Time Complementarity and the Behavior of Asset Returns

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The testable restrictions on the behavior of asset returns are investigated when variable time preference exhibits time complementarity. It is shown that as time complementarity (i.e., temporal risk preference) increases, the volatility of asset prices increases. It is also shown that time complementarity helps explain excessively volatile asset returns and excessively smooth consumption growth. Our empirical study confirms this result. Variable time preference exhibiting time complementarity improves the overall fitness but there is a substantial evidence when the restrictions are imposed simultaneously for different assets. Consumption from services appears to be a major source of the empirical puzzles documented in the literature.

I. Introduction

Many objections have been made to time additive preference in the theoretical grounds (for example, see Hicks 1964; Lucas 1978 among others). Even though this preference provides analytical simplicity, it has a severe restriction that the ratio of marginal utilities of consumption between any two periods is not affected by consumption at other periods. Consequently, the degree of intertemporal dependence is overly limited under this preference.

There is a growing body of empirical literature which examines a representative consumer model with time additive preference. This literature has demonstrated that a standard representative consumer model with time additive preference. This literature has demonstrated that a standard representative consumer model with time additive preference is at least problematic to explain the data. A number of authors including Shiller (1979), Grossman and Shiller (1981), Mankiw, Romer and Shapiro (1985), West (1988) and Campbell and Shiller (1988a, b) have shown that asset prices have been too volatile to be explained by changes in realized dividends for the model with time additive preference.

There has been an observation which has received considerable attention in the literature of asset pricing. Consumption growth during the post World War II period seems to be much too smooth relative to asset real returns for models with time additive preference to explain the data (see Mehra and Prescott 1985; Deaton 1987). For instance, Mehra and Prescott (1985) state "In a private conversation, Fisher Black using the Merton (1973) continuous time model with investment opportunities constructed an example with a curvature parameter of 55". The model with time additive preference needs an unreasonable level of risk aversion such that extreme risk aversion gives enough variability of the marginal rate of substitution to fit the data.

Other studies have shown that the data reject the overidentifying restrictions from the model with time additive preference (for example Hansen and Singleton 1982, 1983; Dunn and Singleton 1986; Mankiw, Rotemberg and Summers 1985).

These studies perhaps suggest that it may be useful to investigate the model with nonseparable preference. In fact, several studies employ nonseparable preference as an alternative to time additive preference. For example, Dunn and Singleton (1986) use a non-time-separable utility function with consumption flows from durables as well as nondurables plus services. Even though this type of nonseparability provides more favorable evidence than time additive preference, they still have to reject the model. Epstein and Zin (1987a, b) employ a multiperiod ordinary certainty equivalent (OCE) utility function which disentangles risk aversion from the elasticity of substitution. The OCE utility function allows state-nonseparability (see Selden 1978). Constantinides (1988) and Sundaresan (1989) use a habit-forming utility function for explaining the equity premium puzzle and the excessively smooth growth in consumption respectively.

In this paper, variable time preference replaces time additive preference. Variable time preference introduces nonseparability by allowing the subjective discount factor to change over time and to depend on the stream of past consumption. Fisher (1930) advocates that relative weights given to future as against present consumption should vary with the level of overall satisfaction derived from a whole consumption program. This motivated Koopmans (1960) to derive recursive utility functions which have such a time-preference structure. Variable time preference belongs to a class of recursive utility functions. It has been widely used in the theory of optimal economic growth by Beals and Koopmans (1969), Lucas and Stokey (1984) and Epstein (1987) among others. Variable time preference allows them to study economies with heterogeneous agents in an interesting way, ensures the uniqueness of the steady state, facilitates the steady-state analysis and eliminates the unappealing feature of the preference with a constant discount rate whereby more than one agent can own capital in a steady state only if those agents share a common discount rate.[1]

Variable time preference satisfies the von Neumann-Morgenstern expected hypothesis. Even though the OCE utility function is a recursive preference and explains the Allais paradox (see Machina 1982), it is not consistent with the expected utility hypothesis. Furthermore, variable time preference is much simpler than the OCE utility function, yet it exhibits time complementarity which time additive preference does not have. Our objective is not to examine the most general form of nonseparable preference. Conversely, it is to investigate whether a very simple form of variable time preference explains the empirical puzzles of the model with time additive preference such as excessively volatile asset returns and the excessively smooth growth in consumption and satisfies the overidentifying restrictions. Variable time preference has been examined by a number of researchers including Uzawa (1968), Epstein (1983, 1987), Lucas and Stokey (1984), Bergman (1985) and Ahn (1989). However, none of the literature on variable time preference has examined the issues described above.

In order to examine the volatility of an asset price, we use a simple general equilibrium model in an exchange economy which is similar to Lucas (1978) with the major exception of variable time preference. A closed form solution of an equity price is derived to examine the effect of time complementarity on the volatility of the equity price. It is known that the volatility of the asset price increases with increasing the usual risk aversion (see LeRoy and La Civita 1981; Michener 1982). Our novel result is that as time com-

[1]Lucas and Stokey (1984) state that using variable time preference, they analyze "economies with heterogeneous agents — economies that do not seem analyzable in an interesting way under the limits imposed by the assumption of time-additive preferences." Epstein (1987) reports the difficulties and limitations arising from the steady-state analysis under time additive preference.

plementarity increases, the volatility increases. The effect of time complementarity on the volatility appears to be much stronger than the effect of the usual risk aversion. An excessive volatility can be obtained under time complementarity while the level of the usual risk aversion is held reasonable. Excessive consumption smoothing can be resolved with time complementarity. If follows that it would be useful to test models with variable time preference with the data.

For testing models with variable time preference, we utilize the restrictions on the data implied by the first order conditions for dynamic optimum which are obtained in not only an exchange economy but also a production economy with two goods. We consider two first order conditions. The one is the trade-off of current and future consumption at a real interest rate measured in terms of nondurables. The other is the trade-off of contemporaneous consumption of nondurables and services at a real price of services measured in terms of nondurables. We introduce nonseparability even across contemporaneous consumption of these goods. The estimation method employed is the nonlinear instrumental variables procedure proposed by Hansen and Singleton (1982). We find that the empirical results under variable time preference are more favorable than those under time additive preference.

Section II introduces variable time preference. Section III develops a theoretical model to examine the volatility of the equilibrium asset price under variable time preference. It investigates whether excessively volatile asset prices can be explained. Section IV describes the empirical methodology to test the model with variable time preference. Section V explains the data and the empirical results. Section VI provides a summary and conclusions.

II. Variable Time Preference

We introduce preference with a time varying discount factor which is called variable time preference. Variable time preference employed in this paper is given by

$$U = \sum_{t=0}^{\infty} u(C_t) \exp(-\sum_{s=0}^{t-1} v(C_s)) \tag{1}$$

where $u(\cdot)$ denotes the within period utility function, $v(\cdot)$ is the discount function and C_t denotes consumption in period t. The subjective discount factor varies through time and depends on the stream of past consumption. The discount functions employed in this paper have the forms of

$$v(C_s) = aC_s + b \tag{2}$$

or

$$v(C_s) = aC_s/\text{trend}_s + b \tag{3}$$

where a and b are constants and "trend" denotes the deterministic trend of consumption in period s. If the discount function $v(\cdot)$ is a constant function (i.e., $a = 0$), then (1) becomes time additively separable. It follows that variable time preference (1) has time additive preference as its special case. Since our objective is to employ the simplest form of variable time preference, we assume that the discount function is linear. Given the smoothness of measured consumption data, this assumption appears to be at least plausible. The ratio of marginal utilities of consumption between any two periods for (1) is affected by consumption at other periods. In other words, (1) is one of nonseparable preferences. In order to ensure that the consumer's optimization problem is well defined, we need to have concavity for variable time preference. Concavity is demonstrated by the following theorem which is similar to Theorem 5 of Epstein (1983).

Theorem 1
If $\ln\text{-}u(C)$ is convex in C, then variable time preference (1) is concave.[2]

Proof: See the Appendix.

