Computations of Cryogenic Cavitating Flows around Turbopump Inducer

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Abstract: This paper deals with the numerical computations of cryogenic cavitating flows around turbopump inducer in liquid rocket. The baseline numerical fluxes for the computations of all-speed two-phase flows (two-phase RoeM and AUSMPW+ schemes) are extended for treating general equation of states, and improved preconditioning techniques are developed for robust and efficient computations in low-speed region. As a validation step for such progress, cryogenic cavitating flows around hydrofoil and ogive are computed. Finally, numerical simulations of three-dimensional KARI turbopump inducer are carried out under various flow conditions with water and cryogenic fluids, and the difference in inducer flow physics depending on the working fluids are examined.

Keywords: Cryogenic Cavitation, All-speed Two-phase Flows, Turbopump Inducer Simulation, Thermal Effect.

1 Introduction

Inducer is one of the key components in liquid rocket propulsion system to achieve high thrust-to-weight ratio. The inducer is placed upstream of the main impeller to improve the cavitation performance. This inducer often suffers from performance breakdown due to serious cavitating conditions at low inlet cavitation number, and some degree of cavitation in inducer flow field is known to be inevitable. Cavitation effects lead to a breakdown of head generation at low inlet cavitation number. The head breakdown rapidly occurs at some critical value of inlet cavitation number. Furthermore, the critical breakdown value can be significantly lower at off-design conditions. From other perspectives, the hydrodynamic vibration caused by cavity fluctuations is also one of the major concerns. It is well-known that cavity fluctuations may yield several dangerous modes such as rotating cavitation, rotating stall cavitation and cavitation surge even in modern high performance turbopumps, such as LE-7 [1], and ARIANE V [2]. Therefore, identifying the critical breakdown value and understanding the flow characteristics of cavitation are crucial in the design process, and they are invaluable in determining the safe operation regime of the propulsion system.

Generally, liquid rocket systems employ cryogenic working fluids. The operating temperature of cryogenic fluids is usually near the critical temperature of the fluids and thus the thermodynamic effects of cavitation play a significant role. At such operating temperatures, the ratio of liquid to vapor density is lower and thus more liquid mass has to vaporize to sustain a cavity. Therefore evaporative cooling effects are more pronounced than water and thus promote lowering the mean fluid temperature in the cavitating region. Since the fluid thermodynamic properties, such as vapor pressure is a strong function of temperature, thermal effects suppress cavitation development and lower the cavity pressure in a mean sense. Typically this results in the improved mean performance of cryogenic pump. Thus, understanding and quantifying the thermodynamic flow characteristics of cryogenic cavitation are

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Thermal effects of cryogenic cavitation were studied extensively by numerous researchers from the 1970s, including Stahl and Stepanoff [3], Hord [4], Ruggeri and Moore [5] and Brennen [6], among others. Stahl and Stepanoff [3] investigated the thermodynamic effect using the so-called B-factor method based on quasi-static theory. More elaborate correlations including dynamic effects were given later by Hord [4], Ruggeri and Moore [5]. Most of these techniques, however, require some degree of empiricism and calibration of coefficients for specific pump geometry.

Regarding the numerical simulations of turbopump inducer, majority of CFD simulations reported in the literature are based on the isothermal incompressible flow conditions, such as Athavale and Singhal [7], Dupont and Okamura [8], Coutier-Delgosha and Reboud [9], and Kiris and Kwak [10]. The effect of temperature variation in fluids is, by definition, not taken in account in these calculations. A few number of groups have been working on developing compressible numerical approach for cavitation problems in liquid rocket pumps, including Uttukar et al. [11], Venkateswaran et al. [12], Hosangadi et al. [13], and our research group [14]. Among these groups, Hosangadi et al. [13] and Uttukar et al. [11] have developed a compressible two-phase numerical method including cryogenic working fluid.

The focus of this paper is to simulate cavitating, cryogenic inducers where variation of real fluid property has a significant impact on the suction performance of turbopump. In order to simulate this class of flows, we first modify the preconditioning techniques which have been developed in the previous work [14] to compute the low-Mach-number region. Then, we extend our baseline numerical schemes – RoeM and AUSMPW+ schemes – into cryogenic fluid flows. To validate the numerical code, computations of cryogenic cavitation problems are carried out under various flow conditions. And, thermal effects are examined by comparing with experimental or other computational results. Finally, numerical simulations of three-dimensional KARI turbopump inducer are carried out under various flow conditions, and thermal effects due to cryogenic cavitation are examined by changing working fluids.

2 Governing Equations

2.1 Homogeneous Mixture Equations

The homogeneous mixture equations with mass fraction is adopted to describe two-phase flows. In homogeneous flow theory, the relative motion between each phases is not independently treated. The mixture is treated as a pseudo-fluid whose properties are suitable averages of each component in the flow. The approach is based on the view that it is sufficient to describe each phase as a continuum obtained from a microscopic description by a suitable averaging process.

