On the Use of Monetary Policy for Moderating Exchange Rate Movements*

Jay H. Levin
Wayne State University

This paper considers the effects of monetary policy undertaken to moderate exchange rate movements under a managed floating exchange rate system. It is motivated by evidence that central banks have at times undertaken such a policy. Using the Dornbusch model as the analytical framework, I consider an initial monetary shock to the economy followed by an attempt by the central bank to use the rate of monetary growth to "lean against the wind." It turns out that monetary intervention increases the initial degree of exchange rate overshooting, increases the deviations of the exchange rate from its new long-run equilibrium level, and intensifies exchange rate movements.

I. Introduction

This paper considers the effects of monetary policy undertaken to moderate exchange rate movements under a managed floating exchange rate system. There is evidence that central banks have at times undertaken such a policy. For example, according to Argy (1982, p. 76):

In general, certainly until late 1977, domestic interest rates (in the United Kingdom) tended to be adjusted in light of external developments,... rising or falling as sterling weakened or strengthened.

More recently, the Staff of the International Monetary Fund (1988, p. 64) has pointed out that the Bank of Canada "places emphasis on..."

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avoiding sharp changes in nominal exchange rates;” and the U.K.
authorities “have on a number of occasions allowed short-term in-
terest rates to adjust substantially to counteract excessive pressure
on the pound.” Finally, Branson (1984) has reported preliminary
econometric evidence that the Bundesbank and the Bank of England
react to movements in their exchange rates by adjusting interest
rates.¹

The objective of moderating exchange rate movements should be
contrasted with attempting to achieve an exchange rate target, about
which some evidence also exists.² However, it is well-known from
the literature on “leaning against the wind,” in addition to the liter-
ature cited above, that central banks are concerned with moderating
exchange rate movements.³ Therefore, the question raised here is
whether monetary policy can be used successfully for this purpose.

In this paper the well-known Dornbusch model (1976) will be used
to analyze the problem. In response to an initial monetary shock to
the economy, the central bank attempts to moderate the exchange
rate movement that subsequently develops by letting the rate of
monetary growth depend on the rate of change of the exchange rate.
Thus, a depreciation of the home currency elicits monetary con-
traction, and an appreciation triggers monetary expansion. On the basis
of these policy reactions, the path of the exchange rate is then
derived. It turns out that the monetary intervention increases the
initial degree of exchange rate overshooting and increases the de-
viations of the exchange rate from its new long-run equilibrium
level. Also, exchange rate movements are intensified as a result of
the monetary intervention. Thus, a rule which looks stabilizing
actually turns out to be destabilizing.

II. The Model

The Dornbusch model can be described by the following three
equations:

\[ r = r^* + \dot{e} \]  \hspace{1cm} (1)

¹For earlier econometric work indicating this policy, see Branson (1981) for the United
States, Artus (1976) and Branson, Haltunen and Masson (1977) for Germany, Amano
(1979) for Japan, and OECD (1977) for the United Kingdom.
²See, for example, Black (1983) and Crockett and Goldstein (1987, p.10).
³See, for example, Argy (1982, pp.68-69), Black (1980) and Wonnacott (1982, p.6).
\[ m - p = -\beta r + \phi \hat{y} \quad (2) \]
\[ \dot{p} = \pi [\mu + \delta (e - p) + (\gamma - 1)\hat{y} - \sigma r] \quad (3) \]

where \( r \) = domestic interest rate; \( r^* \) = foreign interest rate; \( e \) = log of the exchange rate on the foreign currency; \( m \) = log of the domestic money supply; \( p \) = log of the domestic price level; and \( \hat{y} \) = exogenous domestic output. \( \beta, \phi, \delta, \gamma, \pi, \) and \( \sigma \) are positive structural parameters (\( \gamma < 1 \)), and \( \mu \) is a shift parameter. Equation (1) shows perfect asset substitutability between domestic and foreign securities. Assuming rational expectations and no stochastic disturbances, expected \( \hat{e} \) is replaced with actual \( \hat{e} \). Equation (2) is the money market equilibrium condition. Finally, equation (3) shows that the rate of inflation depends on the amount of excess demand in the goods sector.

