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# Dimensional Thinking and the Distance-Decay Parameter in the Gravity Model*

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《**Contents**》



## 1. Introduction

The gravity concept of interaction is expressed in its simple mathematical term as follows:

$$I_{ij}=f(P_i, P_j, d_{ij}). \quad (1)$$

The intensity or volume of interaction between two places $i$ and $j$ ($I_{ij}$) is some function of the mass of the two places($P_i$ and $P_j$) and distance between them ($d_{ij}$).[1] Since interaction is inversely related to distance, the specific form of the distance term in function (1) is expressed as $d_{ij}^{-b}$(or $1/d_{ij}^{b}$) where $b$ is a parameter to be estimated from empirical data. The distance-decay parameter $b$ in the model is obviously a key element to geographers interested in the concepts of distance and space. In spite of geographers' stress on these concepts in the discipline, it is still true that "many of the practical and conceptual difficulties associated with the use of gravity models are related to the distance functicn [particularly the distance decay parameter] which plays such an important part in the model formulation."[2]

Some criticisms about the parameter have been raised intermittently from the point of view of the dimensional analysis since the parameter is simultaneously a geographical measure of the effect of the friction of space and a mathematical exponent related to the expression of the dimension of distance term. These criticisms, however, have largely been

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1) See the following references for the gravity concept. Carrothers, G.P.A., 1956, "A historical review of the gravity and potential concept of human interaction," *Journal of the American Institute of Planners*, vol. 22, pp. 94~102.
Olsson, G., 1965, *Distance and Human Interaction: A Bibliography and Review*, Regional Science Institute, Bibliography Series, No. 2, Philadelphia.
Olsson, G., 1980, *Birds in Egg/Eggs in Bird*, Pion London.
Tocalis, T.R., 1978, "Changing theoretical foundations of the gravity concept of human interaction," Berry, B.J.L. (ed.), *Perspectives in Geography 3: The Nature of Change in Geographical Ideas*, Northern Illinois University Press, Dekalb, Il.

2) Lee, C., 1973, *Models in Planning: An Introduction to the Use of Quantitative Models in Planning*, Pergamon Press, Oxford, pp. 68~69.

ignored in the development of the model, probably because of the difficulty of incorporating them into the geographical interpretation of the model and of the empirical usefulness of the term $b$ in explaining human spatial interaction. What the criticisms have shown is that the parameter has problems when it is thought of in terms of the dimension of the distance term. These problems, however, disappear once the parameter is correctly conceptualized. Furthermore, the proper conceptualization lets the model speak for itself much more about the concept of space than before, even though problems of a lack of a sound theoretical basis of the gravity model remain.

This paper examines problems related to the meaning of the distance-decay parameter from the perspective of dimensional thinking within the Newtonian system of physics and offers a new form of the parameter. In the next section of the paper, dimension and dimensional analysis are briefly introduced. In section 3, dimensional problems of the distance-decay parameter are investigated; the parameter is analyzed in relation to the dimension of the distance term; and a new form of the parameter is derived. The new form is evaluated and interpreted in section 4. The conclusions include a summary.

## 2. Dimensional Analysis

Any spatial phenomenon can be characterized by spatial dimensions such as length, area, or volume and be better understood when considered in terms of its dimensions.[3] This kind of consideration is especially important to the development of mathematical relationships among geographical variables because geographical models expressed as mathematical equations not only must be *balanced* in their dimensions but must have *appropriate* dimensions. Although geographers seldom employ the concept of dimension in an explicit way, it is apparent that various dimensions are used in the analysis of the relationships between spatial variables[4] as "a way of looking at the world".[5]

Dimension is concerned with the units in which quantities are measured and thus is a concept of the unit of measurement.[6] The distance between two cities, for example, is measured by mile or kilometer (km) and dimensionally expressed as a dimension of unit in length $L$ where $L$ is denoted as the dimension of length. Area is measured by the multiplication of one length by the other and expressed as a dimension of 2 in length ($L \times L$ or $L^2$; e.g., $km^2$). Population density, which is one of the most widely used terms in geography, is ex-

3) Kolars, J.F. and Nystuen, J.D., 1974, ***Human Geography: Spatial Design in World Society***, McGraw-Hill, pp. 7~13.

