# A Sorting cum Learning Model with a Moral Hazard Problem\*

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This paper investigates the existence of competitive equilibrium of an information game under the following circumstances: 1) A continuum of test qualities exists where each firm is able to choose one test. Firms offer contracts which are wage/test quality pairs, and they treat contracts of other firms as given. 2) The tests have two kinds of roles. First, as a productivity enhancing device, the tests enhance workers' productivity. Second, as a sorting device, the tests divide workers into those who pass and those who fail. However, tests have a moral hazard problem as a sorting device because workers may choose hidden actions to enhance their test scores. Under the above circumstances, distinctive features of the condition for the nonexistence of the competitive equilibrium are noticed: If a primary purpose of testing workers is to enhance workers' productivity, then the competitive equilibrium of the information game exists. However, if a primary purpose of testing workers is to sort out workers and the tests have a moral hazard problem, the competitive equilibrium does not exist.

#### I. Introduction

In the presence of heterogeneity among the labor force and imperfect information by employers, a common practice for firms is to offer a wage for a given job classification, and to classify applicants in an attempt to ensure a minimum level of performance. A probation period is then established during which the applicants perform-

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ance is carefully observed. On the other hand, the probation period may be useful for some jobs in order to enhance the acquired productivity of workers who have just started their working careers.

This paper develops a testing model which explains the sorting cum learning mechanism of the probation period in firms. The circumstances we consider in this paper is that the personal productivity type (type 1 or type 2) is initially unknown but may be revealed to a certain extent by some costly actions of firms. In our model, firms assign workers by the personal types they declare, but firms may test workers in order to check their declarations with the true types.

The model in this paper considers two types of workers (type 1 and type 2) and firms with two substitutable tasks (task 1 and task 2) in their production process. The efficient mode of production requires firms to have type 1 in task 1 and type 2 in task 2, and the productivity of type 1 in task 1 is greater than the one of type 2 in task 2.

Firms test workers by using the following scheme: Workers who declare themselves as type 1 must pass a test in order to prove that he is actually type 1 and workers who declare themselves as type 2 are considered as type 2 without testing them.

The tests have two kinds of roles. First, as a productivity enhancing device, the tests improve workers' productivity. As the quality of the test increases, workers learn more and their productivity is improved to a greater extent. Second, the tests are a sorting device; the tests divide workers into type 1 and type 2. However, the tests have a moral hazard problem as a sorting device because type 2 may choose hidden actions so that they appear to be type 1. In a test with better quality, type 2 must exert a higher level of hidden actions in order to appear as type 1. A continuum of test qualities exists where each firm is able to choose one test. Better test quality requires a more costly test. Firms are assumed to offer contracts which are wage/test quality pairs and to treat contracts of other firms as given.

The idea of introducing a test as a sorting device is not new. For example, Guash and Weiss (1981) employed a noisy test with measurement error and required the application fee to enhance the effectiveness of the test as a sorting device. This paper imitates the test employed by Guash and Weiss (1981) but the test plays two kinds of role; as a productivity enhancing device and as a sorting device.

The theoretic results in this paper have some similarities with the results obtained by previous insurance models including Stiglitz (1975), Riley (1975), and Rothschild and Stiglitz (1976). For example, Rothschild-Stiglitz showed that the non-existence of competitive equilibrium in the information game is a typical problem if the uninformed is allowed to move first. This paper also obtains Rothschild-Stiglitz type non-existence of competitive equilibrium. However, distinctive conditions for non-existence are noticed: If a primary purpose of testing workers is to enhance workers' productivity, then the competitive equilibrium of the information game exists. However, if a primary purpose of testing workers is to sort out workers and tests have a moral hazard problem, the equilibrium may not exist.

In the first half of the model, it is assumed that a monopolist firm exists, and the optimal testing strategy of the monopolist firm is developed. By comparing the profitability of the optimal testing strategy with the one of the pooling strategy, the best strategy of the monopolist firm is determined. In the second half of the model, firms are in competition with one another, and the existence of competitive equilibrium is investigated.

# II. The Optimal Testing Strategy

Variables used in this paper are described as follows.

i: Individual type i = 1, 2

j: Task type j = 1, 2

 $f_i$ : Productivity when a type i worker is assigned to task  $i(f_1 > f_2)$ 

 $r_i$ : Reservation wage for a type i worker  $(r_1 > r_2)$ 

a: Quality level of test  $(a \ge 0)$ 

 $\tau a$ : Cost of administering the test with quality level a

 $\Theta a$ : A type 2 worker's cost of hidden actions to pass the test with quality a

 $sf_1a$ : Type 1 workers' productivity gain when they pass the test with quality level a. s is the coefficient which determines the effectiveness of test as a productivity enhancing device.