In the Dornbusch model, the money supply is taken to be exogenous. In this paper, however, the money supply is adjusted by the central bank in an attempt to moderate exchange rate movements. Specifically, the following reaction function is assumed:

\[ \dot{m} = \theta \hat{e} \quad (\theta < 0)^4 \quad (4) \]

System (1)-(4) then contains the four endogenous variables, \( e, r, p, \) and \( m \), with the characteristic equation\(^5\)

\[ \beta \lambda^2 + \lambda (\theta + \beta \delta \pi + \pi \sigma) + \delta \pi (\theta - 1) = 0. \quad (5) \]

Since the constant term is negative, the product of the roots is negative. Hence, one root is positive, and the other root is negative. However, the positive root stems from rational exchange rate expectations, so that a stable path to a saddle point exists.

We now consider an expansion of the money supply, as in the Dornbusch model, and ask whether the well-known exchange rate pattern involving overshooting and subsequent direct convergence

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4This intervention rule has been used by Cox (1980), Eaton and Turnovsky (1984), Glick and Hutchinson (1989), Papel (1984), and Roper and Turnovsky (1980) in the level forms \( m - \hat{m} = \theta (e - \hat{e}) \) and \( m = \theta e \). Here \( \hat{e} \) is a target exchange rate, and \( \hat{m} \) is the long-run equilibrium money supply. Note that this reaction function is not based on rational expectations of the central bank but instead on the central bank's belief that such intervention will moderate exchange rate movements. The empirical evidence discussed above is consistent with this reaction function. However, there is an obvious asymmetry in the paper between the behavior of asset holders and the central bank, and this will be addressed at the end of the paper.

5The characteristic equation is actually cubic, but the third root is zero and can be ignored here.
will occur if the central bank follows equation (4). To answer this question, consider the following exchange rate expectations scheme, derived in Appendix A, that is consistent with the stable saddle point path:

$$\dot{e} = \phi_1(\bar{e} - e) + \phi_2(\bar{m} - m)$$  \hspace{1cm} (6)

where $\bar{e}$ is the long-run equilibrium exchange rate; $\bar{m}$ is the long-run equilibrium money supply determined by the model; $\phi_1 = \frac{\pi \beta}{\beta \phi_1 - \theta + \beta \phi_2 \theta - \pi (\delta \beta + \sigma)}$; and $\phi_2 = -\phi_1$. Next, observe from equation (2) that the initial expansion in the money supply will initially cause interest rates to decline because of the lag in the goods sector:

$$dr = (-\frac{1}{\beta})dm_0.$$  \hspace{1cm} (7)

Therefore, from equation (1) $\dot{e}$ is initially given by

$$\dot{e}_0 = (-\frac{1}{\beta})dm_0.$$  \hspace{1cm} (8)

Combining equations (8) and (6) then yields initially

$$e_0 - \bar{e} = \frac{1}{\phi_1 \beta} dm_0 + \frac{\phi_2}{\phi_1} (\bar{m} - m_0).$$  \hspace{1cm} (9)

However, from equation (4)

$$\bar{m} - m_0 = \int_0^\infty \dot{m} dt = \int_0^\infty \theta \dot{e} dt = \theta (\bar{e} - e_0)$$  \hspace{1cm} (10)

Using equation (10) to eliminate $(\bar{m} - m_0)$ from equation (9) then yields

$$e_0 - \bar{e} = \frac{1}{\beta \phi_1 (1 - \theta)} dm_0.$$  \hspace{1cm} (11)

In Appendix B, it is shown that $\phi_1$ is positive, a sensible result indicating regressive expectations with respect to the exchange rate. It follows that monetary expansion, taking the central bank's reaction function into account, initially causes the exchange rate to overshoot its new long-run equilibrium level, as in the Dornbusch model. In essence, since the interest rate initially declines, then from equations (1), (6), and (10) overshooting must occur to produce asset market equilibrium. However, notice from equation (4) that the central bank begins to undertake monetary expansion because the domestic currency starts to appreciate following the overshooting.
Thus, the extent of overshooting is larger because of the induced monetary expansion, which is taken into account in the formulation of exchange rate expectations. In essence, investors anticipate that the monetary expansion will cause the domestic currency to appreciate more slowly, and the latter therefore must overshoot by a larger amount to restore asset market equilibrium.