Throughout this paper, we assume that the condition of Theorem 1 is satisfied, i.e., $\ln\text{-}u(\cdot)$ is convex.

In order to examine variable time preference, we introduce the concept of temporal risk aversion and preference. Lex x and y denote consumption in any two different periods. The consumer is said to be temporally risk averse if he prefers a lottery which has equal probability of $(\underline{x}, \bar{y})$ and $(\bar{x}, \underline{y})$ to one which has equal probability of $(\underline{x}, \underline{y})$ and $(\bar{x}, \bar{y})$ where $\underline{x} < \bar{x}$ and $\underline{y} < \bar{y}$. In other words, a negatively correlated consumption stream is valued more than a positively correlated consumption stream. This result occurs because he is afraid of getting the worst outcome, $(\underline{x}, \underline{y})$. An equal chance of

[2]The condition for the within period utility function, $u(\cdot)$, is satisfied if the function is a negative exponential utility function or a power utility function with a positive relative risk aversion coefficient.

obtaining the worst outcome, (x, y), overrides an equal chance of obtaining the best outcome, $(\bar{x}, \bar{y})$, for a temporally risk averse consumer. Richard (1975) demonstrates that a necessary and sufficient condition for temporal risk aversion is the negativity of the cross partial derivative of the utility function with respect to consumption in two different periods (i.e., $\partial^2 U/\partial x \partial y < 0$). If this partial derivative is zero or positive, the consumer is said to be temporally risk neutral or risk seeking.

We define the concept of time complementarity and substitutability of consumption. The consumer is said to exhibit time complementarity (time substitutability) if and only if $\partial^2 U/\partial x \partial y > 0$ (< 0). This definition is motivated by the notions of factor complementarity and substitutability. Given the definitions, time complementarity is equivalent to temporal risk preference and time substitutability is equivalent to temporal risk aversion.

It is easy to show that time additive preference exhibits temporal risk neutrality. If "a" in (2) or (3) is positive, temporal risk aversion (time substitutability) for variable time preference can be easily established since it is sufficient to show that the cross partial derivative of (1) is negative. This can be done by straightforward differentiation. On the other hand, if "a" is negative, temporal risk preference (time complementarity) can be obtained since the cross partial derivative is positive. Thus variable time preference can exhibit temporal risk aversion or preference (time substitutability or complementarity) depending on the value of "a".

It follows from (1), (2) and (3) that if "a" is positive, as current consumption increases, the discount factor for future consumption decreases. This case is referred as increasing marginal impatience. Uzawa (1968), Epstein (1983) and Lucas and Stokey (1984) assume that $v_C > 0$ which implies a positive "a". In this case, monotonicity is always satisfied, i.e., marginal utility of consumption is positive, given the condition in Theorem 1.

On the other hand, if "a" is negative, as current consumption increases, the discount factor for future consumption increases. It is referred as decreasing marginal impatience which is advocated by Fisher (1930). However, if "a" is a sufficiently big negative value, the discount factor for future consumption may be greater than unity and monotonicity may not be satisfied. Certainly, these results are economically implausible. Thus throughout this paper, we assume that "a" is large enough to yield monotonicity and the discount factor less than unity. Temporal risk aversion as well as preference are consistent with concavity of the utility index as shown in Theorem 1 so that they are compatible with a positive risk premium. It is an empirically interesting issue to investigate whether time complementarity (or temporal risk aversion) is compatible with the data.

For the later analysis, we need a measure for the degree of temporal risk aversion. Define $-(\partial^2 U/\partial C_s \partial C_t)/(\partial U/\partial C_t)$ to be the measure of temporal risk aversion or equivalently time substitutability, where C_t denotes consumption at a later date than C_s. It can be shown that as the value of this measure increases, the magnitude of the risk premium which a consumer would pay to avoid a small gamble would increase, *ceteris paribus*. Since this measure is normalized by marginal utility of consumption, it is invariant to positive affine transformations of U. It follows from differentiation that this measure for (1) is "a" with (2) or "a"/trend with (3). If "a" or "a"/trend are negative, they are measures for temporal risk preference or equivalently time complementarity. In this case, as temporal risk preference increases, the risk premium would decrease, *ceteris paribus*.

III. Volatility of Asset Prices

In this section, we examine whether the excessive volatility of asset prices can be explained by variable time preference in a representative consumer economy. The model here is similar to Lucas (1978) with the major exception of variable time preference.

Consider an exchange economy with a representative consumer who maximizes his (or her) lifetime expected utility

$$E_0\left[\sum_{t=0}^{\infty} -(1/\eta)\exp(-\eta C_t)\exp\left(-\sum_{s=0}^{t-1}(aC_s+b)\right)\right] \tag{4}$$

where η is the absolute risk aversion coefficient of the within period utility function. It is readily seen that (4) is a subclass of utility index (1) where the within utility function is a negative exponential utility function. We assume that there is only one good whose production in each period, y, follows an identically and independent distributed normal process,

$$y \sim N(\mu, \sigma^2). \tag{5}$$

Let z denote the consumer's beginning of period share of the production process and x the consumer's end of period share. Let $p(y)$ denote the equilibrium price of the asset. We assume that no investment and no storage are allowed. Hence production is exogenous. Consequently, it is easy to get equilibrium quantities of consumption and the asset. Namely, in equilibrium, we must have $C = yz$ and $z = x = 1$ where C denotes consumption. By imposing these equilibrium conditions, we would be able to find the behavior of the equilibrium asset price. The consumer faces the budget constraint given by

$$C + p(y)x < yz + p(y)z. \tag{6}$$

Define the indirect utility function, $J(y, z)$, by

$$\begin{aligned} J(y, z) = \max_{C, x} [&-(1/\eta)\exp(-\eta C) \\ &+ \exp(-aC-b)\int J(y', x)dF(y')] \end{aligned} \tag{7}$$

where y and y' denote production in this period and the next period respectively and F is the distribution function of y'. It follows (6) and nonsatiation that

$$\begin{aligned} J(y, z) = \max_x\ [&-(1/\eta)\exp(-\eta\{yz + p(y)(z-x)\}) \\ &+ \exp[-a\{yz + p(y)(z-x)\} - b]\,\mathrm{E}[J(y', x)]] \end{aligned} \tag{8}$$

where E is an expectation operator. Conjecture the indirect utility function in the form of[3]

$$J(y, z) = -(1/\eta)\exp(-\eta yz) + \exp(-ayz)f(z). \tag{9}$$

Substituting (9) into (8) yields

$$\begin{aligned} &-(1/\eta)\exp(-\eta yz) + \exp(-ayz)f(z) \\ &\quad = \max_x [-(1/\eta)\exp(-\eta\{yz + p(y)(z-x)\}) \\ &\qquad + \exp[-a\{yz + p(y)(z-x)\} - b]\cdot \\ &\qquad \{-(1/\eta)\exp(-\eta\mu x + \eta^2\sigma^2x^2/2) \\ &\qquad + \exp(-a\mu x + a^2\sigma^2x^2/2)f(x)\}]. \end{aligned} \tag{10}$$

Using the equilibrium condition, $z = x$, and (10) yields the indirect utility function given by

$$J(y, z) = -(1/\eta)\exp(-\eta yz) + \exp(-ayz - b)A(z) \tag{11}$$

where

[3]In the case of variable time preference, the existence and uniqueness of the indirect utility function have been proved by Lucas and Stokey (1984, Lemma 4).

$$A(z) = \frac{-(1/\eta)\exp(-\eta\mu z + \eta^2\sigma^2 z^2/2)}{1 - \exp(-b - a\mu z + a^2\sigma^2 z^2/2)}. \tag{12}$$

If a is zero, then $J(y, z)$ becomes identical to the one obtained under time additive preference. The first order condition for (10) is

$$\begin{aligned} &-p(y)\exp[-\eta\{yz + p(y)(z - x)\}] \\ &+ ap(y)D(z, x)A(x) + D(z, x)A'(x) = 0 \end{aligned} \tag{13}$$

where $D(z, x)$ is defined as $\exp\{-a\{yz + p(y)(z - x)\} - b\}$ and $A(x)$ is defined as in (12) with the exception of using x rather than z. Using the first order condition (13) and the equilibrium condition, $z = x = 1$, yields the equilibrium asset price given by

$$p(y) = \frac{(\mu - \eta\sigma^2) + (a/\eta)(\mu - a\sigma^2)d/(1 - d)}{(1 - d)\exp(-\eta y + ay + b + \eta\mu - \eta^2\sigma^2/2) + a/\eta} \tag{14}$$

where $d = \exp(-b - a\mu + a^2\sigma^2/2)$.

