
Ki Seong Park
Korea Labor Institute

I specify a model in which younger workers learn from more experienced workers. In particular, the ratio of the effective number of experienced workers to the effective number of newer workers determines how much newer workers learn. Workers pay for their learning by reduced wages, and they receive the benefits from their learning by a growth in their wages with experience. In an industry with a higher employment growth rate, experienced workers as teachers are scarce relative to younger workers as students, and consequently younger workers learn less from experienced workers. Therefore I have an important theoretical implication: an industry with a higher employment growth rate has a flatter experience-wage profile. This hypothesis is supported by an empirical evidence using the 1984 and 1986 Korean Occupational Wage Surveys for workers with 15 (or 20) years or less of experience. I also show that a new-born industry's productivity should increase at a decreasing rate over time under a certain stability condition.

I. Introduction

There have been two ways to model human capital accumulation in the literature: the time-allocation model\(^1\) and the learning-by-doing model.\(^2\) The time-allocation model allows the worker to divide his time between work and learning activities during the day. More time and more market goods allocated to learning increase human capital and result in higher marginal product and income in subsequent periods. In the learning-by-doing or experience model, the amount the worker learns is assumed to increase with the amount of time he

\(^1\)See Ben-Porath (1967) and Heckman (1976).

spends in work activities or with his work experience.

In this paper, I develop a new model by incorporating a new aspect of human capital accumulation. Lucas (1988) emphasized that human capital accumulation is a social activity, involving groups of people, in a way that has no counterpart in the accumulation of physical capital. I study this aspect of human capital accumulation in this paper. My model postulates that younger workers learn from experienced workers and that if younger workers have more or better experienced workers in their firms, they learn more relative to younger workers in other firms. More specifically, the time rate of learning by newer workers is an increasing and concave function of the ratio of the effective number of experienced workers to the effective number of newer workers. This on-the-job learning is incorporated into an industry equilibrium model to see how on-the-job learning is related to the experience-wage profile and the dynamics of an industry.3

I postulate an industry with many identical firms, whose demand is exogenously given and may possibly be growing over time, so that the notation refers interchangeably to both the industry and the firm. A workforce is distinguished by seniority (or age). A crucial assumption is that the supply of inexperienced workers is infinitely elastic at some specific present value of expected lifetime wages.4 The firm chooses its seniority structure and thus its provision for learning. Simultaneously with the firm’s decisions, the market determines the equilibrium wage-learning-seniority structure.

Since younger workers learn from more-experienced workers and since the effective number of experienced workers relative to the effective number of newer workers determines how much newer workers learn, workers pay for their learning by reduced wages, and they receive the benefits from their learning by a growth in their wages with experience. An important implication is that a more rapidly growing industry in terms of employment has a flatter experience-wage profile. In a more rapidly growing industry, experienced workers as teachers are scarce relative to younger workers as students, and consequently younger workers learn less from more-experienced workers. I show that in a more rapidly growing industry the tuition paid by reduced wages by inexperienced work-

3For the industry equilibrium model, see Lucas (1967), Millar (1971), and Lucas and Prescott (1971).
4See equation (10).
ers is smaller and the teaching reward to experienced workers in the form of a growth in their wages with experience is also smaller under a certain condition.

I consider both steady-states and dynamic paths, and discuss both the stability and existence of steady-states. A new-born industry's productivity increases at a decreasing rate over time under a certain stability condition. This provides a basis for the learning curve.

The organization of this paper is as follows. In the next section, I specify an on-the-job learning model. After defining a competitive industry equilibrium with general on-the-job learning, I derive some implications from it. Section III provides a stability condition and investigates the dynamics of the industry. In Section IV, I give an empirical evidence to the hypothesis that a more rapidly growing industry in terms of employment has a flatter experience-wage profile, by using the 1984 and 1986 Korean Occupational Wage Surveys. Section V concludes this paper with a summary of the results and the future research prospect.

II. The Model and Implications

I consider an industry whose representative firm has a linear production function:

\[ Q(t) = A(0, t)n(0, t) + A(1, t)n(1, t), \]  

(1)

where \( Q(t) \) is the output in period \( t \), \( A(0, t) \) is a young worker's human capital, \( n(0, t) \) the number of young workers, \( A(1, t) \) an old worker's human capital, and \( n(1, t) \) the number of old workers in period \( t \). Workers leave the labor market after at most 2 working periods. I assume that \( A(1, t) \) changes according to:

\[ A(1, t + 1) = c \left( \frac{A(1, t)n(1, t)}{A(0, t)n(0, t)} \right)^{\alpha} A(0, t), \]  

(2)

where \( c \) is positive and \( \alpha \) is between 0 and 1. The size of \( c \) may depend on the time allocated to the learning activity, which is assumed to be constant in this paper.\(^5\) A young worker in period \( t \) learns from old workers, and the proportional rate of learning depends on the ratio of the total human capital of teachers, who are old workers to the total human capital of students, who are young

\(^5\)I also consider later the case that the size of \( c \) increases with the time allocated to the learning activity.
workers. I call the ratio the "effective teacher-student ratio" since the usual teacher-student ratio is here adjusted by quality units.\(^6\) The effective teacher-student ratio is composed of two parts; the human capital ratio and the number ratio. Therefore the more human capital the old workers have, the more the young worker learns. And the more there are the old workers with a certain amount of human capital, the more the young worker learns because he can contact his seniors, who are the old workers, more frequently. Equation (2) will be referred to as the human capital accumulation function in this paper.

When an inexperienced worker chooses a firm, he considers not only the current period's wage but also how much he can learn in the firm (i.e., the learning opportunity) because the human capital accumulated through on-the-job learning determines the next period's wage. In this model the learning opportunity in a firm is determined by the effective teacher-student ratio in the firm.

In the next period the inexperienced worker becomes a senior worker. The learning he obtained in the previous period is assumed to be general, so he can move to other firms in the industry, but does not move to other industries because his learning is assumed to be industry-specific.\(^7\) Therefore a firm with a better learning opportunity (i.e., a higher effective teacher-student ratio) can lower the current period's wage it pays relative to other firms because an old worker with a better learning opportunity in the previous period can get higher wage relative to other old workers.

Since it is assumed that there is no long-term contract, the firm cannot prevent workers hired during the last period from leaving it, and it hires newly old workers as well as young workers in each period. From the viewpoint of the firm, old workers have the same amount of human capital in each period because all firms in the industry are identical and provide the same learning opportunity. The firm takes the amount of an old worker's human capital as given. I can now express this idea formally.

The inexperienced worker's human capital in period \(t\), \(\hat{A}(0, t)\), is exogenously given. Let \(\hat{A}(1, t + 1)\) be the industry average human capital of old workers in period \(t + 1\) and \(w(0, t)\) be the effective

\(^6\)See Bowles (1967) for using the teacher-student ratio in formal schooling. Rosen (1975) mentioned the labor market analogue of the teacher-student ratio relevant to formal schooling as an index measuring the size of the learning option connected with the work activity.

