Banks as Suppliers of Medium of Exchange and Optimality of 100% Reserve Banking*

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There are three types of agents — households, firms and banks — in the model economy: Households supply labor to firms and banks, firms supply commodities to households, and banks receive deposits from households and make loans to firms. There are two types of medium of exchange, currency and deposit. Commodites can be purchased both by currency and by deposit at different transaction costs. In this economy, a stationary equilibrium is shown to exist for any given level of the reserve requirement ratio. The stationary equilibrium is optimal, if and only if it is associated with 100% reserve requirement ratio. The equilibrium deposit interest rate, when the reserve requirement ratio is 100%, is the negative of the marginal cost of servicing the deposit balance.

I. Introduction

Recently there has been much progress in analyzing the role of banks in the economy. Following Tobin (1963), these studies have focused on the bank's role of financial intermediation between surplus and deficit units in the economy. They include the "New Monetary Economics" of Black (1970), Fama (1980) and Hall (1982, 1983), and the "legal restrictions theory" of Sargent and Wallace (1982) and Wallace (1983), and more recent studies of banking based on explicit general equilibrium modeling of the economy.[1] These studies generally consider that the government regulations on bank-

*Financial support of Ewha Faculty Research Fund is gratefully acknowledged. This paper was presented at The Third Far Eastern Meeting of the Econometric Society held in June 1991, Seoul. I appreciate the comments of the participants. Also, I have benefited from the comments of an anonymous referee. Any remaining errors or shortcomings of this paper are my responsibility.

[1]For example, Diamond and Dybvig (1983), Romer (1985), Bernanke and Gertler (1985), Freeman (1987), Williamson (1987).

[Seoul Journal of Economics 1991, Vol. 4, No. 3]

ing such as deposit interest rate ceiling and reserve requirement hamper intermediation and lower the efficiency of economy.

In a different direction of research, Englund and Svensson (1988) propose a model in which banks are providers of deposit money which serves as a transactions medium along with the currency supplied by the government. In their model, there are two types of commodities, "cash good" and "check good", such that cash good can be purchased only by currency whereas check good can be purchased either by currency or by deposit. In this environment, they point out that when the deposit interest rate is positive, the equilibrium is inefficient since check good is effectively cheaper than cash good, and more than the optimal amount of the check good is consumed. Thus, they suggest that zero percent deposit interest rate, which would occur if the reserve requirement ratio is 100 percent, is required to attain the efficiency.

This paper generalizes their work in two respects. Firstly, instead of exogenously classifying commodities into cash and check goods, I make all commodities be purchased both by currency and by deposit at different transaction costs. Thus, transactions medium used to purchase a particular commodity is endogenously determined by the transaction costs and the deposit interest rate. Secondly, this paper explicitly takes into account the cost of servicing the deposit balance on the part of a bank (the cost of maintaining the deposit account and providing transactions services through it). This framework indicates another reason for the nonoptimality of positive deposit interest rate — more precisely, the deposit interest rate higher than the negative of the marginal cost of servicing the deposit balance. This is because the economy uses deposit as a transactions medium in purchasing a larger class of commodities and hence uses more real resources for transactions purposes than the optimality requires.

In this paper, I construct a general equilibrium model with a banking sector and show that there exists a stationary equilibrium for any given level of a reserve requirement ratio. And I show that the stationary equilibrium is optimal, if and only if it is associated with 100 percent reserve ratio. The equilibrium deposit interest rate in this case is the negative of the marginal cost of servicing the deposit balance, which accords with the policy prescriptions of earlier advocates of 100 percent reserve.[2]

[2]For earlier proponents of 100 percent reserve banking, see references in Friedman (1960, pp. 65-6) and Fischer (1990, p. 1161-2).

II. The Model

A. Overview

The model is an infinite horizon, competitive economy without uncertainty. It consists of three types of agents: households, firms and banks. A household is endowed with one unit of labor service in each period which it supplies inelastically to firms and banks. With its labor income and dividend received from banks, a household purchases commodities to maximize the utility. A firm borrows from banks and uses the proceeds to purchase labor services. Using labor as a sole input, it produces nonstorable commodities and sells them to households. In each period, a bank receives deposits, makes loans to firms, and pays dividends to its shareholder households. It also purchases labor input to service the deposit balance it maintains. All markets — loan, labor and commodity markets — in the economy are perfectly competitive.

Timing of transactions of this model is as follows. When period t starts, banks are opened and pay interests on the deposit balances carried over from the last period. They collect loans made in the last period with interests, make new loans for this period, and pay dividends to their shareholders. Private loans of the last period are also collected with interests at this stage, and new loans are made at the private loan market of this period. Labor market is also opened at this stage. And households make deposits and withdraw currency from their deposit balances at the banks. At the end of this stage, banks finish their operation for this period and are closed until the start of the next period $(t+1)$.

After the above initial phase of the period, firms produce commodities, and at the end of the period commodity markets are open. Households purchase commodities from firms spending the currency in possession and the deposit balances at banks and consume them. Thus, households are subject to the cash-in-advance type liquidity constraints of currency and deposit balances when they purchase commodities. After the transactions of commodities, the period ends.

B. Optimization

A) Households

There are N nonstorable commodities, indexed by i, in the economy. The representative household has a utility function

$$\sum_{t=0}^{\infty} \delta^t u(x_{1t}, x_{2t}, \ldots, x_{Nt}), \quad 0 < \delta < 1, \tag{1}$$

where δ is the discount factor and $x_{it}\,(> 0)$ is the amount of good i consumed in period t. The function $u: \mathbf{R}^N_{++} \rightarrow \mathbf{R}$ is assumed to be differentiable, strictly concave, increasing, bounded above, and unbounded below in each of its arguments.[3] All commodities are assumed to be normal goods.