Since we are interested in examining the volatility of the asset price, we need to derive its variance from (14). However, it appears to be very difficult to obtain the closed form solution of the variance.[4] The volatility of the asset price has been estimated directly by drawing 1000 values for the production quantity y out of a normal distribution, calculating the prices from (14) and then calculating the sample standard deviation for 1000 draws. The numerical result is provided in Table 1, where the mean value ($\mu = 1.396$) and the standard deviation ($\sigma = 0.155$) of the production quantity are taken from consumption of nondurables during the period January 1959—December 1984.

Table 1 demonstrates a new result that as the degree of time complementarity increases, the volatility increases. As the degree of time substitutability increases, the volatility decreases. The effect of time complementarity on the volatility appears to be much stron-

[4]Using the binomial expansion theorem, we can obtain an analytical approximate formula of the volatility of the asset price. The value of "a" is assumed to be close to zero. The approximate formula is given by

$$\begin{aligned} \mathrm{Var}(p(y)) \approx\ & (k/q)^2\exp\{2(\eta - a)\mu + (\eta - a)^2\sigma^2\}[\exp\{(\eta - a)^2\sigma^2\} - 1] \\ & -(2k^2a/q^3)\exp\{3(\eta - a)\mu + 2.5(\eta - a)^2\sigma^2\}[\exp\{2(\eta - a)^2\sigma^2\} - 1] \end{aligned}$$

where

$$k = (\mu - \eta\sigma^2) + (a/\eta)(\mu - a\sigma^2)d/(1 - d) \quad \text{and}$$

$$q = (1 - d)\exp(b + \eta\mu - \eta^2\sigma^2/2).$$

TABLE 1
VOLATILITY OF THE ASSET PRICE
(exp($-b$) = 0.99, μ = 1.396 and σ = 0.155)*

$a \diagdown \eta$	0.501	0.502	0.503	0.504	0.505
0.0015	49.89	50.22	50.46	50.70	50.94
0.0014	52.33	52.57	52.82	53.07	53.33
0.0013	54.84	55.10	55.36	55.62	55.88
0.0012	57.55	57.82	58.09	58.36	58.63
0.0011	60.46	60.74	61.02	61.30	61.58
0.0010	63.59	63.88	64.18	64.47	64.76
0.0009	66.97	67.28	67.58	67.89	68.20
0.0008	70.63	70.95	71.27	71.59	71.91
0.0007	74.60	74.93	75.27	75.60	75.94
0.0006	78.91	79.26	79.61	79.96	80.31
0.0005	83.61	83.97	84.34	84.70	85.07
0.0004	88.73	89.12	89.50	89.88	90.26
0.0003	94.35	94.75	95.15	95.55	95.95
0.0002	100.51	100.93	101.35	101.77	102.19
0.0001	107.30	107.74	108.18	108.62	109.06
0.0000	114.80	115.26	115.72	116.18	116.65
−0.0001	123.11	123.60	124.08	124.57	125.06
−0.0002	132.37	132.88	133.39	133.90	134.41
−0.0003	142.71	143.25	143.78	144.32	144.86
−0.0004	154.32	154.88	155.45	156.02	156.59
−0.0005	167.41	168.01	168.60	169.20	169.80
−0.0006	182.25	182.88	183.51	184.14	184.77
−0.0007	199.17	199.83	200.49	201.15	201.82
−0.0008	218.58	219.28	219.97	220.67	221.36
−0.0009	241.00	241.73	242.46	243.19	243.92
−0.0010	267.10	267.86	268.62	269.39	270.15
−0.0011	297.73	298.53	299.32	300.11	300.91
−0.0012	334.04	334.86	335.68	336.50	337.32
−0.0013	377.54	378.38	379.21	380.05	380.89
−0.0014	430.32	431.15	431.99	432.82	433.66
−0.0015	495.28	496.08	496.88	497.69	498.51

Note : 1. *: The values of μ and σ were taken from the average and the standard deviation of consumption of nondurables during the period January 1959 — December 1984 respectively.

2. If "a" is negative (or positive), the consumer exhibits temporal risk seeking (or averse). Equivalently, a negative (or positive) "a" implies time complementarity (or substitutability). Here "a" is a measure for temporal risk preference (or aversion). If "a" is zero, the preference becomes identical to time additive preference.

ger than the effect of the usual risk aversion.

The effect of time complementarity on the volatility follows from the desirability of consumption over time. Recently, Ahn (1989) has shown that temporal risk aversion makes consumption smoothing over time less attractive. The direct application of his result implies that time complementarity makes consumption smoothing more attractive. Its underlying intuition follows from the fact that under time complementarity, a current "windfall" income increases marginal utilities of different periods and it gives more incentive to spreading consumption over those periods. This fact is immediate from the positivity of the cross partial derivative of the utility function. The desirability of consumption smoothing increases the volatility of the asset price because spreading the income in the period of good production would increase the asset price and converting the asset into consumption in the period of poor production would decrease the price.

Table 1 displays that the volatility of the asset price increases with increasing the usual risk aversion. This result is compatible with LeRoy and La Civita (1981) and Michener (1982). LeRoy and La Civita (1981) have shown that the high volatility of the asset price may be compatible with extreme risk aversion under time additive preference. There is an overwhelming empirical evidence that the relative risk aversion coefficient is below 10. This evidence implies that extreme risk aversion is not appropriate to explain the observed high volatility of the asset price. Our result suggests that the high volatility of the asset price may be able to be explained by time complementarity given a reasonable level of the usual risk aversion.

Even though Ahn (1989) has not examined the effect of time complementarity on the volatility in consumption growth, it is easy to find this effect from analyzing his result. It is straightforward to show analytically that consumption growth under time complementarity is much smoother than that under time additive preference given a level of the usual risk aversion. This result is anticipated from desirability of consumption smoothing under time complementarity. Consequently, we may conjecture that actual smooth consumption growth would be compatible with actual returns if we employ models with variable time preference exhibiting time complementarity.

Given the fact that time complementarity helps explain the excessively volatile asset prices and the excessively smooth growth in consumption, it is certainly of interest to test models with variable time preference with the data. This job is performed in Section IV and V.

IV. Testable Restrictions

In this section, we will derive testable restrictions on the comovements on consumption and asset returns when variable time preference is employed. These restrictions will be tested in order to find whether the model with variable time preference is compatible with the data. Consider the economy with a representative consumer who maximizes his (or her) lifetime expected utility given by[5]

$$U = \mathrm{E}_0[\sum_{t=0} \frac{1}{\gamma} (C_{t,1}^{\delta} C_{t,2}^{1-\delta})^{\gamma} \exp[-\sum_{s=0}^{t-1} (aC_{s,1}/\mathrm{trend}_{s,1} + b)]] \qquad (15)$$

where $C_{t,1}$ and $C_{t,2}$ denote consumption of nondurables and services in period t respectively and a, b and trends are defined as in (3). In (15), γ and δ are preference parameters where γ is assumed to be less than 0 and δ to be between 0 and 1. For simplicity, a time varying discount factor in (15) is assumed to be affected by nondurables but not services.[6] Detrended consumption of nondurables is also assumed to determine the discount factor.[7] Preference (15) is a nonseparable preference which includes time additive preference as a special case. Furthermore, it is also nonseparable across nondurables and services. Consequently, it leads to richer specifications of real returns than time additive preference. Since the consumer is assumed to optimize over time and variable time preference (15) is proven to be concave in Theorem 1, the following stochastic Euler equation must hold.[8]

$$\mathrm{E}_t[m_{t+1} r_{t+1}] = 1 \qquad (16)$$

[5]Similar utility functions are used by Dunn and Singleton (1986) among others. The major difference between theirs and ours is that ours introduces nonseparability through time varying discount factors and theirs through durability of goods.

