Long-term Contracts and the Optimal Choice of Monetary Instruments*

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This paper studies the implications of long-term contracts for the optimal choice of a monetary instrument. It shows that an increase in the (average) contract length of the economy enables the monetary authority to reduce output variance under either instrument, the interest rate or the money stock. However, an interest rate policy seems to reduce the variance proportionately more than a money stock policy. Nevertheless, the choice of the optimal instrument is invariant with respect to the contract length.

I. Introduction

Despite the intensive research for more than two decades, the optimal choice of monetary instruments still attracts lively studies from monetary economists. In his seminal paper, Poole (1970) showed that the variances of the endogenous variables in a simple IS-LM framework depended on which instrument, the money stock or the interest rate, the policy authority chose to control. He also showed that, when the monetary instrument is observed contemporaneously, i.e., with intermediate information on the monetary instrument, a combination policy is the minimum-variance policy. The instrument choice and intermediate information literature has been reformulated and extended by Kareken, Muench and Wallace (1973), Turnovsky (1975), Friedman (1977), Bryant (1980), Ahn and Jung (1985), Daniel (1986), Fair (1988), Gagnon and Henderson (1988), and others.

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Extending the traditional approach to incorporate the assumption of rational expectations and the Lucas-type supply curve, Sargent and Wallace (1975) showed that the probability distribution of real output is independent of known policy rules, and that under the interest rate regime the expected price level is indeterminate. The role of expectations and/or intermediate information are further studied by Turnovsky (1980), Canzoneri, Henderson and Rogoff (1983), and Dotsey and King (1983).

Previous studies, however, seemed to concentrate on the static problem in the sense that either the choice is to be made for the current period only or, even when the model is dynamic, rational expectations cum single-period contracting frameworks make the dynamic implication of the instrument selected rather trivial.[1] This paper investigates the implications of long-term contracts for the optimal choice of monetary instruments. Because some sectors of the economy are subject to previously set wages and prices, the active role of policy makers is naturally invoked to offset the expectational errors of the supply side as shown by Fischer (1977) and Phelps and Taylor (1977). It will be shown that in such circumstances the choice of a monetary instrument and the optimal setting of the instrument selected must be considered simultaneously.

Section II of the paper sets up an n-period-contract model of the supply side based on Fischer (1977). Combined with the usual demand side represented by the IS-LM, the model is ready to be solved for the equilibrium values of output and price once the monetary instrument, either the money stock or the interest rate, is selected. The novelty here is that the probability distributions of endogenous variables depend on both the instrument selected and the optimal setting of policy rules for the chosen instrument. That is, complete analysis requires more than consideration of the implications of simple policies such as pegging the interest rate or the money stock at some exogenously determined level. Instead, the optimal feedback rules associated with the two regimes must be explicitly considered in order to determine which regime yields the preferred outcome under the assumption that the chosen instrument is manipulated optimally.

Section III examines which instrument would maximize the objec-

[1]An exception is Ahn and Jung (1985) who generalized the instrument choice into a dynamic optimization problem for a small open economy and provided some empirical results (see also Fair 1988).

tive function of the monetary authority. The basic conclusion is that, as the contract length of the economy increases, the relative effectiveness of the interest rate policy is enhanced in the sense that both output and price variances, which are the arguments of the policy objective function, are reduced more with the interest rate policy than with the money stock policy. However, increases in contract length can never cause the interest rate policy to dominate the money stock policy if the money stock policy is optimal with a shorter contract length. That is, the choice of the optimal instrument is invariant with respect to contract length. It is shown that relaxing some of the simplifying assumptions of the model could change this invariance result. The paper ends with some concluding remarks in Section IV.

II. The Model

The model used in our analysis consists of the following relationships:

$$y_t = b_1 y_t + b_2[r_t - ({}_{t+1}p_{t-1} - {}_tp_{t-1})] + v_{1t}, \quad 0 < b_1 < 1,\ b_2 < 0, \tag{1}$$

$$m_t - p_t = c_1 y_t + c_2 r_t + v_{2t}, \quad c_1 > 0,\ c_2 < 0, \tag{2}$$

$$y_t = y^* + d[p_t - (1/n)({}_tp_{t-1} + \cdots + {}_tp_{t-n})] + u_t \quad d > 0, \tag{3}$$

where y_t = real output,
y^* = full-employment level of y_t (assumed to be constant),
r_t = the nominal interest rate,
m_t = the nominal money stock,
p_t = the general price level,

and ${}_{t+j}p_{t-i}$ is defined as the mathematical expectation of p_{t+j} conditional on information set available at $t-i$, that is, ${}_{t+j}p_{t-i} = \mathrm{E}[P_{t+j} \mid I_{t-i}]$. All variables except r_t are measured in natural logarithms.