It is important to recognize that the effect of the monetary rule on the degree of overshooting is due to the rational expectations of asset holders. The rule alters the rate of change of the money supply, but not the level of the money supply, following the overshooting. Thus, there is no further reduction in the interest rate, and this is not the source of instability in the model.

III. The Exchange Rate Path

Now consider the path of the exchange rate following the initial overshooting. It will be useful to present numerical simulations to illustrate the exchange rate path, but the final results do not depend on the particular values chosen for the parameters. This is demonstrated in Appendix C. In the simulations reported in Table 1, all parameters of the model are set equal to unity and θ is varied from 0 to -2. The (log of the) long-run equilibrium exchange rate, prior to the monetary expansion, is normalized at 0, and the money supply is initially expanded by 100 percent. The parameters ϕ₁ and ϕ₂ are the set that satisfies the system’s stability conditions, derived in Appendix B.

The general solution equation for the exchange rate is

\[ e_t - \tilde{e} = A_0 e^{\lambda t} \]  

where \( \lambda \) is the negative characteristic root of equation (5); \( \tilde{e} \) is the new long-run equilibrium exchange rate following the initial monetary expansion; and \( e \) on the right-hand side is the logarithmic base.\(^8\)

\(^6\)It can be easily shown from the definitions of \( \phi_1 \) and \( \phi_2 \) that \( \frac{d\phi_1(1 - \theta)}{d\theta} < 0 \). Therefore, overshooting increases as \( |\theta| \) increases. This result depends on the assumption of rational expectations. It does not occur, for example, when expectations are purely regressive (i.e., \( \phi_1 \) is fixed and \( \phi_2 = 0 \)). In the latter case, however, the central finding of the paper concerning exchange rate deviations remains unchanged.

\(^7\)From equations (6) and (15), \( \dot{e} = \phi_1(1 - \theta)(\dot{e} - e) \). For a given \( (\dot{e} - e) \), \( \dot{e} \) would fall for a larger \( \theta \). Therefore, as \( \theta \) gets larger, \( |(\dot{e} - e)| \) gets larger to restore expected \( \dot{e} \) to the level that satisfies equation (1).

\(^8\)The characteristic root of 0 has no effect here and can be ignored.
TABLE 1
BEHAVIOR OF THE EXCHANGE RATE ($\Delta m_0 = 1$)

<table>
<thead>
<tr>
<th>Case 1</th>
<th>$\theta = 0$, $\phi_1 = 2.41421$, $\int_0^z (e_t - \bar{e}) dt = 0.03553$, $e = 1 + 0.41421 \Delta m_0 e^{-\frac{41421t}{\beta}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t$</td>
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<tr>
<td>-------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>$\theta = -1$, $\phi_1 = 1$, $\int_0^z (e_t - \bar{e}) dt = 0.06250$, $e = 1.5 + 0.5 \Delta m_0 e^{-2t}$</td>
<td>$t$</td>
</tr>
<tr>
<td>$\theta = -2$, $\phi_1 = 0.57735$, $\int_0^z (e_t - \bar{e}) dt = 0.09623$, $e = 2.15470 + 0.57735 \Delta m_0 e^{-\frac{73295}{\beta}}$</td>
<td>$t$</td>
</tr>
</tbody>
</table>

$$e = 2.73205 \quad 2.25685 \quad 2.17277 \quad 2.15790 \quad 2.15527 \quad 2.15480$$

Note: In all exchange rate equations, $e$ on the right-hand side refers to the log-difference base.

From equation (12) we obtain
$$e_0 - \bar{e} = A_0.$$ (13)

Hence, from equation (11) we have
$$A_0 = \frac{1}{\beta \phi_1 (1 - \theta)} dm_0.$$ (14)

In addition, the reaction function (4) can be written in level form as
$$m - \bar{m} = \theta (e - \bar{e}).$$ (15)

Differentiating equation (15) and noting that $d\bar{e} = dm$ from the structural model in the steady state, one obtains initially
$$d\bar{e} = \frac{1}{1 - \theta} dm_0 - \frac{\theta}{1 - \theta} de_0.$$ (16)

But from equation (13)
$$de_0 = d\bar{e} + A_0.$$ (17)

Then
$$d\bar{e} = dm_0 - \theta A_0.$$ (18)
In other words, the long-run equilibrium exchange rate changes by an amount equal to the initial change in the money supply, as in the Dornbusch model, plus an additional amount due to the monetary intervention.\(^9\) Equations (12), (14), and (18) will be used in deriving the exchange rate path in Table 1.