[6]The estimation results obtained with the discount function only depending on nondurables were very similar to the one with the discount function only depending on services. This simplified assumption appears to be innocuous.

[7]The assumption that the discount factor is affected by detrended consumption yields stationary stochastic disturbances so that it facilitates to estimate parameters.

[8]The stochastic Euler equation holds under not only time additive preference (see Lucas 1978; Cox, Ingersoll and Ross 1985) but also variable time preference. The proof for this result can be available upon request from the author.

where m_{t+1} is the consumer's marginal rate of substitution of consumption of nondurables in period t for consumption in period $t+1$. Thus $m_{t+1} = MU^1_{t+1}/MU^1_t$ where MU^1_{t+1} and MU^1_t denote the marginal utilities of nondurables in period $t+1$ and period t respectively. In (16), r_{t+1} denotes the real return between period t and period $t+1$ measured by the implicit price deflator for nondurables and E_t is an expectation operator conditioned on the consumer's information set in period t. Equation (16) is quite general in the sense that it is robust to the specifications of production technologies and distributions of their outputs. Define z_t be a vector of instruments which are in the consumer's information set in period t. From the law of iterated expectation, (16) yields

$$\mathrm{E}[(m_{t+1}r_{t+1} - 1)z_t] = 0 \tag{17}$$

where E is an unconditional expectation operator. Equation (17) implies that from the dynamic optimizing behavior $m_{t+1}r_{t+1}$ is orthogonal to all the instruments in the consumer's information set.

The marginal utilities of nondurables and services in period t from (15) are respectively given by

$$\begin{aligned} MU_t^1 = {} & \delta\,(C_{t,1}^{\delta}C_{t,2}^{1-\delta})^{\gamma}(1/C_{t,1})\cdot \\ & \exp\Big(-\sum_{s=0}^{t-1}(aC_{s,1}/\mathrm{trend}_{s,1} + b)\Big) \\ & -(a/\mathrm{trend}_{t,1})\mathrm{E}_t\Big[\sum_{\tau=t+1}^{\infty}(1/\gamma)(C_{\tau,1}^{\delta}C_{\tau,2}^{1-\delta})^{\gamma}\cdot \\ & \exp\Big(-\sum_{s=0}^{\tau-1}(aC_{s,1}/\mathrm{trend}_{s,1} + b)\Big)\Big] \end{aligned} \tag{18}$$

and

$$\begin{aligned} MU_t^2 = {} & (1-\delta)(C_{t,1}^{\delta}C_{t,2}^{1-\delta})^{\gamma}(1/C_{t,2})\cdot \\ & \exp\Big(-\sum_{s=0}^{t-1}(aC_{s,1}/\mathrm{trend}_{s,1} + b)\Big). \end{aligned} \tag{19}$$

In (18), the marginal utility of nondurables includes the expected value of utility of consumption of goods since nondurables affect the time varying discount factor for future consumption of goods.

The marginal utilities of goods in period $t+1$ are given in an analogous manner. The marginal utilities of nondurables, however, include the expected utility of consumption of goods infinitely far into the future. Consequently, estimating equation (17) directly is not an easy task. Instead, we employ a different approach. For an optimum, the following first order condition must be satisfied

$$MU_t^2 = P_t MU_t^1 \tag{20}$$

where P_t is the relative price of services in terms of nondurables. Equation (20) means that the marginal rate of substitution between nondurables and services should be equal to the relative price. Using (20) and substituting MU_t^1 and MU_{t+1}^1 into (17) yields

$$\begin{aligned} &\mathrm{E}[\{\frac{p_t}{p_{t+1}}[[\frac{C_{t+1,1}}{C_{t,1}}]^{\delta}[\frac{C_{t+1,2}}{C_{t,2}}]^{1-\delta}]^{\gamma}\frac{C_{t,2}}{C_{t+1,2}}\cdot \\ &\exp(-\frac{aC_{t,1}}{\mathrm{trend}_{t,1}}-b)r_{t+1}-1\}z_t]=0. \end{aligned} \tag{21}$$

Equation (21) will be used to set up orthogonality conditions for testing a representative consumer model with variable time preference. The disturbances can be obtained from (21). Define u_t be the disturbance in period t given by

$$\begin{aligned} u_t = &\frac{p_t}{p_{t+1}}[[\frac{C_{t+1,1}}{C_{t,1}}]^{\delta}[\frac{C_{t+1,2}}{C_{t,2}}]^{1-\delta}]^{\gamma}\frac{C_{t,2}}{C_{t+1,2}}\cdot \\ &\exp(-\frac{aC_{t,1}}{\mathrm{trend}_{t,1}}-b)r_{t+1}-1 \end{aligned} \tag{22}$$

where $\mathrm{E}_t\, u_t = 0$ as seen in (17). Consumption of goods is assumed to grow geometrically with a constant growth rate over time

$$C_{t,i} = DC_{t,i}\exp(\alpha_i + \beta_i t) \qquad \text{for } i = 1, 2 \tag{23}$$

where β_i is the constant growth rate of consumption of good i and $DC_{t,i}$ denotes detrended consumption of good i, i.e., $C_{t,i}/\mathrm{trend}_{t,i}$. In (23), α_i is assumed to satisfy $\mathrm{E}\ln DC_{t,i} = 0$. We assume that prices of the goods also grow geometrically with a constant growth rate. The returns are also assumed to be stationary. Consequently, the disturbance in (22) follows stationary stochastic processes because consumption of nondurables in the exponential function is scaled down by the trend. In order to estimate parameters in (22), we will employ the generalized method of moment estimation procedure proposed by Hansen and Singleton (1982). Since the trend of consumption of nondurables should be removed in (22), we adopt the approach suggested by Eichenbaum and Hansen (1985) which adjusts the growth aspect of consumption in estimation. The assumption made in (23) implies the relationship given by

$$\ln C_{t,i} = \alpha_i + \beta_i t + e_t \tag{24}$$

where α_i is introduced to satisfy the relationship given by

$$\mathrm{E}\, e_t = 0. \tag{25}$$

Define b_0 be the vector of parameters to be estimated, i.e., $b_0 = (b_{01}, b_{02})$ where $b_{01} = (\alpha_1, \beta_1)$ and $b_{02} = (\gamma, \delta, a, b)$. The vector of sample orthogonality conditions from the population orthogonality conditions (21) and (25) is given by

$$g_T(b_{01}, b_{02}) = \frac{1}{T} \sum_{t=1}^{t} \begin{bmatrix} \begin{bmatrix} 1 \\ t/T \end{bmatrix} \otimes e_t(b_{01}) \\ z_t \otimes u_t(b_{02}) \end{bmatrix} \tag{26}$$

where T is the number of time series observations and $\otimes$ is the Kronecker product. Hansen (1982) shows that the optimal estimator of b_0 is obtained by minimizing the criterion function

$$J_T(b_0) = g'_T(b_0) S_T^{-1} g_T(b_0) \tag{27}$$

where S_T is the consistent estimator of the weighting matrix of

$$S_0 = \begin{bmatrix} S_0{}^1 & S_0{}^2 \\ S_0{}^{2'} & S_0{}^3 \end{bmatrix}. \tag{28}$$

In (28), the submatrixes are respectively given by

$$S_0{}^1 = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \otimes \sum_{n=-\infty}^{\infty} \mathrm{E}\left[e_t(b_{01}) e'_{t-n}(b_{01})\right] \tag{29}$$

$$S_0{}^2 = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \otimes \sum_{n=-\infty}^{\infty} \mathrm{E}\left[e_t(b_{01}) u'_{t-n}(b_{02}) z'_{t-n}\right] \tag{30}$$

$$S_0{}^3 = \sum_{n=-\infty}^{\infty} \mathrm{E}\left[z_t u_t(b_{02}) u'_{t-n}(b_{02}) z'_{t-n}\right]. \tag{31}$$

For details, see Hansen and Eichenbaum (1985). It follows from (26), (27) and (28) that the parameters in the trend equation (24) as well as the preference parameters are estimated simultaneously. If the model is valid, we may guess that the minimized value of the criterion function must be close to zero. In fact, Hansen (1982) shows that the minimized value of the criterion function multiplied with the number of time series observations is distributed asymptotically as a chi-square random variable with degrees of freedom equal to the difference between the dimension of vector $g_T(b_0)$ and the dimension of the parameter vector b_0. The value is used to test the validity of the overidentifying restrictions implied by the model.