At the start of period t, a household collects the amount $(1 + i_{t-1})m_{t-1}$ as the loan made in the previous period, m_{t-1}, at the interest rate of $i_{t-1}\,(> 0)$ matures. It also sells its labor service at the wage rate of $w_t\,(\geq 0)$ and receives dividend $v_t\,(\geq 0)$ from the banks. I assume that bank shares are owned uniformly by households and are not traded.[4] The household uses these proceeds to hold currency c_t (≥ 0) and deposit $d_t\,(\geq 0)$ at a bank, and to make a loan m_t. (If $m_t <$ 0, then the household is a net borrower.) Thus, a household's budget constraint is[5]

$$\begin{gathered} c_t + d_t + m_t = w_t + v_t + (1 + i_{t-1})m_{t-1}, \\ c_t \geq 0,\ d_t \geq 0,\ \forall t \geq 0, \\ m_{-1} \text{ is given.} \end{gathered} \tag{2}$$

At the end of the period, a household purchases commodities. Each commodity can be purchased both by currency and by deposit (by writing a check) at different transaction costs. Specifically, I assume that a household has to buy α_i [β_i, resp.] $(1 \leq \alpha_i, \beta_i)$ units of commodity i to consume one unit of commodity i when it pays by currency [deposit, resp.]. Thus, to consume a unit of commodity i, a household pays a transaction cost of $(\alpha_i - 1)$ units of commodity i when it pays by currency, and $(\beta_i - 1)$ units of commodity i when it pays by deposit. Let x^c_{it} [x^d_{it}, resp.] $(x^c_{it}, x^d_{it} \geq 0)$ denote the amount of

[3]The assumption of "unbounded below in each of its arguments," with the monotonically increasing property of the utility function, implies that, as $x_{it} \rightarrow 0$, $u(x_{1t}, \ldots, x_{it}, \ldots, x_{Nt}) \rightarrow -\infty$ $\forall i = 1, \ldots, N$.

[4]To avoid inessential complications, I do not explicitly consider the trade in bank shares in the model. Nevertheless, it can be easily accommodated in the model, which will yield the equilibrium price of a bank share. Arbitrage between lending and bank share holding would dictate that the share price is (v/i) in a stationary equilibrium where v and i are the dividend and the loan interest rate respectively in the equilibrium.

[5]Independent of household's optimization, the loan interest rate (i_t) is assumed to be positive. And it is larger than the deposit interest rate (r_t). (See (30).) Thus we need not consider the household's carryover of currency and deposit from a period to the next.

commodity i a household consumes and was paid by currency [deposit, resp.]. Then

$$\begin{aligned} &x_{it}^c + x_{it}^d = x_{it}, \\ &x_{it}^c, x_{it}^d \geq 0, \ x_{it} > 0, \ \forall i = 1, 2, \ldots, N \text{ and } \forall t \geq 0. \end{aligned} \tag{3}$$

And given the price p_{it} (> 0) of commodity i and the deposit interest rate r_t (> -1) in period t, the household is subject to the following liquidity constraints:

$$\begin{aligned} &\sum_{i=1}^{N} p_{it} \alpha_i x_{it}^c \leq c_t, \\ &\sum_{i=1}^{N} p_{it} \beta_i x_{it}^d \leq (1 + r_t) d_t, \quad \forall t \geq 0. \end{aligned} \tag{4}$$

A household's optimization problem is to maximize (1) subject to (2), (3), and (4) given the sequences of w_t, v_t, p_{it}, r_t, and i_{t-1} for $t \geq 0$. To exclude the possibility of arbitrarily large spending financed by the Ponzi scheme of ever increasing borrowing, I assume that, given bounded sequences of w_t, v_t, p_{it}, r_t, and i_{t-1}, the household is constrained to choose a sequence of m_t that is also bounded. With an appropriate choice of the bound, I assume that it is not binding at the household's optimal choice. Then the Lagrangian function L^h of this problem, ignoring the bounds on m_t, is

$$\begin{aligned} &L^h(\{c_t, d_t, \{x_{it}^c, x_{it}^d\}_{i=1}^{N}, m_t, \lambda_t, \mu_t, \gamma_t\}_{t=0}^{\infty}) \\ &= \sum_{t=0}^{\infty} \delta^t [u(x_{1t}^c + x_{1t}^d, \ldots, x_{Nt}^c + x_{Nt}^d) \\ &\quad + \lambda_t (c_t - \sum_{i=1}^{N} p_{it} \alpha_i x_{it}^c) \\ &\quad + \mu_t \{(1 + r_t) d_t - \sum_{i=1}^{N} p_{it} \beta_i x_{it}^d\} \\ &\quad + \gamma_t \{w_t + v_t + (1 + i_{t-1}) m_{t-1} - c_t - d_t - m_t\}]. \end{aligned}$$

Assuming that the transversality condition for m_t is satisfied,[6] the following first-order conditions are necessary and sufficient for a solution to the household's problem:

$$c_t : \lambda_t - \gamma_t \leq 0, \ c_t(\lambda_t - \gamma_t) = 0, \tag{5}$$

$$d_t : \mu_t(1 + r_t) - \gamma_t \leq 0, \ d_t\{\mu_t(1 + r_t) - \gamma_t\} = 0, \tag{6}$$

$$\begin{aligned} &x_{it}^c : u_i(x_{1t}^c + x_{1t}^d, \ldots, x_{Nt}^c + x_{Nt}^d) - \lambda_t p_{it} \alpha_i \leq 0, \\ &\quad x_{it}^c \{u_i(x_{1t}^c + x_{1t}^d, \ldots, x_{Nt}^c + x_{Nt}^d) - \lambda_t p_{it} \alpha_i\} = 0, \end{aligned} \tag{7}$$

$$x_{it}^d : u_i(x_{1t}^c + x_{1t}^d, \ldots, x_{Nt}^c + x_{Nt}^d) - \mu_t p_{it} \beta_i \leq 0, \tag{8}$$