Case 1 in Table 1 involves no monetary intervention by the central bank to moderate exchange rate movements. The money supply is initially expanded, and the results conform exactly to the Dornbusch model. Overshooting initially occurs, as \(e\) rises above its new long-run equilibrium level of 1, and the foreign currency then gradually depreciates to the new equilibrium. The measure of exchange rate divergence, \(\int_0^\infty (e_t - \bar{e})^2 dt\), based on a quadratic loss function, is 0.03553.\(^{10}\)

In Case 2, the central bank undertakes monetary intervention (\(\theta = -1\)) to moderate the exchange rate movement. Again, monetary expansion initially produces overshooting, and as expected, the latter is more than in Case 1 because of the anticipated monetary reaction of the central bank. Thereafter, the foreign currency depreciates toward the new long-run equilibrium level. Because of the larger initial overshooting, starting at \(t = 0\), the exchange rate is always farther from its long-run equilibrium level (now 1.50000) than in Case 1. Hence, the measure of exchange rate divergence has increased to 0.06250.

In Case 3, the monetary policy parameter, \(\theta\), is changed to \(-2\) to determine the effects of stronger monetary intervention. As expected, the initial degree of overshooting is increased even further, and the exchange rate always diverges from its long-run equilibrium level (here 2.15470) by an even greater amount. Therefore, the measure of exchange rate divergence rises even further to 0.09623.

Figure 1 shows how these results are generated in the model. The schedule labelled PPP shows combinations of \(p\) and \(e\) for which long-run purchasing power parity prevails. The schedule labelled \(\hat{p} = 0\) shows combinations of \(e\) and \(p\) for which the exchange rate is constant.\(^{11}\) The phase diagram shows that the system is dynamically

\(^9\)Alternatively, note that
\[de = dm = dm_0 + (\hat{m} - m_0)\]
\[= dm_0 + \theta (\hat{e} - e_0) = dm_0 - \theta A_0.\]

\(^{10}\)Notice that this measure takes into account the initial exchange rate gap caused by monetary expansion and the speed of adjustment of the system since it reduces to \(\frac{-A_0}{2\lambda}\).

\(^{11}\)The \(\hat{p} = 0\) schedule is obtained by setting \(\hat{p} = 0\) in equation (3), eliminating \(r\) by use
unstable, but a path to a saddle point will exist. Starting at point $a$, where long-run purchasing power parity is assumed to prevail, monetary expansion causes the exchange rate to overshoot at point $b$ and in the absence of monetary intervention to subsequently converge to point $c$. (The $\dot{p} = 0$ and $\dot{e} = 0$ schedules for this case are not drawn to avoid cluttering the diagram.) In the case of monetary intervention, however, the exchange rate overshoots all the way to point $d$ and subsequently converges to point $e$. Not only does the diagram indicate greater overshooting in the case of monetary intervention, but it also suggests that the exchange rate deviations will be larger as illustrated in the numerical simulations.

Another possible criterion for judging the effectiveness of monetary intervention is the impact on $\dot{e}$ since the central bank’s objective is to moderate $\dot{e}$. By differentiating (12) and applying (12), (14), and (A12), one obtains

of equation (2), and eliminating $m$ by use of the central bank reaction function (15) in level form. Its slope is $-\beta \hat{\beta} + \sigma \bar{\sigma}$. The $\dot{e} = 0$ schedule is obtained by setting $\dot{e} = 0$ in equation (1), setting $r = r^n$ in equation (2), and eliminating $m$ by using the central bank reaction function in level form. Its slope is $\frac{1}{\hat{\beta}}$. 