Equations (1) and (2) describe the economy's IS and LM curves, respectively. As written, the IS curve depends on the expected real rate of interest, where ${}_{t+1}p_{t-1} - {}_tp_{t-1}$ is the expected rate of inflation. Individuals decide the optimal level of current versus future consumption as a function of the real interest rate (the relative price of current versus future consumption) and a change in the real rate triggers a decision to rearrange their optimal consumption path. These decisions may also depend on unobservable variables, such as anticipated inflation, in which case the agents must form expectations.[2]

The aggregate supply curve, equation (3), is a straightforward generalization of Fischer's long-term contract model. Here we assume the economy is divided into n identical sectors each of which is characterized by an n-period nominal wage contract.[3] Following the same procedure as in Fischer (1977), we arrive at the supply behavior of the economy, equation (3).[4]

A. The Money Stock Policy

When the money stock is chosen as the instrument of the monetary authority, the system is fully integrated. Aggregate demand depends on both the price level and real income, so that real output is given by the solution of the aggregate demand and supply equations. Eliminating r_t from equations (1) and (2), and combining the resulting equation with equation (3) we obtain

$$\begin{aligned} y^* &+ d[p_t - (1/n)({}_tp_{t-1} + \cdots + {}_tp_{t-n})] \\ &= (a_2/a_1)(m_t - p_t) - (b_2/a_1)({}_{t+1}p_{t-1} - {}_tp_{t-1}) \\ &\quad + (1/a_1)[v_{1t} - (b_2/c_2)v_{2t}] - u_t \end{aligned} \tag{4}$$

where $a_1 = [1 - b_1 + (c_1b_2/c_2)] > 0$ and $a_2 = (b_2/c_2) > 0$.

To solve equation (4) in terms of p_t without the expectational terms we use the standard method of undetermined coefficients.

[2]For an alternative formulation of IS equation see McCallum (1981), in which he justifies the adoption of $r_t - ({}_{t+1}p_{t-1} - p_t)$ as a measure of the expected real rate of interest.

[3]For a more satisfactory treatment of contracts in macro models and justifications for their existence, see Taylor (1980) and Hall and Taylor (1988). The purpose of this paper is to investigate the implications of long-term contracts for the instrument choice problem. For simplicity, we assume a fixed contract length. Endogenizing the contract length is a very challenging problem and, as alluded in Section III, the variable contract length might make the choice problem much more complicated. This is especially so when the length affects other behavioral parameters. However, the contract determination is beyond the scope of the current paper.

[4]The slope of the supply curve and the disturbance term may depend on the length of contract. For the purpose of the current paper these inherently interesting relationships are assumed nonexistent. See the brief discussion in Section III for possible implications of this dependency.

Assume

$$p_t = p^* + \sum_{i=0}^{\infty} \alpha_i u_{t-i} + \sum_{i=1}^{2} \sum_{j=0}^{\infty} \beta_{ij} v_{it-j}, \tag{5}$$

$$m_t = m^* + \sum_{i=1}^{\infty} s_i u_{t-i} + \sum_{i=1}^{2} \sum_{j=1}^{\infty} q_{ij} v_{it-j}, \tag{6}$$

$$u_t = \phi u_{t-1} + \varepsilon_t, \tag{7}$$

$$v_{it} = \tau_i v_{it-1} + \mu_{it}, \qquad i = 1, 2, \tag{8}$$

where p^*, α_j, β_{ij}, $i = 1, 2$, $j = 0, 1, \cdots$, are parameters to be determined.

Equation (6) is the money supply rule in feedback form. s_j and q_{ij}, $i = 1, 2$, $j = 1, 2, \cdots$, are set to maximize the objective function of the monetary authority as discussed below. Note that monetary policy does not respond contemporaneously to the exogenous variables, but rather with a one-period lag. ε_t, μ_{1t}, and μ_{2t} are taken as mutually independent white noises with constant variances σ_ε^2, $\sigma_{\mu_1}^2$ and $\sigma_{\mu_2}^2$ respectively.[5]

Taking the mathematical expectations of equation (5) conditional on $I_{t-1}, I_{t-2}, \cdots, I_{t-n}$, substituting the resulting expressions of ${}_tp_{t-1}$, ${}_tp_{t-2}, \cdots, {}_tp_{t-n}$ into (4), and comparing coefficients on both sides of the equation result in the following solution for the price level:

$$\begin{aligned} p_t = m^* &- (a_1/a_2)y^* + \hat{\alpha}_0 \varepsilon_t + (\hat{\alpha}_0 \phi + \hat{\alpha}_1) \varepsilon_{t-1} + \cdots \\ &+ (\hat{\alpha}_0 \phi^{n-1} + \hat{\alpha}_1 \phi^{n-2} + \cdots + \hat{\alpha}_{n-1}) \varepsilon_{t-(n-1)} \\ &+ (\hat{\alpha}_0 \phi^n + \cdots + \hat{\alpha}_n) u_{t-n} \\ &+ \sum_{i=1}^{2} [\hat{\beta}_{i0} \mu_{it} + (\hat{\beta}_{i0} \tau_i + \hat{\beta}_{i1}) \mu_{it-1} + \cdots \\ &\qquad + (\hat{\beta}_{i0} \tau_i^{n-1} + \hat{\beta}_{i1} \tau_i^{n-2} + \cdots + \hat{\beta}_{in-1}) \mu_{it-(n-1)} \\ &\qquad + (\hat{\beta}_{i0} \tau_i^n + \hat{\beta}_{i1} \tau_i^{n-1} + \cdots + \hat{\beta}_{in}) v_{it-n}] \end{aligned} \tag{9}$$

where $\hat{\alpha}_0 = -a_1/(a_1 d + a_2)$, $\hat{\beta}_{10} = 1/(a_1 d + a_2)$, and $\hat{\beta}_{20} = -a_2/(a_1 d + a_2)$. Three sets of coefficients, $(\hat{\alpha}_1, \cdots, \hat{\alpha}_n)$, $(\hat{\beta}_{i1}, \cdots, \hat{\beta}_{in})$, $i = 1, 2$, are complicated functions of ϕ, τ_i's, and parameters of the monetary rule (6) and the structural model, because each set is the solution of n simultaneous equations resulting from the method of undetermined coefficients.[6] However, as we see below, these coeffi-

[5]Equation (7) and (8) take a simplistic dynamic form whose justifications are quite standard in empirical macroeconomic models. For example, the usual investment equation and the aggregate supply which depends on the current and/or lagged capital stock could easily generate the specification.