4) Haynes, R.M., 1978, "A note on dimensions and relationships in human geography," ***Geographical Analysis***, vol. 10(3), pp. 288~292.

5) Kolars, J.F. and J.D. Nystuen, 1974, *op. cit.*, p. 9. There are several excellent reviews of dimensional analysis: Haynes, R.M., 1975, "Dimensional analysis: some applications in human geography," ***Geographical Analysis***, vol. 7(1), pp. 51~68.
Haynes, R.M., 1982, ***An Introduction to Dimensional Analysis for Geographers***, Concepts and Techniques in Modern Geography, No. 33, Geo Abstracts, Norwich.
Huntley, H.E., 1952, ***Dimensional Analysis***, Dover Publications, New York.
de Jong, F.T., 1967, ***Dimensional Analysis for Economists***, North-Holland Publishing Company, Amsterdam.
Schepartz, B., 1980, ***Dimensional Analysis in the Biomedical Sciences***, Charles C. Thomas Publisher, Springfield, Il.

6) de Jong, F.T., 1967, *op. cit.*, p. 1.

pressed as (population/area) and has a dimension of $N/L^2$ where $N$ refers to the dimesion (the unit of measurement) of population.

There are two kinds of dimensions, primary and secondary. Primary dimensions are those not divisible into or measurable in terms of other units or operations.[7] For example, in mathematics, the meaning of certain terms must be accepted as intuitively given in order to define or describe other terms. These undefined terms, upon which the subsequent logical structures of mathematics are built, are called primary dimensions.[8] Three primary dimensions, length, mass and time symbolized by $L$, $M$, and $T$, respectively, are considered in the Newtonian system of physics, and additional primary dimensions are required to describe different or more complex systems. Examples include value, population and information, which are are used in the analysis of spatial systems.[9] Secondary dimensions are expressible as specific combinations of primary ones. Population density is an example of secondary dimensions which is expressed as the combination of the primary dimensions of population and length.

Dimensional analysis is a technique used in physics and engineering for deriving theoretical equations, checking empirical formulae, designing experiments, interpreting results from scale models and converting between different systems of units.[10] In dimensional analysis, the principle of dimensional homogeneity is required from an operational point of view since, in mathematical equations, only identical operations of measurement or mathematical combinations of such operations can be equated to both sides of the equation.[11]

For example, the Newtonian Gravitational Model and its dimensional equation are expressed as equations (2) and (3), respectively:

$$F = G\ m_1 m_2 / d_{12}^2 \quad (2)$$

$$LMT^{-2} = (L^3 M^{-1} T^{-2}) MM/L^2 \quad (3)$$

where $F$ is the force with which each mass pulls the other, $m_1$ and $m_2$ are the sizes of the two masses concerned, $d_{12}$ is the distance between the two masses 1 and 2, and means "dimensional equivalent." to The dimensions of both sides of the equation (2) are balanced by the dimensional constant $G$ which is expressed as $(L^3 M^{-1} T^{-2})$ in equation (3). The gravity model as a direct analogy of the equation (2) and its dimensional equation are represented as equations (4) and (5), respectively:

$$I_{ij} = k\ P_i P_j / d_{ij}^2 \quad (4)$$

$$N^2 = (L^2) NN/L^2 \quad (5)$$

where $k$ is a constant. Both sides of equation (4) are equated in their dimensional expressions in equation (5) by the constant k which is expressed as $L^2$.