Assuming fully compressible flows including thermal effects, the governing equation consist of mixture mass-, momentum-, and energy-conservation laws, together with a one-phase mass-conservation law. A system preconditioning is then introduced to cover the low-Mach-number region. The preconditioned form of the three-dimensional Navier-Stokes system can be written in computational coordinates as follows:

$$\frac{1}{J} \frac{\partial Q}{\partial t} + \frac{\Gamma}{J} \frac{\partial Q_p}{\partial \tau} + \frac{\partial E}{\partial \xi} + \frac{\partial F}{\partial \eta} + \frac{\partial G}{\partial \zeta} = \frac{\partial E_x}{\partial \xi} + \frac{\partial F_x}{\partial \eta} + \frac{\partial G_x}{\partial \zeta} + S_{cav} + S_{rotation}, \tag{1}$$

where $J$ indicates the Jacobian matrix. The primitive variable vector $Q_p$ and the $\xi$-directional inviscid flux vector $E$ are defined by

$$Q_p = [p \quad u \quad v \quad w \quad T \quad Y_i]^T, \tag{2}$$

2
\[ E = \left[ \rho_m U \rho_m uU + \xi_x p \rho_m vU + \xi_y p \rho_m wU + \xi_z p \rho_m H_m U \rho_m Y[U] \right]^T. \]  

Here, \( p, \rho_m, h_m \) and \( H_m \) are the pressure, mixture density, mixture enthalpy, and mixture total enthalpy, respectively; \( Y_1 \) stands for the mass fraction of the gas phase; \( U = \xi_x u + \xi_y v + \xi_z w \) is the contravariant velocity component normal to a control interface; and \( \xi_x, \xi_y \) and \( \xi_z \) are directional cosines. For viscous equations, the viscous term is added as in single-phase flow. The mixture viscosity is computed based on volume fraction. For rotational problems, relative velocity formulation is used to express equations for a rotating reference frame. So, the rotating source term with the rotational speed \( \Omega_z \) along the z-axis is given by

\[ S_{\text{rotation}} = -\frac{\rho_m}{J} \begin{bmatrix} 0 & -2\Omega_z v - x\Omega_x^2 & 2\Omega_z u - y\Omega_x^2 \end{bmatrix}^T. \]  

**2.2 Equation of State**

The definition of the mixture density \( \rho_m \) plays the role of the mixture equation of state (EOS):

\[ \rho_m(p, T, Y) = \frac{1}{Y \rho_g(p, T) + (1 - Y) \rho_l(p, T)}. \]  

In Eq. (5), \( \rho_g, \rho_l \) is the density of each phase on the occupied computational mesh, respectively, and \( \rho_m \) is the mixture density defined on the whole computational mesh.

The mixture enthalpy in Eq. (3) is defined as

\[ h_m = h_g Y_1 + h_l (1 - Y_1). \]  

For rotating reference frame, the rothalpy \( (I_m) \) is used instead of the total enthalpy:

\[ I_m = H_m - \frac{\Omega_z^2 \rho_m}{2}. \]  

For viscous equations, the mixture viscosity and heat conductivity are computed based on volume fraction.

The properties of cryogenic fluids are sensitive to temperature variation compared to other fluids. Accurate equation of state is then an essential part to examine thermal effects in cryogenic cavitation flow. All thermodynamic properties (density, vapor pressure, viscosity, enthalpy, etc.) were generated as a function of the local pressure and temperature. In the present work, these properties were generated from the standard thermodynamic database 12 available from National Institute of Standard and Technology (NIST) for pure fluids [15]. For computational efficiency, modeling of all thermodynamic properties is conducted using regression analysis of Helmholtz (cryogenic fluid) or Gibbs free energy (water) equation of states.

Finally, from the dynamic and thermal equilibrium within the same computational mesh, the total governing system is closed as follows:

\[ p = p_l, T = T_l = T_g. \]
2.3 System Preconditioning

The preconditioning matrix $\Gamma$ has the following form:

$$
\Gamma = \begin{pmatrix}
\frac{1}{\beta} & 0 & 0 & 0 & \frac{\partial \rho_m}{\partial T} & \frac{\partial \rho_m}{\partial Y_1} \\
\frac{u}{\beta} & \rho_m & 0 & 0 & \frac{\partial \rho_m}{\partial T} & \frac{\partial \rho_m}{\partial Y_1} \\
\frac{v}{\beta} & 0 & \rho_m & 0 & \frac{\partial \rho_m}{\partial T} & \frac{\partial \rho_m}{\partial Y_1} \\
\frac{w}{\beta} & 0 & 0 & \rho_m & \frac{\partial \rho_m}{\partial T} & \frac{\partial \rho_m}{\partial Y_1} \\
\frac{H_m + \rho_m \frac{\partial H_m}{\partial \rho} - 1}{\beta} & \rho_m u & \rho_m v & \rho_m w & \frac{\partial \rho_m}{\partial T} + \rho_m \frac{\partial H_m}{\partial T} + \frac{\partial H_m}{\partial Y_1} & \frac{\partial \rho_m}{\partial Y_1} + \rho_m \frac{\partial H_m}{\partial Y_1} \\
\frac{Y_1}{\beta} & 0 & 0 & 0 & \frac{Y_1}{\beta} \frac{\partial \rho_m}{\partial T} + \frac{\partial \rho_m}{\partial Y_1} & \rho_m + Y_1 \frac{\partial \rho_m}{\partial Y_1} \\
\end{pmatrix}
$$

(9)

In Eq. (9), the pseudo-compressibility parameter $\beta$ has a square value of the local velocity magnitude in a preconditioned form. If $1/\beta = \partial \rho_m / \partial p$, then $\Gamma$ goes back to $\Gamma_e = \partial Q / \partial Q_p$, resulting in the non-preconditioned system in the primitive form. The eigenvalues of the preconditioned Euler system are

$$
\lambda \left( \Gamma^{-1} \frac{\partial E}{\partial Q_p} \right) = U, U', \frac{\partial U}{\partial T} + D, U' - D,
$$

(10)

where

$$
U' = \frac{1}{2} \left( 1 + \frac{c'^2}{c^2} \right) U, \quad D = \frac{1}{2} \left( \frac{1 - \frac{c'^2}{c^2}}{c^2} \right) U^2 + 4c'^2,
$$

(11)

$$
c'^2 = \frac{\rho_m \frac{\partial h_m}{\partial T}}{\rho_m \frac{1}{\beta} \frac{\partial h_m}{\partial T} + \frac{\partial \rho_m}{\partial \rho} \left( 1 - \rho_m \frac{\partial h_m}{\partial \rho} \right)}.
$$