[6] I will be primarily interested in a stationary equilibrium, and it is easily seen that the transversality conditions are satisfied in this type of equilibrium.

$$x_{it}^{d}\{u_i(x_{1t}^{c}+x_{1t}^{d},\ldots,x_{Nt}^{c}+x_{Nt}^{d})-\mu_t p_{it}\beta_i\}=0,$$

$$m_t: -\gamma_t+\delta\gamma_{t+1}(1+i_t)=0, \tag{9}$$

$$\lambda_t: c_t-\sum_{i=1}^{N}p_{it}\alpha_i x_{it}^{c}\geq 0,\ \ \lambda_t(c_t-\sum_{i=1}^{N}p_{it}\alpha_i x_{it}^{c})=0, \tag{10}$$

$$\mu_t: (1+r_t)d_t-\sum_{i=1}^{N}p_{it}\beta_i x_{it}^{d}\geq 0,$$

$$\mu_t\{(1+r_t)d_t-\sum_{i=1}^{N}p_{it}\beta_i x_{it}^{d}\}=0, \tag{11}$$

$$\gamma_t: \{w_t+v_t+(1+i_{t-1})m_{t-1}-c_t-d_t-m_t\}=0,$$

$$c_t,\ d_t,\ x_{it}^{c},\ x_{it}^{d}\geq 0,\ \ \lambda_t,\ \mu_t\geq 0, \tag{12}$$

$$\forall i=1,\ldots,N \text{ and } \forall t\geq 0.$$

A household's choice of a transactions medium to purchase a particular commodity is determined by the following rule which can be easily proved by the above conditions.

Proposition 1

If $\alpha_i<\beta_i/(1+r_t)$, then $x_{it}^{c}>0$ and $x_{it}^{d}=0$. If $\alpha_i>\beta_i/(1+r_t)$, then $x_{it}^{c}=0$ and $x_{it}^{d}>0$.

This result is intuitive in that α_i and $\beta_i/(1+r_t)$ are the effective real costs of consuming commodity i that a houehold faces when it uses respectively currency and deposit as a transactions medium.

B) Firms

In period t, the representative firm borrows $l_t\,(\geq 0)$ from banks and in the private loan market to purchase the labor service $n_t^f(\geq 0)$, which is the only input in the production process, and uses it to produce $x_{it}^{f}\,(\geq 0)\,(i=1,\ldots,N)$ units of commodities. I assume that one unit of labor service is transformed into one unit of any of the commodities on any scale. Thus, a firm's production function in period t is

$$\sum_{i=1}^{N}x_{it}^{f}=n_t^f. \tag{13}$$

The profit of a firm in period t is

$$\sum_{i=1}^{N}p_{it}x_{it}^{f}-(1+i_t)l_t \text{ where } l_t=w_t n_t^f. \tag{14}$$

A firm is free to enter and to exit in each period. Hence, a firm's optimization is to maximize (14) subject to (13) given p_{it}, w_t, and i_t for all $t\geq 0$. The Lagrangian function L^f of this problem is

$$L^f(n_t^f,\ \{x_{it}^{f}\}_{i=1}^{N},\ \varphi_t)=\sum_{i=1}^{N}p_{it}x_{it}^{f}-(1+i_t)w_t n_t^f + \varphi_t(n_t^f-\sum^{N}x_{it}^{f}).$$

The following first-order conditions are necessary and sufficient for a solution to this problem:

$$n_t^f: -(1+i_t)w_t + \varphi_t \leq 0,\ n_t^f\{-(1+i_t)w_t + \varphi_t\} = 0, \tag{15}$$

$$x_{it}^f: p_{it} - \varphi_t \leq 0,\ x_{it}^f(p_{it} - \varphi_t) = 0, \tag{16}$$

$$\begin{aligned} &\varphi_t: n_t^f - \sum_{i=1}^{N} x_{it}^f = 0, \\ &\quad n_t^f \geq 0,\ x_{it}^f \geq 0, \\ &\quad \forall i = 1,\ldots, N \text{ and } \forall t \geq 0. \end{aligned} \tag{17}$$

Now I state a proposition which can be easily proved by these conditions.

Proposition 2

(1) There exists a solution to the firm's optimization problem at which n_t^f and x_{it}^f $\forall i = 1,\ldots, N$ are (finitely) positive, if and only if

$$(1+i_t)w_t = p_{it}\ \forall i = 1,\ldots, N. \tag{18}$$

(2) At a solution, if it exists, the profit is zero.

The equation (18) simply shows the equality of marginal cost and marginal revenue of the firm at its optimization. Since only the case where a firm produces finitely positive amounts of commodities as its optimal behavior is of interest, and as the firm's profit is zero in this case by the above proposition, I have not explicitly considered in the model the disbursement of a firm's profit to its owners.