V. Data and Empirical Results

The data during the period January 1959 — December 1984 were used for testing the model. We have two sub-testing periods, i) January 1959 — December 1978 and ii) January 1979 — December 1984, because of several reasons. First, we like to have a consistency for the testing period with the previous literature (for instance, Hansen and Singleton 1982; Dunn and Singleton 1986; Epstein and Zin 1987b). Second, monthly consumption data are unavailable before this period and the Federal Reserve changed its operating target from the level of interest rates to the rate of growth in the money supply in October 1979.

The monthly real consumption data of nondurables and services were obtained from the Citibase data tape. The real per capita consumption series for nondurables and services were constructed by dividing each consumption series by the monthly population series obtained from the tape. The price series were computed by dividing the implicit price deflator of services by the counterpart of nondurables. The montly nominal returns on the NYSE value-weighted index as well as the NYSE equally-weighted index were obtained from the CRSP tapes. The nominal returns on Treasury bills were taken from Ibbotson (1985). These nominal returns were transformed to real returns by dividing them by the implicit deflator of nondurables.

We implicitly assume that the decision interval of the consumer is a month which is identical to the interval of the sample data.[9] The vector of instruments included unity, the lagged values of the growth rates of real consumption of nondurables and services and the lagged values of real returns.[10] Therefore, The vector, z_t, was defined as

$$z_t = (1, \frac{C_{t,1}}{C_{t-1,1}} - 1, \cdots, \frac{C_{t-NLAG,1}}{C_{t-NLAG-1,1}} - 1,$$

[9]As Dunn and Singleton (1986) indicate, if the actual decision interval is different from a month, estimation and inference made in this paper may produce incorrect conclusions. The assumption allows us to avoid problems related to temporal aggregation.

[10]The inclusion of the lagged values of the relative prices as an instrument did not make a significant difference. We had the similar results with this inclusion.

$$\frac{C_{t,2}}{C_{t-1,1}} - 1, \cdots, \frac{C_{t-NLAG,2}}{C_{t-NLAG-1,2}} - 1,\ r_{1t}, \ldots, r_{mt-NLAG}) \tag{32}$$

where $NLAG$ denotes the number of the lagged values of variables employed as instruments and m denotes the number of securities whose returns were considered.

Table 2 displays the estimates of parameters obtained when re-

TABLE 2
ESTIMATES OF PARAMETERS FOR RETURN: NYSE VALUE-WEIGHTED

$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{a}$	$\hat{b}$	χ^2	$NLAG$	dF
			(Jan. 1959 — Dec. 1978)					
0.1274	0.0013	10.4690	−0.0452	−0.1121	0.1401	3.2318	2	3
(0.0027)	(0.00002)	(12.7510)	(0.2603)	(1.3280)	(1.3445)	(0.6428)		
0.1283	0.0013	10.1206	−0.0045	−0.4928	0.5190	3.5796	3	6
(0.0027)	(0.00002)	(9.9362)	(0.1637)	(0.8152)	(0.8295)	(0.2666)		
0.1296	0.0013	0.8000	−0.7252	−0.4524	0.4554	10.1284	4	9
(0.0027)	(0.00002)	(2.9490)	(3.2333)	(0.4463)	(0.4488)	(0.6598)		
0.1304	0.0013	1.0089	−0.5908	−0.4763	0.4798	10.5466	5	12
(0.0027)	(0.00002)	(2.6906)	(2.0267)	(0.3638)	(0.3656)	(0.4319)		
			(Jan. 1959 — Dec. 1984)					
0.1373	0.0012	7.8682	−0.0389	−0.2782	0.2988	3.1162	2	3
(0.0025)	(0.00001)	(15.9612)	(0.3023)	(0.8907)	(0.9183)	(0.6259)		
0.1381	0.0012	1.8829	−0.3848	−0.2286	0.2355	5.2888	3	6
(0.0025)	(0.00001)	(6.8496)	(1.7355)	(0.4287)	(0.4365)	(0.4927)		
0.1393	0.0012	−0.4933	1.9065	−0.3014	0.3033	8.4740	4	9
(0.0025)	(0.00001)	(3.4201)	(12.4323)	(0.3507)	(0.3542)	(0.5128)		
0.1404	0.0012	−0.1327	7.5193	−0.4272	0.4300	9.2687	5	12
(0.0025)	(0.00001)	(3.0269)	(169.1228)	(0.2720)	(0.2741)	(0.3202)		
			(Jan. 1979 — Dec. 1984)					
0.4333	0.0009	4.8358	0.1423	−1.2091	1.2225	0.5007	2	3
(0.0039)	(0.0001)	(7.6452)	(0.3397)	(1.6502)	(1.6605)	(0.0812)		
0.4330	0.0009	2.1424	−0.0419	−0.6885	0.6964	1.6851	3	6
(0.0039)	(0.0001)	(4.5776)	(0.5698)	(0.7146)	(0.7184)	(0.0537)		
0.4333	0.0009	2.6639	−0.0122	−0.8951	0.9039	2.1325	4	9
(0.0039)	(0.0001)	(3.8206)	(0.4208)	(0.6593)	(0.6621)	(0.0108)		
0.4344	0.0009	−0.9640	0.6375	−0.8776	0.8813	5.0326	5	12
(0.0039)	(0.0001)	(3.4162)	(2.3041)	(0.6452)	(0.6487)	(0.0431)		

Note : Standard errors of the parameter estimates are given in parentheses from column 1 through 6 and probability values of the test statistics are given in parentheses in column 7. The last column shows degrees of freedom.

turns on the NYSE value-weighted index were used. The chi-square test statistic of the overidentifying restrictions implied by the model is provided in column 7 of the table. The probability values of the statistics are given in parentheses. None of the test statistics show the rejection of the model even at the ten percent significance level. This result is a contrast to the unfavorable evidence against the model with time additive preference obtained by Hansen and Singleton (1982, 1984). Variable time preference (15) is significantly different from their time additive preference in several respects. First, our preference exhibits time complementarity or substitutability depending on the value of "a". Our preference is non-time-separable since the discount rate for future consumption is affected by past consumption. Second, our preference includes consumption from two goods whereas Hansen and Singleton (1982) considered the preference with consumption from a single good. Further, our preference introduces nonseparability across consumption from the two goods. Our result is supporting the conclusion of Dunn and Singleton (1986) that the nonseparability across consumption of two goods at a point of time is an important ingredient in the better fit of two good models. However, Dunn and Singleton (1986) introduce a different type of non-time-separability from ours through the durability of goods and the technology for producing service flows from the goods.

The point estimates of parameters related to the trend of nondurables, $\hat{\alpha}_1$ and $\hat{\beta}_1$, are reported in the first and second column respectively. They are statistically significant. The point estimates of $\hat{\beta}_1$ in the pre-1979 period are larger than the point estimates of $\hat{\beta}_1$ in the post-1979 period. Thus consumption growth appears to be smaller after 1979. Most of the point estimates of the curvature parameters for nondurables, $\hat{\gamma}\hat{\delta}$, are negative and range from -0.6145 to 0.6881. Most of the point estimates of the curvature parameters for services, $\hat{\gamma}(1-\hat{\delta})$, are positive and range from -0.3494 to 10.9422. For all the cases, the point estimates for nondurables are smaller than the point estimates for services. This result implies that the consumer exhibits risk aversion for nondurables and risk preference for services.

