

Estimation of Respiratory Impedance and Source Pressure Using a Thevenin Equivalent Circuit Model

E.J. Cha,* B.G. Min,* C.W. Lee,*** K.S. Park,* and K.Y. Kim**

Departments of Biomedical Engineering and Internal Medicine,** Seoul National
University Hospital and Department of Electronics Engineering**, College of
Engineering, Seoul National University.*

INTRODUCTION

Equivalent linear network models of the respiratory mechanics have been used to estimate the lung resistance and compliance for early diagnosis of the obstructive lung diseases (Butler et al., 1960; Grimby et al., 1968; Macklem, 1972; Pimmel et al., 1978; Schwaber et al., 1967). Among those methods, the forced oscillation methods have been most extensively studied by various investigators (Dubois et al., 1956; Franetzki et al., 1979; Goldman et al., 1970; Hyatt et al., 1976; Landser et al., 1976; Michaelson et al., 1975; Peslin et al., 1975).

In the forced oscillation method, the effects of the active respiration force on the accuracy of computing respiratory impedance were neglected. However, one recent report (Delavault et al., 1980) has shown in a theoretical and simulation study that the effect of the parasitic signals is an important source of error. In their study, the measured impedance value was shown to vary from the true respiratory impedance value of the subject to the impedance value of apparatus, depending upon the amplitude of the source signal produced by the subject. Also, the complexity of the procedure and instrumentation has limited clinical application of the forced oscillation method. Hence, it is desirable to

develop a new clinical method which can provide the estimated values of both the active respiration force and the lung impedance in a single clinical test.

In this paper, we have used a Thévenin equivalent circuit model of the respiratory mechanics to estimate simultaneously the respiratory impedance and the source pressure during spontaneous and maximal breathing procedures.

The computed respiratory resistances of six normal human subjects were compared with the measured airway resistance values using the plethysmographic method.

THEORY

The Thévenin equivalent network theorem shows that any complex linear time-invariant network can be replaced by a simple equivalent circuit of a single pressure source and a source impedance, whatever the output loads may be connected to the network (Desoer, 1969). Thus, if the ventilatory function of the lung can be considered as a linear time-invariant network, replacement of the whole ventilatory mechanics by an equivalent source (fig.1) would completely represent the original ventilatory function, as far as the respiratory pressure and flow outputs are concerned.

Using the phasor notations for the Thévenin equivalent network of the ventilatory functions, the phasors of the source pressure, $\bar{V}_s(\omega)$, and the source impedance, $\bar{Z}_s(\omega)$, at the fundamental

Address: Byoung G. Min, Department of Biomedical Engineering, Seoul National University Hospital, 28, Yeongun-Dong Chongro-Koo, Seoul, Korea, 110.

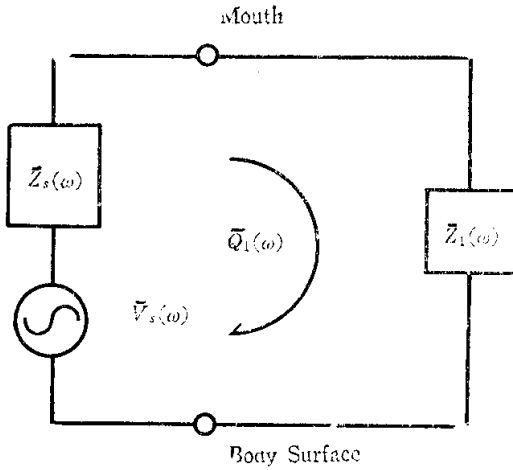


Fig. 1. Thévenin equivalent source model of the ventilatory function.

angular respiratory frequency, ω , are related to the output respiratory flow, $\bar{Q}_l(\omega)$, and the output load impedance, $\bar{Z}_l(\omega)$, as follows;

$$\frac{\bar{V}_s(\omega)}{\bar{Q}_l(\omega)} = Z_s(\omega) + Z_l(\omega) \quad (1)$$

From eq. 1, the magnitudes of the fundamental component of the source pressure, V_s , and the flow Q_0 , in control state are related by

$$\frac{1}{Q_0^2} = \left(\frac{R_s}{V_s} \right)^2 + \left(\frac{X_s}{V_s} \right)^2 \quad (2)$$

where R_s is the resistive component of the source impedance, Z_s , and X_s is the reactive component of Z_s .