[6]See Appendix A. Appendix A also demonstrates that $(\alpha_{n+1}, \alpha_{n+2}, \cdots)$ and $(\beta_{in+1}, \beta_{in+2}, \cdots)$, $i = 1, 2$, do not affect the equilibrium solution.

cients do not play any significant role in the choice of instrument.

Using equation (9) to evaluate ${}_{t}p_{t-i}$, $i = 1, 2, \cdots, n$, we rewrite equation (3) as

$$\begin{aligned} y_t - y^* = {} & (d\hat{\alpha}_0 + 1)\varepsilon_t + d\hat{\beta}_{10}\mu_{1t} + d\hat{\beta}_{20}\mu_{2t} + \phi^n u_{t-n} \\ & + \{\text{terms involving } \varepsilon_{t-i}\text{'s, } \mu_{1t-i}\text{'s, and } \mu_{2t-i}\text{'s,} \\ & \quad i = 1, 2, \cdots, n-1\}. \end{aligned} \tag{10}$$

B. The Interest Rate Policy

When the interest rate is selected as the monetary instrument, the model becomes recursive and the current probability distribution of real output is independent of the other current endogenous variables.[7] Real output is demand-determined and the IS equation gives current aggregate demand.

Equilibrium values of real output and the general price are obtained by combining equations (1) and (3) and applying the method of undetermined coefficient as in the previous subsection, except that an equation for the interest rate feedback is needed instead of equation (6):[8]

$$r_t = r^* + \sum_{i=1} k_i u_{t-i} + \sum_{i=1} g_{1i} v_{1t-i}. \tag{11}$$

Once again, the policy instrument responds to the exogenous variables with a one-period lag. The price level solution is

$$\begin{aligned} p_t = p^* & + \tilde{\alpha}_0 \varepsilon_t + (\tilde{\alpha}_0\phi + \tilde{\alpha}_1)\varepsilon_{t-1} + \cdots \\ & + (\tilde{\alpha}_0\phi^{n-1} + \tilde{\alpha}_1\phi^{n-2} + \cdots + \tilde{\alpha}_{n-1})\varepsilon_{t-(n-1)} \\ & + (\tilde{\alpha}_0\phi^n + \tilde{\alpha}_1\phi^{n-1} + \cdots + \tilde{\alpha}_n)u_{t-n} \\ & + \tilde{\beta}_{10}\mu_{1t} + (\tilde{\beta}_{10}\tau_1 + \tilde{\beta}_{11})\mu_{1t-1} + \cdots \\ & + (\tilde{\beta}_{10}\tau_1^{n-1} + \cdots + \tilde{\beta}_{1n-1})\mu_{1t-(n-1)} \\ & + (\tilde{\beta}_{10}\tau_1^n + \tilde{\beta}_{11}\tau_1^{n-1} + \cdots + \tilde{\beta}_{1n})v_{1t-n} \end{aligned} \tag{12}$$

where $\tilde{\alpha}_0 = -1/d$, $\tilde{\beta}_{10} = 1/(1-b_1)d$ and two sets of coefficients, $(\tilde{\alpha}_1, \cdots, \tilde{\alpha}_n)$ and $(\tilde{\beta}_{11}, \cdots, \tilde{\beta}_{1n})$, are functions of ϕ, τ_1 and para-

[7]The recursive structure is broken if current prices enter the IS equation (e.g., through the real balance effect). Turnovsky (1980) examines an alternative specification which is nonrecursive.

[8]Note that the feedback rule for the interest rate does not contain v_{2t-i}'s, $i = 1, 2, \cdots$. This is because the recursive structure of the model with the interest rate as the policy instrument makes the disturbance term v_{2t-i}'s of the LM curve inactive in equilibrium equations of p_t and y_t.

meters of the interest rate rule and the structural model.[9] The output solution in this case is

$$y_t - y^* = [1/(1 - b_1)]\,\mu_{1t} + \phi^n u_{t-n} + \{\text{terms involving } \varepsilon_{t-i}\text{'s and } \mu_{1t-i}\text{'s}, \quad i = 1, 2, \cdots, n-1\}\,. \tag{13}$$

III. Optimal Monetary Rules and the Instrument Choice

The basic monetary decision problem in a static setting is to choose a monetary instrument, either the interest rate or the money stock, and optimal values for the selected instrument that maximize the objective function of the policy authority. In most of the instrument choice studies these decisions are treated as if they are separable. Given the usual objective function of minimizing $E(y_t - y^*)^2$, the output variance determines the choice of instrument and the model parameters the instrument value.

To illustrate this point further, consider the model of the previous section for the case of $n = 1$, which then becomes a rational expectations cum single-period contracting model. For such a model, the equilibrium output equation becomes a function of noncontrollable error terms; for example, in equations (10) and (13) the terms in braces disappear and no room exists for the policy authority to influence output because policy parameters, s_i's, q_{ji}'s, k_i's and g_{1i}'s, do not affect the equilibrium equations at all. However, when contract length is extended (i.e., $n > 1$), policy parameters play a definite role in affecting the output and price solutions. As will be demonstrated below, only certain monetary rules would make terms in braces disappear so as to make the resulting processes have minimum unconditional variances. Thus the choice of an instrument and setting of the monetary rules for the instrument selected must be determined simultaneously.