 $W_1$ : Wage for a worker who declares himself to be type 1

 $W_2$ : Wage for a worker who declares himself to be type 2

N: Total number of workers

v: Total number of firms (v > 1)

n: n = N/v

The following assumptions are made:

## Assumption 1

Two types of workers exist; type 1 and type 2. Workers' true types are only known by the workers themselves. Type 1 and type 2 workers have comparative advantage in task 1 and task 2 respectively. Each type has 0 productivity in the alternative task.

## Assumption 2

A continuum of positive test quality exists where the firm chooses one test. The testing cost increases with the quality (i.e.,  $\tau > 0$ ).

## Assumption 3

Type 1 workers' productivity in task 1 is augmentable through a learning process in the test (i.e., s > 0). On the contrary, type 2 workers' productivity in task 2 relies on their innate attributes and cannot be modified even if they pass the test.

## Assumption 4

Type 2 workers are allowed to choose hidden actions in order to appear as type 1. Cost of type 2's hidden actions increases with test quality (i.e.,  $\Theta > 0$ ).

# Assumption 5

$$\tau > sf_1$$

With better test quality, testing cost increases more rapidly than the productivity gain of type 1 workers in the test.

## Assumption 6

Any strategy can be withdrawn costlessly if it becomes unprofitable after the firm makes its own strategy.

# Assumption 7

Equal number of type 1 and type 2 workers (N/2) exists.

For the time being, it is assumed that a monopolist firm exists. The monopolist firm's testing strategy is as follows: If a worker declares himself as type 1, he must prove it by passing the test. If a worker declares himself as type 2, he is considered as type 2.

The monopolist firm's optimal testing strategy is specified by the optimal choice of  $(W_1, W_2, a)$ . Let  $\alpha = (W_1^*, W_2^*, a^*)$  denote the optimal testing strategy where  $W_1^*, W_2^*$  and  $a^*$  are the optimal values of  $W_1, W_2$  and a respectively.

 $W_1^*, W_2^*$  and  $a^*$  are determined by the following problem.

$$W_1 - \Theta a < W_2. \tag{2}$$

In the above minimization problem,  $^1$  the firm minimizes the testing cost ( $\tau a$ ) minus the productivity gain ( $sf_1a$ ) plus wage cost ( $W_1$ ) per type 1 worker. Constraint (1) specifies that the wage for a type i worker must be greater than his reservation wage. If constraint (1) is violated, no type i worker (i=1,2) will work for the firm. Constraint (2) is a truth revealing constraint of a type 2 worker. In constraint (2), ( $W_1 - \Theta a$ ) is the net wage for a type 2 worker when he declares himself as type 1. The truth revealing constraint specifies that the net wage for a type 2 worker pretending to be type 1 must be smaller than or equal to the wage if he reveals his true type.

The above minimization problem is solved by setting the optimal  $W_1$  and  $W_2$  as  $W_1^* = r_1$  and  $W_2^* = r_2$  beforehand and modifying the above minimization problem in the following way:

Min 
$$(\tau - sf_1)a$$
  
s.t.  $\Theta a > r_1 - r_2$ .

The solution of the above minimization problem is:

$$a^* = (r_1 - r_2)/\Theta. (3)$$

The interesting insights about  $a^*$  are obtained as follows:

- 1) As  $(r_1 r_2)$  increases, a type 2 worker is more inclined to pretend to be type 1. To discourage a type 2 worker from feigning to be type 1, the testing quality must be raised.
- 2) As  $\Theta$  increases, cost of a type 2 worker's appearing as type 1 increases, and a type 2 worker is less inclined to pretend to be type 1. Then, the firm is able to induce a type 2 worker to reveal his

 $<sup>^{1}\</sup>mathrm{By}$  assumption 5, the model is restricted to the case with  $\tau > sf_{1}$ . A reader may consider the case with  $\tau \leq sf_{1}$ . If  $\tau \leq sf_{1}$ ,  $a^{*}$  is unbounded and the truth revealing constraint is always satisfied with a strict inequality. In this case, the informational aspect of the problem becomes trivial.