(12)

The main purpose of the system preconditioning is to scale the system eigenvalues so that they have the same order of magnitude, which leads to convergence enhancement in the low-Mach-number region. The preconditioned sound speed $c'$, has been chosen to avoid unstable behaviors at near-stagnation regions and to recover the original governing equations at supersonic flow regions [16, 17]. As a result, the following definition of $c'$ is used:

$$
c' = \min(c, \max(V, V_{ref})).
$$

(13)


2.4 Cavitation Model

The cavitation source term is defined via a simplified non-equilibrium finite rate form as follows:

\[ S_{\text{cav}} = \frac{1}{f} \left[ 0 \quad 0 \quad 0 \quad 0 \quad \dot{m}_{\text{evaporation}} - \dot{m}_{\text{condensation}} \right] , \quad (14) \]

where the term \( \dot{m}_{\text{evaporation}} \) is the evaporation rate of vapor being generated from liquid at a region in which the local pressure is less than the vapor pressure. Conversely, \( \dot{m}_{\text{condensation}} \) is the condensation rate for reconversion of vapor back to liquid regions in which the local pressure exceeds the vapor pressure. Here, several models proposed by Merkle [18], Kunz [19], Singhal [20], and the Mushy IDM [21] model are employed. The formulation of each cavitation model is presented in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Cavitation source term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merkle’s model</td>
<td>( \dot{m}<em>{\text{evaporation}} = \frac{C_e \max(p_v - p, 0) \rho \alpha}{0.5 \rho u</em>{\text{e}}^2 \tau} ), ( \dot{m}<em>{\text{condensation}} = \frac{C_e \max(p - p_v, 0) \rho \alpha \rho_l}{0.5 \rho u</em>{\text{e}}^2 \tau} )</td>
</tr>
<tr>
<td>Kunz’s model</td>
<td>( \dot{m}<em>{\text{evaporation}} = \frac{C_e u \rho_1 \rho_v (1 - Y)}{\sigma} \left[ \frac{2 \max(p_v - p, 0)}{3 \rho_l} \right] ), ( \dot{m}</em>{\text{condensation}} = \frac{C_e \rho \alpha \alpha_l^2}{\tau} )</td>
</tr>
<tr>
<td>Singhal’s model</td>
<td>( \dot{m}<em>{\text{evaporation}} = \frac{[\rho + (1 - \rho) e^{-\alpha_l / \rho_v}] \rho_v}{\rho_l} \left[ \frac{\max(p_v - p, 0) \rho \alpha_l}{(U</em>{m, n} - U_{l, n}) \rho_l - \rho \alpha_l Y_{\tau}} \right] ), ( \dot{m}<em>{\text{condensation}} = \frac{\rho_l \max(p - p_v, 0) \rho \alpha_l}{\rho_m (U</em>{m, n} - U_{l, n}) \rho_l - \rho \alpha_l Y_{\tau}} )</td>
</tr>
<tr>
<td>Mushy IDM</td>
<td></td>
</tr>
</tbody>
</table>

3 Numerical Methods

3.1 Generalization of the Shock-Discontinuity Sensing Term

The two-phase RoeM and AUSMPW+ schemes which have been developed in the previous research [14] have some control functions that monitor the following pressure function \( \bar{p} \) around a cell interface:

\[ \bar{p}_{L,R} = \frac{1 + \alpha/l}{1 - \alpha/l} \], \( \Pi_{1/2}^* = \min \left( \frac{\bar{p}_L}{\bar{p}_R}, \frac{\bar{p}_R}{\bar{p}_L} \right) , \quad (15) \]

where, \( a_{1/2} \) is the volume fraction of the gas phase at a cell interface that can be obtained from
geometrical information or from cell interface pressure, temperature, and mass fraction. Eq. (15) is valid for various type of equation of state formulation, for example, ideal gas EOS, Tait’s EOS and Peng-Robinson EOS.

With this information, both schemes are able to detect shock discontinuity in two-phase region and control the amount of numerical diffusions to enhance the stability and/or accuracy of the schemes. However, pressure function $\tilde{p}$ is not applicable for general cryogenic fluids because they require different formulations of EOS. To extend general cryogenic fluids, the pressure function is newly proposed as follows:

$$p_{L,R}^* = p_{L,R} + \rho_{m,1/2} c_{m,1/2}^2, \quad \Pi_{1/2}^* = \min\left(\frac{p_{L}^*}{p_R^*} \frac{p_R}{p_L^*}\right). \quad (16)$$

To ensure the capability of the new shock-discontinuity sensing term, the behavior of $\Pi_{1/2}^*$ is examined for one-dimensional mixture shock conditions. Table 2 compare the inverse values of the shock-discontinuity sensing terms ($\Pi_{1/2}^*$, $\Pi_{1/2}^*$ and $\Pi_{1/2}^*$) in terms of the mass fraction of the gas phase with $M=1.5$, 2.0 and 6.0. Even for the lower Mach number case ($M=1.5$), $\Pi_{1/2}^*$ near the liquid phase is very large due to the large density and high speed of sound. $\Pi_{1/2}^*$ and $\Pi_{1/2}^*$, however, provide well-scaled values throughout the whole range of mass fraction, which indicates that $\Pi_{1/2}^*$ and $\Pi_{1/2}^*$ can be used consistently for all mixture flows regardless of the mixture density and speed of sound. In contrast with the previous shock discontinuity sensing term $\Pi_{1/2}^*$, $\Pi_{1/2}^*$ only use mixture density and speed of sound which do not depend on the type of equation of state. Thus, $\Pi_{1/2}^*$ can be applicable to general equation of state formulation, i.e., independent of type of state formulation.