Consider the money stock policy with the policy objective of minimizing the unconditional output variance. A tedious but straightforward calculation shows that terms in the braces of equation (10) can be set equal to zero by determining optimal values of policy para-

[9]Note that since p^* is not expressed in terms of model parameters, interest rate policy has the well-known problem of price level indeterminacy. However, p^* may be connected to the target growth rate of money (m^*), so that the equilibrium price level becomes properly determined (see McCallum 1981; Daniel 1986; Gagnon and Henderson 1988).

meters, s_t's and q_{jt}'s of equation (6).[10] The resulting process then becomes a function of uncontrollable error terms, ε_t, μ_{1t}, μ_{2t}, and u_{t-n}. The term u_{t-n} appears in the solution because in any period t the contract which was signed at period $t-n$ is still operative. The subsequent real shocks ε_{t-n+1}, $\varepsilon_{t-n+2}, \cdots$, ε_{t-1} can be successfully offset by the monetary authority and hence have no effect on current aggregate output. The objective function evaluated with optimal feedback rules becomes

$$\begin{aligned}\sigma^2_{y|m} &= \mathrm{E}(y_t - y^*)^2 \mid_{\text{optimal money stock rule}} \\ &= [(\frac{b_2}{dc_2(1-b_1)+c_1b_2d+b_2})^2 + \phi^{2n}/(1-\phi^2)]\,\sigma^2_\varepsilon \\ &\quad + (\frac{dc_2}{dc_2(1-b_1)+c_1b_2d+b_2})^2\sigma^2_{\mu_1} \\ &\quad + (\frac{db_2}{dc_2(1-b_1)+c_1b_2d+b_2})^2\sigma^2_{\mu_2}.\end{aligned} \tag{14}$$

A similar procedure for the interest rate policy gives us

$$\begin{aligned}\sigma^2_{y|r} &= \mathrm{E}(y_t - y^*)^2 \mid_{\text{optimal interest rate rule}} \\ &= [\phi^{2n}/(1-\phi^2)]\,\sigma^2_\varepsilon + (\frac{1}{1-b_1})^2\sigma^2_{\mu_1}.\end{aligned} \tag{15}$$

One selection criterion of instrument is the comparison of $\sigma^2_{y|m}$ and $\sigma^2_{y|r}$. Whichever instrument yields a lower unconditional variance would be the preferred policy tool.[11] Looking at the difference between the two variances

$$\begin{aligned}\sigma^2_{y|m} - \sigma^2_{y|r} &= (\frac{b_2}{dc_2(1-b_1)+c_1b_2d+b_2})^2\sigma^2_\varepsilon \\ &\quad + (\frac{db_2}{dc_2(1-b_1)+c_1b_2d+b_2})^2\sigma^2_{\mu_2} \\ &\quad + [(\frac{1}{(1-b_1)+(c_1b_2d+b_2)/dc_2})^2 \\ &\quad - (\frac{1}{1-b_1})^2]\,\sigma^2_{\mu_1}\end{aligned} \tag{16}$$

[10]See Fischer (1977) for a simple case of 2-period contract. Appendix B also provides a detailed example for the case of $n=3$.

[11]However, since prices also are endogenous, a more comprehensive criterion would be based on the moments of the bivariate distribution of price and output.

$$= (\text{positive term}) + (\text{positive term}) + (\text{negative term}),$$

we confirm Poole's result that when disturbances come mainly from the financial sector ($\sigma^2_{\mu_2} \gg \sigma^2_{\mu_1}$), the interest rate policy tends to have a better output stability. We also confirm findings of Craine and Havenner (1981) and Turnovsky (1980) that the presence of supply shocks might favor the interest rate policy. Equation (16) shows clearly that, if the major source of disturbances lies in the supply side ($\sigma^2_\varepsilon \gg \sigma^2_{\mu_i}$, $i = 1, 2$), the interest rate policy results in a smaller output variance.[12]

One interesting result of our model is the effect of contract length on output variances. As we can see in equations (14) and (15), an increase in the overall contract length (n) reduces output variance under either instrument.[13] However, this variance-reducing role of contract length is enhanced more by the interest rate policy than by the money stock policy. Equation (15) shows that under the interest rate regime the entire output variance due to supply shocks tends to decrease as the contract length n increases. On the other hand, when the money stock is the policy instrument only a part of the output variance from the same disturbances decreases. This is because the interest rate policy makes the aggregate demand curve vertical in the price–output plane and so the output variance consists of independent impacts from the supply side and the IS curve (no effect from the money market). Therefore, an increase in n will directly affect the output variance coming from the supply side. On the other hand, the money stock policy lets the negatively sloped aggregate demand interact with the supply curve and hence all three sources of disturbances are intermingled in their impacts on the output variance.[14] The contract length affects only the direct impact of the supply side error, not the indirect impact (the first term in the bracket of equation (14)).

The reason why the contract length reduces the output variance is quite clear. Equilibrium output equations (10) and (13) contain only one term that varies with n. This term, $\phi^n u_{t-n}$, represents the

[12] However, Craine and Havenner (1981) show that the interest rate policy implies a larger price variance.

[13] It can be shown that price variance is also reduced by an increase in n. Note that the existence of supply shocks is crucial for the contract length to affect the output variance.

[14] Note that the three coefficients of structural error variances in equation (14) are functions of parameters of all three structural equations.

effect on equilibrium output of the supply-side innovation that is known at time $t-n$ to all workers as well as the policy maker. This error term need not be included in the feedback rules because it will be correctly anticipated by workers and hence included in wage contracts. But the longer the contract length becomes the more flexibility has the policy maker in offsetting subsequent real shocks.[15] Put in another way the decrease in output variance due to an increase in n represents an improvement in output stability arising from stickier nominal wage. Of course, trying to take short-run advantage of this sticky wage might lead to changes in the structure of labor contracts as Lucas (1976) pointed out. Also note that if u_t is a random walk (i.e., $\phi = 1$), the impact of observed shocks prior to $t-n$ on current output does not deteriorate over time with an increase in n and hence the result that longer contracts imply less output variability no longer obtains.

