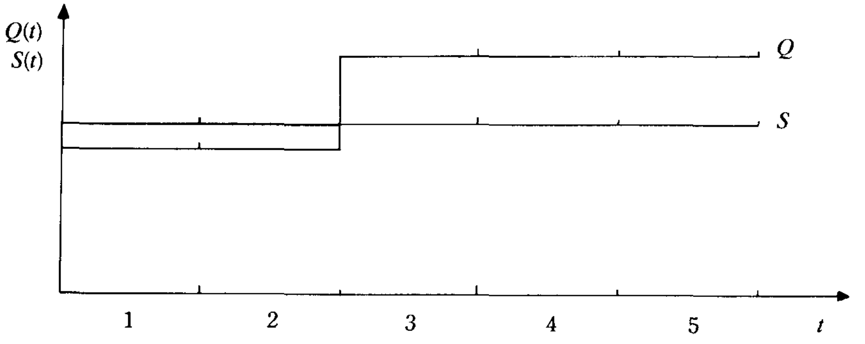

FIGURE 3
SUPERIORITY
Note: In this example Q is superior.

C. Superiority and Roundaboutness

We now return to the general T-period case and introduce the central concepts of superiority and roundaboutness, which will be used later in the discussion of the sign of the interest rate. As a prerequisite, we define:

Stationarity
A consumption path $Q \equiv (Q(1), \cdots, Q(T))$ is called stationary if $Q(1) = Q(2) = \cdots = Q(T)$.

There is not more than one efficient stationary program. For $K(1) = 0$, this program is given by

$$S \equiv (S(1), \cdots, S(T)) = (L/l_1, \cdots, L/l_1).$$

We can now define:

Superiority
A consumption path Q is superior to the efficient stationary program S (or simply superior), if

$$\sum_{t=1}^{T} Q(t) > \sum_{t=1}^{T} S(t).$$

Roundaboutness
A consumption path Q is called more roundabout than the efficient stationary path S (or simply, more roundabout) if there exists a t' such that $1 < t' < T$ and

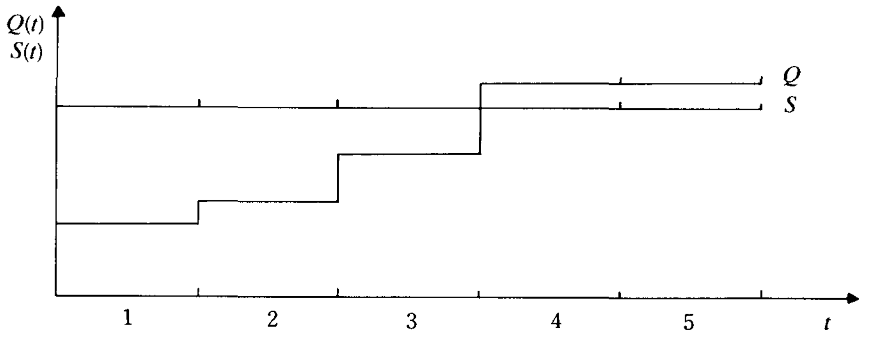

FIGURE 4
ROUNDABOUTNESS
Note: Q is more roundabout than S

$$Q(t) \leq S(t) \text{ for every } t \in \{1, \cdots, t'\};$$

$$Q(\mathrm{t}) < S(t) \text{ for at least one } t \in \{1, \cdots, t'\};$$

$$Q(t) \geq S(t) \text{ for every } t \in \{t' + 1, \cdots, T\};$$

$$Q(t) > S(t) \text{ for at least one } t \in \{t' + 1, \cdots, T\}.$$

D. A Criterion for Superiority

We now give a criterion for superiority of roundabout programs. We start with the case $T = 2$. Suppose S is the stationary path. For a more roundabout path to be superior, it must be possible to achieve more than 1 unit of additional second-period consumption by reducing first-period consumption by 1 unit, starting from the stationary path. Hence, the absolute value of the slope of the transformation curve has to be greater than 1.

Hence, more roundabout programs are superior if and only if

$$(l_1 - l_2)/l_3k_2 > 1; \text{ i.e. } l_1 > (l_2 + l_3k_2). \tag{10}$$

This condition has a straightforward interpretation: The labor input necessary to produce 1 unit of the consumption good with technique T_1 has to be greater than the corresponding quantity for T_2, which contains direct labor input (l_2) and indirect labor input (l_3k_2). This condition can be generalized to T periods. As before, we assume that $l_1 > l_2$. What is needed is a condition that guarantees that one unit of consumption in period 1 allows an additional total consumption of at least one unit in periods $2, \cdots, T$.