Also, when the source pressure has increased by k times from the control state with a pure resistive load, R_l , connected to the source, the magnitude of the loaded flow, Q_{lk} , can be expressed as follows in eq. 2;

$$\frac{1}{Q_{lk}^2} = \left(\frac{R_l + R_s}{kV_s} \right)^2 + \left(\frac{X_s}{kV_s} \right)^2 \quad (3)$$

where the subscript l in Q_{lk} indicates that the flow is measured with the added loading and the subscript k indicates that the source pressure is kV_s as compared with the control source pressure of V_s . For unloaded condition with the same magnitudes of source parameters (kV_s and R_s), the output flow, Q_{0k} , become

$$\frac{1}{Q_{0k}^2} = \left(\frac{R_s}{kV_s} \right)^2 + \left(\frac{X_s}{kV_s} \right)^2 \quad (4)$$

Then, it can be shown from eqs. 2, 3, and 4 that

$$\left(\frac{k}{Q_{lk}} \right)^2 = \left(\frac{R_l + R_s}{V_s} \right)^2 + \left(\frac{X_s}{V_s} \right)^2 \quad (5)$$

and

$$k = \frac{Q_{0k}}{Q_0} \quad (6)$$

Eq. 6 shows that the magnitude of the increase in the source pressure from V_s to kV_s can be externally measured by the ratio of the unloaded output flow at kV_s condition to the unloaded output flow measured in control state. Also, eq. 5 shows that the measurable quantities of $(k/Q_{lk})^2$ is parabolically related to the load resistance, R_l , for constant source parameters (V_s , R_s , and X_s) corresponding to one subject. From this parabolic relation, the source parameters can be estimated using the least square error method.

Once the reactive component (X_s) is computed in eq. 5, the total dynamic compliance, C_s , can be estimated in eq. 7, as the inductive component was known to be negligible in low frequency ranges (Mead et. al., 1952, 1964; Otis et. al., 1950);

$$C_s = \frac{1}{2\pi f X_s} \quad (7)$$

where f is the fundamental respiration frequency.

MATERIALS AND METHODS

A. CLINICAL EXPERIMENTS

Clinical experiments were performed for six normal male subjects using a Hewlett-Packard pulmonary computer system (H.P. 47804S) and a body plethysmograph (H.P. 47604A). A Fleisch pneumotachograph (H.P. 21073B), a differential pressure transducer (H.P. 270), and various amplifiers (H.P. 47304A, 8802A, 8805C) were used for the measurement of the respiratory pressures and flows with and without addition of the resistive external loading.

Experiments were performed using the following protocols; After the system calibration, the subjects performed full inspiration and expiration at unloaded condition until the subjects got used to the pneumotachograph and the setup. Then, after two cycles of the control measurements, the loaded respiration was performed for two full cycles and returned to the unloaded condition. These measurements were repeated seven to nine times for each subject using various magnitudes of the loads in one experiment. The loading and unloading were produced almost instantaneously at the FRC level. Before the measurement was repeated with different magnitudes of the load, the subjects took three minutes of resting period and started the next unloaded measurements. At the end of the experiment, the subject performed the total body plethysmograph test for measurement of the airway resistance.

The respiratory pressure and flow signals were sampled and digitized at a sampling rate of 40 (samples/sec) using a DEC Co's MINC-11 computer during experiment and were used for the parameter estimation program. All amplifiers were calibrated using a constant flow generator and a water manometer for the flow ranges of 0-4(1/s) and the pressure ranges of 0-10 (cmH₂O). The static linearity of the amplifiers has the error range of $\pm 5\%$ in the calibration test.

B. PARAMETER ESTIMATION

For the computation of source parameters, Q_{lk} of eqs. 3 and 5 was determined as the pneumotach flow measured during loading condition, and Q_{0k} was given as the flow measured after elimination of the load resistance. Also, the control measurement before the first loading was used as Q_0 value in eqs. 2 and 6.

Average values of the fundamental components of the Fourier series of the two conse-

cutive pressure and flow measurements were used for computation. The load resistances were computed from the mean values of the mouth pressure and flow. Then, for various measurements with different loadings, the corresponding k values were computed from eq. 6, and the source parameters (R_s , X_s , V_s) were estimated using the least-square-error method.

RESULTS

Fig. 2 shows the measured respiratory pressure and flow waveforms before, during and after releasing the resistive loading. From the mouth pressure and flow during loading, the resistive load (R_l) was computed using the mean values of pressure and flow. Seven to nine different loadings were used for each experiment with the range of the load resistance of 0.4~10.0 (cmH₂O/1/s).

Figs. 3-8 show the relationship of the measured values of $k^2/Q_{lk}^2(1/1^2/s^2)$ to the applied load resistance (cmH₂O/1/s) in six normal subjects during maximal breathing. An average respiration frequency was 0.4(Hz) during spontaneous breathing and 1.2(Hz) during maximal breathing.