## 3. Dimensional Thinking and the Distance-Decay Parameter

### (1) Dimensional Problems of the Parameter

In the gravity model, interaction is expressed as relationships among the variables of the population size of two places and the distance between them. Equation (6) is a more flexible form of the model than equation (4) in expressing the distance term.[12]

$$I_{ij} = kP_i P_j / d_{ij}^b, \quad (6)$$

7) Schepartz, B., 1980, *op. cit.*, p. 11.
8) Kolars, J.F. and J.D. Nystuen, 1974, *op. cit.*, p. 10.
9) Haynes, R.M., 1975, *op. cit.*, pp. 52~53.
10) *Ibid.*, p. 51.
11) Schepartz, B., 1980, *op. cit.*, p. 15.
12) The parameters in population terms are not used because the interest in this paper is only in the

With the comparison of the two distance terms, $d_{ij}^2$ and $d_{ij}^b$ in equations (4) and (6) respectively, it is clear from dimensional considerations that the parameter $b$ is the reciprocal of 2 and thus can have the same kind of meaning as 2. However, $b$ is much more flexible than 2 in measuring and explaining the effect of the friction of space on interation empirically because it can take any positive real number. This is one of the reasons that the gravity model has evolved from equation (4) to equation (6) and been widely used in interaction studies.

In the process of this evolution, there have been some arguments about the form of the distance-decay parameter in terms of dimensional considerations. These arguments fall into three categories: 1) the meaning of the change of the parameter in the model; 2) the meaning of the parameter in relation to the distance term; and 3) the role of the parameter in relation to the constancy of $k$ as a dimensional constant.

A note of skepticism related to the first category came from Stewart and Warntz[13] nearly three decades ago. They raised doubts about the arbitrary adjustment of the exponent of distance (the distance-decay parameter) in the gravity model from a theoretical persepective. Although they did not state explicitly the meaning of the distance-decay parameter, it is clear that the exponent was considered as the dimension of the distance term and thus they thought that:

> The recent rapid increase in the number of papers and articles employing the so-called "gravity model"...... has unfortunately brought with it much confusion concerning the exponent of distance to be employed. Whereas the 'weight' assigned to people must be adjusted to fit observations, the function of distance is not such an arbitrarily adjustable parameter.
>
> (Stewart and Warntz, 1958, 116)

The change of the exponent of distance had brought confusion about the meaning of the model as well as of the exponent itself.[14] One confusion is that the change in the unit of measurement (dimension) in some variable(s) in a model brings a different concept to the model. For example, population density is equal to the concept of population divided by area and is dimensionally represented as $(N/L^2)$. However, if population is divided by length such as miles or km, it has a dimension of $(N/L)$ and represents not the concept of population ***density*** but population ***potential*** in terms of its dimension. The change of dimension from area $(L^2)$ to distance $(L)$ has brought a change of a basic concept in the model: from population density to population potential. Thus the change of some dimension(s) in a model makes the model a different one.

This logic can be extened to the analysis of the gravity model. In equation (6), the nominator $P_iP_j$ is divided by $d_{ij}^2$, $d_{ij}^{1.5}$ or distance with some other exponent according to the results of the parameter estimation. Mathematically, if the value of the parameter $b$ is changed, then the dimension of distance term is also changed and the meaning of the

parameter of the distance term. In the same reason, the dimension of interaction is simply defined as $N^2$.

13) Stewart, J.Q., and W. Warntz, 1958, "Physics of population distribution," *Journal of Regional Science*, vol. 1, pp. 99~123.

14) The suggestion by Stewart and Warntz for retaining the exponent as 2 appears sound from a mathematical point of view, but any further comments in support of this idea have not been followed up by them.

whole model must be changed too. A gravity model having, for example, a dimension of 2 in length in its distance term ($P^2/L^2$) must be a different model from that having a dimension of 1.5 in length in the distance term($P^2/L^{1.5}$). If we admit this reasoning, then we can say little in general about results produced by gravity models since almost all models estimated have different and unique dimensions and can not be compared. In order to be interpreted as same model, the model must retain the same dimension for the distance variable.