As shown above, reducing consumption by 1 unit allows the consumption of additional $(l_1 - l_2)/l_3k_2$ units in period 2. As capital deteriorates at rate c, the possible additional consumption in period 3 is only $(1 - c)(l_1 - l_2)/l_3k_2$ and, more generally, $(1 - c)^{t-2}(l_1 - l_2)/l_3k_2$ in periods t $(t = 3,\cdots, T)$.

Total consumption in periods $2,\cdots, T$ therefore increases by

$$\sum_{t=0}^{T-2}(1 - c)^t(l_1 - l_2)/l_3k_2,$$

while it decreases by 1 unit in period 1. As it follows from the formula for geometric series that

$$\sum_{t=0}^{T-2}(1 - c)^t(l_1 - l_2)/l_3k_2 = [(l_1 - l_2)/l_3k_2]\ \frac{1 - (1 - c)^{T-1}}{c},$$

we obtain the following condition for superiority

$$\frac{l_1 - l_2}{l_3k_2}\ \frac{1 - (1 - c)^{T-1}}{c} > 1$$

or, equivalently,

$$l_1 > l_2 + \frac{cl_3k_2}{1 - (1 - c)^{T-1}} \tag{11}$$

This condition is easily seen to be equivalent to (5.11) in Faber (1979, p. 94).

The boundary cases $T = 2$, $T = \infty$ are of special interest. In the first case, we have

$$l_1 > l_2 + l_3k_2,$$

as was to be expected.

In the second case, we obtain the much weaker inequality

$$l_1 > l_2 + cl_3k_2.$$

For $2 < T < \infty$ and $0 < c < 1$, the expression on the right hand side of (11) is a monotonically decreasing function of T. Hence, the longer the planning horizon, the more likely the technique is to be superior.

E. Replacement of a Technique

The model we sketched above is useful for the understanding of the process of replacement of an old technique by a new one and thus of structural change. To see this, it is useful to assume that the capital good which is needed for T_1 is not available at the beginning of the time horizon. The introduction of the new technique is impossible without a (temporary) reduction in consumption, because inputs have to move from process R_1 to process R_3 before the first unit of the consumption good can be produced in R_2.

Whether the new technique is actually introduce, depends on

i) the degree of superiority (and hence on the length of the planning horizon, because we saw above that this degree depends positively on the planning horizon) and

ii) the time-preference of the decision maker, which will be discussed in the next section.

III. The Demand Side

A. The Optimization Problem

Suppose that a Central Planning Agency maximizes an aggregated continuous, strictly concave and monotonic welfare function $W(Q(1), \cdots, Q(T))$. In particular, we shall assume that its form is

$$W(Q(1),\cdots, Q(T))= \sum_{t=1}^{T} (1+ \delta)^{1-t} W_1(Q(t))$$

where W_1 is spot-welfare function and δ is the rate of time preference.

The set of feasible consumption paths that is given by (5), (6) and (7) is clearly compact and convex, so that a unique solution path exists that is characterized by the Kuhn-Tucker conditions.

The Lagrangean function is given by

$$\begin{aligned} L = {} & W(Q(1),\cdots, Q(T)) + \sum_{t=1}^{T} p_{tQ}(x_1(t) + x_2(t) - Q(t)) \\ & + \sum_{t=1}^{T} p_{tL}(L - l_1x_1(t) - l_2x_2(t) - l_3x_3(t)) \\ & + \sum_{t=1}^{T} p_{tK}[K(1) + \sum_{s=1}^{t-1} x_3(s) - ck_2 \sum_{s=1}^{t-1} x_2(s) - k_2x_2(s)], \end{aligned}$$

where p_{tQ}, p_{tL}, p_{tK} $(t = 1,\cdots, T)$ are the multipliers.