\(^7\)See Becker (1983) for the definition of general and specific human capital.
wage of a young worker in period \( t \), whose work and learning activities will provide him with \( A(1, t + 1) \) human capital in period \( t + 1 \). Since workers are free to seek employment in other firms after each period, if a young worker with \( A(0, t) \) human capital is hired in period \( t \) in a firm where work and learning activities provide him with \( A(1, t + 1) \) human capital in period \( t + 1 \), his wage in period \( t \) is:

\[
A(0, t)w(0, t) - \beta (1 - \mu) |A(1, t + 1) - \bar{A}(1, t + 1)| w(1, t + 1), \tag{3}
\]

where \( \beta \) is the discount factor, \( \mu \) is the death rate for young workers, and \( w(1, t + 1) \) is an old worker’s effective wage in period \( t + 1 \). Confronted with the market-determined trade-off between the current period’s wage, (3), and the learning opportunity, \( A(1, t + 1) \), he is indifferent to which firm in the industry hires him.\(^8\) The firm hires newly old workers with the industry average human capital.

Therefore, the production function for the firm is:

\[
Q(t) = A(0, t)n(0, t) + \bar{A}(1, t)n(1, t). \tag{4}
\]

The human capital accumulation functions for the firm are:

\[
A(1, t + 1) = c \left| \frac{\bar{A}(1, t)n(1, t)}{A(0, t)n(0, t)} \right|^\alpha A(0, t), \tag{5}
\]

where \( t = 0, 1, 2, \cdots \). The firm maximizes the present value of its net revenue stream:

\[
\sum_{t=0}^{\infty} \beta^t [p(t) |A(0, t)n(0, t) + \bar{A}(1, t)n(1, t)| - |A(0, t)w(0, t)
- \beta (1 - \mu)[A(1, t + 1) - \bar{A}(1, t + 1)] w(1, t + 1)| n(0, t)
- \bar{A}(1, t)w(1, t)n(1, t)|],
\]

where \( p(t) \) is the output price in period \( t \). The constraint is equation (5). In equilibrium, the following four equations (7)–(10) hold.

\[
A(1, t) = \bar{A}(1, t), \tag{7}
\]

where \( t = 1, 2, \cdots \). Equation (7) means that the amount of learning the firm provides is equal to the industry average. Hence workers do not move. The amount of an old worker’s human capital in period 0, \( \bar{A}(1, 0) \), is exogenously given. The output market clearing condition is given by:

\[
Q(t) = \lambda \frac{\partial}{\partial \beta} (p(t)), \tag{8}
\]

\(^8\)The second term in (3) is an equalizing wage differential. See Rosen (1975) for details.
where \( t = 0, 1, \cdots \), and \( \lambda - 1 \) is the output demand growth rate. The labor market clearing conditions are equations (9) and (10).

\[
n(1, t + 1) = (1 - \mu)n(0, t),
\]

\[(9)\]

where \( t = 0, 1, \cdots \). A young worker becomes an old worker in the next period if he is still alive. The number of old workers in period 0, \( n(1, 0) \), is exogenously given. I assume that the supply of inexperienced workers is infinitely elastic at some present value of expected lifetime wages (\( W(t) \)), which is given by the present value in other industries. Therefore I have:

\[
W(t) = A(0, t)w(0, t) - \beta (1 - \mu) |A(1, t + 1) - \tilde{A}(1, t + 1)| w(1, t + 1) + \beta (1 - \mu)A(1, t + 1)w(1, t + 1),
\]

\[(10)\]

where \( W(t) \) is the present value, which is exogenously given. Then \( |n(0, t)|_{t=0}^\infty \) is demand-determined. I summarize the above competitive equilibrium in the following definition:

**Definition 1**

An industry equilibrium for initial state \( |A(1, 0), n(1, 0)| \) is a set of nonnegative sequences \( \{Q^*(t), n^*(0, t), n^*(1, t + 1), A^*(1, t + 1), w^*(0, t), w^*(1, t), p^*(t)\}_{t=0}^\infty \) such that equations (7)--(10) are satisfied and such that the present value (6) is maximized for all sets of nonnegative sequences \( \{Q(t), n(0, t), n(1, t + 1), A(1, t + 1)\}_{t=0}^\infty \) satisfying equations (4) and (5).

The Lagrangian for the firm is:

\[
L = \sum_{t=0}^\infty \beta^t \{ p(t)Q(t) - A(0, t)w(0, t)n(0, t) - A(1, t)w(1, t)n(1, t) + \beta (1 - \mu) |A(1, t + 1) - \tilde{A}(1, t + 1)| w(1, t + 1)n(0, t) + \psi(t) |g(t)A(0, t) - A(1, t + 1)| \},
\]

where

\[
\[Q(t) = A(0, t)n(0, t) + \tilde{A}(1, t)n(1, t),
\]

\[
g(t) = \frac{\tilde{A}(1, t)n(1, t)}{A(0, t)n(0, t)}^a,
\]

\( \psi(t)(t = 0, 1, \cdots) \) is the Lagrangian multiplier for equation (5).

The choice variables are \( [n(0, t), n(1, t + 1), A(1, t + 1)]_{t=0}^\infty \). Then the first order conditions are:

\[
0 = p(t)A(0, t) - [A(0, t)w(0, t) - \beta (1 - \mu) |A(1, t + 1)|] A(1, t + 1)
\]
\[-\bar{A}(1, t + 1)|w(1, t + 1)] \\
- \psi(t) \sigma r(t) \sigma^{-1} n(0, t)^{-2} \bar{A}(1, t) n(1, t), \quad (11)\]

\[0 = p(t) \bar{A}(1, t) - \bar{A}(1, t) w(1, t) \\
+ \psi(t) \sigma r(t) \sigma^{-1} \bar{A}(1, t) n(0, t)^{-1}, \quad (12)\]

\[0 = \beta (1 - \mu) w(1, t + 1)n(0, t) - \psi(t), \quad (13)\]

where \(r(t)\) is the effective teacher–student ratio for the young worker in period \(t\). Equation (11) shows that, in equilibrium, the young worker’s wage equals his value of marginal product less his tuition for learning from his seniors. Equation (12) shows that the old worker’s wage equals his value of marginal product plus his reward for teaching his juniors, who are the young workers. Equation (13) shows that the marginal revenue from providing a learning opportunity equals the marginal cost of doing that.

The transversality condition\(^9\) is:

\[\lim_{t \to \infty} \beta^{t+1}(1 - \mu) \bar{A}(1, t + 1) w(1, t + 1) n(0, t) = 0. \quad (14)\]

I first consider the steady-state paths. I assume that \(A(0, t)\) and \(W(t)\) increase at the rates of \(\nu - 1\) and \(\pi - 1\), respectively. \(\sigma\) is the price elasticity of the output demand. Then from the first-order and equilibrium conditions, I have:

**Proposition 1**

The steady-state employment, output, wages, and price increase at the rates of \(\lambda \pi^{-\sigma} \nu^{\sigma^{-1}} - 1\), \(\lambda \pi^{-\sigma} \nu^{\sigma} - 1\), \(\pi - 1\), and \(\frac{\pi}{\nu} - 1\), respectively.