Although the contract length has different impacts on output variance under each policy regime, it does not affect the choice itself. To see this, note that equation (16) is invariant with respect to the contract length. However, this may be the result of some simplifying assumptions of the model. For example, d may not remain constant as the contract length varies. If the labor market is subject to longer contracts, then the unanticipated aggregate price elasticity (d) of suppliers is very likely to increase because a larger fraction of the economy has a fixed nominal wage at any point in time. Such being the case, $\sigma^2_{y|m} - \sigma^2_{y|r}$ is definitely influenced by n through d. Appendix C demonstrates that the effect of this unanticipated price response (d) on $\sigma^2_{y|m} - \sigma^2_{y|r}$ depends on relative magnitudes of behavioral error variances.

As in Poole (1970) we may consider a combination policy. Let the money stock feedback rule be modified in the following way:

$$m_t = m^* + \sum_{i=1}^{\infty} s_i u_{t-i} + \sum_{i=1}^{2} \sum_{j=1}^{\infty} q_{ij} v_{it-j} + f(r_t - r^*). \qquad (17)$$

Note that m_t responds contemporaneously to r_t. Equation (17) together with equations (1)-(3) now form a four-equation model. The decision variables are s_i's, q_{ij}'s and f. Solving r_t in terms of m_t, error terms, and parameters of the model and substituting into equation (17), we express m_t as

[15]See Footnote 10. We could make the error processes (7) and (8) more complicated, but there would not be substantive changes in the results of the paper.

$$\begin{aligned} m_t = {} & (a/\Pi)[m^* + \sum_{i=1}^{\infty} s_i u_{t-i} + \sum_{i=1}^{2} \sum_{j=1}^{\infty} q_{ij} v_{it-j}] - (afr^*/\Pi) \\ & + (f/\Pi)[(c_1 d + 1)(-b_{2\ t+1}p_{t-1} + b_{2\ t}p_{t-1}) \\ & - (1 - b_1)\{y^* - (d/n)({}_t p_{t-1} + \cdots + {}_t p_{t-n})\}] \\ & + (f/\Pi)[(c_1 d + 1)v_{1t} + d(1 - b_1)v_{2t} - (1 - b_1)u_t] \end{aligned} \tag{18}$$

where $a = -b_2 c_1 d - b_2 - (1 - b_1)dc_2$ and $\Pi = a + fd(1 - b_1)$. This equation simply says that the monetary authority can utilize the current information on u_t, v_{1t}, and v_{2t} revealed in r_t by optimally setting decision variables (reaction parameters) to minimize output variance. The exact process of deriving optimal solutions is conceptually straightforward but the algebra is excruciatingly cumbersome. What is noteworthy is that the resulting output variance is smaller than either of $\sigma^2_{y|m}$ and $\sigma^2_{y|r}$.[16]

IV. Conclusion

This paper examined the implications of long-term contracts for the optimal choice of monetary instruments. Since the model is dynamic and the objective function is to minimize unconditional variance of output, the choice of a monetary instrument and the optimal rules of the instrument selected must be determined simultaneously.

This rational expectations cum multiperiod contract model confirms Poole's results that when disturbances come mainly from the financial sector, the interest rate policy tends to minimize output variance more than the money stock policy. It also supports findings of Craine and Havenner (1981) and Turnovsky (1980) that the presence of supply shocks might favor the interest rate policy. More importantly, the model shows that an increase in (average) contract length of the economy enables the monetary authority to reduce output variance under either instrument. However, the interest rate policy seems to reduce the variance proportionately more than the money stock policy. Although the model structure prohibits the variable contract length from reversing the choice from one instrument to the other, we showed that relaxing some of the simplifying assumptions could easily produce such a reversal.

[16]This is simply because the conditional variance is always less than or equal to the unconditional one. The algebraic detail of the combination policy is available from the authors upon request.

Appendix A

We derive three sets of equation systems under the money stock policy, one for each disturbance term, by the method of undertermined coefficients. For example, in the case of the supply side disturbance, the following equation system can be obtained by comparing coefficients of both sides of equation (4):

$$[(n-1)d/n](\alpha_0\phi + \alpha_1) = a_2 s_1/a_1 - a_2(\alpha_0\phi + \alpha_1)/a_1$$
$$- b_2\{\alpha_0\phi^2 + \alpha_1\phi + \alpha_2$$
$$- (\alpha_0\phi + \alpha_1)\}/a_1 - \phi,$$

$$[(n-2)d/n](\alpha_0\phi^2 + \alpha_1\phi + \alpha_2) = a_2(s_1\phi + s_2)/a_1$$
$$- a_2(\alpha_0\phi^2 + \alpha_1\phi + \alpha_2)/a_1$$
$$- b_2\{(\alpha_0\phi^3 + \alpha_1\phi^2 + \alpha_2\phi + \alpha_3)$$
$$- (\alpha_0\phi^2 + \alpha_1\phi + \alpha_2)\}/a_1 - \phi^2,$$
$$\cdots$$