From this, we derive the Kuhn-Tucker conditions as

$$\partial L/ \partial Q(t) = \partial W/ \partial Q(t) - p_{tQ} \leq 0 \tag{12}$$

$$\partial L/ \partial x_1(t) = p_{tQ} - l_1 p_{tL} \leq 0 \tag{13}$$

$$\partial L/ \partial x_2(t) = p_{tQ} - l_2p_{tL} - ck_2 \sum_{s=t+1}^{T} p_{sQ} - k_2p_{tK} \leq 0 \tag{14}$$

$$\partial L/ \partial x_3(t) = -l_3p_{tK} + \sum_{s=t+1}^{T} p_{sK} \leq 0 \tag{15}$$

$$(t = 1,\cdots, T).$$

If we regard the multipliers as shadow prices as usual, conditions (12)–(15) can be interpreted as follows (assuming for simplicity that the $Q(t)$, $x_j(t)$ are all positive so that (12)–(15) are equalities). (12) means that the (shadow) price of the consumption good equals the marginal welfare of consumption in each period. (13) and (14) imply that the price of the consumption good equals the average costs of production in processes R_1 and R_2, respectively. Here $l_1 p_{tL}$ and $l_2 p_{tL}$ are labor costs, $k_2 p_{tk}$ are the costs of capital services for the production of one unit of the consumption good. The term

$$ck_2 \sum_{s=t+1}^{T} p_{sQ}$$

stands for opportunity costs due to capital deterioration. Finally, (15) indicates that the (labor) costs of production of one unit of the capital good equal the sum of the prices of its future services.

B. Prices and the Rate of Interest

The multiplier p_{tQ} of the consumption restriction

$$x_1(t) + x_2(t) - Q(t) \geq 0$$

is equal to the marginal welfare of consumption for an optimal path, i.e. to the shadow price of consumption. Hence $p_{tQ}/p_{(t+1)Q}$ can be seen as an interest factor, i.e.

$$p_{tQ}/p_{(t+1)Q} \equiv 1 + r_t \tag{16}$$

where r_t is the rate of interest in period t. One goal of Neo-Austrian capital theory is to provide reasonable conditions for the positivity of the rate of interest. We shall illustrate this in Section IV for the two-period model. To this end, we first need to discuss the notion of time preference.

C. Time Preference

We now introduce a measure for the "impatience" of a society. For simplicity, we start with the case $T = 2$. Intuitively, we want the measure to tell us how many units of consumption society has to get in period 2 in order to be willing to give up one unit of consumption in period 1.

However, this definition is not unique because it depends on the consumption bundle that is used as a reference point. (cf. points A,

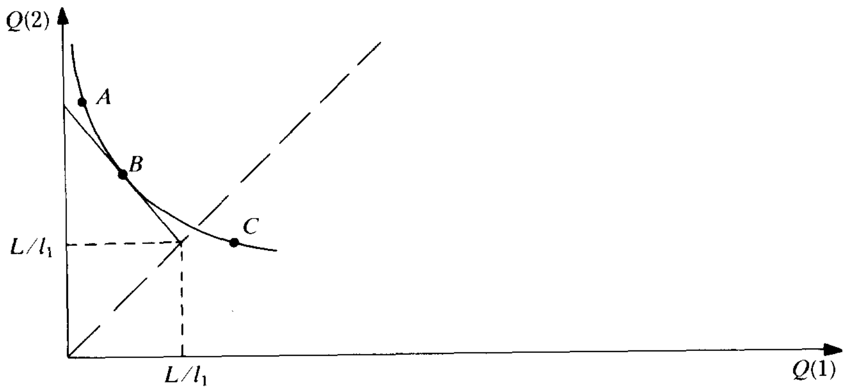

FIGURE 5
DEFINITION OF THE RATE OF TIME PREFERENCE

B, C in Figure 5).

To obtain uniqueness, the reference bundle has to be specified. Firstly, it has to lie on the diagonal so that one has a situation where equal amounts of the two goods are consumed. Therefore, one defines δ, *the rate of time preference* by means of the equation

$$-(1+\delta) \equiv \frac{dQ(2)}{dQ(1)}\bigg|_{W(Q(1),Q(2))=\text{const.},\ Q(1)=Q(2)}, \tag{17}$$

i.e. $1+\delta$ is the absolute value of the slope of an indifference curve at its intersection with the diagonal. An equivalent formulation of (17) is

$$1+\delta \equiv \{\partial W/\partial Q(1)\}/\{\partial W/\partial Q(2)\}\Big|_{Q(1)=Q(2)}, \tag{18}$$

as the total differentiation of $W(Q(1), Q(2)) = \text{const.}$ shows. Strictly speaking, a rate of time preference that is defined in (17) is only reasonable for welfare functions such that the slope of the indifference curve is constant along the diagonal. Such functions are for instance homothetic and additively separable functions, i.e.

$$\sum_{t=1}^{2}(1+\delta)^{1-t}W_1(Q(t)) \tag{19}$$

with a spot-welfare function W_1 and a constant $\delta > -1$.