The second argument concerns the meaning of the distance term itself, especially when the parameter has a real value. When, for example, the parameter is 1.5 in its value, the distance term becomes ${d_{ij}}^{1.5}$ and the dimension is $L^{1.5}$. What does a distance term in dimension of $L^{1.5}$(e.g., $km^{1.5}$) mean?[15] A dimension of unit in length ($L$) means length expressed in km; the square ($L^2$) means area, $km^2$; and the cube ($L^3$) means volume. The dimension of length has a discrete integer value in these cases and there are no real values such as 1.5 or 0.75. In this regard the meaning of distance terms having real value parameter such as 1.5 or 0.75 (e.g., $km^{0.75}$) are bizarre and puzzling, at least in interaction model.

An argument about the third category is that the dimensional constancy in the constant $k$ must be retained. The gravity model expressed as in equation (6) has to follow the mathematical rule of the dimensional constancy in constant $k$ and the principle of dimensional homogeneity to become an equation. However, in order to follow the principle of dimensional homogeneity of the equation, "the 'constant' $k$ must somehow change its very unit of measurement"[16] according to the values of $b$, the distance-decay parameter, as shown below. The dimensional expression of equation (6) is as follows:

$$N^2=(L^b)\ NN/L^b \qquad (7a)$$

The constant $k$ is expressed as $L^b$ in its dimension to balance both sides of equation (7a). However, the dimension of $k$ becomes $L^2$ when $b=2$ and $L^{1.5}$ when $b=1.5$ as in the following equations (7b) and (7c), respectively:

$$N^2=(L^2)\ \mathrm{NN}/L^2 \qquad (7b)$$

$$N^2=(L^{1.5})\ NN/L^{1.5} \qquad (7c)$$

The constant $k$, which is also a dimensional constant, no longer has the concept of constancy in equations (7b) and (7c) because it changes its unit of measurement (dimension) according to the results of the estimation of $b$ value. In this sense, "Haynes's analysis casts serious doubts on the theoretical acceptability of conventional gravity models, whatever their very considerable practical value in flow estimation".[17]

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15) Stewart, J.Q. and W. Warntz, 1958, *op. cit.*, p. 119.
Taaffe, J. and H.L. Gauthier, 1973, *Geography of Transportation*, Prentice-Hall, Englewood Cliffs, N.J., p. 98.

16) Haynes, R.M., 1975, *op. cit.*, p. 58.

17) Haggett, P., A.D. Cliff, and A. Frey, 1977, *Locational Analysis in Human Geography*, London, Edward Arnold, p. 37.
Haynes (1975) has discussed this problem in the (pareto) gravity model such as equation (6), and suggested negative exponential distance-decay functions as the alternatives. The function and its dimensions are expressed as follows, respectively:

$$I_{ij}=kP_iP_j\exp(-mD_{ij})$$

$$N^2=1\ N\ N\ 1L^{-1}\ L$$

In the negative exponential model, both sides of the dimensional equation are balanced and the distance-decay coefficient, $m$, is expressed as $L^{-1}$ in its dimension and interpreted as the proportion of interaction decrease per unit distance. The negative exponential model has no problem in its dimensionality, but the meaning of $m$ is different from that of $b$ in the gravity model.

### (2) Dimensional Thinking and the Parameter

If there are no answers to the preceding arguments, then the problem is far beyond of Haggett, et. al.'s comment because Geography has employed this mathematically suspicious equation in one of its core areas. The arguments can be reduced to the following statement: the gravity model is empirically useful but mathematically suspicious. The arguments so far have in common that the distance-decay parameter is thought of not geographically but rather mathematically the dimension of the distance term. In the discussion of distance-decay parameter as the dimension of the distande term, the three arguments share one element: they are related to the distance-decay parameter whice determines the dimension of the distance term. If a model is "dimensionally unbalanced this is a clear signal that *something* is wrong"[18] (italics mine). What is, then, the 'something' in the gravity model: is it in the model itself or in the process of interpretation?