C) Banks

When period t starts, the representative bank is holding currency R_{t-1} as a reserve which has been carried over from the period $(t-1)$. During the initial stage of period t, the bank collects the loan it made in the last period with interest, $(1+i_{t-1})L_{t-1}$, makes this period's loan $L_t\,(\geq 0)$ to firms at the interest rate i_t, pays dividend $v_t\,(\geq 0)$ to households, and purchases the labor service $n_t^b\,(\geq 0)$ at the wage rate w_t to service the deposit balance of this period. It also redeems the deposit it supplied (or accepted in common parlance) in the previous period with interest, $(1+r_{t-1})D_{t-1}$, and supplies deposit $D_t\,(\geq 0)$ of this period. Then the bank is closed for the rest of the period t.

I assume that the labor service demanded by a bank, n_t^b, is a fixed proportion $z\,(>0)$ of the real value of commodities that the deposit supplied by the bank can purchase. If the deposit supplied by a bank is D_t, the nominal value of commodities that the deposit can purchase is $(1+r_t)D_t$, and its real value is $(1+r_t)D_t/(1+i_t)w_t$ since the price level is $(1+i_t)w_t$ in equilibrium by Proposition 2. Thus, the following equation is obtained:

$$z\frac{(1+r_t)D_t}{(1+i_t)w_t} = n_t^b. \tag{19}$$

Let $k_t = z(1+r_t)/(1+i_t)$. Then the labor cost $w_t n_t^b$ of a bank is a proportion $k_t\,(>0)$ of the deposit balance D_t, i.e. $w_t n_t^b = k_t D_t$, and k_t is the marginal cost of servicing the deposit balance.

The bank's budget constraint is

$$\begin{aligned} R_t &= R_{t-1} + (1+i_{t-1})L_{t-1} - (1+r_{t-1})D_{t-1} \\ &\quad + D_t - L_t - v_t - w_t n_t^b, \\ &\quad w_t n_t^b = k_t D_t, \\ &R_t,\ L_t,\ D_t,\ v_t,\ n_t^b \geq 0,\ \forall t \geq 0, \\ &R_{-1},\ L_{-1},\ D_{-1}\ (\geq 0) \text{ are given.} \end{aligned} \tag{20}$$

The bank is also subject to the legal reserve requirement. The reserve it holds should not be less than the fraction $\rho\ (0 \leq \rho \leq 1)$ of the deposit balance it supplies:

$$R_t \geq \rho D_t,\ \forall t \geq 0. \tag{21}$$

And I assume that a bank maximizes the present value of its current and future dividends stream. Thus, its objective function is

$$v_0 + \frac{v_1}{(1+i_0)} + \cdots + \frac{v_t}{(1+i_0)\cdots(1+i_{t-1})} + \cdots. \tag{22}$$

The bank's optimization is to maximize (22) subject to (20) and (21) given the sequences of w_t, r_{t-1}, i_{t-1}, and k_t for $t \geq 0$. Similarly to the case of a household's optimization, to exclude the possibility of arbitrarily large dividends financed by ever increasing deposit supply, I assume that, given bounded sequences of w_t, r_{t-1}, i_{t-1}, and k_t, the bank is constrained to choose the sequence of D_t that is also bounded. Again I assume that the bound on D_t is nonbinding at the bank's optimal choice. Then, ignoring the bound on D_t, the Lagrangian function L^b of this problem is

$$\begin{aligned} &L^b(\{v_t, R_t, L_t, D_t, \eta_t, \xi_t\}_{t=0}^{\infty}) \\ &= \sum_{t=0}^{\infty} \frac{1}{(1+i_0)\cdots(1+i_{t-1})} \end{aligned}$$

$$\cdot[v_t + \eta_t\{R_t - R_{t-1} - (1 + i_{t-1})L_{t-1} + (1 + r_{t-1})D_{t-1} + L_t - (1 - k_t)D_t + v_t\} + \xi_t\{R_t - \rho D_t\}].$$

Assuming that the transversality conditions for R_t, L_t, and D_t are satisfied, the following first-order conditions are necessary and sufficient for a solution to this problem:

$$v_t : 1 + \eta_t \leq 0,\ v_t(1 + \eta_t) = 0, \tag{23}$$

$$\begin{aligned} R_t &: \eta_t + \xi_t - \eta_{t+1}/(1 + i_t) \leq 0, \\ &R_t\{\eta_t + \xi_t - \eta_{t+1}/(1 + i_t)\} = 0, \end{aligned} \tag{24}$$

$$L_t : \eta_t - \eta_{t+1} \leq 0,\ L_t(\eta_t - \eta_{t+1}) = 0, \tag{25}$$

$$\begin{aligned} D_t &: -\eta_t(1 - k_t) - \xi_t\rho + \eta_{t+1}(1 + r_t)/(1 + i_t) \leq 0, \\ &D_t\{-\eta_t(1 - k_t) - \xi_t\rho \\ &\quad + \eta_{t+1}(1 + r_t)/(1 + i_t)\} = 0, \end{aligned} \tag{26}$$

$$\begin{aligned} \eta_t &: R_t - R_{t-1} - (1 + i_{t-1})L_{t-1} \\ &+ (1 + r_{t-1})D_{t-1} + L_t \\ &- (1 - k_t)D_t + v_t = 0, \end{aligned} \tag{27}$$

$$\begin{aligned} \xi_t &: R_t - \rho D_t \geq 0,\ \xi_t(R_t - \rho D_t) = 0, \\ &v_t, R_t, L_t, D_t, \eta_t \geq 0, \\ &\forall t \geq 0. \end{aligned} \tag{28}$$

Now the following proposition can be easily proved by these conditions.