<sup>&</sup>lt;sup>2</sup>A reader may consider a rewarding solution such that  $W_2 > r_2$ . With  $W_2 > r_2$ , the monopolist rewards the honesty of type 2 workers and thereby can save the testing cost. However, as we extend our analysis to the perfectly competitive market whose equilibrium is one of main concerns in this paper, the rewarding solution cannot be sustained because other non-rewarding firms (i.e.,  $W_2 = r_2$ ) take advantage of the rewarding of a firm and can reduce their testing cost. Here, we limit ourselves to the case where  $W_2 = r_2$ .

true type at a low test quality.

## III. The Determination of the Best Strategy

In the previous part, the optimal testing strategy for the monopolist firm  $\alpha$  was determined. The profit of the firm with  $\alpha$  is specified by  $\pi(\alpha)$ :

$$\pi(\alpha) = (N/2) \{ (1 + sa^*)f_1 - \tau a^* - r_1 \} + (N/2)(f_2 - r_2)$$
 (4)

Now, the best strategy for the firm<sup>3</sup> is determined. Then, it is necessary to compare the profit from the testing strategy with the one from the following non-testing strategies.

 $\beta$ : The pure pooling strategy where the identical wage  $r_1$  is offered to both types of workers and no one is tested. Under the pure pooling strategy, all workers are assigned to task 1 since  $f_1 > f_2$ .

The expected productivity of a worker under  $\beta$  is  $f_1/2$ , and the expected profit of  $\beta$  is:

$$\pi(\beta) = N[(f_1/2) - r_1]. \tag{5}$$

Between  $\alpha$  and  $\beta$ , the best strategy is the one which gives a maximum profit. Let us examine the inequality  $\pi(\alpha) > \pi(\beta)$  which is the condition where  $\alpha$  is the best strategy. Substituting (4) and (5) for  $\pi(\alpha) - \pi(\beta)$  gives:

$$\frac{(N/2)f_2 + (N/2)sf_1(r_1 - r_2)/\Theta}{+ (N/2)(r_1 - r_2) - (N/2)\tau(r_1 - r_2)/\Theta}.$$
 (6)

(6) is decomposed into four terms. The first term  $(N/2)f_2$  is the expected efficiency gain of  $\alpha$  from assigning type 2 workers to task 2 where they have comparative advantages. The second term  $(N/2)sf_1(r_1-r_2)/\Theta$  is the productivity gain of type 1 workers as they learn from the test. The third term  $(N/2)(r_1-r_2)$  is the savings of wage cost of type 2 workers under  $\alpha$ . The last term  $(N/2)\tau$   $(r_1-r_2)/\Theta$  is the testing cost.

Let  $\delta$  be  $\pi(\alpha) - \pi(\beta)$ . Then, the comparative statics with  $\delta$  are as follows:

<sup>&</sup>lt;sup>3</sup>The first term of the right hand side in (4) (i.e.,  $(N/2)[(1+sa^*)f_1-\tau a^*-r_1]$ ) should be positive. Otherwise, the firm would hire only type 2 workers and assign them in task 2, which becomes a trivial case.

$$d\delta / df_1 = (N/2) s(r_1 - r_2)/\Theta > 0$$

$$d\delta / df_2 = N/2 > 0$$

$$d\delta / ds = (N/2) f_1(r_1 - r_2)/\Theta > 0$$

$$d\delta / d\tau = -(N/2) (r_1 - r_2)/\Theta < 0$$

$$d\delta / dr_1 = (N/2) - (N/2) (\tau - sf_1)$$

$$d\delta / dr_2 = (N/2) (\tau - sf_1) - N/2.$$

The above comparative statics give some interesting insights.

- 1) As  $f_1$  or  $f_2$  increases, the efficiency loss from wrongly assigned personal types increases. Then, the value of sorting personal types increases, and the relative profitability as  $\alpha$  increases.
- 2) Wage cost under  $\beta$  is  $r_1N$  and wage cost under  $\alpha$  is  $N(r_1+r_2)/2$ . As  $r_1$  increases, wage cost increases more under  $\beta$  than under  $\alpha$ . Thus in terms of saving in wage cost,  $\alpha$  is better than  $\beta$ . However, as  $r_1$  increases, a type 2 worker is more inclined to pretend to be type 1, and better testing quality is required to discourage the type 2 worker's pretension. Therefore, as  $r_1$  increases,  $\alpha$  saves more wage cost but requires greater testing cost associated with better testing quality. On the contrary, as  $r_2$  increases,  $\alpha$  saves less wage cost but requires less wage cost but requires less testing cost.
- 3) As s increases, the productivity gain of type 1 workers passing the test increases and the relative profitability of  $\alpha$  increases.
- 4) As  $\tau$  increases, the testing cost increases and the relative profitability of  $\alpha$  decreases.