<table>
<thead>
<tr>
<th>$P_L=1$ atm</th>
<th>$M_L=1.5$</th>
<th>$M_L=2.0$</th>
<th>$M_L=6.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass fraction</td>
<td>$1/\Pi_{1/2}$</td>
<td>$1/\Pi^{*}_{1/2}$</td>
<td>$1/\Pi^{**}_{1/2}$</td>
</tr>
<tr>
<td>$Y_1=0.0$</td>
<td>6565.68</td>
<td>3.18750</td>
<td>1.62494</td>
</tr>
<tr>
<td>$Y_1=10^{-8}$</td>
<td>4836.94</td>
<td>2.61162</td>
<td>1.53927</td>
</tr>
<tr>
<td>$Y_1=10^{-7}$</td>
<td>8.24318</td>
<td>1.22860</td>
<td>1.00187</td>
</tr>
<tr>
<td>$Y_1=10^{-6}$</td>
<td>2.42609</td>
<td>2.06508</td>
<td>1.00246</td>
</tr>
<tr>
<td>$Y_1=10^{-5}$</td>
<td>2.26608</td>
<td>2.22898</td>
<td>1.02029</td>
</tr>
<tr>
<td>$Y_1=10^{-4}$</td>
<td>2.25124</td>
<td>2.24752</td>
<td>1.16414</td>
</tr>
<tr>
<td>$Y_1=10^{-3}$</td>
<td>2.24981</td>
<td>2.24943</td>
<td>1.59168</td>
</tr>
<tr>
<td>$Y_1=10^{-2}$</td>
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<td>1.80068</td>
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<td>2.25426</td>
<td>2.25426</td>
<td>1.83092</td>
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<td>$Y_1=1.0$</td>
<td>2.45833</td>
<td>2.45833</td>
<td>1.85784</td>
</tr>
</tbody>
</table>

3.2 Derivation of the Preconditioned Two-Phase RoeM Scheme

In the previous research [14], the HLLC (Harten-Lax-van Leer with Contact restoration)-type precondition strategy by Luo et al. [22] has been implemented into the two-phase RoeM scheme. By directly scaling its numerical dissipation of using the preconditioned eigenvalues, the preconditioned two-phase RoeM can be obtained as follows:
\[ E_{1/2} = \frac{1}{2} \left[ E_L + E_R - \tilde{M}^* \Delta E + \tilde{D} (\tilde{M}^* - 1) \Delta Q + g \tilde{D} (1 - |\tilde{M}^*|) B \Delta Q^* \right]. \]  

(17)

\[ B \Delta Q^* = \left( \Delta \rho_m - f \frac{\Delta p}{D^2} \right) \begin{bmatrix} 1 & \hat{u} & \hat{v} & \hat{w} & \hat{H}_m & \hat{Y}_m \end{bmatrix} + \left( \begin{array}{cccccc} 0 & \Delta u - n_x \Delta U & \Delta v - n_y \Delta U & \Delta w - n_z \Delta U & \Delta H - \frac{\Delta p}{\hat{\rho}_m} - \hat{U} \Delta U & \Delta Y \end{array} \right), \]  

(18)

where, \( \tilde{M} = \text{sign}(\tilde{M}) \times \text{min}(1,|\tilde{M}|) \) and \( \tilde{M}^* = \tilde{U}/\tilde{D} \). \( \tilde{U} \) and \( \tilde{D} \) have the same form as \( U' \) and \( D \) in Eq. (11), but with Roe-averaged values. However, numerical dissipations of Eq. (17) are different from the preconditioned two-phase Roe scheme. Although it yields a little effect on the solution accuracy, the preconditioned two-phase Roe scheme is well-known for avoiding the checkerboard problem [23]. So, a consistent preconditioned two-phase Roe scheme can be directly derived from the preconditioned two-phase Roe scheme.

The derivation process of the preconditioned two-phase RoeM is almost the same as that of gas-dynamic flow [24]. The first step is to convert the Roe scheme into a HLLE-like form.

For the supersonic region, the preconditioned two-phase Roe scheme becomes

\[ E_{1/2} = \frac{1}{2} \left[ E_L + E_R - \tilde{M} A_p \Delta Q_p \right], \]  

(19)

where, \( \tilde{M} = |\tilde{U}/\tilde{D}| \). The properties with hat symbol indicate Roe-averaged values. The remaining numerical dissipation part is

\[ \left( \tilde{M} A_p - \hat{X} \hat{X}^{-1} \hat{X}^{-1} \right) \Delta Q. \]  

(20)

Then, the eigenvalues of \( \left( \tilde{M} A_p - \hat{X} \hat{X}^{-1} \hat{X}^{-1} \right) \) for the subsonic region are as follows:

\[ \lambda_{1,6} = c^* \left( \tilde{M} \frac{\hat{U}}{c^*} (1 - \alpha) - 1 \right), \]  

(21)

\[ \lambda_{2,3,4,5} = \tilde{M} \hat{U} - |\hat{U}| = c^* \left( \tilde{M} \frac{\hat{U}}{c^*} (1 - \alpha) - 1 \right) + c^* \left( 1 - \frac{|\hat{U}|}{c^*} + \alpha \tilde{M} \frac{\hat{U}}{c^*} \right), \]  

(22)

where, \( c^* = |\tilde{U}/\tilde{D}|\). With the two common parts \( c^* \left( \tilde{M} \frac{\hat{U}}{c^*} (1 - \alpha) - 1 \right) \) and \( c^* \left( 1 - |\hat{U}|/c^* + \alpha \tilde{M} \frac{\hat{U}}{c^*} \right) \) of the eigenvalues, the numerical dissipation of Eq. (20) can be grouped into two parts, resulting in the following HLLE-like form:

\[ E_{1/2} = \frac{1}{2} \left[ E_L + E_R - \tilde{M} A_p \Delta Q_p + c^* \left( \tilde{M} \frac{\hat{U}}{c^*} (1 - \alpha) - 1 \right) \Delta Q + c^* \left( 1 - \frac{|\hat{U}|}{c^*} + \alpha \tilde{M} \frac{\hat{U}}{c^*} \right) B \Delta Q \right], \]  