This proposition shows that the growth rates are independent of the human capital accumulation function.\(^10\) Since experienced workers leave the labor market with their human capital and inexperienced young workers enter firms with exogenously-given human capital, human capital increases over lifetime of workers due to on-the-job learning, but does not change across generations in the steady-state.\(^11\)

I next show how the slope of the steady-state experience-wage

---

\(^9\)If Condition (14) is not satisfied, the maximization problem for the firm is not well-defined.

\(^10\)In Park (1987), a model with firm-specific on-the-job learning has the same result as Proposition 1.

\(^11\)Suppose that the proportional rate of learning is linear in the human capital ratio and concave in the number ratio:
profile is affected by growth rates of the output demand and the young worker’s human capital. From the first-order and equilibrium conditions, I have in the steady-state:

\[
A(1, t)w(1, t) / A(0, t)w(0, t) = c \frac{1}{1-\sigma} (1 - \mu) \frac{\sigma}{1-\alpha} / [\eta \frac{1}{1-\sigma} \nu \frac{1}{1-\sigma} - \alpha \beta \pi |c(1 - \mu)\frac{1}{1-\sigma}|] 
\]

(15)

where \( \eta \) is the employment growth rate, which is \( \lambda \nu^{\sigma-1} \pi^{-\sigma} \).

Suppose that there are two industries which have the same human capital accumulation function and the same growth rate of the young worker’s human capital but which have different output demand growth rates. In the industry with a higher output demand growth rate (hence with a higher employment growth rate), old workers as teachers are scarce relative to young workers as students, and thus the amount of learning is lower. With the human capital accumulation function, (2), both the tuition paid by reduced wages by young workers and the teaching reward to old workers in the form of a growth in their wages with experience are also lower. Hence the slope of the steady-state experience–wage profile is flatter in the industry with a higher output demand growth rate. Equation (15) shows this result with \( \beta \pi \eta < 1 \), which is implied by the transversality condition (14).

For comparison, consider the time-allocation model in which an old worker’s human capital is determined by:

\[
A(1, t + 1) = m(\theta_t)A(0, t),
\]

where \( \theta_t \) is the proportion of a young worker’s time allocated to learning in period \( t \) and \( m(\theta) \) is increasing and strictly concave in \( \theta \). The production function is concave in \( A(0, t)n(0, t), A(1, t)n(1, t) \). In the industry with a higher output demand growth rate (hence with a higher employment growth rate), old workers are scarce relative to young workers, thus the proportion allocated to learning is higher. The old–to–young human capital ratio \( (m(\theta)) \) is therefore higher, and the steady-state experience–wage profile should be steeper in the industry with a higher output demand growth rate.

\[
A(1, t + 1) = cA(1, t) / A(0, t) [n(1, t) / n(0, t)]^\alpha A(0, t).
\]

Then human capital accumulates across generations in the steady-state, and the accumulation rate depends inversely on the industry employment growth rate. See Park (1987) for some details.
This implication is contrary to that from my model. (If young and old workers are perfect substitutes in the production function, the slope of the experience-wage profile is independent of the output demand growth rate.)

Back to my model, suppose that the above two industries have different growth rates of the young worker's human capital but have the same output demand growth rate and the same price elasticity of the output demand. In the industry with a higher growth rate of the young worker's human capital, the teacher-student ratio is lower, thus the amount of learning, which is measured by the old-to-young human capital ratio, is accordingly lower although the absolute level of the old worker's human capital is higher. This tends to lower the slope of the experience-wage profile in the industry with a higher growth rate of the young worker's human capital. If the price elasticity of the output demand is greater than or equal to 1, then the slope of the experience-wage profile in the industry with a higher growth rate of the young worker's human capital is lower. If the price elasticity is less than 1, the industry with a higher growth rate of the young worker's human capital has a lower employment growth rate, which tends to raise the slope of the experience-wage profile by the above argument. The following proposition summarizes the arguments more precisely.

**Proposition 2**

Assume that the production function is linear as in equation (1), and that the human capital accumulation function is a power function as in equation (2).

1) The steady-state experience-wage profile becomes flatter as the output demand growth rate (hence the employment growth rate) increases.

2) If the price elasticity of the output demand is greater than or equal to 1, then the steady-state experience-wage profile becomes flatter as the growth rate of the young worker's human capital increases.

From equation (15), the followings are also derived. First, the slope increases as the scale parameter of the human capital function \( c \) increases. Second, the slope increases as the discount factor \( (\beta) \) increases. Third, the slope increases as the survival rate \( (1 - \mu) \) increases. Finally, the slope increases as the present value of lifetime wage \( (\pi - 1) \) increases.
III. Dynamics

In this section, I first show that the resource allocation of Definition 1 solves a certain consumer-surplus maximization problem. Then I investigate the dynamics, using the solution to the problem.

Define the function \( s(Q(t), t) \) by:

\[
s(Q(t), t) = \int_0^{Q(t)} d^{-1}(z/\lambda) dz,
\]

where \( d^{-1}(Q(t)/\lambda) = p(t) \), so that for given \( t \), \( s(Q(t), t) \) is a strictly increasing, strictly concave, and continuously differentiable function of \( Q(t) \). Then \( s(Q(t), t) \) is the area under the demand curve of the industry at an output of \( Q(t) \) in period \( t \). Then define a discounted consumer surplus, \( DCS \), for the industry by:

\[
DCS = \sum_{t=0}^{\infty} \beta^t |s(Q(t), t) - W(t)n(0, t)|.
\]

By a trivial argument\(^{12}\) I have:

**Proposition 3**

Suppose, for given \( |A(1, 0), n(1, 0)|, [Q^*(t), n^0(0, t), n^0(1, t), A^*(1, t + 1), w^*(0, t), w^*(1, t), p^*(t)]_{t=0}^{\infty} \) satisfies Definition 1. Then \( |Q^*(t), n^0(0, t)|_{t=0}^{\infty} \) maximizes \( DCS \) for all \( |Q(t), n(0, t)|_{t=0}^{\infty} \), satisfying (1)-(2) and (9).

I use this consumer-surplus maximization problem to derive a condition under which the steady-state is stable in a generalized model. For simplicity, I set \( W(t) = W \) and \( A(0, t) = 1 \). I assume that \( \beta \lambda < 1 \). In the generalized model the output is assumed to be produced by:

\[
Q(t) = A(1, t)n(1, t)f\left(\frac{A(0, t)n(0, t)}{A(1, t)n(1, t)}\right).
\]

The old worker’s total human capital is assumed to change according to:

\[
A(1, t + 1)n(1, t + 1) = A(1, t)n(1, t)h\left(\frac{A(0, t)n(0, t)}{A(1, t)n(1, t)}\right).
\]

Functions \( f \) and \( h \) are strictly increasing and continuously differentiable. Function \( f \) is concave and function \( h \) is strictly concave.\(^{13}\)

\(^{12}\)See Debreu (1983) or Park (1987).