We say that there is *impatience to consume* if the absolute value of the slope of the indifference curve at the diagonal is greater than 1 i.e. $1+\delta > 1$, or if $\delta > 0$ ($\delta < 0$: patience; $\delta = 1$: neutrality).

To define a rate of time preference δ for more general functions

$W(Q(1),\cdots, Q(T))$, it is natural to take

$$(1+\delta) \equiv (\partial W/\partial Q(t))/(\partial W/\partial Q(t+1)) \Big|_{Q(t)=Q(t+1)}.$$

To make it sure that this is well-defined, we will confine ourself to functions, such as:

$$W \equiv \sum_{t=1}^{t} (1+\delta)^{1-t} W_1(Q(t)).$$

For such functions

$$(\partial W/\partial Q(t))/(\partial W/\partial Q(t+1)) = (1+\delta)\frac{dW_1/dQ(t)}{dW_1/dQ(t+1)}$$

is equal to $(1+\delta)$ whenever $Q(t) = Q(t+1)$, so that without risk of ambiguity, we can speak of the time preference factor $(1+\delta)$ and the rate of time preference δ.

IV. Positivity of the Rate of Interest

We are now in a position to give sufficient conditions for the positivity of the rate of interest.

We shall prove the following theorem, which can, with somewhat more effort, be generalized to the T-period case.

Theorem

If there exists a superior and more roundabout program and there is impatience to consume or neutrality, the rate of interest is positive.

Proof: The maximization problem is given as

$$\text{Max } W(Q(1), Q(2)),$$

$$\text{subject to, } H_{tQ}: x_1(t) + x_2(t) - Q(t) \geq 0 \tag{5}$$

$$H_{tL}: -l_1x_1(t) - l_2x_2(t) - l_3x_3(t) + L \geq 0 \tag{6}$$

$$H_{tK}: -k_2x_2(t) - ck_2\sum_{s=1}^{t-1} x_2(s) + K(1) + \sum_{s=1}^{t-1} x_3(s) \geq 0 \tag{7}$$

$$\text{for } t = 1, 2.$$

The corresponding Lagrangean function is:

$$L = W + \sum_{t=1}^{2} (p_{tQ}H_{tQ} + p_{tL}H_{tL} + p_{tK}H_{tK})$$

The Kuhn-Tucker conditions are

$L1$: $\partial L/\partial Q(1) = \partial W/\partial Q(1) - p_{1Q} \leq 0$
$L2$: $\partial L/\partial Q(2) = \partial W/\partial Q(2) - p_{2Q} \leq 0$
$L3$: $\partial L/\partial x_1(1) = p_{1Q} - l_1 p_{1L} \leq 0$
$L4$: $\partial L/\partial x_2(1) = p_{1Q} - l_2 p_{1L} - k_2 p_{1K} - ck_2 p_{2K} \leq 0$
$L5$: $\partial L/\partial x_3(1) = -l_3 p_{1L} + p_{2K} \leq 0$
$L6$: $\partial L/\partial x_1(2) = p_{2Q} - l_1 p_{2L} \leq 0$
$L7$: $\partial L/\partial x_2(2) = p_{2Q} - l_2 p_{2L} - k_2 p_{2K} \leq 0$
$L8$: $\partial L/\partial x_3(2) = -l_3 p_{2L} \leq 0$.

These conditions are equalities if the respective parameters are positive. Since the welfare function is monotonic, the maximum is on the transformation curve, i.e. in A (case 1), B (case 2) or inbetween (case 3) in Figure 2. For simplicity, we shall assume in the following that the rate of depreciation is zero.