(23)
Following the idea of the RoeM scheme for gas dynamics [24], we introduce the Mach-number-based control functions $f$ and $g$ to balance the feeding and damping rates of pressure-density coupling in the continuity equation:

$$\begin{align*}
    B\Delta Q &= \left( \Delta \rho_m - \frac{c^* - (1 - 2\alpha) \hat{U} - \alpha \hat{M} \Delta p}{c^* - \hat{U}} + \alpha \hat{M} \frac{\Delta \rho}{c^*} \right) \begin{pmatrix}
        1 \\
        \hat{u} \\
        \hat{v} \\
        \hat{w} \\
        \hat{H}_m \\
        \hat{Y}_\Lambda
    \end{pmatrix} + \hat{\rho}_m \begin{pmatrix}
        0 \\
        \Delta u - n_x \Delta U \\
        \Delta v - n_y \Delta U \\
        \Delta w - n_z \Delta U \\
        \Delta H_m - \frac{\Delta p}{\hat{\rho}_m} - \hat{U} \Delta U \\
        \Delta Y_\Lambda
    \end{pmatrix}.
\end{align*}$$  \tag{24}

Eq. (25) and (26) are the two-phase shock-stable RoeM scheme. The control functions $f$ and $g$ are defined as follows:

$$
\begin{align*}
    f &= \begin{cases}
        1 & \hat{M}^h = 1 \text{ and } \hat{M}^l = 0 \\
        1 & \hat{M}^h \neq 1 \text{ or } \hat{M}^l \neq 0
    \end{cases},
    \quad
    g &= \begin{cases}
        1 & \hat{M}^l = 1 \text{ and } \hat{M}^h = 0 \\
        1 & \hat{M}^l \neq 1 \text{ or } \hat{M}^h \neq 0
    \end{cases}.
\end{align*}$$  \tag{27, 28}

where, $h = \min(\Pi^*_{1/2}, \Pi^*_{L,1/2}, \Pi^*_{L,L,1/2}, \Pi^*_{R,1/2}, \Pi^*_{R,R,1/2})$. And the new Shock-discontinuity sensing term is used in $f$ and $g$.

In order for numerical flux to preserve the total enthalpy, the following condition is required:

$$D_{\text{energy}} = D_{\text{continuity}} \times H_m.$$  \tag{29}

The dissipation of the continuity equation and energy equation on subsonic region is given by
\[ D_{\text{continuity}} = \Delta \rho_m \left[ \frac{1}{H_m} \right] + f \left( c^* - (1 - 2\alpha) \left| \tilde{U} \right| - \alpha \tilde{U} \tilde{M} \right) \frac{\Delta \rho}{c^*} \left[ \frac{1}{H_m} \right] + \tilde{M} \tilde{p} \Delta U \left[ \frac{1}{H_m} \right] + \dot{\rho}_m \left[ \frac{0}{\Delta H_m} \right] + \Delta \rho \left[ \tilde{M} \tilde{U} - \left| \tilde{U} \right| \right] + \dot{\rho}_m \tilde{U} \Delta U \left[ c^* - \left| \tilde{U} \right| + \alpha \tilde{U} \tilde{M}' \right]. \] (30)

Eq. (30) shows that the last two terms are the error sources. Although the fourth term on the right hand side in Eq. (30) does not satisfy the condition of Eq. (29), \( \Delta H \) becomes zero when the total enthalpy is conserved. Thus, the total enthalpy conservation is satisfied by eliminating the last two terms. Then, the scheme preserving the total enthalpy can be expressed as

\[ E_{1/2} = \frac{1}{2} \left[ E_L + E_R - \tilde{M}' \tilde{A}_{\rho} \Delta Q_{\rho} + c' \left( \tilde{M}' \frac{\tilde{U}}{c} (1 - \alpha) - 1 \right) \Delta Q' + g c' \left( 1 - \frac{\tilde{U}}{c} + \alpha \tilde{M}' \frac{\tilde{U}}{c} \right) B \Delta Q'' \right], \] (31)

\[ \Delta Q' = \begin{bmatrix} \Delta(\rho_m) \\ \Delta(\rho_m u) \\ \Delta(\rho_m v) \\ \Delta(\rho_m w) \\ \Delta(\rho_m H_m) \\ \Delta(\rho_m Y) \end{bmatrix}, \quad B \Delta Q'' = \begin{bmatrix} 1 \\ 0 \\ \Delta \rho_m - f \left( c^* - (1 - 2\alpha) \left| \tilde{U} \right| - \alpha \tilde{U} \tilde{M}' \right) \frac{\Delta \rho}{c^*} \left[ \frac{1}{H_m} \right] + \dot{\rho}_m \left[ \frac{0}{\Delta H_m} \right] + \Delta \rho \left[ \tilde{M} \tilde{U} - \left| \tilde{U} \right| \right] + \dot{\rho}_m \left[ \tilde{U} \Delta U \right] \end{bmatrix}. \] (32)

By introducing the signal velocities to prevent expansion shock without compromising the capturing capability of contact discontinuity, the final form of the all-speed two-phase RoeM scheme is summarized as follows:

\[ E_{1/2} = \frac{1}{b_1 - b_2} \begin{bmatrix} b_1 E_L - b_2 E_R + b_1 b_2 \Delta Q' - b_1 \beta_2 \frac{g}{\tilde{M} \tilde{U} - \left| \tilde{U} \right|} - B \Delta Q'' \right] \] (33)

where, \( b_1 = \max(\tilde{U}' + \tilde{D}, \tilde{U}' + \tilde{D}, 0) \) and \( b_2 = \min(\tilde{U}' + \tilde{D}, \tilde{U}' + \tilde{D}, 0) \).