$$(d/n)(\alpha_0\phi^{n-1} + \alpha_1\phi^{n-2} + \cdots + \alpha_{n-1})$$
$$= a_2(s_1\phi^{n-2} + s_2\phi^{n-3} + \cdots + s_{n-1})/a_1$$
$$- a_2(\alpha_0\phi^{n-1} + \alpha_1\phi^{n-2} + \cdots + \alpha_{n-1})/a_1$$
$$- b_2\{(\alpha_0\phi^n + \alpha_1\phi^{n-1} + \cdots + \alpha_n)$$
$$- (\alpha_0\phi^{n-1} + \alpha_1\phi^{n-2} + \cdots + \alpha_{n-1})\}/a_1 - \phi^{n-1},$$

$$0 = a_2(s_1\phi^{n-1} + s_2\phi^{n-2} + \cdots + s_n)/a_1$$
$$- a_2(\alpha_0\phi^n + \alpha_1\phi^{n-1} + \cdots + \alpha_{n-1}\phi + \alpha_n)/a_1$$
$$- b_2\{(\alpha_0\phi^{n+1} + \alpha_1\phi^n + \cdots + \alpha_n\phi + \alpha_{n+1})$$
$$- (\alpha_0\phi^n + \alpha_1\phi^{n-1} + \cdots + \alpha_n)\}/a_1 - \phi^n,$$

$$\alpha_{i+1} = a_2 s_i/b_2 + (b_2 - a_2)\alpha_i/b_2, \quad i = n+1,\ n+2, \cdots$$

The first n equations have $n+1$ unknowns $(\alpha_1, \alpha_2, \cdots, \alpha_{n+1})$. However, the stability condition of the equilibrium price equation enables us to set $\alpha_{n+i} = 0$, $i = 1, 2, \cdots$. To see this substitute the equations for α_{i+1}, $i = n+1, \cdots$, into the price equation to obtain

$$p_t = m^* - a_1 y^*/a_2 + \alpha_0\varepsilon_t + (\alpha_0\phi + \alpha_1)\varepsilon_{t-1}$$
$$+ (\alpha_0\phi^2 + \alpha_1\phi + \alpha_2)\varepsilon_{t-2} + \cdots$$
$$+ (\alpha_0\phi^n + \alpha_1\phi^{n-1} + \cdots + \alpha_{n-1}\phi + \alpha_n)u_{t-n}$$
$$+ \alpha_{n+1}\sum^{\infty}[(b_2 - a_2)/b_2]^{i-(n+1)}u_{t-i}$$

$$+ (a_2/b_2) \sum_{j=n+1}^{\infty} s_j \sum_{i=j+1}^{\infty} [(b_2 - a_2)/b_2]^{i-(j+1)} u_{t-i}$$
$$+ \{\text{similar terms of } \mu_{1t-i}\text{'s}, \ \nu_{1t-j}\text{'s and } \mu_{2t-i}\text{'s}, \ \nu_{2t-j}\text{'s}, \ i = 0, 1, \cdots, n-1, \ j = n, \ n+1, \cdots\}.$$

For the variance of price level to be finite, α_{n+1} and s_j, $j = n + 1, \cdots$, should be zero because $(b_2 - a_2)/b_2 > 1$. This enables us to solve n simultaneous equations to get $\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_n$ in terms of ϕ and parameters of the monetary rule and the structural model. The same procedure applies to $\{\hat{\beta}_{i1}, \hat{\beta}_{i2}, \cdots, \hat{\beta}_{in}\}$, $i = 1, 2$, and the two sets of coefficients under the interest rate regime.

One might object to using a finite variance condition for the price level to solve the system (this is particularly so when one worries about the nonstationarity of macro-variables). However, alternative solutions are possible as long as one is willing to allow one degree of freedom in setting α_{n+1} at an arbitrary value. Or, one could invoke the concept of collective rationality in solving the undetermined coefficients as in Taylor (1977). In the latter case, the minimum variance solution would be identical to ours. Regardless of the solution method adopted, the instrument choice remains substantively intact.

Appendix B

For the result of the optimal policy rules, we examine the three-period contract case under the money stock policy. When $n = 3$, the equation (10) in the text can be explicitly written as follows:

$$\begin{aligned} y_t - y^* = {} & (d\hat{\alpha}_0 + 1)\varepsilon_t + [(2d/3)(\hat{\alpha}_0\phi + \hat{\alpha}_1) + \phi]\varepsilon_{t-1} \\ & + [(d/3)(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) + \phi^2]\varepsilon_{t-2} \\ & + d\hat{\beta}_{10}\mu_{1t} + [(2d/3)(\hat{\beta}_{10}\tau_1 + \hat{\beta}_{11})]\mu_{1t-1} \\ & + [(d/3)(\hat{\beta}_{10}\tau_1^2 + \hat{\beta}_{11}\tau_1 + \hat{\beta}_{12})]\mu_{1t-2} \\ & + d\hat{\beta}_{20}\mu_{2t} + (2d/3)(\hat{\beta}_{20}\tau_2 + \hat{\beta}_{21})\mu_{2t-1} \\ & + (d/3)(\hat{\beta}_{20}\tau_2^2 + \hat{\beta}_{21}\tau_2 + \hat{\beta}_{22})\mu_{2t-2} \\ & + \phi^3 u_{t-3}. \end{aligned}$$