Proposition 3
There exists a solution to the bank's optimization problem at which v_t, R_t, L_t, and D_t are (finitely) positive for all $t \geq 0$, if and only if

$$1 + r_t = (1 + i_t)(1 - k_t) - i_t\rho,\ \forall t \geq 0. \tag{29}$$

By the definition of k_t, this equation can be expressed also as follows:

$$1 + r_t = \frac{1 + i_t(1 - \rho)}{1 + z} \tag{30}$$

The equation (29) has a simple interpretation. If a bank supplies one unit of deposit, it should hold ρ of the proceeds as a reserve and spend k_t to service the deposit balance. Thus, its loanable funds amount to $(1 - \rho - k_t)$. At the end of the period, the revenue is $(1 - \rho - k_t)(1 + i_t) + \rho$, and the cost is $(1 + r_t)$. As these should be equal at the bank's optimum, the equation (29) is obtained. When there is no cost in servicing the deposit balance ($k_t = 0$), the equation becomes $r_t = i_t(1 - \rho)$ as is commonly derived in the literature (Romer 1985; Englund and Svensson 1988).

C. Equilibrium and Optimality

In this section, first, I define the equilibrium of the model and show that a stationary equilibrium exists in this model. Then I consider the conditions of optimality and draw some policy implications.

Since both firms and banks are subject to the constant returns to scale, I normalize the sizes of the representative firm and the bank such that henceforth all quantity variables in their optimization problems are measured in "per household" units. In addition, I assume that the stock of currency (high power money) in the economy, $H(>0)$ per household, is constant throughout. Now I define the equilibrium of the economy.

Definition 1

An equilibrium is a bounded sequence $\{c_t, d_t, m_{t-1}, \{x^c_{it}, x^d_{it}\}^N_{i=1}, n^f_t, \{x^f_{it}\}^N_{i=1}, l_t, v_t, R_{t-1}, D_{t-1}, n^b_t, \{p_{it}\}^N_{i=1}, w_t, r_{t-1}, i_{t-1}, k_t\}^\infty_{t=0}$, where all variables are nonnegative except for $m_{t-1} \gtreqless 0, p_{it} > 0, r_{t-1} > -1$ and $i_{t-1} > 0 \ \forall t \geq 0$, and numbers $\rho \in [0, 1]$ and $H\,(>0)$ such that the following conditions, (a) through (h), are satisfied:

(1) Optimization
- (a) $\{c_t, d_t, \{x^c_{it}, x^d_{it}\}^N_{i=1}, m_t\}^\infty_{t=0}$ is a solution to the household's optimization problem, given m_{-1} and $\{w_t, v_t, \{p_{it}\}^N_{i=1}, r_t, i_{t-1}\}^\infty_{t=0}$.
- (b) $\{n^f_t, \{x^f_{it}\}^N_{i=1}, l_t\}^\infty_{t=0}$ is a solution to the firm's optimization problem, given $\{\{p_{it}\}^N_{i=1}, w_t, i_t\}^\infty_{t=0}$.
- (c) $\{v_t, R_t, L_t, D_t, n^b_t\}^\infty_{t=0}$ is a solution to the bank's optimization problem, given $R_{-1}, L_{-1}, D_{-1}, \{w_t, r_{t-1}, i_{t-1}, k_t\}^\infty_{t=0}$, and ρ.

(2) Market Clearance
- (d) (commodity) $\alpha_i x^c_{it} + \beta_i x^d_{it} = x^f_{it} \ \forall i = 1,\dots, N$ and $\forall t \geq 0$.
- (e) (labor) $n^f_t + n^b_t = 1 \ \forall t \geq 0$.
- (f) (loan) $l_t = L_t + m_t \ \forall t \geq 0$.
- (g) (deposit) $d_t = D_t \ \forall t \geq 0$.
- (h) (currency) $R_t + c_t = H \ \forall t \geq 0$.

A stationary equilibrium is an equilibrium in which all variables are invariant over time. And, hereafter, the variables in a stationary equilibrium will be denoted by the corresponding variables in the above definition without time subscripts. In the following, I make a technical assumption which guarantees positive demands for currency and deposit by households, and then I show that there exists a stationary equilibrium in this model.

Assumption 1
There exists a commodity that can be purchased only by currency. Also, there exists a commodity that can be purchased only by deposit.[7]

Proposition 4
For any given $H\,(>0)$ and $\rho\,(0 \leq \rho \leq 1)$, a stationary equilibrium exists in this model.

Proof: The proof is done by constructing a candidate for a stationary equilibrium (Step 1) and then by confirming that this candidate actually satisfies the conditions of equilibrium in Definition 1 (Step 2).

Step 1: Choose i, r and y such that $\delta = 1/(1+i)$, $1+r = \{1+i(1-\rho)\}/(1+z)$ (These are the values i and r necessarily take in a stationary equilibrium by (9) and (30), when δ, ρ and z are exogenously given.), and $y > 0$. Given y and r, solve the following problem:

$$\max\ u(x_1^c + x_1^d, \ldots, x_N^c + x_N^d) \text{ with respect to } \{x_i^c, x_i^d\}_{i=1}^N$$
$$\text{subject to } \sum_{i=1}^N \alpha_i x_i^c + \sum_{i=1}^N \beta_i x_i^d/(1+r) \leq y.$$

There exists a solution of $\{x_i^c, x_i^d\}_{i=1}^N$ to this problem. If there exists $j \in \{1, \ldots, N\}$ such that $\alpha_j = \beta_j/(1+r)$, there are multiple solutions. (A househould is indifferent between currency and deposit in choosing a transactions medium to purchase commodity j.) In that case, choose a solution satisfying $x_j^c \geq 0$ and $x_j^d = 0$. Since the utility function is unbounded below in each of its arguments, every commodity is purchased at a solution ($x_i^c + x_i^d > 0$ for all $i = 1, \ldots, N$). And by the above choice of a solution and Proposition 1, every commodity is purchased by a unique transactions medium (either by currency or by deposit, but not by both, i.e. $x_i^c x_i^d = 0$ for all $i = 1, \ldots,$

[7]That is, there exist $i, j \in \{1, \ldots, N\}$ such that $\alpha_i < \infty$, $\beta_i = \infty$ and $\alpha_j = \infty$, $\beta_j < \infty$. This is not an unrealistic assumption. In general, we do not write a check to ride a bus, nor do we pay currency to buy a house.