# IV. The Competitive Equilibrium

In this part, it is examined how the competition forces the firms to choose a zero profit strategy. It is examined that firms choose the strategy maximizing their profits, given strategic structure of the other firms.

First, the zero profit pooling strategy and the zero profit testing strategy is specified. Secondly, it is shown that the zero profit strategies specified in the first part are the only candidate for the competitive equilibrium strategy.

$$a_0 = (W_1^{\alpha} - W_2^{\alpha})/\Theta.$$

From (7), (7)' is obtained such that

$$W_1^{\alpha} = [\Theta f_1 + f_2(\tau - sf_1)]/(\Theta - sf_1 + \tau)$$

$$W_2^{\alpha} = f_2$$

$$a_0 = (f_1 - f_2)/(\Theta - sf_1 + \tau).$$
(7)

#### Theorem 1

1) If  $\pi(\alpha) > 0$ , there always exists a deviant strategy which breaks down  $\alpha$ , and 2) among the zero profit testing strategies,  $\alpha_0$  is the only candidate for the competitive equilibrium strategy.

*Proof*: 1) Suppose that all firms choose  $\alpha = (W_1, W_2, a)$  where a = $(W_1 - W_2)/\Theta$ . Note that  $\pi(\alpha) = (n/2)[(1 + sa)f_1 - \tau a - W_1]$  $+ (n/2)(f_2 - W_2)$ . If  $\pi(\alpha) > 0$ , it is necessary to prove that there exists  $\alpha'$  which breaks down  $\alpha$ . Suppose that a deviant firm offers  $W_1$  and  $W_2$  which satisfies  $W_1 > W_1$ ,  $W_2 > W_2$  and  $(W_1 - W_2) =$  $(W_1 - W_2)$ . At the same level of test quality, a type 2 worker will reveal his true type because  $a = (W_1 - W_2)/\Theta = (W_1 - W_2)/\Theta$ . If the deviant strategy  $\alpha'$  can be offered, both types of workers will work for the deviant firm. Let  $\pi_d(\alpha')$  as the profit of a deviant strategy  $\alpha'$  given that all the other firms choose  $\alpha^5$ . Then,  $\pi_d(\alpha')$  $= (N/2)\cdot[(1+sa)f_1 - \tau a - W_1] + (N/2)\cdot(f_2 - W_2)$ . In order to prove the existence of  $\alpha'$ , it should be shown that there are values of  $W_1$  and  $W_2$  satisfying  $\pi_d(\alpha') > \pi(\alpha)$ . Suppose that  $W_1 = W_1$ +  $\phi_1$  and  $W_2 = W_2$   $\phi_2$  where  $\phi_1 > 0$ ,  $\phi_2 > 0$ , and  $(W_1 - W_2)/\Theta$ =  $[(W_1 + \phi_1) - (W_2 + \phi_2)]/\Theta$ . Then,  $[\pi_d(\alpha') - \pi(\alpha)] = (v - \phi_1)$ 1)  $\pi(\alpha) - (N/2) \cdot (\phi_1 + \phi_2)$ . If  $\phi_1 = \phi_2 = 0$ ,  $[\pi_d(\alpha') - \pi(\alpha)]$  $= (\nu - 1) \cdot \pi(\alpha) > 0$  since  $\nu > 1$ . By choosing  $\phi_1$  and  $\phi_2$  sufficiently close to 0, it still yields  $\pi_d(\alpha') > \pi(\alpha)$ . Therefore, if  $\pi(\alpha)$ > 0, there always exists a deviant strategy  $\alpha'$  which breaks down

2) Consider a zero profit strategy  $\alpha = (W_1, W_2, a)$  where  $\pi(\alpha) = 0$ . It was proven that if  $\alpha$  is the equilibrium strategy,  $W_2 = f_2$  (=  $W_2^{\alpha}$ ). Then, it naturally follows that  $W_1 = W_1^{\alpha} = (1 + sa_0)f_1 - \tau a_0$  where  $a_0 = (W_1^{\alpha} - W_2^{\alpha})/\Theta$ . Suppose that  $W_2 < f_2$ . Then, the deviant firm which employs only type 2 workers earns positive profit.