In order to find the answer of the above question, let us first think of the square term within the distance variable in equation (4) since the gravity model has evolved from equation (4) to equation (6) and has been named after the Newtonian Gravitational Model in equation (2) which is closer to equation (4) than to equation (6) in its form. The square term in equation (4) is related to the dimension of the distance term. Distance with a square exponent, $d_{ij}^2$, is an area term which describes the 'size' of an 'area' and has a dimension of 2 in length ($L^2$). For example, if $d_{ij}$ is 5km, then the $d_{ij}$ term represents neither ($5^2$km)

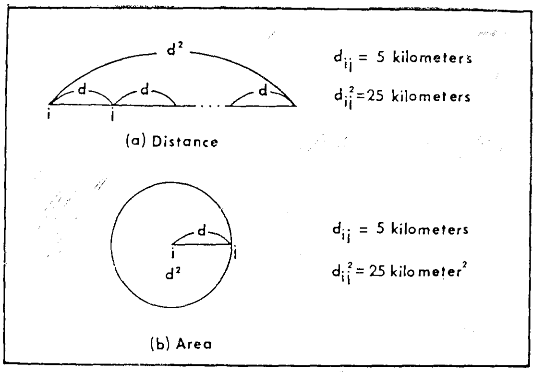


**Fig. 1.** The Concept of $d_{ij}^2$

nor (25km) but $(5\text{km})^2$, ($5^2\text{km}^2$) or ($25\text{km}^2$), which indicates the size of an area of a circle of radius 5km(Fig. 1).[19]

In the two different graphic presentations in Fig. 1, (a) represents $d_{ij}^2$ as a concept of distance and, therefore, its dimension becomes $L$; and (b) represents it as concept of area and its dimension becomes $L^2$. It is clear in equation (2), the Newtonian Gravitational Model, that the $d_{ij}^2$ term has the meaning of case (b) because the dimension is expressed as $L^2$ only in the latter case. Therefore, the 2 in the $d_{ij}^2$ term in equation (2) or (4) has a specific meaning if we think of it in relation to the dimension of distance term as (b) in Fig. 1: it is an exponent which makes the distance term the area of a circle of radius $d_{ij}$. This exponent makes the model work for an area because movements do not take place along one dimensional lines linking pairs of points but over a two-dimensional area.[20] Therefore, the gravity model postulates that "the interaction between two masses is...inversely proportional to the *space* over which the interaction must take place"[21] (italics mine).

18) Haynes, R.M., 1982, *op. cit.*, p. 18.
19) The shape of an interaction field is assumed as a circle in Fig. 1 and $\pi$ is deleted in the expression of the size of the area because it can be absorbed into the constant $k$ term in the gravity model.
20) Haggett, A.D. P. Cliff, and A. Frey, 1977, *op. cit.*, p. 47.
21) Yeates, M., 1974, *An Introduction to Quantitative Analysis in Human Geography*, New York, McGraw-Hill, p. 125.

Although $d_{ij}^2$ can be divided into '$d_{ij} \times d_{ij}$' mathematically, it has a unit concept itself in representing interaction: the 'space.'[22] The term $d_{ij}^2$ represents the 'stage' on which people play out their interaction. Even when $b$ has the value of 1.5, it is hard to deny that the interaction occurs over a two-dimensional area or space. Even when the exponent is changed from 2 to some other real value, there is no change in the situation in which interaction takes place. The spatial dimension of interaction is same whether the exponent remains as 2 or takes other values.