N). And, due to strict concavity of the utility function, the solution is unique. And the solutions of x_i^c and x_i^d are continuous functions of y. Let $\{x_i^c(y), x_i^d(y)\}_{i=1}^N$ denote the solution given y. Now define functions c', d', n^f and n^b of y such that

$$c'(y) = \sum_{i=1}^{N} \alpha_i x_i^c(y), \quad d'(y) = \sum_{i=1}^{N} \beta_i x_i^d(y)/(1+r),$$
$$n^f(y) = \sum_{i=1}^{N} \alpha_i x_i^c(y) + \sum_{i=1}^{N} \beta_i x_i^d(y), \text{ and } n^b(y) = 1 - n^f(y).$$

Since all commodities are normal goods, and since x_i^c and x_i^d are continuous functions of y, n^f is a strictly increasing, continuous function of y. Thus, n^b is a strictly decreasing, continuous function of y. Now define a function $\hat{n}^b = z(1+r)d'$. Note that d' is a positive, strictly increasing, continuous function of y. (By Assumption 1, $c'(y)$ and $d'(y)$ are positive for all $y > 0$). Thus, $\hat{n}^b$ is also a strictly increasing, continuous function of y. In fact, n^b is the amount of labor available for banks' employment, and $\hat{n}^b$ is the amount of labor that banks demand. If $y \downarrow 0$, then $n^b \uparrow 1$ and $\hat{n}^b \downarrow 0$, and if $y \uparrow \infty$, then $n^b \downarrow -\infty$ and $\hat{n}^b \uparrow \infty$. Thus, there exists (unique) $y'(>0)$ such that $n^b(y') = \hat{n}^b(y') \in (0, 1)$. Given this y', choose w', $m'(\lesseqgtr 0)$ and $v'(\geq 0)$ such that $w' = 1/(1+i)$ (This is the value that the real wage rate necessarily takes in a stationary equilibrium by Proposition 2, (1).) and $y' = w' + v' + im'$. Now define the following functions of y : $x_i^f = \alpha_i x_i^c + \beta_i x_i^d$ for $i = 1, ..., N$, $l' = w'n^f$, $L' = l' - m'$, $D' = d'$, $R' = \rho D'$, $H' = R' + c'$, and $p_i = H/H'$ for $i = 1, ..., N$. And let $k = z(1+r)/(1+i)$. Functions with the prime symbol evaluated at y' show the real values of the respective variables in a stationary equilibrium. Their nominal values are the products of these real values and the price level H/H'. Similarly, multiply each of m', v', and w' by the price level H/H' to obtain their respective nominal values in a stationary equilibrium. Denote the nominal values by omitting the prime symbol. For instance, $c(y') = (H/H')c'(y')$. Then the set $\{c(y'), d(y'), m, \{x_i^c(y'), x_i^d(y')\}_{i=1}^N, n^f(y'), \{x_i^f(y')\}_{i=1}^N, l(y'), v, R(y'), L(y'), D(y'), n^b(y'), \{p_i(y')\}_{i=1}^N, w, r, i, k\}$ constitutes a candidate for a stationary equilibrum.

Step 2: To show that this is actually an equilibrium, first, note that all of the market clearing conditions (Definition 1 (d)–(h)) are satisfied. Next, I show that budget constraints of agents are satisfied. Obviously, the household's budget constraint $c(y') + d(y') = w + v + im$ and the firm's budget constraint $l(y') = wn^f(y')$ are satisfied. Now note that the following equation holds in a stationary equilibrium:

$$c(y^\frown) + (1+r)d(y^\frown) = (1+i)l(y^\frown) \tag{31}$$

It is satisfied since $c(y^\frown) + (1+r)d(y^\frown) = \sum_{i=1}^{N} p_i(y^\frown)\,\alpha_i x_i^c(y^\frown) + \sum_{i=1}^{N} p_i(y^\frown) \cdot \beta_i x_i^d = (1+i)wn^f(y^\frown) = (1+i)l(y^\frown)$. It shows that total revenue of the firm is equal to its loan obligation (including interest payment) in an equilibrium, and it is the zero-profit condition of the firm. Now, it can be checked that if the budget constraints of households and firms and all of the market clearing conditions are satisfied along with equation (31), then the bank's budget constraint in a stationary equilibrium, $iL(y^\frown) - rD(y^\frown) - wn^b(y^\frown) = v$, is satisfied. (This equation shows that in a stationary equilibrium, the bank's dividend is equal to its interest income less the deposit interest payment and the cost of servicing the deposit balance.)

Finally, it can be easily checked that all the other first order conditions of each agent's maximization problem ((5)–(12), (15)–(17), (23)–(28)) are satisfied.

Q.E.D.