 $<sup>{}^4</sup>a_0$  in  $\alpha_0$  is calculated in Appendix.

 $<sup>^{5}\</sup>pi_{d}(\alpha')$  needs to be distinguished from  $\pi(\alpha')$ .  $\pi_{d}(\alpha')$  is the profit of the deviant firm given that all the other firms choose  $\alpha$ .  $\pi(\alpha')$  is the profit of the firm when all firms choose the same strategy  $\alpha'$ .

On the other hand, suppose that  $W_2 > f_2$ . Then, the deviant firm which employs only type 1 workers makes positive profit. In order for  $\alpha$  to be the equilibrium strategy,  $W_2 = f_2$ . Therefore,  $\alpha_0$  is the only candidate for the equilibrium strategy.

Q.E.D.

 $eta_0$ : The zero profit pooling strategy which offers  $W^{eta}$  for both types of workers and  $W^{eta}$  is

$$W^{\beta} = f_1/2. \tag{8}$$

Theorem 2

If  $\pi(\beta) > 0$ , there always exists deviant strategy which breaks down  $\beta$ . Then, among the pooling strategies  $\beta_0$  is the only candidate for the competitive equilibrium strategy.

Proof: Suppose that the firm with a pooling strategy chooses  $W < W^{\beta}$  (i.e.  $\pi(\beta) > 0$ ). Now, it is proven that there exists  $\beta'$  which breaks down  $\beta$ . Suppose that a deviant firm chooses W'(>W). Let  $\pi_d(\beta')$  as profit of the firm with the deviant strategy  $\beta'$ , given that all the other firms choose  $\beta$ . Then, since W'>W, all the workers will work for the deviant firm. To prove the existence of  $\beta'$ , it is necessary to show that there always exist values of W' with  $\pi_d(\beta') > \pi(\beta)$ . Suppose that  $W' = W + \phi$  where  $\phi > 0$ .  $\pi_d(\beta') - \pi(\beta) = (\nu - 1) \cdot \pi(\beta) > 0$  since  $\nu - 1$ . By choosing  $\phi$  sufficiently close to 0, it is obtained that  $\pi_d(\beta') > \pi(\beta)$ . Therefore, if  $\pi(\beta) > 0$ , there always exists a deviant strategy which breaks down  $\beta$ .

Q.E.D.

Theorem 1 and 2 showed that if the competitive equilibrium exists, it is characterized by a zero profit strategy.

Theorem 3

If  $W_1^{\alpha} > W^{\beta}$ ,  $\alpha_0$  is the competitive equilibrium strategy.

Proof: If  $W_1^{\alpha} > W_2^{\alpha} > W^{\beta}$ , both types of workers prefer  $\alpha_0$  to  $\beta_0$  as  $\alpha_0$  offers the higher wage for both types of workers. Given that the other firms choose  $\alpha_0$ , there exists no further incentive to choose  $\beta_0$  as a deviant strategy. If  $W_1^{\alpha} > W^{\beta} > W_2^{\alpha}$ , type 1 workers prefer  $\alpha_0$  to  $\beta_0$  but type 2 workers prefer  $\beta_0$  to  $\alpha_0$ . Given that the other firms choose  $\alpha_0$ , the profit of the firm choosing  $\beta_0$  as a deviant strategy becomes  $\pi_d(\beta_0) = -(N/2) W^{\beta}$  which is negative, and thereby there exists no incentive to choose  $\beta_0$  as a

deviant strategy. Therefore, if  $W_1^{\alpha} > W^{\beta}$ ,  $\alpha_0$  is the competitive equilibrium strategy.

It may be inferred that if  $W^{\beta} > W_1^{\alpha} > W_2^{\alpha}$ ,  $\beta_0$  dominates  $\alpha_0$  as both types of workers prefer  $\beta_0$  to  $\alpha_0$ , and thus  $\beta_0$  is the equilibrium strategy. However, it is shown that  $\beta_0$  cannot be the equilibrium strategy.

#### Theorem 4

There always exists a deviant testing strategy  $\alpha$  which breaks down  $\beta_0$ .