\(^{13}\)With these assumptions, equations (1) and (2) become:
It is assumed that $h(\delta) = \lambda$ for some $\delta > 0$. $A(1, t)n(1, t)$ is the number of young workers which will yield $\lambda A(1, t)n(1, t)$ units of old worker's total human capital in the next period. The next proposition gives a condition under which the steady-state is stable.\textsuperscript{14}

**Proposition 4**

If the firm employs young workers for any positive $A(1, t)$ and if

$$f'(z)/f(z) \geq h'(z)/h(z), \quad (20)$$

then a positive steady-state, if it exists, is stable.\textsuperscript{15}

**Proof:** The functional equation associated with the consumer surplus maximization problem is:

$$v(k) = \max_{x_t \geq 0} \left[ \int_{0} \left( k f(x/k), 0 \right) - Wx + \beta \lambda v \left( \lambda^{-1}kh(x/k) \right) \right], \quad (21)$$

where $x_t = \lambda^{-1}n(0, t)$ and $k_t = \lambda^{-1}A(1, t)n(1, t)$. The numbers of young and old workers are normalized by the output demand growth rate ($\lambda$).

From Theorems 7, 8, and 10 in Lucas, Prescott and Stokey (1985), I have the followings. This functional equation (21) is satisfied by a unique continuous bounded function $v(k)$ on $(0, \infty)$. The function $v(k)$ is continuous, strictly increasing, and strictly concave in $k$. For any $k$, $v(k)$ is attained by a unique policy function $x(k)$ and $x(k)$ is a continuous function in $k$. The function $v(k)$ is continuously differentiable if $x(k) > 0$ for $k > 0$.

The policy function $x(k)$ must satisfy:

$$s' k f(x(k)/k), 0 | f'(x(k)/k) - W$$

$$+ \beta v' \lambda^{-1}kh(x(k)/k)h'(x(k)/k) = 0. \quad (22)$$

Suppose that $x(k)/k$ is a nondecreasing function of $k$. The left-hand side of (22) decreases as $k$ increases. Hence $x(k)/k$ is a strict-

$$Q(t) = A(1, t)n(1, t) | 1 + \frac{A(0, t)n(0, t)}{A(1, t)n(1, t)} \quad (N1)$$

$$A(1, t + 1)n(1, t + 1) = A(1, t)n(1, t)(1 - \mu)\frac{n(0, t)}{A(1, t)n(1, t)}^{1-\alpha} \quad (N2)$$

\textsuperscript{14}Appendix provides a condition under which there exists a unique steady-state.

\textsuperscript{15}For production and human capital functions (N1)-(N2), Condition (20) becomes:

$$z \geq \frac{a}{1 - a}.$$
ly decreasing function of \( k \). Suppose that \( \lambda^{-1}k\lambda(x(k)/k) \) is a nonincreasing function of \( k \). The left-hand side of (22) increases as \( k \) increases if (20) is satisfied. Hence \( \lambda^{-1}k\lambda(x(k)/k) \) is a strictly increasing function of \( k \). Therefore, the proposition follows (see Figure 1).

Condition (20) means that the elasticity of the period \( t \) output with respect to young workers in period \( t' \) is greater than or equal to the elasticity of old workers' total human capital in period \( t + 1 \) with respect to young workers in period \( t \). This condition basically excludes the oscillation of the normalized total human capital of old workers \( (k_i) \), which, under the condition, approaches monotonically its stationary-state value.

Suppose that there is a new-born industry with a very small amount of the old workers' total human capital. The old worker's human capital increases over time from on-the-job learning along the path in Figure 1. This appears like a learning curve; the industry's productivity increases at a decreasing rate over time (see
Figure 2.
A Learning Curve

Figure 2.

Figure 3 shows a level effect. Suppose the industry output demand shifts upwards from period 0 on, without change in its growth rate. Then more than $\lambda n(0, -1)$ young workers are hired at 0, so that an old worker’s human capital at 1 becomes lower than $A(1, 0)$, and $A(1, t)$ increases over time to $A(1, 0)$. The slope of the experience-wage profile in the steady-state, however, is not affected by the shift of demand.

Figure 4 demonstrates a growth effect. Suppose the output demand growth rate shifts upwards from period 1 on, from $\lambda$ to $\lambda'$. Then the number of young workers hired at 0 is greater than $\lambda n(0, -1)$, but less than the number that will yield $\lambda' A(1, 0) n(1, 0)$ units of old workers’ total human capital at 1. Therefore, an old worker’s human capital at 1 becomes lower than $A(1, 0)$, and $A(1, t)$ decreases over time to a level lower than $A(1, 0)$. 
FIGURE 3
LEVEL EFFECT

FIGURE 4
GROWTH EFFECT
IV. Empirical Test

In this section, I test the first part of Proposition 2 that an industry with more rapidly growing employment has a flatter experience-wage profile.

Data used for the estimation of wage functions are drawn from the 1984 and 1986 Korean Occupational Wage Surveys. Only manufacturing workers are sampled and the sampling rates are different according to scales of establishments because the raw data were biased toward workers in large establishments (see Table B1). From this sample male workers of ages 15–54 are drawn. The average annual employment growth rates of 3-digit industries from 1973 to 1983 are used as employment growth rates (see Table B2).

The form of the estimated wage function is:

\[
\ln w = a_0 + a_1 E + a_2 E^2 + a_3 X + a_4 X^2 + a_5 T + a_6 T^2 + a_7 M + a_8 S + a_9 W + a_{10} L + a_{11} L \cdot X, \tag{23}
\]

where \( W \) is the hourly wage rate, which is defined as \( \text{monthly wage} + (\text{yearly bonus}/12) / \text{monthly working hours} \). \( E \) is years of schooling, which is defined as 6 years if workers are primary school graduates or lower, 9 years if middle school graduates, 12 years if high school graduates, and 16 years if college/university graduates or over. Total work experience \( (X) \) is calculated as the worker’s age minus his years of schooling minus 6. \( T \) is years of tenure in the firm, \( M \) is the marital dummy, \( S \) is the scale dummy, which is defined as 0 if the number of workers in the establishment is less than or equal to 299 and 1 if greater than 299, and \( W \) is the manual worker’s dummy. \( L \) is the employment growth rate of the worker’s industry, and \( L \cdot X \) is the interaction term of the employment growth rate and experience.

By differentiating function (23), I have:

\[
\frac{1}{w} \frac{\partial w}{\partial X} = a_3 + 2a_4 X + a_{11} L, \tag{24}
\]

where \( a_{11} \) shows the effect of the employment rate on the slope of the experience-wage profile. If the first part of Proposition 2 is correct, then \( a_{11} \) must be negative.