Then the output variance becomes

$$\sigma_y^2 = [(d\hat{\alpha}_0 + 1)^2 + (4d^2/9)(\hat{\alpha}_0\phi + \hat{\alpha}_1)^2$$

$$
\begin{aligned}
&+ \phi^2 + (4/3)\phi d(\hat{\alpha}_0 \phi + \hat{\alpha}_1) \\
&+ (d^2/9)(\hat{\alpha}_0 \phi^2 + \hat{\alpha}_1 \phi + \hat{\alpha}_2)^2 \\
&+ \phi^4 + (2/3)d\phi^2(\hat{\alpha}_0 \phi^2 + \hat{\alpha}_1 \phi + \hat{\alpha}_2) \\
&+ \{\phi^6/(1-\phi^2)\}]\,\sigma_\varepsilon^2 \\
&+ [(d\hat{\beta}_{10})^2 + (4/9)d^2(\hat{\beta}_{10}\tau_1 + \hat{\beta}_{11})^2 \\
&+ (d^2/9)(\hat{\beta}_{10}\tau_1^2 + \hat{\beta}_{11}\tau_1 + \hat{\beta}_{12})^2]\,\sigma_{\mu_1}^2 \\
&+ [(d\hat{\beta}_{20})^2 + (4/9)d^2(\hat{\beta}_{20}\tau_2 + \hat{\beta}_{21})^2 \\
&+ (d^2/9)(\hat{\beta}_{20}\tau_2^2 + \hat{\beta}_{21}\tau_2 + \hat{\beta}_{22})^2]\,\sigma_{\mu_2}^2.
\end{aligned}
$$

The monetary authority determines the parameters of the policy rule (s_i, q_{ji}, $i = 1, 2, 3$, $j = 1, 2$) by setting $\partial\sigma_y^2/\partial s_i = \partial\sigma_y^2/\partial q_{1i} = \partial\sigma_y^2/\partial q_{2i} = 0$ ($i = 1, 2, 3$). First, $\partial\sigma_y^2/\partial s_i = 0$, $i = 1, 2, 3$, yields

$$
\begin{aligned}
\partial\sigma_y^2/\partial s_i = {} & [(8/9)d^2(\hat{\alpha}_0\phi + \hat{\alpha}_1)(\partial\hat{\alpha}_1/\partial s_i) \\
& + (4/3)\phi d(\partial\hat{\alpha}_1/\partial s_i) \\
& + (2/9)d^2(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) \\
& \cdot\{(\partial\hat{\alpha}_1/\partial s_i)\,\phi + \partial\hat{\alpha}_2/\partial s_i\} \\
& + (2/3)d\phi^2\{(\partial\hat{\alpha}_1/\partial s_i)\,\phi + \partial\hat{\alpha}_2/\partial s_i\}\,\sigma_\varepsilon^2 = 0
\end{aligned}
$$

Rewriting this, we get

$$
\begin{aligned}
&[(8/9)d^2(\hat{\alpha}_0\phi + \hat{\alpha}_1) + (4/3)\phi d + (2/9)d^2\phi \\
&\quad\cdot(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) + (2/3)d\phi^3](\partial\hat{\alpha}_1/\partial s_i) \\
&+ [(2/9)d^2(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) \\
&\quad + (2/3)d\phi^2](\partial\hat{\alpha}_2/\partial s_i) = 0.
\end{aligned}
$$

Since $\partial\hat{\alpha}_1/\partial s_i \neq 0$ and $\partial\hat{\alpha}_2/\partial s_i \neq 0$, we obtain

$$
\begin{aligned}
&(8/9)d^2(\hat{\alpha}_0\phi + \hat{\alpha}_1) + (4/3)\phi d \\
&+ (2/9)d^2\phi(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) + (2/3)d\phi^3 = 0,
\end{aligned}
$$

$$
(9/2)d^2(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) + (2/3)d\phi^2 = 0.
$$

Simplifying these gives us the desired result:

$$
\begin{aligned}
(d/3)(\hat{\alpha}_0\phi^2 + \hat{\alpha}_1\phi + \hat{\alpha}_2) + \phi^2 &= 0, \\
(2/3)d(\hat{\alpha}_0\phi + \hat{\alpha}_1) + \phi &= 0.
\end{aligned}
$$

The same procedure can be applied to show that the coefficients on μ_{it-j}, $i, j = 1, 2$, also vanish.

Appendix C

It is strightforward to show that

$$\begin{aligned}\partial(\sigma^2_{y|m} - \sigma^2_{y|r})/\partial d &= \partial\sigma^2_{y|m}/\partial d \\ &= \underset{(+)}{2\sigma^2_\varepsilon(b_2/A)}\underset{(-)}{[-b_2\{c_2(1-b_1)+c_1b_2\}/A^2]} \\ &\quad + 2\sigma^2_{\mu_1}\underset{(+)}{(dc_2/A)}\underset{(+)}{(c_2b_2/A^2)} \\ &\quad + 2\sigma^2_{\mu_2}\underset{(+)}{(db_2/A)}\underset{(+)}{(b_2^2/A^2)}\end{aligned}$$

where $A = dc_2(1-b_1) + c_1b_2d + b_2 < 0$. When the supply shocks dominate ($\sigma^2_\varepsilon \gg \sigma^2_{\mu_i}$, $i = 1, 2$), an increase in d will flatten the supply curve in the price-output plane and hence reduce output variance (see Figure C1). On the other hand, when demand shocks are much larger than the supply shocks ($\sigma^2_\varepsilon \ll \sigma^2_{\mu_i}$, $i = 1, 2$), the reverse happens (see Figure C2).