### 3.3 Derivation of the Preconditioned Two-Phase AUSMPW+ Scheme

According to the previous studies on the preconditioning of AUSM-type schemes by Edwards and Liou [25] for gas dynamics and by Edwards et al. [26] for two-phase flows, there are three issues in preconditioning AUSM+ and AUSMDV. The first is the scaling issue to reflect the preconditioning system eigenvalues. In the previous research [14], we have adopted the same scaling technique by Edwards and Liou [25]:

\[ E_{1/2} = M_{L}^{*} c_{1/2} Q_{L} + M_{R}^{*} c_{1/2} Q_{R} + \left( p_{L}^{*} + p_{R}^{*} \right) \] (34)
In Eq. (34), $\tilde{M}_{L,R}$ and $p_{*L,R}$ indicate the Mach number splitting functions using the following scaled Mach number:

$$\tilde{M}_{L,R}^* = \frac{1 + M^2_{r,1/2}}{2} \frac{M_{L,R}}{\phi_{1/2}} + \frac{1 - M^2_{r,1/2}}{2} \frac{M_{R,L}}{\phi_{1/2}},$$  \hspace{1cm} (35)$$

where, $M^2_{r} = c^2 / c^2$ and

$$\phi_{1/2} = \sqrt{\left(1 - M^2_{r,1/2}\right) M^2_{1/2} + 4 M^2_{r,1/2}}. \hspace{1cm} (36)$$

Eq. (36) is introduced to reflect the preconditioned system eigenvalues. The scaled interfacial speed of sound and preconditioned Mach number are:

$$c^*_1 = c_{1/2} \times \phi_{1/2}, \hspace{0.5cm} M^*_1 = U^*_1 / D_{1/2} = U^*_1 / c_{1/2} / \phi_{1/2} = M_{1/2} / \phi_{1/2}. \hspace{1cm} (37)$$

While the above scaling method is successful in most of all-speed flow regions, the scaled Mach number (Eq. (36)) and the preconditioned Mach number (Eq. (37)) make it inefficient to compute the Mach number splitting function. Following the scaling technique proposed in the AUSM+ up scheme [27], we thus employ the original Mach number and pressure fluxes as follows:

$$p_{1/2} = p^*_{L} p_{L} + p^*_{R} p_{R} - k_u p^*_L p^*_R \left(\rho_{m,R} + \rho_{m,L}\right) \phi_{1/2} \alpha_{1/2} \left(u_R - u_L\right), \hspace{1cm} (38)$$

where, $0 \leq k_u \leq 1$.

The second issue is the treatment of the pressure-velocity coupling at low speed. Since AUSMPW+ already has a pressure-velocity coupling term through the control function, $f_{L,R}$, a simple scaling for $f_{L,R}$ is enough. For $0 \leq M_{1/2} \leq 1$, the numerical dissipation of AUSMPW+ in the mass flux can be expressed by

$$D_{AUSMPW+}^{(\rho_m)} = \left\{U^r_R + \frac{c_{1/2}}{2} \left(M_R - 1\right)^2 \alpha_{1/2} \left(1 + f_L\right)\right\} \Delta \rho_{m} + \frac{U^R_R + U^L_L}{2 c_{1/2}} \rho_{m,R} \Delta U + \frac{c_{1/2}}{2} \left(M_R - 1\right)^2 \Delta f. \hspace{1cm} (39)$$

From Eq. (39), the numerical dissipation related to $\Delta p$ in the mass flux becomes smaller as the Mach number is lowered:

$$\frac{c_{1/2}}{2} \left(M_R - 1\right)^2 \Delta f \rightarrow O(U) \times O(\rho_m) \times O(\rho_{m,R}) \times O(1) \times O(U) \times \frac{\Delta p}{O(c^2_m)} \times O(1). \hspace{1cm} (40)$$

In low Mach number flows, $O(c^2_m)$ is too large compared with $\Delta p$ and the pressure-velocity coupling could be too small. So, scaling the control function $f_{L,R}$ is conducted as in AUSM+ up scheme.

$$f_{L,R} = \left(\frac{p^*_{L,R}}{p^*_s} - 1\right) \times (1 - \omega) \times \frac{1}{\phi_{1/2}}. \hspace{1cm} (41)$$
The last issue is about the calculation of the liquid phase. For the two-phase AUSMPW+ scheme, however, the scaling problem arising from the calculation of different phases, is already cured by $\Pi^{1/2}$ and $\bar{p}^*$ in the pressure-based weight function $\omega$ and $f_{L,R}$.

With the preceding modifications, the consistent extension of AUSMPW+ scheme for all-speed two-phase computations is obtained.

### 3.4 Time Integration and Turbulence Model

To enhance the computation efficiency, the implicit LU-SGS [28] time integration method is successfully implemented for the preconditioned Navier-Stokes system. For the numerical treatment of cavitation source terms in flow, we identify a source and sink component of cavitation model and treat sink term implicitly and the source term explicitly.