Table 1 shows the estimation results of function (23), using the 1986 sample. On the first column, which is the estimation results
TABLE 1
WAGE FUNCTIONS FOR THE 1986 SAMPLE

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>$X \leq 15$</th>
<th>$X &gt; 15$</th>
<th>Age $\leq 35$</th>
<th>Age $&gt; 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.0506**</td>
<td>0.0690**</td>
<td>0.0230**</td>
<td>0.0685**</td>
<td>0.0381**</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0031)</td>
<td>(0.0046)</td>
<td>(0.0022)</td>
<td>(0.0060)</td>
</tr>
<tr>
<td>$X^2$</td>
<td>-0.0011**</td>
<td>-0.0017**</td>
<td>-0.0005**</td>
<td>-0.0018**</td>
<td>-0.0007**</td>
</tr>
<tr>
<td></td>
<td>(0.00003)</td>
<td>(0.0002)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$L$</td>
<td>0.8714**</td>
<td>1.4552**</td>
<td>1.0330**</td>
<td>1.1898</td>
<td>1.7592**</td>
</tr>
<tr>
<td></td>
<td>(0.1033)</td>
<td>(0.1609)</td>
<td>(0.3566)</td>
<td>(0.1373)</td>
<td>(0.4676)</td>
</tr>
<tr>
<td>$L \cdot X$</td>
<td>-0.0162**</td>
<td>-0.0867**</td>
<td>-0.0189</td>
<td>-0.0534**</td>
<td>-0.0449*</td>
</tr>
<tr>
<td></td>
<td>(0.0059)</td>
<td>(0.0165)</td>
<td>(0.0146)</td>
<td>(0.0121)</td>
<td>(0.0179)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6366</td>
<td>0.6544</td>
<td>0.5604</td>
<td>0.6141</td>
<td>0.5772</td>
</tr>
<tr>
<td>$N$</td>
<td>27187</td>
<td>16140</td>
<td>11047</td>
<td>18864</td>
<td>8323</td>
</tr>
</tbody>
</table>

Notes: 1.* and ** mean that the coefficient is significant at 5 percent and 1 percent significance levels, respectively.
2. Numbers in parentheses are standard errors.

for the whole sample, the estimated coefficient of $L \cdot X$ is negative. This means that an industry with a higher employment growth rate has a flatter experience-wage profile. The sample is divided into two groups by 15 years of experience and the estimation results for the two groups are on the second and third columns, respectively. While the negative effect of the employment growth on the slope is statistically significant for the lower experience group, the negative effect is statistically insignificant for the higher experience group. The more the worker’s experience becomes, the more difficult he is to accumulate human capital. Rather his human capital is likely to become obsolescent or to diminish. Since on-the-job learning which is postulated in this paper is not well attained by workers with higher experience, the negative effect of the employment growth on the slope is weak for them. Another reason may be that the 10 year employment growth rates do not well reflect the growth rates of the number of highly experienced workers.

The sample is also divided into two groups by age 35, for which the results are on the fourth and fifth columns. The estimated coefficients of $L \cdot X$ for the two groups are not statistically different.\(^{17}\)

\(^{16}\)For the whole samples in Tables 1 to 4, the condition indices for multicollinearity are less than 20, which means that multicollinearity is no problem. See Johnston (1984, p. 250).

\(^{17}\)If errors of two groups are independent.
Table 2
Wage Functions for the 1984 Sample

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>$X \leq 15$</th>
<th>$X &gt; 15$</th>
<th>$Age \leq 35$</th>
<th>$Age &gt; 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.0521**</td>
<td>0.0689**</td>
<td>0.0270**</td>
<td>0.0678**</td>
<td>0.0465**</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.0034)</td>
<td>(0.0050)</td>
<td>(0.0023)</td>
<td>(0.0065)</td>
</tr>
<tr>
<td>$X^2$</td>
<td>-0.0011**</td>
<td>-0.0014**</td>
<td>-0.0006**</td>
<td>-0.0017**</td>
<td>-0.0009**</td>
</tr>
<tr>
<td></td>
<td>(0.00003)</td>
<td>(0.0002)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$L$</td>
<td>0.4291**</td>
<td>1.1893**</td>
<td>0.6820</td>
<td>0.7729**</td>
<td>1.1053**</td>
</tr>
<tr>
<td></td>
<td>(0.1083)</td>
<td>(0.1629)</td>
<td>(0.3996)</td>
<td>(0.1420)</td>
<td>(0.5131)</td>
</tr>
<tr>
<td>$L \times X$</td>
<td>-0.0022</td>
<td>-0.0981**</td>
<td>-0.0066</td>
<td>-0.0422**</td>
<td>-0.0232</td>
</tr>
<tr>
<td></td>
<td>(0.0064)</td>
<td>(0.0169)</td>
<td>(0.0163)</td>
<td>(0.0128)</td>
<td>(0.0197)</td>
</tr>
<tr>
<td>$\tilde{R}^2$</td>
<td>0.6433</td>
<td>0.6663</td>
<td>0.5569</td>
<td>0.6233</td>
<td>0.5822</td>
</tr>
<tr>
<td></td>
<td>(25342)</td>
<td>(15579)</td>
<td>(9763)</td>
<td>(18045)</td>
<td>(7297)</td>
</tr>
</tbody>
</table>

Note: See notes of Table 1.

Table 2 shows the estimation results for the 1984 sample. The estimated coefficients of $L \times X$ are all negative, but only the coefficients for workers of 15 years or less of experience and 35 years or less of age are statistically significant. These two coefficients are not statistically different from those for the 1986 sample. This means that the effect of the employment growth on the slope is pretty stable for workers of lower experience or of young age.

Table 3 shows the estimation results for the joint sample of 1984 and 1986. The estimated coefficient of $L \times X$ for the whole sample is negative and significant at 5 percent. I calculate the percentage slope of the experience-wage profile, plugging the estimated coefficients into equation (24). The percentage slope for the whole manufacturing, for which the employment growth rate is 6.63 percent, is 1.81 percent at the average experience of 14.8 years. The slope is 1.89 percent for the wood and cork industry, which has the lowest employment growth rate (−2.18 percent), and 1.74 percent for the electrics and electronics industry, which has the highest employment growth rate (14.71 percent). The difference between the two

\[
\frac{|\hat{\beta}_1 - \hat{\beta}_1^2| - |\beta_1 - \beta_1^2|}{\sqrt{|SE(\hat{\beta}_1)|^2 + |SE(\hat{\beta}_1^2)|^2}}
\]

where superscripts indicate groups and $SE$ is standard error, has asymptotically the standard normal distribution. The value of the test statistic is 0.3934.

\(^{18}\)See footnote 17. The values of test statistic for workers of 15 years of experience and for workers of 35 years of age are 0.4827 and 0.6359, respectively.
### Table 3
**Wage Functions for the Joint Sample**

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>$X \leq 15$</th>
<th>$X &gt; 15$</th>
<th>Age $\leq 35$</th>
<th>Age $&gt; 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.0513**</td>
<td>0.0685**</td>
<td>0.0247**</td>
<td>0.0678**</td>
<td>0.0416**</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0022)</td>
<td>(0.0034)</td>
<td>(0.0016)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>$X^2$</td>
<td>-0.0011**</td>
<td>-0.0015**</td>
<td>-0.0005**</td>
<td>-0.0017**</td>
<td>-0.0008**</td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$L$</td>
<td>0.6480**</td>
<td>1.2963**</td>
<td>0.8738**</td>
<td>0.9666**</td>
<td>1.4500**</td>
</tr>
<tr>
<td></td>
<td>(0.0748)</td>
<td>(0.1146)</td>
<td>(0.2663)</td>
<td>(0.0988)</td>
<td>(0.3458)</td>
</tr>
<tr>
<td>$L \cdot X$</td>
<td>-0.0093*</td>
<td>-0.0897**</td>
<td>-0.0137</td>
<td>-0.0466**</td>
<td>-0.0350**</td>
</tr>
<tr>
<td></td>
<td>(0.0044)</td>
<td>(0.0118)</td>
<td>(0.0109)</td>
<td>(0.0088)</td>
<td>(0.0133)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6435</td>
<td>0.6638</td>
<td>0.5622</td>
<td>0.6233</td>
<td>0.5821</td>
</tr>
<tr>
<td>$N$</td>
<td>52529</td>
<td>31719</td>
<td>20810</td>
<td>36909</td>
<td>15620</td>
</tr>
</tbody>
</table>

Notes: 1. The 1986 dummy variable is added to the right-hand side of regression equation (23). The 1984 wage rates are converted into the 1986 terms by using the consumer price index.
2. See notes of Table 1.