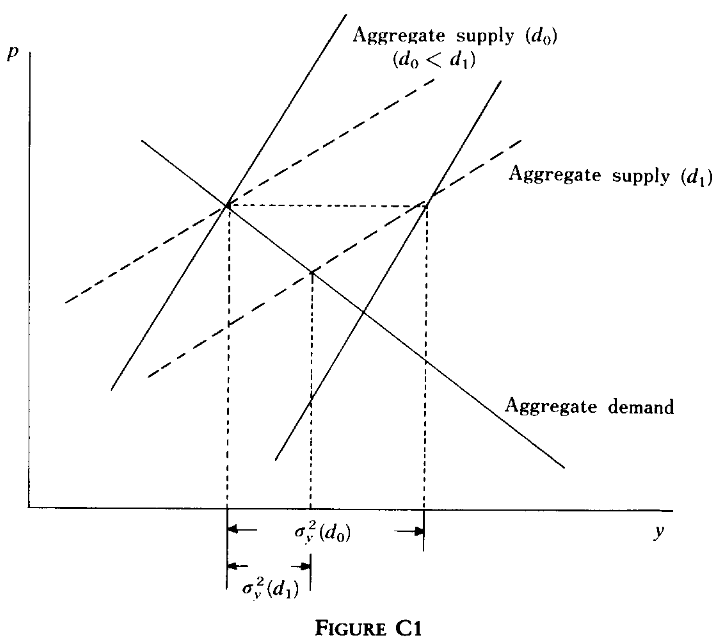

FIGURE C1

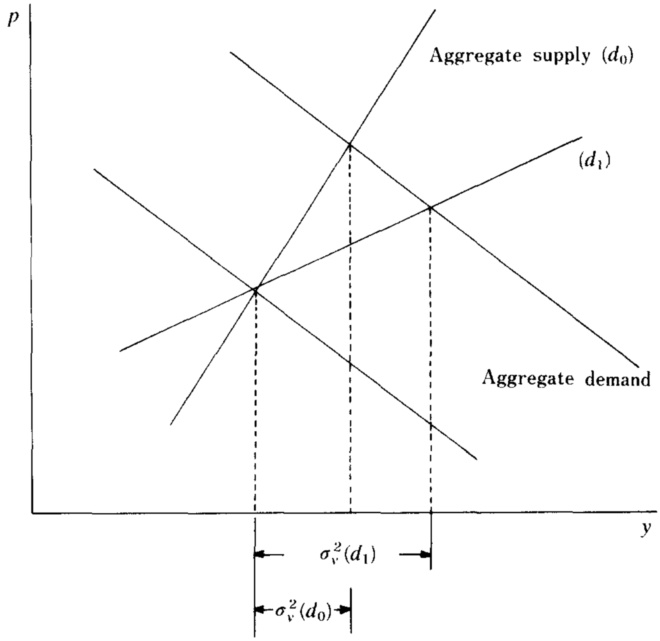

FIGURE C2

References

Ahn, C., and Jung, W. "The Choice of Monetary Instrument in a Small Open Economy: The Case of Korea." *Journal of International Money and Finance* 4 (December 1985) : 469-84.

Bryant, R. *Money and Monetary Policy in Interdependent Nations.* The Brookings Institution, 1980.

Canzoneri, M., Henderson, D., and Rogoff, K. "The Information Content of the Interest Rate and Optimal Monetary Policy." *Quarterly Journal of Economics* 98 (November 1983) : 545-66.

Craine, R., and Havenner, A. "Choosing a Monetary Instrument: The Case of Supply-side Shocks." *Journal of Economic Dynamics and Control* 3 (August 1981) : 217-34.

Daniel, B. "Monetary Aggregate versus Interest Rate Rules." *Journal of Macroeconomics* 8 (Winter 1986) : 75-86.

Dotsey, M., and King, R. "Monetary Instruments and Policy Rules in a Rational Expectations Environment." *Journal of Monetary Economics* 12 (September 1983) : 357-82.

Fair, R. "Optimal Choice of Monetary Policy Instruments in a Macroeconometric Model." *Journal of Monetary Economics* 22 (September 1988) : 301-15.

Fischer, S. "Long-term Contracting, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy* 85 (February 1977): 191-205.

Friedman, B. "The Inefficiency of Short-run Monetary Targets for Monetary Policy." *Brookings Papers on Economic Activity* 2 (1977): 293-335.

Gagnon, J., and Henderson, D. "Nominal Interest Rate Pegging under Alternative Expectations Hypotheses." International Finance Discussion Papers No. 336, Board of Governors of the Federal Reserve System, 1988.

Hall, R., and Taylor, J. *Macroeconomics*. Norton, 1988.

Kareken, J., Muench, T., and Wallace, N. "Optimal Open Market Strategy: The Use of Information Variables." *American Economic Review* 63 (March 1973): 156-72.

Lucas, R. "Econometric Policy Evaluation: A Critique." *Carnegie-Rochester Conference Series on Public Policy* 1 (1976): 19-46.

McCallum, B. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations." *Journal of Monetary Economics* 8 (November 1981): 319-29.

Phelps, E., and Taylor, J. "Stabilizing Power of Monetary Policy under Rational Expectations." *Journal of Political Economy* 85 (February 1977): 163-90.

Poole, W. "Optimal Choice of Monetary Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics* 84 (May 1970): 197-216.

Sargent, T., and Wallace, N. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *Journal of Political Economy* 83 (April 1975): 241-54.

Taylor, J. "Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations." *Econometrica* 45 (September 1977): 1377-85.

———. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (February 1980): 1-23.

Turnovsky, S. "Optimal Choice of Monetary Instrument in a Linear Model with Stochastic Coefficients." *Journal of Money, Credit and Banking* 7 (February 1975): 51-80.

———. "The Choice of Monetary Instrument under Alternative Forms of Price Expectations." *Manchester School* 48 (March 1980): 39-62.