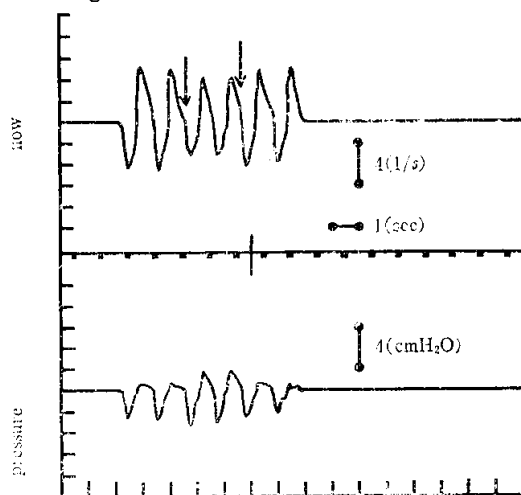


Fig. 2. Respiratory flow(upper) and mouth pressure (lower) waveforms with the applied load. Load was applied for the duration between two arrows.

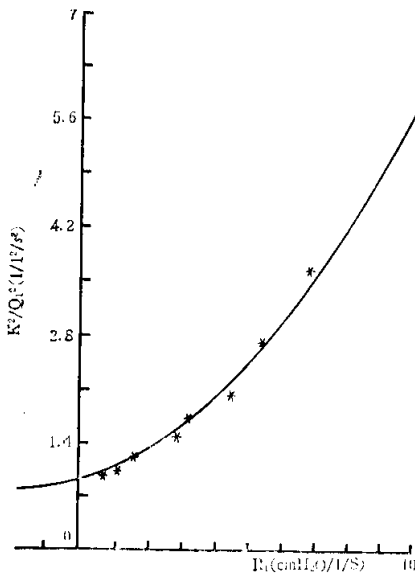


Fig. 3. The measured data (*) in relating $(k/Q_{ik})^2$ and the load resistance (R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{ik})^2 = 0.03 R_l^2 + 0.14 R_l + 0.92$ (optimally fitting the measured data) is shown by a connected line (subject 1).

It is shown in figs. 3-8 that the measured data are approximated within the range of one standard deviation by a parabolic function in the $(k/Q_{ik})^2$ vs. R_l graph, as indicated in eq.5. Using these measured data and eqs. 5 and 6, the optimal values of the source parameters are estimated by the least-square error method by best fitting a parabolic equation to the measured data, and are summarized in table 1. For the estimation of source parameters, we used the measured data having the applied load resistance less than $6.0(\text{cmH}_2\text{O}/\text{l/s})$.

As shown in table 1, the estimated total respiratory resistance during maximal breathing has an average value of $3.46(\text{cmH}_2\text{O}/\text{l/s})$, as compared to the airway resistance value of $1.86(\text{cmH}_2\text{O}/\text{l/s})$ measured by the plethysmographic method in six subjects. The relation between

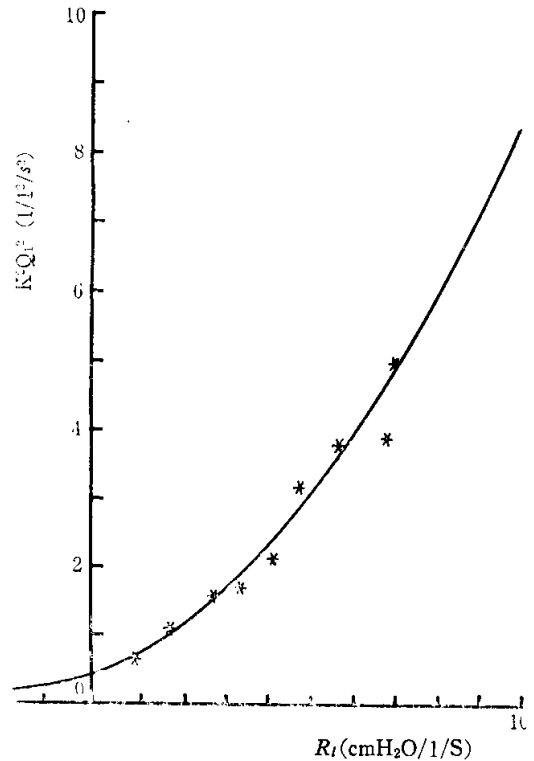


Fig. 3. The measured data (*) in relating $(k/Q_{ik})^2$ and the load resistance (R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{ik})^2 = 0.05 R_l^2 + 0.45 R_l$ (optimally fitting the measured data) is shown by a connected line (subject 2).

the total respiratory resistance (R_s) and the airway resistance (R_a) is shown in fig. 9, where R_s and R_a is related by an equation of $R_s = 2.55 R_a + 1.29$ with a correlation coefficient of 0.98.