Figure 1
Exchange Rate Behavior with and Without Monetary Intervention
\[ \dot{\varepsilon} = -\frac{1}{\beta} \varepsilon^{\lambda \varepsilon} \]  

which is absolutely larger for larger values of \(|\theta|\) since \(|\lambda|\) decreases with \(|\theta|\). Thus, at each point in time, monetary intervention increases the rate of change of the exchange rate, contrary to the central bank’s intentions, as shown in Figure 2.\(^{12}\) The reason is that at each point in time, the gap between \(\varepsilon_t\) and \(\bar{\varepsilon}\) is larger for larger values of \(|\theta|\). This gap is the dominant factor determining the expected and hence actual rate of change of the exchange rate, given the assumption of rational expectations.\(^{13}\)

\[ \begin{align*} 
\varepsilon & \quad \theta < 0 \\
\theta = 0 & 
\end{align*} \]

\textbf{FIGURE 2}

\textbf{EXCHANGE RATE PATHS}

Finally, it is important to observe that the model contains an asymmetry. Whereas asset holders have rational exchange rate expectations, the central bank simply follows its reaction function. An alternative model would treat asset holders and the central bank symmetrically. Specifically, while the central bank still follows its reaction function, asset holders no longer forecast exchange rates accurately. A well-known example would be the case of regressive expectations.

\(^{12}\)Another possible criterion is \(\int_{t_0}^{t_1} \dot{\varepsilon} dt\) if one wishes to consider the entire time horizon. However, this is equivalent to \(\varepsilon - \varepsilon_0\), which increases in absolute value with \(|\theta|\).

\(^{13}\)From footnote 7, \(\dot{\varepsilon} = \phi_t (1 - \theta) (\varepsilon_t - \bar{\varepsilon})\). As \(|\theta|\) gets larger, \(\phi_t (1 - \theta)\) becomes smaller and \((\varepsilon_t - \bar{\varepsilon})\) becomes larger, but the second factor outweighs the first.
\[ \dot{e} = k(\hat{e} - e), \]  
(20)

where \( \hat{e} \) is the expected \( e \), and \( k \) is a constant. However, even in this case it remains true that \( \int_0^\infty (e_t - \hat{e})^2 dt \) increases with \( \theta \) \( .^{14} \)

IV. Conclusions

In summary, monetary intervention to moderate exchange rate movements increases the initial degree of exchange rate overshooting in the model, increases the deviations of the exchange rate from its long-run equilibrium level, and intensifies exchange rate movements. Also, stronger monetary intervention increases these effects. On all counts, one has to conclude that monetary policy used to moderate exchange rate movements under the managed floating exchange rate system, has undesirable effects on exchange rate behavior.\( .^{15} \)

Appendix A

*Derivation of the Exchange Rate Expectations Scheme*

Consider exchange rate expectations scheme (6). Then from equation (1) we obtain

\[ r = r^* + \phi_1(\hat{e} - e) + \phi_2(\hat{m} - m). \]  
(A1)

Differentiating (A1) then yields

\[ \dot{e} = -\frac{1}{\phi_1} \dot{r} - \frac{\phi_2}{\phi_1} \dot{m}. \]  
(A2)

But from equation (2)

\[ \dot{m} - \hat{p} = -\beta \dot{r} \]  
(A3)

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\( .^{14} \)As noted in footnote 6, however, it is no longer true that overshooting increases with \( \theta \). Also, in contrast to the case of rational expectations, \( e \) increases up to some point in time (in comparison with the Dornbusch case) and then decreases.

\( .^{15} \)A counter-intuitive monetary intervention would involve \( \theta > 0 \). Depreciation of the home currency would then lead to monetary expansion, and appreciation would produce monetary contraction. However, for \( \theta > 1 \), two stable \( \phi_1, \phi_2 \) sets exist. One produces a superior exchange rate path, and the other an inferior path, compared to no intervention. Moreover, it is not obvious which path the system would take. In the case of regressive expectations, it is easily shown that the system is unstable when \( \theta \) lies within a specific range. In general, then, such intervention seems infeasible.
of from equations (3) and (4)
\[ \theta \dot{e} - \pi \left[ \delta (e - \bar{e}) - \delta (p - \bar{p}) - \sigma (r - \bar{r}) \right] = -\beta \dot{r}. \]  \hspace{1cm} (A4)

But from equation (2)
\[ p - \bar{p} = m - \bar{m} + \beta (r - \bar{r}). \]  \hspace{1cm} (A5)