If one admits that $d_{ij}^2$ in equation (4) is an area term and that interaction must be understood on the basis of the area term in its spatial perspective, the same argument can be extended to the interpretation of $d_{ij}^b$ in equation (6). The parameter $b$ in $d_{ij}^b$ is the reciprocal of 2 in $d_{ij}^2$. In order to understand the meaning of the parameter $b$, it is necessary to investigate the variable $d_{ij}$ in detail. The variable $d_{ij}$ is consisted of two parts: one is the magnitude of the variable and the other is the dimension of the variable. For example, the distance between Seoul and Pusan is roughly 400km, which is constituted of both '400' (the magnitude) and 'km' (the dimension). The unit of measurement (dimension) is automatically defined by the word 'distance' in our present system of knowledge. In contrast, the magnitude is determined by the actual measurement of distance. Therefore, it is important to distinguish whether the parameter is raised in either the magnitude or the dimension, or both of them when we raise an exponent on the variable in empirical formula. From the above discussion it is reasonable to think that interaction takes place over space, more specifically over two-dimensional area. This means that the basic dimension of spatial interaction is space and, therefore, there must be a term which indicates the concept of space in the model. This is the very square term in the dimension. Therefore, as long as the spatial interaction takes place over space, the square term must be kept. If the exponent 2 can not be changed the focus is on the magnitude. It is important at this point to understand that what the gravity model says is simply not the tautology, that is, the interaction is negatively proportional to distance, but, more importantly, the spatial relation of two interacting places in terms of their interaction. This relation could be expressed as distance,[23] and it is safe to say that it is this magnitude of distance we look for by raising a parameter in the distance term. A parameter, therefore, can be raised on the magnitude term to measure the spatial relation in regard to interaction.

The distance-decay parameter $b$ which is estimated from empirical studies is actually a combination of two exponents: one for dimension and the other for spatial relation. The former is fixed and gives mathematical rigor to the model. The latter is flexible and allows the empirical utility of the model. Thus the parameter, $b$, can be expressed as follows;

$$b=2c \qquad (8)$$

where $2c$ is the new distance-decay parameter, and 2 is a fixed and $c$ is a flexible parameter which will be interpreted later. The new distance term is expressed as the right-hand sides of equations (9a) or (9b):

$$d_{ij}^b=d_{ij}^{2c} \qquad (9a)$$

$$=(d_{ij}^c)^2. \qquad (9b)$$

Therefore, the new gravity model becomes:

$$I_{ij}=kP_iP_j/d_{ij}^{2c} \qquad (9c)$$

22) Strictly speaking, it represents the area or surface.

23) For the concept of distance as spatial relation, see Gatrell, A., 1983, *Distance and Space: A Geographical Perspective*, Clarendon Press, Oxford.

The expression in equation (9b) shows the concept of $c$ more clearly than in equation (9a): $c$ is related not to the expression of dimension but only to the magnitude of the distance term. It is worthwhile mentioning that the equation (9c) is nothing but a mathematical expression of Yeates' statement quoted above.

The concept of $c$ can be more easily understood if both sides of the equation (9b) take an actual dimension. First, let us think of the term on the left-hand side of equation (9b). If the distance is measured as ***km***, $d_{ij}^b$ can be expressed as $(d_{ij}\ km)^b$ which is equal to $(d_{ij}^b\ km^b)$.[24] Therefore, if $b$=1.5, this becomes $(d_{ij}\ km)^{1.5}$ or $(d_{ij}^{1.5}\ km^{1.5})$. This is the method traditionally used to interpret the distance-decay parameter $b$. Problems related to this procedure have been discussed above. Second, let us think of the term on the right-hand side. With the example above, $(d_{ij}^c)^2$ can be expressed as $(d_{ij}^c\ km)^2$ which is equal to $((d_{ij}^c)^2\ km^2)$.[25] If $b$=1.5, the value of $c$ becomes 0.75 by equation (8) and the new distance term becomes $(d_{ij}^{0.75}\ km)^2$ or $((d_{ij}^{0.75})^2 km^2)$. In this case, the dimension of the distance term is 2 of length ($L^2$), which is a constant integer *regardless* of the results of the parameter estimates. In this example, the term $(d_{ij}^{0.75})^2$ represents the size of an area of a circle of radius $(d_{ij}^{0.75}km)$. Thus it is easily understood in the gravity model expressed in equation (9c) that the dimension of the distance term is 2 in length ($L^2$) and that the magnitude of distance itself is represented as $d_{ij}^c$. Therefore the gravity model, adapted by the distance exponent $2c$ as in equation (9c), has flexibility in measuring the effect of the friction of space as does $b$ in equation (6) but it does so without any problem in its dimensions in mathematical terms.