The negative effect of the employment growth on the slope is large for workers of 15 years or less of experience. The slope at 9.0 years of experience, which is the average for workers of 15 years or less experience, is 3.56 percent for the whole manufacturing, 4.56 percent for the wood and cork industry, and 2.83 percent for the electrics and electronics industry. The difference between the latter two slopes is 1.52 percent point, which is 42.7 percent of the slope for the whole manufacturing. The estimated coefficients of $L \cdot X$ for workers of 35 or less years of age and for worker of 36 years or more of age are statistically significant and not different. This result also holds for the 1986 sample. These results mean that the negative effect of the employment growth on the slope is less affected by age than by experience. The estimated coefficient of $L \cdot X$ for workers of 16 years or more of experience is not statistically significant.

Since the effect is significant for workers of lower experience, I introduce a spline variable into the regression equation instead of $L$ and $L \cdot X$:

---

19See footnote 17. The value of the test statistic is 0.7274.
ON-THE-JOB LEARNING

\[ SPLAN = (1 - \frac{X}{x'})L \cdot D, \]

where \( x' \) is assumed to be exogenous and is given a value of 10, 15, or 20. \( D = 1 \) if \( X < x' \) and \( D = 0 \) otherwise. The SPLANE variable limits the effect to workers of lower experience than \( x' \). The percentage slope of the experience-wage profile is:

\[
\frac{1}{w} \frac{\partial w}{\partial X} = \alpha_3 + 2 \alpha_4 X - \frac{\alpha_{12}}{x'} L \cdot D,
\]

(25)

where \( \alpha_{12} \) is the coefficient of the SPLANE variable.

Table 4 shows the estimation results with the SPLANE variable. The estimated coefficients of the SPLANE variables with \( x' = 10 \) are not significant, but the coefficients with \( x' = 15 \) and \( x' = 20 \) are positive and significant. The effect of the employment on the slope is measured by \( -\frac{\alpha_{12}}{x'} \), of which the estimated values with \( x' = 15 \) is not significantly different from that with \( x' = 20 \). For example, the estimated values of \( -\frac{\alpha_{12}}{x'} \), are \(-0.0463\) with \( x' = 15 \) and \(-0.0497\) with \( x' = 20 \) for the joint sample. Plugging the estimated coefficients with \( x' = 20 \) for the joint sample into equation (25), the percentage slopes of the experience-wage profile are 1.78 percent at 14.8 years of experience for the whole manufacturing, 2.22 percent for the wood and cork industry, and 1.38 percent for the electrics and electronics industry. The difference between the latter two slopes is 0.84 percent point, which is 47.2 percent of the slope of the whole manufacturing.

From investigating Tables 1 to 4, it is concluded that an industry with a higher employment growth rate has a flatter experience-wage profile for workers with 15 years or less of experience. Although

\[
\sqrt{SE(\frac{\hat{\alpha}_{12}}{x'})^2 + SE(\frac{\hat{\alpha}_{12}}{x'})^2 + COV(\frac{\hat{\alpha}_{12}}{x'}, \frac{\hat{\alpha}_{12}}{x'})}
\]

where \( x' = 15 \) and \( x' = 20 \), has the standard normal distribution asymptotically. Since

\[
|COV(\frac{\hat{\alpha}_{12}}{x'}, \frac{\hat{\alpha}_{12}}{x'})| \leq \sqrt{VAR(\frac{\hat{\alpha}_{12}}{x'}) \cdot VAR(\frac{\hat{\alpha}_{12}}{x'})},
\]

the minimum standard error is calculated. By using this minimum, the maximum of the test statistic is calculated. The values are 0.8686 for the 1984 sample, 0.1910 for the 1986 sample, and 0.5913 for the joint sample.
### Table 4
Wage Functions with Spline

<table>
<thead>
<tr>
<th></th>
<th>1984 and 1986</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1984</td>
<td>1986</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>0.0495**</td>
<td>0.0550**</td>
<td>0.0566**</td>
<td>0.0502**</td>
<td>0.0548**</td>
<td>0.0565**</td>
</tr>
<tr>
<td>(0.0010)</td>
<td>(0.0010)</td>
<td>(0.0009)</td>
<td>(0.0015)</td>
<td>(0.0015)</td>
<td>(0.0014)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>$X^*$</td>
<td>-0.0011**</td>
<td>-0.0012**</td>
<td>-0.0012**</td>
<td>-0.0011**</td>
<td>-0.0012**</td>
<td>-0.0012**</td>
</tr>
<tr>
<td>(0.00002)</td>
<td>(0.00002)</td>
<td>(0.00002)</td>
<td>(0.00004)</td>
<td>(0.00003)</td>
<td>(0.00003)</td>
<td>(0.00003)</td>
</tr>
<tr>
<td>Spline (10)</td>
<td>-0.0615</td>
<td>-0.1685</td>
<td></td>
<td></td>
<td></td>
<td>0.0820</td>
</tr>
<tr>
<td>(0.1054)</td>
<td>(0.1497)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.1485)</td>
</tr>
<tr>
<td>Spline (15)</td>
<td>0.6950**</td>
<td>0.4554**</td>
<td>0.9655**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.0973)</td>
<td>(0.1390)</td>
<td>(0.1360)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spline (20)</td>
<td>0.9939**</td>
<td></td>
<td></td>
<td>0.7484**</td>
<td>1.2570*</td>
<td></td>
</tr>
<tr>
<td>(0.0824)</td>
<td></td>
<td></td>
<td></td>
<td>(0.1185)</td>
<td>(0.1145)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6422</td>
<td>0.6426</td>
<td>0.6432</td>
<td>0.6426</td>
<td>0.6427</td>
<td>0.6431</td>
</tr>
<tr>
<td>$N$</td>
<td>52529</td>
<td>52529</td>
<td>52529</td>
<td>25342</td>
<td>25342</td>
<td>25342</td>
</tr>
</tbody>
</table>

Notes: 1. The 1986 dummy variable is added to the right hand side of regression equation (23) for the joint sample.
2. See notes of Table 1.
this hypothesis is derived from my model, I consider another model in which the hypothesis is said to be derived; a bid-up model in which the industry with unexpectedly high employment demand would bid up young workers' wages to attract them. However, there are some problems in deriving the hypothesis from this bid-up model. First, the increase of young workers' wages is a short-period phenomenon, which is difficult to be sustained for a long period such as 10 years. If this phenomenon is sustained for 2 or 3 years, the wages of the enterers in this period are relatively high not only for inexperienced workers but also for experienced workers. Rather employers may scout for experienced workers who has a lot of industry-specific human capital, so that the slope of the experience-wage profile may increase with the employment growth rate. And if the phenomenon is sustained for a long period such as 10 years or more, the experienced workers' wages are higher than those of other industries when the inexperienced workers become experienced workers, so that the level of the experience-wage profile goes up but the slope is not changed. Second, if inexperienced workers' human capital has a distribution, the industry with an unexpectedly high employment hires workers of relatively lower human capital, so that rather the average of the inexperienced workers' wages may be lower. Because of these two reasons the above hypothesis can be said to be derived only from my model.