Now I turn to the question of optimality. An allocation is defined to be optimal, if the utility of the representative household is maximized given the per–household resource constraint of the economy. Since the utility function of a household is additively seperable, and commodities are nonstorable, the optimality of the economy is achieved when $x_{it}^c = x_i^{c*}$ and $x_{it}^d = x_i^{d*}$ for all $i = 1,\ldots, N$ and $t \geq 0$ where $\{x_i^{c*}, x_i^{d*}\}_{i=1}^{N}$ is a solution to the following maximization problem:

$$\begin{aligned} &\max\ u(x_1^c + x_1^d, \ldots, x_N^c + x_N^d) \text{ with respect to } \{x_i^c\ x_i^d\}_{i=1}^{N} \\ &\text{subject to } \sum_{i=1}^{N} \alpha_i x_i^c + (1+z)\sum_{i=1}^{N} \beta_i x_i^d \leq 1. \end{aligned} \tag{32}$$

Here $\sum_{i=1}^{N} \alpha_i x_i^c [\sum_{i=1}^{N} \beta_i x_i^d$ resp.] is the amount of labor input needed to produce commodities purchased by currency [deposit, resp.]. Total amount of labor input available in the economy is used in part to provide consumption of commodities $(\sum_{i=1}^{N} x_i^c + \sum_{i=1}^{N} x_i^d)$ and in part to provide transaction services $(\sum_{i=1}^{N}(\alpha_i - 1)x_i^c + \sum_{i=1}^{N}(\beta_i - 1)x_i^d)$. This transaction cost is borne by households. When deposits are used as a transactions medium, there is an additional transaction cost, $z(\sum_{i=1}^{N} \beta_i x_i^d)$, which is the cost of servicing deposit balances and is borne by banks. From the maximization problem (32), the following proposition is obtained.

Proposition 5

At optimum, if $\alpha_i < (1+z)\beta_i$, *then* $x_i^{c*} > 0$ and $x_i^{d*} = 0$. And if α_i

$> (1+z)\beta_i$, then $x_i^{c*} = 0$ and $x_i^{d*} > 0$.

This result is intuitively interpretable, since α_i and $(1+z)\beta_i$ are the real social costs of consuming one unit of commodity i using respectively currency and deposit as transactions mediums. The next proposition shows the relation between a stationary equilibrium and the optimality.

Proposition 6
A stationay equilibrium is optimal, if and only if it is associated with $\rho = 1$.

Proof: Let $\{\bar{x}_i^c, \bar{x}_i^d\}_{i=1}^N$ be the consumptions of commodities purchased by currency and deposit in a stationary equilibrium. Then it is necessarily a solution to the following one-period maximization problem of a household in stationary condition:

$$\begin{aligned} &\max\ u(x_1^c + x_1^d, \ldots, x_N^c + x_N^d) \text{ with respect to } \{x_i^c, x_i^d\}_{i=1}^N \\ &\text{subject to } p\sum_{i=1}^N \alpha_i x_i^c + \{p/(1+r)\} \sum_{i=1}^N \beta_i x_i^d \leq w + v + im. \end{aligned} \tag{33}$$

From the budget constraint of a bank in (20), $v = iL - (r+k)D$. By Proposition 3, $r + k = -ik + i(1-\rho)$. Thus, $v = iL - \{-ik + i(1-\rho)\} D$. And from $L + m = l$ (See Definition 1, (f).) $= wn^f$ (See (14).) and $kD = wn^b$ (See (19).), it follows that $L + m + kD = w(n^f + n^b) = w$. Thus, in (33), $w + v + im = w + i(L + kD + m) - i(1-\rho)D = (1+i)w - i(1-\rho)D = p - i(1-\rho)D$ (See Proposition 2, (1).).

Now to prove sufficiency, suppose that $\rho = 1$. Then, $w + v + im = p$. And, from (30), it holds that $1/(1+r) = 1 + z$, if $\rho = 1$. Thus, maximization problem (33) is identical to the maximization problem (32). Therefore, $\{\bar{x}_i^c, \bar{x}_i^d\}_{i=1}^N$ solves (32), and the sufficiency is proved.

To prove necessity, first note that any equilibrium sequence of consumption, $\{\{x_{it}^c, x_{it}^d\}_{i=1}^N\}_{t=0}^\infty$, satisfies the budget constraint of (32) with equality for all $t \geq 0$. This is because, in equilibrium,

$$\begin{aligned} \sum_{i=1}^N \alpha_i x_{it}^c + \sum_{i=1}^N \beta_i x_{it}^d &= \sum_{i=1}^N x_{it}^f \text{ (See Definition 1, (d).)} \\ &= n_t^f \text{ (See (13).)} \\ &= 1 - n_t^b \text{ (See Definition 1, (e).)} \\ &= 1 - z\frac{(1+r_t)D_t}{(1+i_t)w_t} \text{ (See (19).)} \end{aligned}$$

$$= 1 - z\frac{\sum_{i=1}^{N} p_{it}\beta_i x_{it}^d}{(1+i_t)w_t}$$

(See Definition 1, (g) and the liquidity constraint of deposit in (4) which holds as an equality in equilibrium since $i_t > r_t$ in equilibrium.)

$$= 1 - z\sum_{i=1}^{N} \beta_i x_{it}^d \text{ (See Proposition 2.).}$$

Thus, we have $\sum_{i=1}^{N} \alpha_i x_{it}^c + (1+z)\sum_{i=1}^{N} \beta_i x_{it}^d = 1$, and $\{\{x_{it}^c, x_{it}^d\}_{i=1}^{N}\}_{t=0}^{\infty}$ satisfies the budget constraint of (32) for all $t \geq 0$. Now, if $\rho < 1$, then, by (30), we have $1/(1+r_t) < (1+z)$ for all $t \geq 0$. Then, for all $t \geq 0$, the cost of a unit consumption of commodity i purchased by currency relative to that of commodity j purchased by deposit that a household faces ($\alpha_i / \{\beta_j/(1+r_t)\}$) is different from (in fact, higher than) the relative cost the society faces ($\alpha_i / \{\beta_j(1+z)\}$, see (32).) for all $i, j = 1,..., N$. Hence, the solution to a household's optimization problem cannot be equal to the solution to the social optimization problem (32).