## 4. Evaluation and Interpretation of the New Distance-Decay Parameter

The $2c$ is evaluated and interpreted on the basis: 1) the dimensional stability of the gravity model; 2) 2c as a substitute for $b$; and 3) the interaction distance.

### (1) The Dimensional Stability of the Gravity Model

The new form of the gravity model (equation 9c) is expressed dimensionally as follows:

$$N^2=(L^2)NN/L^2. \qquad (10)$$

In equation (10), both sides of the equation are balanced (dimensional homogeneity), and the three arguments discussed earlier have been satisfied: there is no change in the dimension of any variable by empirical application (the first category of the three arguments); there are no unclear or puzzling dimensions such as $L^{1.5}$ or $L^{0.75}$(the second category of argument); and the constant $k$ is always expressed in its dimension as $L^2$ (constancy of the dimension of $k$; the third category of argument). Therefore, the equation (9c) is not only dimensionally stable but clear in the meaning of its dimensions. This stability indicates that the model can always be interpreted as encompassing the same concepts.

### (2) 2*c* as a Substitute for *b*

The distance-decay parameter $b$ is, as the name implies, an indicator of the effect of the friction of space on spatial interaction. This

24) In the latter expression, $d$ represents only the magnitude in contrast to the former which represents both the magnitude and the dimension.

25) $(d_{ij}^c)^2$ will not be represented as $(d_{ij}^c km^c)^2$ because the dimension of the distance term in the gravity model is $(L^2)$.

meaning is represented mathematically as the slope of the regression line indicating the relationship between the interaction and the distance when the gravity model is transformed into natural logarithmic form in order to estimate its parameters as in equation (11):

$$\ln I_{ij} = \ln k + \ln P_i + \ln P_j - b \ln d_{ij}. \quad (11)$$

Therefore, the greater the value of $b$, the less the interaction between the two places, $i$ and $j$, is likely to be, and, at the same time, the steeper the slope. The new parameter $2c$ must also have this meaning to substitute for $b$. In order to represent this meaning, the value of $2c$ must be equal to that of $b$, and this is already defined in equation (8). Therefore, $2c$ can be used as a substitute for $b$ without losing any information about the effect of the friction of space on spatial interaction.

### (3) The "Interaction Distance"

The flexibility which $b$ has in equation (6) is represented through the value of $c$ in the new distance-decay parameter in equation (9c). Since $c$ has nothing to do with the dimension of distance term, the changes of the value of $c$ do affect, in the interpretation of the new distance term and the new gravity model, not the dimension but only the magnitude of distance itself. This means that what is changed in the process of spatial interaction is only the magnitude of distance itself. The distance between two places $i$ and $j$ is transformed from $d_{ij}$ to $d_{ij}^c$ by their interaction, and the change of distance is represented by the value of $c$. If the value of $c$ is 1, then $d_{ij}^c$, the distance transformed, is the same as that originally measured, $d_{ij}$; if it is less than 1, $d_{ij}^c$ is also less than $d_{ij}$, which means the transformed distance is interpreted as shrunken from the original; and if it is greater than 1, $d_{ij}^c$ is also greater than $d_{ij}$, which means that the transformed is stretched from the original.

The new distance $d_{ij}^c$, which is transformed from the originally measured distance $d_{ij}$, implies the concept of relative distance which is perceived in terms of spatial interaction. However, this distance is slightly different in its concept from that of relative distance (such as time or cost) because it can be derived from the absolute distance (when the distance is measured as miles or *km*) or from relative distance (when the distance is measured as time or cost). There are no concepts in geography to represent the concept of a distance which is transformed from either absolute or relative distance in the context of interaction. The new distance form, $d_{ij}^c$, therefore, may be called ***interaction distance***. It measures the spatial relation between the origin and the destination in terms of interaction between them. When $c$ is less than 1.0 the two places are "actually 'closer' than they are in simple geographical space."[26] Put differently, the interaction distance ($d_{ij}^c$) is perceived as shorter than the actual distance ($d_{ij}$) between them. If $c$ is greater than 1.0, the opposite interpretation applies. If $c$ is equal to 1.0, the interaction distance is the same as the actual one.