A time varying discount factor in (15) is $\exp(-\hat{a}C_{t,1}/\text{trend}_{t,1}-\hat{b})$. The average values of this discount factor range from 0.9726 to 0.9927. The point estimates of $\hat{a}$ are negative for all the cases. As shown earlier, a negative $\hat{a}$ implies that the consumer exhibits time complementarity (i.e., temporal risk preference). This result is con-

TABLE 3
ESTIMATES OF PARAMETERS FOR RETURN: T-BILL

$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{a}$	$\hat{b}$	χ^2	*NLAG*	DF
			(Jan. 1959 — Dec. 1978)					
0.1276	0.0013	1.2223	−0.0189	−0.0038	0.0046	2.1411	2	3
(0.0027)	(0.00002)	(0.5248)	(0.0532)	(0.0430)	(0.0430)	(0.4564)		
0.1286	0.0013	1.1561	−0.0207	−0.0030	0.0036	3.5817	3	6
(0.0027)	(0.00002)	(0.4632)	(0.0491)	(0.0316)	(0.0319)	(0.2669)		
0.1301	0.0013	1.0923	−0.0252	−0.0030	0.0035	6.3709	4	9
(0.0027)	(0.00002)	(0.1256)	(0.0312)	(0.0208)	(0.0209)	(0.2977)		
0.1312	0.0013	1.0959	−0.0165	−0.0077	0.0082	6.7927	5	12
(0.0027)	(0.00002)	(0.1241)	(0.0281)	(0.0180)	(0.0181)	(0.1290)		
			(Jan. 1959 — Dec. 1984)					
0.1374	0.0012	2.8725	0.0536	−0.0857	0.0906	0.0711	2	3
(0.0025)	(0.00001)	(2.6430)	(0.0456)	(0.0588)	(0.0563)	(0.0049)		
0.1383	0.0012	1.9214	0.0382	−0.0739	0.0767	1.8699	3	6
(0.0025)	(0.00001)	(0.7549)	(0.0358)	(0.0276)	(0.0274)	(0.0679)		
0.1394	0.0012	1.0141	0.0111	−0.0749	0.0758	10.5446	4	9
(0.0025)	(0.00001)	(0.2039)	(0.0434)	(0.0163)	(0.0163)	(0.6918)		
0.1404	0.0012	1.0877	0.0034	−0.0652	0.0662	14.6138	5	12
(0.0025)	(0.00001)	(0.1800)	(0.0365)	(0.0144)	(0.0144)	(0.7368)		
			(Jan. 1979 — Dec. 1984)					
0.4318	0.0009	1.5750	0.0697	−0.1418	0.1453	2.9754	2	3
(0.0036)	(0.0001)	(0.7669)	(0.0884)	(0.0895)	(0.0901)	(0.6046)		
0.4318	0.0009	1.6099	0.0182	−0.0590	0.0627	5.2654	3	6
(0.0039)	(0.0001)	(0.4630)	(0.0564)	(0.0483)	(0.0485)	(0.4897)		
0.4321	0.0009	1.2960	−0.0059	−0.0401	0.0435	8.0227	4	9
(0.0039)	(0.0001)	(0.2431)	(0.0536)	(0.0334)	(0.0335)	(0.4681)		
0.4323	0.0010	1.2203	−0.0270	−0.0397	0.0431	8.2890	5	12
(0.0039)	(0.0001)	(0.1988)	(0.0490)	(0.0303)	(0.0304)	(0.2370)		

Note : Standard errors of the parameter estimates are given in parentheses from column 1 through 6 and probability values of the test statistics are given in parentheses in column 7. The last column shows degrees of freedom.

sistent with the prediction of the analysis in Section III that time complementarity helps explain excessively volatile asset prices and excessively smooth consumption growth. In fact, time complementarity improves the overall fit of the model. Even though all the estimates of $\hat{a}$ are negative, none of them appear to be significantly different from zero.

Table 3 shows the estimates of parameters obtained when returns of Treasury bills were employed. The parameter estimates in Table

3 are qualitatively similar to those in Table 2. The point estimates of the curvature parameters for nondurables, $\hat{\gamma}\hat{\delta}$, range from -0.0329 to 0.1540. The point estimates of the curvature parameters for services, $\hat{\gamma}(1 - \hat{\delta})$, range from 1.0028 to 2.7185. Thus variations of the estimates of the curvature parameters in Table 3 are much smaller than those in Table 2. This result follows from statistically significant parameter $\hat{\gamma}$. However, the consumer exhibits risk preference for services. The average values of the time varying discount factor range from 0.9953 to 0.9995. All the estimates of $\hat{a}$ are negative and some of them are statistically significantly different from zero. Thus time complementarity is obtained.

Table 4 provides the estimates of parameters obtained with returns on the NYSE equally-weighted index. Even though qualitative results are similar to Table 2 and 3, unfavorable evidences based on the overidentifying test appeared when the data from January 1959 to December 1984 were used. The point estimates of the curvature parameters for nondurables, $\hat{\gamma}\hat{\delta}$, range from -1.1356 to 2.5763. The point estimates of the curvature parameters for services, $\hat{\gamma}(1 - \hat{\delta})$, range from -5.8134 to 18.5580. The mean values of the time varying discount factor range from 0.9504 to 1.0083. The value of the discount factor appears to be greater than unity when we use the data from January 1959 to December 1984. All the estimates of $\hat{a}$ are negative. Thus time complementarity is implied.

Table 5 reports the estimates of parameters when returns on several securities were used simultaneously in estimation. The fit of the model appears to be unfavorable when the longterm data from January 1959 to December 1978 or December 1984 were used. Our result in Table 5 is consistent with Dunn and Singleton (1986) in that the strong evidence against models are reported in the case of multiple returns. The point estimates of the curvature parameters for nondurables, $\hat{\gamma}\hat{\delta}$, range from -14.2224 to 0.3260. The point estimates of the curvature parameters for services, $\hat{\gamma}(1 - \hat{\delta})$, range from -172.6475 to 16.9126. The mean values of the time varying discount factor range from 0.9641 to 1.4131. Most of the point estimates of $\hat{a}$ are negative.

Table 6 and 7 show the estimates of parameters when the model is simplified with $\hat{a}$ set to zero such that variable time preference (15) becomes time additive preference. Thus the preference does not exhibit time complementarity. They report the results only for the time period January 1959 to December 1984. Table 6 includes the two goods, nondurables and services, whereas Table 7 admits a