Q.E.D.

In fact, the necessity part of the above proof yields even a stronger result: Any equilibrium, stationary or nonstationary, associated with $\rho < 1$ is not optimal.

Proposition 6 is illustrated in Figure 1. In this economy, there are two commodities, 1 and 2, which are purchased by currency and deposit respectively. The solid line is the social budget constraint of (32), and the broken line is the household's budget constraint of (33) in a stationary equilibrium. Point A shows the household's optimal choice, and point B shows the social optimum. Note that A is also on the social budget constraint. If $\rho = 1$, two budget constraints coincide, and the social optimum is achieved. But, if $\rho < 1$, as in Figure 1, the cost of consuming commodity 2 relative to the cost of consuming commodity 1 becomes lower, and the household consumes larger amount of commodity 2 ($\bar{x}_2^d > x_2^{d*}$) and smaller amount of commodity 1 ($\bar{x}_1^c < x_1^{c*}$) than the social optimum requires. And by strict concavity of the utility function, B is preferred to A.

These results can be recapitulated as follows. When the reserve requirement ratio is less than one, the private cost, in real terms, of consuming a commodity using deposit as a transactions medium is less than its social cost ($\beta_i/(1+r_t) < (1+z)\beta_i$). In particular, the lower reserve requirement ratio is associated with the higher deposit interest rate in an equilibrium (See (30).), and this increases

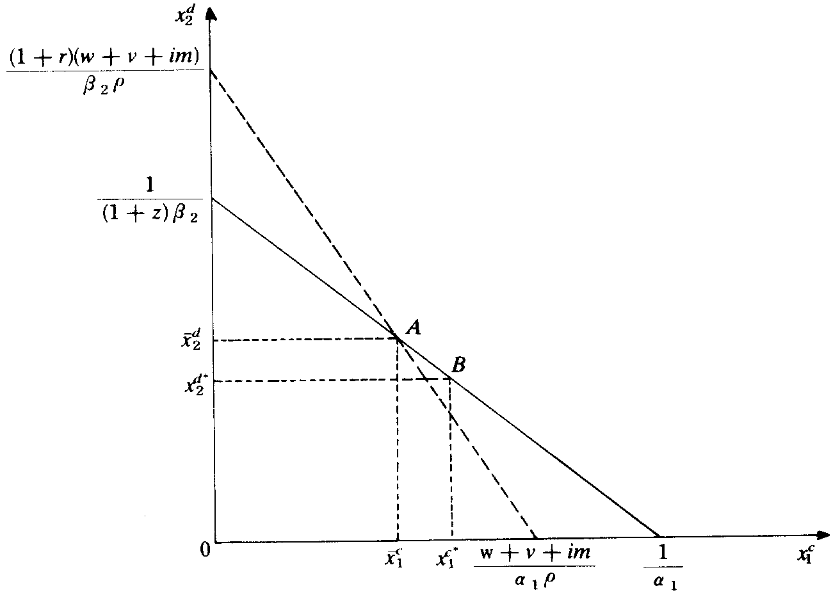

FIGURE 1
THE NONOPTIMALITY OF $\rho < 1$

inefficiency in two respects: firstly, households consume more of the commodities which are purchased by deposit than the optimality requires since they are effectively cheaper for households than before (See Figure 1.), and secondly, households switch to deposit as a means of payment in purchasing some of the commodities that have previously been purchased more efficiently by currency. (Compare Propositions 1 and 5.)

Proposition 6 suggests the desirability of 100 percent reserve requirement as an optimal regulation of banking. If $\rho = 1$, then $r_t = -(1+i_t)k_t$ by Proposition 3. Since the deposit interest is negative in this case, depositors pay service charge of $(1+i_t)k_t$ at the end of the period for each unit of deposit they made at the beginning of the period. If the service charge is collected at the time the deposit is made, as it seems more practical, then the charge should be equal to the marginal cost of servicing the deposit balance ($k_t = -r_t/(1+i_t)$).

III. Concluding Remarks

The 100 percent reserve banking has been advocated usually on two grounds: it shields the quantity of money in the economy from the influence of shifts in the composition of currency and deposit in the hands of public and, thereby, enhances the price level stability, and it protects effectively the banking system from bank runs and, thereby, enhances the financial stability of the economy.[8]

This paper investigated yet another reason for the desirability of 100 percent reserve banking: under the competitive equilibrium, it results in a deposit interest rate which correctly reflects the social cost of providing transaction services through deposits and brings about an optimal allocation of resources.

There are many issues to resolve for the actual implementation of this system. One of them is the scope of deposits that should be subject to the reserve requirements. Friedman (1960) asserts that reserve requirements should be uniform for all types of deposits. While this paper does not explicitly deal with this subject, the model suggests that the criterion should be whether the deposit can serve as a means of payment. Since the time deposit with a positive maturity does not perform such a function, and it is effectively a loan from a household to a bank, the time deposit may not be subject to the reserve requirement, contrary to Friedman's argument.

Another difficult issue involves the dynamic problem concerning the transition from the present fractional system to the 100 percent reserve system. Friedman (1960) proposes that the reserve requirement ratio be raised in a sequence of preannounced steps up to 100 percent. Another way may be to make the marginal reserve requirement ratio 100 percent such that any demand deposit made after a certain point of time should carry 100 percent reserve. These and other aspects of the 100 percent reserve banking remain the subjects of further research.

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[8]For a general discussion, see Friedman (1960).

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