Proof: Given that all firms choose  $\beta_0$ , the profitability of the deviant testing strategy is examined. Consider the deviant testing strategy  $\alpha = (W_1, W_2, a)$  where  $W_1 > W_1^{\alpha}$ ,  $W_2 = W_2^{\alpha}$  ( $< W^{\beta}$ ) and  $a = (W_1 - W^{\beta})/\Theta$ . If  $W_1 = W^{\beta}$ , the testing strategy  $\alpha$  is degenerated into the zero profit pooling strategy  $\beta_0$ .  $\pi_d(\alpha)$  is specified as  $\pi_d(\alpha) = (N/2) [f_1 - (\tau - sf_1)a - W_1]$ . Suppose that  $W_1 = W^{\beta} + \phi$  where  $\phi > 0$ . Then,  $\pi_d(\alpha)$  decreases with  $\phi$ . By choosing  $\phi$  sufficiently close to 0,  $\pi_d(\alpha) > 0$  is always obtained.

Q.E.D.

Theorem 3 and 4 show that  $W_1^{\alpha} > W^{\beta}$  is a necessary and sufficient condition for the existence of the competitive equilibrium.

## Proposition

If 1) the primary purpose of testing workers is to sort out workers rather than to enhance workers' productivity (i.e., s is small), 2) tests have a moral hazard problem as a sorting device (i.e.,  $\Theta$  is small) because type 2 workers may choose hidden actions so that they appear to be type 1, the competitive equilibrium is not likely to exist.

Proof: From (7)' and (8), it is obtained;

$$dW_1^{\alpha}/ds = \left[\Theta f_1(f_1 - f_2)\right]/(\Theta - sf_1 + \tau)^2 > 0$$

$$dW_1^{\alpha}/d\Theta = \left[(f_1 - f_2)(\tau - sf_1)\right]/(\Theta - sf_1 + \tau)^2 > 0$$

$$dW^{\beta}/ds = dW^{\beta}/d\Theta = 0.$$

Therefore, as s or  $\Theta$  decreases (increases), it is less (more) likely to have  $W_1^{\alpha} > W^{\beta}$  and the competitive equilibrium less (more) likely

<sup>&</sup>lt;sup>6</sup>If  $\phi = 0$ , a = 0,  $\pi_d(\alpha) = 0$  and the deviant testing strategy  $\alpha$  degenerates into the zero profit pooling strategy.

to exist.

Q.E.D.

#### V. Conclusion

This paper investigates the existence of competitive equilibrium for an information game under the following circumstances: 1) Two types of workers exist; type 1 and type 2. Firms hire workers by the productivity types they declare, but they test workers to determine the workers' true types. 2) A continuum of test qualities exists where each firm is able to choose one test. Firms offer contracts which are wage/test quality pairs, and they treat contracts of other firms as given. 3) The tests have two kinds of roles. First, as a productivity enhancing device, the tests improve workers' productivity. For a better quality test, workers learn more and their productivity is improved to a greater extent. Second as a sorting device, the tests divide workers into type 1 and type 2. However, the tests have a moral hazard problem as a sorting device because type 1 may choose hidden actions so that he appears to be type 2. For a better quality test, type 2 must exert a higher degree of hidden actions in order to pretend to be type 1.

The theoretic results of the paper have some similarities with the results obtained by previous insurance models including Rothschild and Stiglitz (1976). Rothschild-Stiglitz showed that the non-existence of competitive equilibrium is a typical problem when the uninformed is allowed to move first. This paper also obtains Rothschild-Stiglitz type non-existence of competitive equilibrium. However, some distinctive features for the non-existence of equilibrium are noticed: If a primary purpose of testing workers is to enhance workers' productivity, then the competitive equilibrium of the information game exists. However, if a primary purpose of testing workers is to sort out workers and tests have a moral hazard problem, the equilibrium may not exist.

## Appendix

The Proof of the Optimal Testing Quality

 $W_1$  and  $W_2$  are the wage offers for both types of workers. The optimal testing strategy is specified by the choice of a which minimizes the testing cost.

$$\operatorname{Min}_{a} (\tau - sf_{1})a$$
s.t.  $W_{1} - \Theta a < W_{2}$ 

The optimal testing quality of the above problem satisfy:

$$\Theta a = (W_1 - W_2) \text{ or } a = (W_1 - W_2)/\Theta$$

For example, under the zero profit testing strategy  $\alpha_0$ ,  $W_1 = W_1^a$  and  $W_2 = W_2^a$ . Then, the optimal testing quality  $a_0$  satisfies  $a_0 = (W_1^a - W_2^a)/\Theta$ .

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