TABLE 4
ESTIMATES OF PARAMETERS FOR RETURN: NYSE EQUALLY-WEIGHTED

$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{a}$	$\hat{b}$	χ^2	*NLAG*	dF
			(Jan. 1959 — Dec. 1978)					
0.1276	0.0013	21.1343	0.1219	−1.9664	2.0221	4.5740	2	3
(0.0027)	(0.00002)	(29.0913)	(0.1038)	(2.2834)	(2.3403)	(0.7942)		
0.1284	0.0013	17.8066	0.0630	−1.0722	1.1201	6.6777	3	6
(0.0027)	(0.00002)	(19.0599)	(0.0985)	(1.1652)	(1.1967)	(0.6483)		
0.1296	0.0013	2.8611	0.0247	−0.7388	0.7504	14.3931	4	9
(0.0027)	(0.00002)	(4.3016)	(0.3135)	(0.5688)	(0.5732)	(0.8910)		
0.1306	0.0013	1.2152	−0.0095	−0.6385	0.6456	17.3765	5	12
(0.0027)	(0.00002)	(3.6686)	(0.7397)	(0.4390)	(0.4420)	(0.8640)		
			(Jan. 1959 — Dec. 1984)					
0.1372	0.0012	−6.6393	0.1244	−0.2768	0.2693	12.0129	2	3
(0.0025)	(0.00001)	(23.1712)	(0.2211)	(1.0928)	(1.1373)	(0.9927)		
0.1382	0.0012	−6.5234	0.1385	−0.1948	0.1876	13.5060	3	6
(0.0025)	(0.00001)	(11.1318)	(0.1839)	(0.5512)	(0.5676)	(0.9643)		
0.1392	0.0012	−2.8670	0.3741	−0.3620	0.3636	15.5495	4	9
(0.0025)	(0.00001)	(4.2240)	(0.4423)	(0.4082)	(0.4127)	(0.9231)		
0.1403	0.0012	−3.7175	0.3122	−0.4165	0.4158	16.6011	5	12
(0.0025)	(0.00001)	(3.8306)	(0.2752)	(0.3226)	(0.3259)	(0.8348)		
			(Jan. 1979 — Dec. 1984)					
0.4332	0.0009	−1.2639	0.7090	−0.7511	0.7614	1.6767	2	3
(0.0039)	(0.0001)	(8.7443)	(3.7296)	(1.5675)	(1.5775)	(0.3579)		
0.4329	0.0009	−2.9564	0.3841	−0.7645	0.7718	2.9275	3	6
(0.0039)	(0.0001)	(5.5412)	(0.6100)	(0.9152)	(0.9196)	(0.1821)		
0.4332	0.0009	−1.5536	0.5744	−0.9847	0.9941	3.3072	4	9
(0.0039)	(0.0001)	(3.9595)	(1.4477)	(0.6818)	(0.6832)	(0.0491)		
0.4337	0.0009	−2.6312	0.4283	−0.9639	0.9712	3.7802	5	12
(0.0040)	(0.0001)	(3.2448)	(0.5904)	(0.6330)	(0.6353)	(0.0129)		

Note: Standard errors of the parameter estimates are given in parentheses from column 1 through 6 and probability values of the test statistics are given in parentheses in column 7. The last column shows degrees of freedom.

single good, nondurables. Comparing the test statistics in Table 6 with those corresponding in Table 2 through Table 5, we can see that including $\hat{a}$ in the estimation significantly improves the overall fit of the model in nearly every case. Thus variable time preference appears to be supported by the data. In Table 7, most of the point estimates of $\hat{b}$ are significant but the test statistics suggest that the model with time additive preference is not consistent with the data.

TABLE 5
ESTIMATES FOR COMBINATIONS OF SECURITIES

$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{a}$	$\hat{b}$	χ^2	*NLAG*	dF
Estimates of Parameters for Return: NYSE-VW & T-Bill								
(Jan. 1959 — Dec. 1978)								
0.1274	0.0013	−4.6286	0.0822	0.0968	−0.1100	15.2541	1	6
(0.0027)	(0.00002)	(7.5354)	(0.1122)	(0.4007)	(0.4052)	(0.9816)		
0.1285	0.0013	1.2260	−0.0335	0.0047	−0.0039	19.8214	2	14
(0.0027)	(0.00002)	(0.3373)	(0.0444)	(0.0278)	(0.0282)	(0.8641)		
(Jan. 1959 — Dec. 1984)								
0.1367	0.0012	2.9049	0.0501	−0.0829	0.0878	12.1793	1	6
(0.0025)	(0.00001)	(1.7762)	(0.0478)	(0.0587)	(0.0582)	(0.9419)		
0.1379	0.0012	2.6460	0.0441	−0.0849	0.0893	13.3482	2	14
(0.0025)	(0.00001)	(0.8420)	(0.0402)	(0.0421)	(0.0427)	(0.5007)		
(Jan. 1979 — Dec. 1984)								
0.4335	0.0009	2.1508	0.1489	−0.2708	0.2750	2.5738	1	6
(0.0038)	(0.00009)	(1.3984)	(0.1148)	(0.2168)	(0.2182)	(0.1399)		
0.4326	0.0009	2.0648	0.0648	−0.1252	0.1297	7.2140	2	14
(0.0035)	(0.00009)	(0.6171)	(0.0674)	(0.0980)	(0.0986)	(0.0739)		
Estimates of Parameters for Return: NYSE-VW & NYSE-EW								
(Jan. 1959 — Dec. 1978)								
0.1269	0.0013	−6.9764	0.1631	−0.1488	0.1326	19.7355	1	6
(0.0027)	(0.00002)	(15.6527)	(0.1316)	(1.2028)	(1.2313)	(0.9969)		
0.1285	0.0013	1.2260	−0.3355	0.0047	−0.0039	19.8214	2	14
(0.0027)	(0.00002)	(0.3373)	(0.0444)	(0.0278)	(0.0282)	(0.8641)		
(Jan. 1959 — Dec. 1984)								
0.1365	0.0012	−164.5309	0.0775	2.2900	−2.5442	10.4123	1	6
(0.0025)	(0.00001)	(146.0106)	(0.0698)	(8.1404)	(8.2240)	(0.8917)		
0.1377	0.0012	−13.9880	0.0961	0.4849	−0.5139	23.7072	2	14
(0.0025)	(0.00001)	(10.6647)	(0.0666)	(0.6856)	(0.7000)	(0.9503)		
(Jan. 1979 — Dec. 1984)								
0.4348	0.0009	17.0800	0.0098	−1.6174	1.6498	7.2088	1	6
(0.0038)	(0.00009)	(9.7989)	(0.2322)	(2.2516)	(2.2557)	(0.6980)		
0.4349	0.0009	11.3005	0.0123	−1.3142	1.3393	13.0027	2	14
(0.0038)	(0.0001)	(4.9160)	(0.1604)	(1.0598)	(1.0623)	(0.4737)		

TABLE 5
(CONTINUED)

$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{a}$	$\hat{b}$	χ^2	*NLAG*	dF
Estimates of Parameters for Return: NYSE-EW & T-Bill								
(Jan. 1959 — Dec. 1978)								
0.1264	0.0013	−176.9111	0.0241	−9.1613	8.8635	13.1211	1	6
(0.0027)	(0.00002)	(69.4401)	(0.0662)	(7.9770)	(7.9986)	(0.9588)		
0.1283	0.0013	1.3662	−0.0343	0.0176	−0.0164	26.5282	2	14
(0.0027)	(0.00002)	(0.4884)	(0.0485)	(0.0303)	(0.0310)	(0.9778)		
(Jan. 1959 — Dec. 1984)								
0.1369	0.0012	3.2441	0.0515	−0.0755	0.0811	23.9659	1	6
(0.0025)	(0.00001)	(1.5869)	(0.0460)	(0.0632)	(0.0633)	(0.9995)		
0.1380	0.0012	3.1948	0.0398	−0.0727	0.0782	25.7225	2	14
(0.0025)	(0.00001)	(1.1937)	(0.0399)	(0.0494)	(0.0503)	(0.9719)		
(Jan. 1979 — Dec. 1984)								
0.4343	0.0009	2.3107	0.1411	−0.2837	0.2882	7.5604	1	6
(0.0038)	(0.00009)	(1.8061)	(0.1153)	(0.2427)	(0.2444)	(0.7279)		
0.4335	0.0009	2.5253	0.0620	−0.1319	0.1372	14.6138	2	14
(0.0035)	(0.00009)	(0.8634)	(0.0632)	(0.1181)	(0.1189)	(0.5949)		
Estimates of Parameters for Return: NYSE-VW & NYSE-EW & T-BILL								
(Jan. 1959 — Dec. 1978)								
0.1279	0.0013	1.5423	−0.0107	0.0314	−0.0299	32.3302	1	14
(0.0027)	(0.00002)	(0.8990)	(0.0943)	(0.0719)	(0.0735)	(0.9964)		
(Jan. 1959 — Dec. 1984)								
0.1364	0.0012	−179.3489	0.0793	−1.7560	1.4960	15.7240	1	14
(0.0025)	(0.00001)	(64.0026)	(0.0648)	(4.5893)	(4.5993)	(0.6695)		
(Jan. 1979 — Dec. 1984)								
0.4332	0.0009	3.0861	0.0283	−0.3648	0.3701	18.9084	1	14
(0.0035)	(0.00009)	(1.3503)	(0.0953)	(0.1849)	(0.1855)	(0.8315)		

Note : Standard errors of the parameter estimates are given in parentheses from column 1 through 6 and probability values of the test statistics are given in parentheses in column 7. The last column shows degrees of freedom.