V. Conclusions

This paper has developed a new model of on-the-job learning, based on the teacher-student ratio. The model postulates that younger workers learn from more-experienced workers and that if younger workers have more or better more-experienced workers in their firms, they learn more relative to younger workers in other firms. This gives a different interpretation of wage than other human capital models; a young worker's wage equals his value of marginal product less his tuition for learning from his seniors, and an old worker's wage equals his value of marginal product plus his reward for teaching his juniors. The shape of the experience-wage profile is affected by the industry employment growth rate. Assuming a linear production function and a power human capital function, a more rapidly growing industry in terms of employment has a flatter experience-wage profile.
This hypothesis is supported by the 1984 and 1986 Korean Occupational Wage Surveys at least for workers of 15 (or 20) years or less of experience.

The human capital function is found to have no effect on the steady-state output growth rate if the proportional rate of learning is assumed to be strictly concave in the effective teacher-student ratio.

I provide a stability condition of a generalized model, under which a new-born industry’s productivity increases at a decreasing rate.

According to my model, the parameters of the model such as $c$ or $\alpha$ should be improved in order to improve productivity through on-the-job learning. Therefore the future research should be done on what organization, what industrial relations, and what communication channels in the firm improve $c$ or $\alpha$. For this purpose the human capital accumulation function should be estimated at the levels of firms and nations as well as industries, and the relationships between the function and organization, industrial relations, and communication channels should be studied.

Appendix A

The time path of the normalized total human capital of old workers follows a difference equation:  
\[ k_{t+1} = \lambda^{-1} k_t h(x(k_t)/k_t). \]  
(A1)

A normalized total human capital stock of old workers $k > 0$ is a stationary solution to (A1) if and only if it is a solution to:
\[ x(k) = \delta k, \]  
(A2)
since $h(\delta) = \lambda$. The solutions to (A2) are described in the next proposition. I assume that $x(k) > 0$ for all $k > 0$. Define the function $U(x, k)$, $(x, k) \in [0, \infty) \times [0, \infty)$, by:
\[ U(x, k) = s(kf(x/k), 0), \]
where the function $s$ is from (16), so that $U$ is a strictly increasing, strictly concave, and continuously differentiable function of $(x, k)$.  

\[ v(k) = \max_{x > 0} [U(x, k) - Wx + \beta \lambda v \lambda^{-1}kh(x/k)] \]  
(N3)

\[ \text{See the proof of Proposition 4 for the normalization.} \]

\[ \text{The functional equation (21) becomes:} \]

\[ v(k) = \max_{x > 0} [U(x, k) - Wx + \beta \lambda v \lambda^{-1}kh(x/k)] \]  
(N3)
Proposition A1
Suppose the following (A3)-(A5) are satisfied:

\[ |1 - \beta \lambda + \beta \delta h'(\delta)| \delta U_{xx}(\delta k, k) + U_{kk}(\delta k, k) \]
\[ + \beta h'(\delta) U_{kk}(\delta k, k) + \delta U_{xk}(\delta k, k) < 0, \text{ for all } k > 0, \quad (A3) \]

\[ \lim_{k \to \infty} |1 - \beta \lambda + \beta \delta h'(\delta)| U_x(\delta k, k) + \beta h'(\delta) U_k(\delta k, k) \leq W |1 - \beta \lambda + \beta \delta h'(\delta)|, \quad (A4) \]

\[ U(x, k) \geq 0 \text{ for all } (x, k) \in [0, \infty) \times [0, \infty). \quad (A5) \]

Then equation (A2) has a solution \( k^c > 0 \) if and only if

\[ |1 - \beta \lambda + \beta \delta h'(\delta)| U_x(0, 0) + \beta h'(\delta) U_k(0, 0) \]
\[ > W |1 - \beta \lambda + \beta \delta h'(\delta)|. \quad (A6) \]

A positive stationary solution, if it exists, must satisfy:

\[ |1 - \beta \lambda + \beta \delta h'(\delta)| U_x(\delta k, k) + \beta h'(\delta) U_k(\delta k, k) \]
\[ = W |1 - \beta \lambda + \beta \delta h'(\delta)|, \quad (A7) \]

so there is at most one positive solution to (A2).\(^{23}\)

Proof: I first show that any solution to (A2) satisfies (A7). From (21), I have:

\[ v(k) \geq U(\delta k, k) - \delta kW + \beta \lambda v(k) \]

(since \( x = \delta k \) is always feasible) for all \( k \) with equality if and only if \( k \) satisfies (A2). Then collecting terms,

\[ v(k) \geq (1 - \beta \lambda)^{-1} |U(\delta k, k) - \delta kW| \quad (A8) \]

with equality if and only if \( k \) satisfies (A2). If \( k \) satisfies (A2), (21) implies:

\[
U(\delta k^c, k^c) - \delta k^cW + \beta \lambda v(k^c) \geq U(x, k^c) - Wx + \beta \lambda v(\lambda^{-1}k^c h(x/k^c))
\]

for all \( x \geq 0 \). Now applying (A8), which holds with equality at \( k^c \),

\[
U(\delta k^c, k^c) - \delta k^cW + \beta \lambda \{U(\delta k^c, k^c) - \delta k^cW\} \geq U(x, k^c) - Wx + \beta \lambda \{U(\lambda^{-1}k^c h(x/k^c), \lambda^{-1}k^c h(x/k^c)) - \delta \lambda^{-1}k^c h(x/k^c)W\} \geq U(x, k^c) - Wx + \beta \lambda \{U(\lambda^{-1}k^c h(x/k^c), \lambda^{-1}k^c h(x/k^c)) - \delta \lambda^{-1}k^c h(x/k^c)W\} / (1 - \beta \lambda)
\]

\(^{23}\)The proof of this proposition follows the proof of Lemma 5 in Lucas and Prescott (1971).
for all $x \geq 0$. At $x = \delta k^c$, this inequality holds with equality. At any other value of $x$, the right-hand side takes on a smaller value, which is to say that $\delta k^c$ maximizes the expression on the right. Then the first-order condition:

$$
|1 - \beta \lambda + \beta \delta h(\delta)| U_\delta(\delta k, k) + \beta h(\delta) U_k(\delta k, k) = W |1 - \beta \lambda + \beta \delta h(\delta)|
$$

is satisfied. If (A6) does not hold, no $k^c > 0$ satisfies this condition, so the necessity of (A6) is proved. Further, I have proved that positive stationary solutions satisfy (A7) and that since the left-hand side of (A7) is strictly decreasing in $k$ by (A3) and converges to a number not larger than $W |1 - \beta \lambda + \beta \delta h(\delta)|$ by (A4), there is at most one such solution. To see that (A6) is sufficient for the existence of a positive solution to (A2), I must rule out the possibilities that $x(k) > \delta k$ or $x(k) < \delta k$ for all $k > 0$. To rule out the former possibility, recall that $x(k)$ is bounded, so that for $k$ sufficiently large, $x(k) < \delta k$. To rule out the latter possibility, suppose that (A7) has a positive solution $k^24$ and that $x(k) < \delta k$ for all $k > 0$. Define the function $z(k)$ by:

$$
z(k) = \max[(1 - \beta \lambda)^{-1} |U(\delta k, k) - \delta kW|, 0]
$$

so that from (A8), $z(k) < \nu(k)$ for all $k > 0$. Now, define the operator $H_x$ on bounded continuous functions on $(0, \infty)$ by:

$$
H_x y(k) = U(x(k), k) - W x(k) + \beta \lambda y |\lambda^{-1} k h(x(k)/k)|
$$