Specifically, with Eq. (1) we have

$$
\frac{\partial S_{\text{cav}}}{\partial Q_p} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial S_{\text{cav}}}{\partial p} & 0 & 0 & \frac{\partial S_{\text{cav}}}{\partial T} & \frac{\partial S_{\text{cav}}}{\partial Y_i}
\end{bmatrix}.
$$

(42)

So, maximum eigenvalue of Eq. (42) is follows:

$$
\lambda_{\max} \left( \Gamma^{-1} \frac{\partial S_{\text{cav}}}{\partial Q_p} \right) = \frac{\partial h_m}{\partial Y_i} \frac{\partial \rho_m}{\partial T} \frac{1}{\rho_m \left( \frac{1}{c_T^2} + \frac{1}{c^2} \right)} \frac{\partial S_{\text{cav}}}{\partial p}
$$

$$
- \frac{\partial h_m}{\partial Y_i} \rho_m \left( \frac{1}{c_T^2} \frac{1}{c^2} + \frac{\partial \rho_m}{\partial p} \frac{1}{\rho_m} \left( 1 - \frac{\partial h_m}{\partial p} \rho_m \right) \right) \frac{\partial S_{\text{cav}}}{\partial T} + \frac{1}{\rho_m} \frac{\partial S_{\text{cav}}}{\partial Y_i}
$$

(43)

The effects of turbulent mixing are taken into account by employing $k-\omega$ SST model. Rotation and curvature effect are considered by Spalart-Shur correction [29].

### 4 Numerical Result

#### 4.1 Numerical Computations of Cryogenic Cavitating Flows

As a validation case, numerical simulations of experiments by Hord [4] for both liquid nitrogen and liquid hydrogen are presented. Since the considerable blockage effects are expected, it is necessary to model details of tunnel and geometries. The error of temperature measurement is 0.2(K) and is presented at the temperature variation curves in the present study. Detailed information can be found in the Reference [4, 30] and numerical conditions are summarized at Table 3.
Table 3. Numerical conditions of validation problem

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Working fluid</th>
<th>Run number</th>
<th>T[K]</th>
<th>V[m/s]</th>
<th>Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrofoil</td>
<td>Liquid nitrogen</td>
<td>289C</td>
<td>88.64</td>
<td>23.5</td>
<td>1.55</td>
</tr>
<tr>
<td>(two-dimensional planar)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid hydrogen</td>
<td>231C</td>
<td>20.63</td>
<td>51.4</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>Ogive</td>
<td>Liquid nitrogen</td>
<td>312D</td>
<td>83</td>
<td>23.5</td>
<td>0.46</td>
</tr>
<tr>
<td>(axi-symmetric)</td>
<td>Liquid hydrogen</td>
<td>349B</td>
<td>21.33</td>
<td>63.9</td>
<td>0.38</td>
</tr>
</tbody>
</table>

The computed pressure contour, Fig. 1-(a), indicates strong interaction between the cavity and the tunnel wall. And the temperature contour, Fig. 1-(b), shows strong temperature depression at the cavitation region due to condensation process. Figure 1-(c) represents the Mach number contour, which shows incompressible and compressible flow fields are co-exists which explains the necessity of all-speed numerical methods, such as preconditioning and shock-stable spatial discretization scheme.

The pressure depression in Fig. 2 shows the thermal effects of cryogenic fluids. If working fluid has no thermal effects, minimum pressure depression values would be zero. But, local temperature decrement due to thermal effects of cryogenic fluids lowers the local vaporization pressure. Thus, negative value can be seen at pressure depression \((p - p_{v,\infty})\), where \(p\) is the local pressure and \(p_{v,\infty}\) is the free stream vaporization pressure.

Comparing cavitation models in Fig. 2, Merkle’s model, Kunz’s model and Mushy IDM similarly showed steep pressure recovery at the cavity closure region. But, only Singhal’s model showed smoother results. The reason for such difference is considered to be the formulation of cavitation source terms. Singhal’s model contains pressure difference term \((p - p_{v,\infty})\) in root square, but others are not. Therefore, Singhal’s model yields smeared evaporation and condensation process than other models at cavity closure region. This tendency is clearly seen in Fig. 3 which shows condensation rate for each cavitation model. The region of condensation in Singhal’s model is smeared and wide compared to other models.

Figure 1. Numerical results of Hord’s experiments: hydrofoil(upper, Run 289C) and ogive(lower, Run 349B)

The temperature range from triple point to critical point of liquid hydrogen is relatively narrow than that of liquid nitrogen, but the range of vaporization pressure is similar. Hence, thermal effects of liquid hydrogen are more pronounced than liquid nitrogen. This point can be clarified by comparing the maximum pressure depression to the free stream vaporization pressure ratio. For example, Run 289C showed 25.4% and Run 231C showed 38.8%. This tendency is similar to study of Hosangadi et al. [31]. For ogive problem, numerical results were smoother compared to hydrofoil results due to the axi-symmetric effect, but showed similar tendency.
Figure 2. Pressure depression (upper) and temperature variation (lower) comparison

Figure 3. Comparison of condensation rate for cavitation models
4.2 Numerical Computations of Cavitating Flows around Turbopump Inducer

In this section, numerical results of turbopump inducer will be presented. The target inducer was experimented at KARI (Korea Aerospace Research Institute) with cold water [32]. In this study, the experimental study is simulated in design flow rate and in off-design flow rate. Additionally, numerical computations with liquid hydrogen and liquid oxygen will be presented to examine the thermal effects of cryogenic fluids. The geometry information of inducer is summarized in Table 4, and overall configuration and its surface mesh are presented in Fig. 4.