In sum, time complementarity (i.e., a negative estimate of $\hat{a}$) helps explain the puzzles of the model with time additive preference that asset prices have been too volatile to be explained by changes in realized dividends and that consumption growth during the postwar period seems to be much too smooth relative to asset real returns to

TABLE 6
ESTIMATES OF THE MODEL WITH $\hat{\alpha} = 0$ (TWO GOODS)

Securities	$\hat{\gamma}$	$\hat{\delta}$	$\hat{b}$	χ^2	*NLAG*	dF
		(Jan. 1959 — Dec. 1984)				
NYSE-VW	−1.3762	1.1143.	0.0003	11.7500	5	13
	(2.9635)	(2.2670)	(0.0067)	(0.4518)		
T-BILL	0.9853	−0.0660	0.0006	30.1294	5	13
	(0.1380)	(0.0358)	(0.0003)	(0.9955)		
NYSE-EW	−5.3393	0.3019	−0.0042	17.6776	5	13
	(3.7980)	(0.1923)	(0.0084)	(0.8298)		
NYSE-VW & T-BILL	1.9832	−0.0268	0.0031	21.8069	2	15
	(0.5258)	(0.0328)	(0.0012)	(0.8870)		
NYSE-VW & NYSE-EW	−8.6143	0.0826	−0.0178	25.4816	2	15
	(6.4420)	(0.0721)	(0.0146)	(0.9562)		
NYSE-EW & T-BILL	2.5233	−0.0072	0.0042	29.8109	2	15
	(0.8017)	(0.0343)	(0.0018)	(0.9874)		
NYSE-VW & NYSE-EW & T-BILL	2.6864	0.0190	0.0045	33.9014	1	15
	(1.0893)	(0.0353)	(0.0024)	(0.9965)		

Note : Standard errors of the parameter estimates are given in parentheses from column 2 through 4 and probability values of the test statistics are given in parentheses in column 5. The last column shows degrees of freedom.

explain the data. This is the reason why the model with variable time preference improves the overall fitness and the model is not rejected. However, it is very interesting to note that the curvature parameter for services has the values of 18.5580 or even −172.6475. It implies that the consumer is extremely risk averse or seeking. This result occurs since consumption flows from services are too smooth to be compatible even with the model with variable time preference. This suggests that the major source for the problem is consumption flows from services. This observation may be useful to develop a model which is compatible with the data. Allowing a durable component for the flow of utility from services (for example, as mentioned by Epstein and Zin (1987b), the utility derived from going to the dentist) may resolve the problem of services. Even though the separation between nondurables and services are examined here, the separation between nondurables and durables certainly warrants a future study. Temporal aggregation problems are

TABLE 7
ESTIMATES OF THE MODEL WITH $\hat{a} = 0$ (SINGLE GOOD: NONDURABLE)

Securities	$\hat{\gamma}$	$\hat{b}$	χ^2	*NLAG*	dF
	(Jan. 1959 — Dec. 1984)				
NYSE-VW	−1.2894 (0.6636)	0.0037 (0.0023)	6.9520 (0.3579)	5	9
T-BILL	−0.1391 (0.0705)	0.0008 (0.0002)	41.1059 (1.0000)	5	9
NYSE-EW	−0.8802 (0.8260)	0.0086 (0.0029)	12.5702 (0.8170)	5	9
NYSE-VW & T-BILL	−0.1664 (0.0709)	0.0007 (0.0002)	45.2149 (1.0000)	2	12
NYSE-VW & NYSE-EW	−0.1535 (0.5827)	0.0033 (0.0022)	24.9807 (0.9851)	2	12
NYSE-EW & T-BILL	−0.1863 (0.0711)	0.0006 (0.0002)	54.0969 (1.0000)	2	12
NYSE-VW & NYSE-EW & T-BILL	−0.1832 (0.0009)	0.0836 (0.0002)	51.0801 (1.0000)	1	13

Note : Standard errors of the parameter estimates are given in parentheses from column 2 through 3 and probability values of the test statistics are given in parentheses in column 4. The last column shows degrees of freedom.

also worthy of investigation.

VI. Conclusion

We have theoretically demonstrated that the introduction of variable time preference exhibiting time complementarity helps explain excessively volatile asset prices and excessively smooth consumption growth for which the model with time additive preference is at least problematic.

Our empirical study confirms the theoretical result. In nearly every case, the point estimates of $\hat{a}$ are negative. A negative value of $\hat{a}$ implies time complementarity for variable time preference examined in this paper. Time complementarity provides a significantly better fitness than in the case of time additive preference. However, our test results provide substantial evidence against our model when the restrictions on the behavior of asset returns were simultaneously tested with returns on different assets. We have found that services is the major source of the puzzles. Our future research needs a model, where consumption flows from services are treated differently from those of nondurables.

Appendix

Proof of Theorem 1

Suppose the discount function has the following form: $v(C_s) = aC_s/\text{trend}_s + b$. Define $\phi(\cdot) = \log - u(\cdot)$. It follows that $-\exp[\phi(\cdot)] = u(\cdot)$. Then $U(\theta) = -\sum_{t=0} \exp[\phi(C_t) - \sum_{s=0}^{t-1} (aC_s/\text{trend}_s + b)]$ where θ denotes an infinite dimensional vector of multiperiod consumption. Let $V(\theta) = -U(\theta)$. If we demonstrate convexity of V, it follows that $U(\cdot)$ must be concave. Thus we will show the convexity of V, i.e., $V(\theta^{(1)} + (1-\lambda)\theta^{(2)}) < \lambda V(\theta^{(1)}) + (1-\lambda)V(\theta^{(2)})w$ here $0 < \lambda < 1$ and $\theta^{(1)}$, $\theta^{(2)} \in \overline{R}^{\infty}$.

$$
\begin{aligned}
& V(\theta^{(1)} + (1-\lambda)\theta^{(2)}) \\
&= \sum_{t=0}^{\infty} \exp[\phi(\lambda C_t^{(1)} + (1-\lambda)C_t^{(2)} \\
&\quad - \sum_{s=0}^{t-1} \{a(\lambda C_s^{(1)} + (1-\lambda)C_s^{(2)}/\text{trend}_s + b\}] \\
&\le \sum_{t=0}^{\infty} \exp[\lambda\, \phi(C_t^{(1)}) + (1-\lambda)\phi(C_t^{(2)}) \\
&\quad - \sum_{s=0}^{t-1} \{\lambda (aC_s^{(1)}/\text{trend}_s + b) \\
&\quad + (1-\lambda)(aC_s^{(2)}/\text{trend}_s + b)\}] \\
&= \sum_{t=0}^{\infty} \exp[\lambda \{\phi(C_t^{(1)}) - \sum_{s=0}^{t-1}(aC_s^{(1)}/\text{trend}_s + b)\} \\
&\quad + (1-\lambda)\{\phi(C_t^{(2)}) - \sum_{s=0}^{t-1}(aC_s^{(2)}/\text{trend}_s + b)\}] \\
&\le \sum_{t=0}^{\infty} [\lambda \exp\{\phi(C_t^{(1)}) - \sum_{s=0}^{t-1}(aC_s^{(1)}/\text{trend}_s + b)\} \\
&\quad + (1-\lambda)\exp\{\phi(C_t^{(2)}) - \sum_{s=0}^{t-1}(aC_s^{(2)}/\text{trend}_s + b)\}] \\
&= \lambda V(\theta^{(1)}) + (1-\lambda)V(\theta^{(2)})
\end{aligned}
$$

The first inequality follows from convexity of ϕ and that the exponential function is an increasing function. The second one follows from convexity of the exponential function. Thus U is concave. The proof that U is concave with the form of the discount function, $v(C_s) = aC_s + b$, follows from similar arguments.

Q.E.D.

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