It is readily verified that for any $y, z$ in the domain of $H_x$, $|H_x y - H_x z| \leq \beta \lambda \|y - z\|$. Also $H_x \nu(k) = \nu(k)$ where $\nu$ is the solution to (21), and for any $y$,

$$
\lim_{n \to \infty} H^n_x y = \nu(k). \quad (A9)
$$

I next show that for $z(k)$ as defined above, $H_x z(k) < z(k)$ for all $k \in (0, k]$. I have, directly from the definitions of $z$ and $H_x$, and using (A5),

$$\begin{align*}
H_x z - z & \leq U(x(k), k) - U(\delta k, k) \\
& + \beta \lambda [U |\lambda^{-1} \delta k h(x(k)/k), \lambda^{-1} k h(x(k)/k)| - U(\delta k, k)](1 - \beta \lambda) + W |-x(k) - \beta \delta k h(x(k)/k)/(1 - \beta \lambda) \\
& + \delta k/(1 - \beta \lambda)|.
\end{align*}
$$

If (A6) is satisfied, (A7) has a positive solution $k$ because I assume (A3)-(A4).
The strict concavity of $U$ and $h$ implies:

$$U \left| \lambda^{-1} \frac{\partial}{\partial u} \frac{kh(x(k)/k)}{\lambda^{-1}kh(x(k)/k)} \right| - U(\delta k,k) \leq |U_x(\delta k,k) \delta + U_k(\delta k,k)k| \lambda^{-1}h(x(k)/k) - 1|, \quad (A11)$$

$$\lambda^{-1}h(x(k)/k) - 1 < \lambda^{-1}h'(\delta) |x(k)/k - \delta|, \quad (A12)$$

$$U_x(\delta k,k) - W < U_x(x(k),k) - W < 0, \quad (A13)$$

where the second inequality in (A13) comes from the first-order condition of (21)\(^{25}\). Combining (A10)–(A13), (A5), and (A7), then gives, for $k \in (0, k^c]$,

$$H_x z < z. \quad (A14)$$

One may also verify that $y(k) < z(k)$ over the interval $(0, k^c]$ implies $H_x y(k) < H_x z(k)$ on this interval. Thus (A9) and (A14) together implies for $k \in (0, k^c]$

$$v(k) = \lim_{n \to \infty} H_x^n z(k) < z(k),$$

which contradicts (A8). This completes the proof.

Under conditions (A3)–(A4), the left-hand side of (A7), which is the first-order condition of maximization problem (21)(or (N3)) with the envelope theorem applied in the steady-state, decreases and converges to a number not larger than $W |1 - \beta \lambda + \beta \delta h'(\delta)|$ as $k$ goes to infinity.

Appendix B

<table>
<thead>
<tr>
<th>TABLE B1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NUMBER RATIOS OF WORKERS BY ESTABLISHMENT SCALES</strong></td>
</tr>
<tr>
<td>(Unit: %)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>10–29</td>
</tr>
<tr>
<td>30–99</td>
</tr>
<tr>
<td>100–299</td>
</tr>
<tr>
<td>300–499</td>
</tr>
<tr>
<td>500–</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: *OWS* is the Occupational Wage Survey and *SROELC* is the Survey Report

\(^{25}\)See equation (22).
on Establishment Labor Conditions. Sample is selected such that it has similar number ratios of workers to those in SROELC.


**Table B2**

**EMPLOYMENT GROWTH RATES OF INDUSTRIES**

<table>
<thead>
<tr>
<th>Industry</th>
<th>KSIC Number</th>
<th>Employment Growth Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foods</td>
<td>311, 312</td>
<td>3.16</td>
</tr>
<tr>
<td>Beverages</td>
<td>313</td>
<td>3.48</td>
</tr>
<tr>
<td>Textiles</td>
<td>321</td>
<td>5.27</td>
</tr>
<tr>
<td>Apparel</td>
<td>322</td>
<td>3.49</td>
</tr>
<tr>
<td>Leather</td>
<td>323</td>
<td>11.49</td>
</tr>
<tr>
<td>Footwear</td>
<td>324</td>
<td>6.82</td>
</tr>
<tr>
<td>Wood and Cork</td>
<td>331</td>
<td>-2.18</td>
</tr>
<tr>
<td>Furniture</td>
<td>332</td>
<td>10.39</td>
</tr>
<tr>
<td>Paper</td>
<td>341</td>
<td>6.23</td>
</tr>
<tr>
<td>Printing</td>
<td>342</td>
<td>3.39</td>
</tr>
<tr>
<td>Industrial Chemicals</td>
<td>351</td>
<td>6.79</td>
</tr>
<tr>
<td>Other Chemical Products</td>
<td>352</td>
<td>10.20</td>
</tr>
<tr>
<td>Petroleum Refineries</td>
<td>353</td>
<td>1.70</td>
</tr>
<tr>
<td>Miscellaneous Products of Petroleum and Coal</td>
<td>354</td>
<td>1.31</td>
</tr>
<tr>
<td>Rubber</td>
<td>355</td>
<td>10.31</td>
</tr>
<tr>
<td>Plastics, n.e.c.</td>
<td>356</td>
<td>10.64</td>
</tr>
<tr>
<td>Pottery, China, and Earthenwares</td>
<td>361</td>
<td>7.81</td>
</tr>
<tr>
<td>Glass</td>
<td>362</td>
<td>8.61</td>
</tr>
<tr>
<td>Other Nonmetallic Minerals</td>
<td>369</td>
<td>4.61</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>371</td>
<td>10.54</td>
</tr>
<tr>
<td>Non-ferrous Metals</td>
<td>372</td>
<td>11.10</td>
</tr>
<tr>
<td>Fabricated Metals</td>
<td>381</td>
<td>9.41</td>
</tr>
<tr>
<td>Machinery</td>
<td>382</td>
<td>11.24</td>
</tr>
<tr>
<td>Electrics and Electronics</td>
<td>383</td>
<td>14.71</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>384</td>
<td>10.65</td>
</tr>
<tr>
<td>Medical, Photographic and Optical,...</td>
<td>385</td>
<td>9.52</td>
</tr>
<tr>
<td>Other Manufacturing</td>
<td>390</td>
<td>6.86</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>3</td>
<td>6.63</td>
</tr>
</tbody>
</table>


**References**


Ben-Porath, Y. “The Production of Human Capital and the Life Cycle of Earn-
ON-THE-JOB LEARNING