Case 1: *maximum in A*

In this case, $x_2(1) = x_3(1) = 0$; $x_2(2) = x_3(2) = 0$, while $Q(1) > 0$, $Q(2) > 0$, $x_1(1) > 0$, $x_1(2) > 0$. $L1$, $L2$, $L3$, $L6$ are therefore equalities. From $L3$ and $L6$, we get

$$p_{1Q} = l_1 p_{1L} \tag{C1}$$

$$p_{2Q} = l_1 p_{2L} \tag{C2}$$

From $L5$, $p_{1L} \geq p_{2K}/l_3$ and hence, from (C1):

$$p_{1Q} = l_1 p_{1L} \geq p_{2K}\, l_1/l_3. \tag{C3}$$

On the other hand, by $L7$,

$$p_{2Q} \leq l_2 p_{2L} + k_2 p_{2k} \tag{C4}$$

and hence by (C2) and (C3)

$$p_{2Q} \leq l_2 p_{2Q}/l_1 + k_2 l_3 p_{1Q}/l_1.$$

Rearrangement of terms shows that $(l_1 - l_2)p_{2Q} \leq k_2 l_3 p_{1Q}$ and hence

$$p_{1Q}/p_{2Q} \geq (l_1 - l_2)/l_3 k_2. \tag{20}$$

If the technique is roundabout, the right hand side of (20) is greater than 1 by (10). As $p_{1Q}/p_{2Q} = 1 + r$ by (12), it follows that $1 + r > 1$ and hence $r > 0$.

Case 2: maximum between A and B

In this case, $L5$ and $L7$ are equalities because both $x_3(1)$ and $x_2(2)$ are positive. Hence, (20) becomes an equality so that

$$1 + r = p_{1Q}/p_{2Q} = (l_1 - l_2)/l_3k_2$$

Hence, superiority implies $r > 0$.

Case 3: maximum in B

As $L2$ is now an equality, $p_{1Q}/p_{2Q} \geq (\partial W/\partial Q(1))/(\partial W/\partial Q(2))$ from $L1$ and $L2$, so that $1 + r$ is bounded below by the absolute value of the slope of the indifference curve in the optimal point. By impatience (neutrality), the absolute value of the slope at its intersection with the diagonal is greater than or equal to 1. Because of the strict concavity of the welfare function it is therefore greater than 1 to the left of the diagonal and in particular in B. Hence $r > 0$.

V. Generalizations and Applications

The model has been generalized in different respects.

1) Economies with arbitrary (finite) numbers of primary factors, capital goods and consumption goods have been considered (see Berhnolz, Faber and Reiß 1978; Reiß 1981). It has been shown that with appropriate modifications of the concepts of superiority and roundaboutness, the theorem on the positivity of the interest rate expands to economies of this type.
2) Infinite time horizons have been allowed (see Stephan 1983, 1985). To guarantee the existence of competitive prices and efficiency prices in infinite horizon economies, the neoclassical literature usually makes resort to rather unappealing assumptions, such as "nontightness" and "reachability". It turns out that appropriate generalizations of superiority and roundaboutness are adequate substitutes for these assumptions in the sense that they can be used to obtain sufficient conditions for the existence of the desired price systems which contain the neoclassical conditions as special cases and are considerably less restrictive.
3) The model has been reformulated for a competitive market economy (Faber and Bernholz 1973) in an intertemporal Arrow-Debreu framework. With utility-maximizing households and profit-maximizing firms whose respective technology sets are described via the production processes T_1, T_2, T_3, the result on the

positivity of the interest rate remains valid.

4) The model has been extended to a game-theoretic framework to investigate situations of imperfect competition (see Böge, Faber and Güth 1982; Thadden 1986). In particular, the Neo-Austrian approach has been used to study the influence of the market structure on macroeconomic variables.
5) The main applications, however, have been to the theory and empirical analysis of problems of (renewable and non-renewable) natural resources and the environment (see Niemes (1981), Faber, Niemes and Stephan (1983, 1987), Maier (1984), Faber and Wagenhals (1988), Faber, Stephan, and Michaelis (1989), Stephan (1989), Faber, Proops, Ruth and Michaelis (1990)).

In these models and the corresponding empirical studies, the technology is usually extended in one or both of the following directions:

(1) It is assumed that for the production of the capital good an exhaustible resource is required which has to be extracted in the fourth, time-consuming process.
(2) In the consumption good processes, pollutants appear as additional outputs which negatively influence welfare: the welfare function contains additional arguments $U(t)$ for environmental quality in period t, which depend negatively on the concentration of the pollutants, which in turn depends positively on the emissions of pollutants in earlier periods. Furthermore, a pollution-abatement process is added to the model.

With models of this type, shadow-prices for natural resources can be calculated. These can be used to compute charges and taxes for the use of the environmental goods, which in turn allows to formulate environmental policy recommendations (see Faber, Stephan and Michaelis 1989). Furthermore, the explicit treatment of the production process allows for the investigation of the effects of switching from one technique to another on consumption, capital stocks, resource stocks and environmental quality.

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