<table>
<thead>
<tr>
<th>Table 4. Geometry parameter of target inducer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry parameter</td>
</tr>
<tr>
<td>Blade number</td>
</tr>
<tr>
<td>Solidity at tip</td>
</tr>
<tr>
<td>Blade tip angle</td>
</tr>
<tr>
<td>Inlet(β₁bt)</td>
</tr>
<tr>
<td>Outlet(β₂bt)</td>
</tr>
<tr>
<td>Radical tip clearance</td>
</tr>
<tr>
<td>Angle variation on line normal to hub</td>
</tr>
</tbody>
</table>

4.2.1 Cavitating Flows in Cold Water

At first, numerical results in cold water is presented. Figure 5 shows the normalized head-rise coefficient as a function of cavitation number. Although there exists small discrepancies around head breakdown point, numerical results were very well matched with experimental data. Figure 6 shows surface pressure distribution with volume fraction iso-surface of α=0.2 in various cavitation numbers. Even though cavitation number is sufficiently high, cavitation region can be locally existing as in Fig. 5-(a), but it does not affect the overall performance. As cavitation number is decreased, cavitation region is further expanded and pressure increment is gradually decreased. Finally, cavitation region covers the entire suction surface as in Fig. 6-(f), and the inducer performance is hardly expected. This tendency can be clearly seen in Fig. 7 and 8, which show the pressure and volume fraction contour at blade tip, respectively.

Figure 4. Surface grid and topology near the leading edge and trailing edge

Figure 5. Normalized head-rise coefficient comparison
Figure 6. Pressure distribution on inducer surface and volume fraction iso-surface at 0.2 in cold water

Figure 7. Pressure contour on cylindrical cut at blade tip for various cavitation number in cold water

Figure 8. Volume fraction contour on cylindrical cut at blade tip for various cavitation number in cold water
4.2.2 Cavitating Flows in Cryogenic Fluids

Numerical results in cryogenic fluids are presented to examine the thermal effects. Many studies were conducted on thermal effects of cryogenic fluids. Especially, Brennen [6] introduced thermodynamic parameter as a measure of thermodynamic characteristics. The definition of parameter and its non-dimensional form are given by

\[ \sum(T_\infty) = \frac{(\rho_c L)^2}{\rho^2 C_p T_\infty \eta_i}, \quad \Sigma^* = \sum \sqrt{\frac{C}{\nu_{tip}}} . \] (44)

Figure 9 represents the variation of thermodynamic parameters in cryogenic fluids and cold water. When the working fluid has a large value, the flow yields a strong thermodynamic effect, and growth of cavitation is suppressed. In this context, liquid hydrogen is likely to have the strongest thermodynamic effect among the three cryogenic fluids. The thermal effects between cold water and cryogenic fluids is performed with liquid oxygen for KARI inducer.

Figure 10 compares the normalized head-rise coefficient near the head breakdown point. Even though thermodynamic parameters between cold water and liquid oxygen are well matched, the latter one shows an improved cavitating performance. This difference is due to the thermal effects of cryogenic fluids. Indeed, by adjusting the thermodynamic parameter, cavitating performance without thermal effects can be similar to that of cryogenic flow which contains considerable thermal effects.

Due to the thermal effects of cryogenic fluids, cavitation is noticeably suppressed and overall performance is enhanced in liquid oxygen. This point can be clearly seen in Fig. 11, which compares the temperature variation near the leading edge at the same cavitation number. Because of the thermal effects, temperature decrement in liquid oxygen is more pronounced than the case of cold water. This tendency agrees well with the experimental research of Franc et al. [33], which has been conducted with the refrigerant R-114 and cold water.

Figure 9. Thermodynamic parameter comparison
Figure 10. Normalized head-rise coefficient near the head breakdown point

Figure 12 shows the surface pressure distribution with volume fraction iso-surface of \( \alpha = 0.2 \) for liquid hydrogen in various cavitation numbers. The equivalent figure for the cold water is presented at Fig. 6. Comparing with the cold water results, cavitation in liquid hydrogen shows frosty interface with significant less-vapor contents. In liquid hydrogen case, the order of dynamic pressure is smaller than cold water. Hence, liquid hydrogen shows a weaker pressure variation around cavitation region, and leads to the frothy cavitation characteristics. This point can be clearly seen in Fig. 13, showing the volume fraction contour on cylindrical cut at blade tip. Comparing with the cold water case in Fig. 8,
cavitation in liquid hydrogen is more smeared and spread out.

Figure 11. Comparison of temperature variation near the tip in cold water and in liquid oxygen

Figure 12. Pressure distribution on inducer surface and volume fraction iso-surface at 0.2 in liquid oxygen

Figure 13. Volume fraction contour on cylindrical cut at blade tip for various cavitation number in liquid oxygen
5 Conclusion and Future Work

Numerical simulations of the flow fields around turbopump inducer are performed. To order to simulate cryogenic cavitating flows, modeling of cryogenic equation of state using the Standard thermodynamic database 12 available from NIST16 for pure fluids is successfully implemented by a regression analysis for the NIST database. Next, two-phase numerical methods which have already been developed for water-gas two phase flows is extended into cryogenic flow fields by modifying the shock discontinuity sensing term. Through this modification, the proposed numerical schemes can be applicable to any types of equation of state. In order to enhance robustness and accuracy in the low-Mach-number region, the preconditioned two-phase RoeM scheme is successfully modified in a manner consistent with the gas dynamic RoeM scheme. The preconditioned two-phase AUSMPW+ scheme is also obtained for more efficient and robust computations. Other numerical sub-components are successfully applied for cryogenic all-speed two-phase flow computations.

To validate the proposed numerical methods, experimental studies of Hord [4, 29] are computed with different cavitation models. Numerical results are quite reasonable compared to experimental data and other researcher’s results. Finally, numerical simulation of three-dimensional KARI turbopump inducer is performed. Computed results with cold water at three flow rates are turned out to be quite reliable compared to experimental data. To examine the thermal effects in cryogenic fluids, cavitating performance of liquid oxygen are compared to that of cold water. Even though thermodynamic parameter is matched for similarity purpose, thermal effects of liquid oxygen make overall performance better than the cold water case. In addition, characteristics of cryogenic cavitation are examined by comparing the numerical results of liquid hydrogen and cold water.

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