## 5. Conclusions

When the distance-decay parameter has been considered in the context of the dimension of distance term, the new distance-decay parameter has been derived. The parameter is divided into two parts: one is fixed and the other flexible, each of which has meaning geographically or mathematically. The new distance-

26) Abler, R., J.S. Adams and P. Gould, 1971, *Spatial Organization*, Englewood Cliffs, N.J., Prentice-Hall, p. 392.

decay parameter, $2c$, maintains the dimensional stability of the gravity model and shows a new kind of relative distance called the interaction distance.

Although this analysis has not given any theoretical soundness to the gravity model, it has implied some new concepts related to distance and space and cast doubts on the traditional interpretation of the distance-decay parameter in the gravity model. In addition, the derivation of the concept of interaction distance has shown that the gravity model is not a simple expression of a tautological relation between population and distance but a useful measure of spatial relation of interacting two places.

However, there are several problems left unsolved or suggested in this study. One of them is the interpretation of the meaning of each of the three terms in the distance exponent; $2c$, 2 and $c$. Some of the meanings are investigated here. However, because the distance-decay parameter as $2c$ or $c$ is known to include the effect of the spatial structure as well as the effect of the friction of space,[27] the above three terms should be further investigated in regard to both of these effects theoretically and empirically. The other is about the philosophical system which is related to the interpretation. In this study, the interpretation of the dimension of the model has strictly followed the Newtonian system of physics. However, it will be also fruitful to interpret the dimension or the model itself in regard to other systems, for example, the Einstein's theory of gravitation[28] or the concept of Riemannian space.

The idea of the gravity concept is relatively extensively applied to the study of human spatial interaction. However, to investigate the meaning of distance and space through the gravity concept in regard to human aspects remaines as one of the diffcult things we have to deal with.

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27) Curry, L., 1972, "A spatial analysis of gravity flows," *Regional Studies*, 6, pp. 131～147.
Curry, L., D. Griffith and E. Sheppard, 1975, "Those gravity parameters again," *Regional Studies*, 9, pp. 289～296.
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# 重力모델의 距離遞減指數의 次元分析的 硏究

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重力모델(the gravity model)은 지리학에서 가장 많이 사용되고 있는 모델들 중의 하나로서, 空間的 相互作用이 거리의 증가에 따라 어떻게 감소하는 가를 측정할 수 있는 유용한 모델이다. 본 연구는 이 모델의 실제적용 및 그 결과의 해석에 있어서 가장 관심을 끌어온 距離遞減指數(the distance-decay parameter)를 거리 변수의 次元(dimension)이라는 관점에서 그 문제점을 분석하고 새로운 형태의 지수를 도출하여 이를 평가분석한 것이다.

중력모델에 나타나는 차원상의 문제점은 모두 3가지로 요약될 수 있으며, 모두 거리체감지수의 변화에 관계되어 있다. 이 지수를 물리학의 Newton 모델에 비추어 해석하였을때, 지리학에서 사용되는 거리체감지수는 서로 다른 2개의 지수로 분리되며, 따라서 중력모델은 $I_{ij}=k\frac{P_i P_j}{d^{2c}}$로 표현될 수 있다. 새로운 거리체감지수(2c)의 유용성은 다음과 같이 요약될 수 있다.

첫째, 중력모델에 차원상의 安定性을 부여하여 준다.

둘째, 과거의 지수가 가지는 정보량을 상실함이 없이 대체 사용될 수 있다.

마지막으로, 거리변수와 다른 한 지수의 결합($d^c$)은 相互作用距離(the interaction distance)라는 두 지점간의 공간적 관계를 측정할 수 있는 개념을 제공하여 준다.

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