Therefore,
\[ \theta \dot{e} - \pi \left[ \delta (e - \bar{e}) - \delta (m - \bar{m}) - (\delta \beta + \sigma)(r - \bar{r}) \right] = -\beta \dot{r}. \]  \hspace{1cm} (A6)

Using equation (1) to eliminate \((r - \bar{r})\) and (A2) and equation (4) to eliminate \(\dot{r}\) yields
\[ \dot{e} = \frac{\pi \delta}{\beta \phi_1 - \theta + \beta \phi_2 \theta - \pi (\delta \beta + \sigma)} (e - \bar{e}) \]  \hspace{1cm} (A7)
\[ - \frac{\pi \delta}{\beta \phi_1 - \theta + \beta \phi_2 \theta - \pi (\delta \beta + \sigma)} (m - \bar{m}). \]

Comparing (A7) with equation (6) shows that \(\phi_1\) is given by the coefficient of \((\bar{e} - e)\) in (A7) and \(\phi_2\) by the coefficient of \((\bar{m} - m)\) in (A7).

**Appendix B**

**Proof that \(\phi_1 > 0\)**

The solution for \(\phi_1\) yields
\[ \beta \phi_1^2 (1 - \theta) - \phi_1 \left[ \theta + \pi (\delta \beta + \sigma) \right] - \pi \delta = 0. \]  \hspace{1cm} (A8)

Since the constant term is negative, one root of \(\phi_1\) is positive, and one root is negative. But the characteristic equation of system (1)-(4) and (6) is given by
\[ \lambda \left[ \theta (\beta \phi_2 - 1) + \beta \phi_1 \right] + \theta \left[ \pi \delta (\beta \phi_2 - 1) + \pi \sigma \phi_2 \right] + (\pi \delta \beta \phi_1 + \pi \delta + \pi \sigma \phi_1) = 0. \]  \hspace{1cm} (A9)

For \(\phi_1 > 0\) and hence \(\phi_2 < 0\), the coefficient of \(\lambda\) and the constant term are both positive. Therefore, the stability conditions are satisfied, and this must be the stable \(\phi_1, \phi_2\) set. It can be shown that the negative root of \(\phi_1\) does not satisfy the stability conditions.

*Q.E.D.*
Appendix C

Proof that \( \frac{d}{d|\theta|} \int_0^\infty (e_t - \bar{e})^2 dt > 0 \)

Let \( C = \theta + \pi (\beta \delta + \sigma) \)

From equation (5),
\[
\lambda = -\frac{C - \sqrt{C^2 + 4 \beta \delta \pi (1 - \theta)}}{2 \beta} \tag{A10}
\]

From equation (A8),
\[
\phi_1 = -\frac{C + \sqrt{C^2 + 4 \beta \delta \pi (1 - \theta)}}{2 \beta (1 - \theta)} \tag{A11}
\]

Therefore,
\[
\phi_1 = -\frac{\lambda}{1 - \theta} \tag{A12}
\]

From equation (12)
\[
\int_0^\infty (e_t - \bar{e})^2 dt = \frac{A_0^2}{-2 \lambda} \tag{A13}
\]
or
\[
\int_0^\infty (e_t - \bar{e})^2 dt = \frac{1}{-2 \lambda \beta^2 \phi_1^2 (1 - \theta)^2} \tag{A14}
\]
using equation (14). Then from equation (A12)
\[
\int_0^\infty (e_t - \bar{e})^2 dt = \frac{1}{-2 \lambda^3 \beta^2} \tag{A15}
\]

But from equation (A10), letting \( D = C^2 + 4 \beta \delta \pi (1 - \theta) \)
\[
\frac{d}{d|\theta|}(-\lambda^3) < 0
\]

iff \( 3(C + \sqrt{D})^2[1 + \frac{1}{\sqrt{D}}(\theta - \beta \delta \pi + \pi \sigma)] > 0. \tag{A16} \)

The last bracketed expression is positive if and only if
\[
\sqrt{D} > \pi (\beta \delta - \sigma) - \theta \tag{A17}
\]

where reduces to
\[ 4 \pi^2 \delta \beta \sigma + 4 \beta \delta \pi > 0 \]  

(A18)

which holds.

\[ Q.E.D. \]

References