Table 1 also shows that the variation of the total dynamic compliance (C_s) among six subjects is larger than those of the source pressure (V_s) and the source resistance (R_s). But the reproducibility of the measurements for a given subject is good in maximal breathing, as the standard deviation is within $\pm 10\%$ range of average values for all three estimated source parameters.

Estimation of source parameters during spontaneous breathing is more unreliable as compared with the maximal breathing data. The reproducible source parameters could be obtained only in two subjects (4 and 5), as shown in table 2.

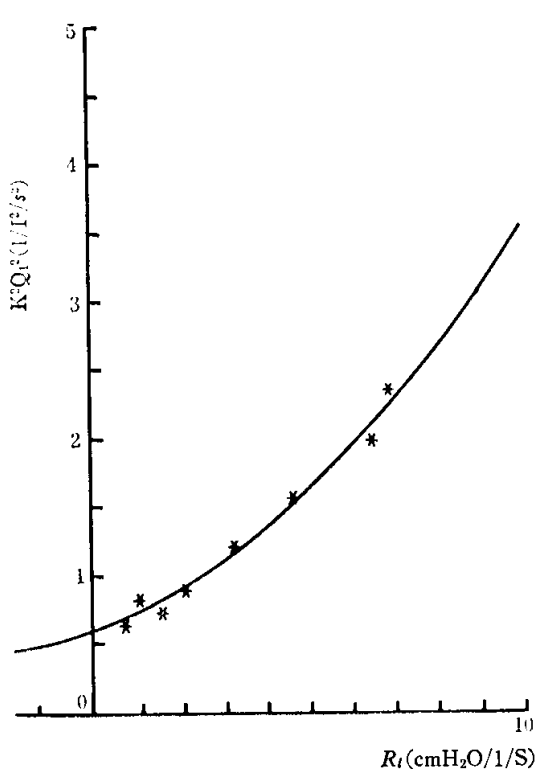


Fig. 5. The measured data(*) in relating $(k/Q_{ik})^2$ and the load resistance (R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{ik})^2 = 0.02 R_l^2 + 0.12 R_l + 0.62$ (optimally fitting the measured data) is shown by a connected line (subject 3).

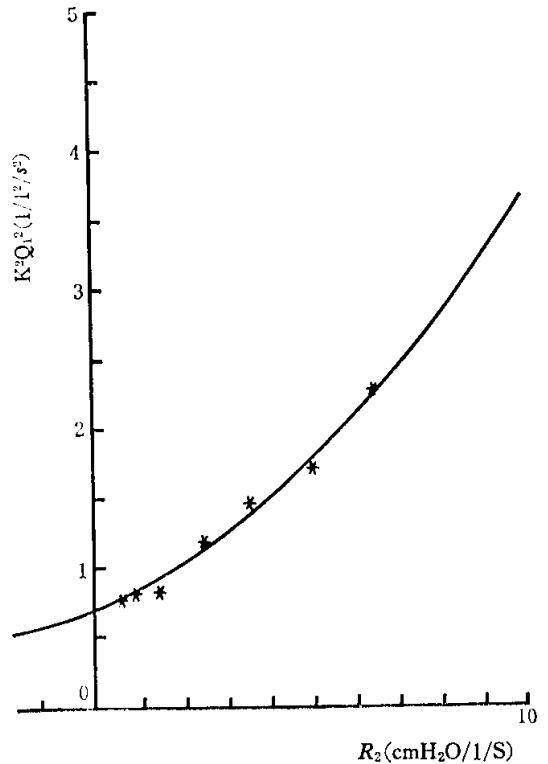


Fig. 6. The measured data(*) in relating $(k/Q_{ik})^2$ and the load resistance (R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{ik})^2 = 0.02 R_l^2 + 0.13 R_l + 0.71$ (optimally fitting the measured data) is shown by a connected line (subject 4).

Table 1. The estimated source parameters and the measured airway resistance during maximal breathing in six normal subjects

Subject No.	Total Respiratory Resistance (cmH ₂ O/1/s)	Total Dynamic Compliance (1/cmH ₂ O)	Respiratory Source Pressure (cmH ₂ O)	Airway Resistance (cmH ₂ O/1/s)
1	2.02±0.15	0.072±0.008	5.43±0.47	1.25±0.20
2	2.20±0.20	0.163±0.010	4.28±0.36	1.43±0.09
3	3.01±0.11	0.054±0.009	7.42±0.68	1.69±0.15
4	3.87±0.40	0.047±0.005	7.77±0.54	2.03±0.10
5	4.79±0.42	0.090±0.004	7.12±0.43	2.45±0.10
6	4.88±0.35	0.042±0.008	8.34±0.92	2.33±0.21
Mean±S.D.	3.46±1.14	0.078±0.041	6.73±1.41	1.86±0.44

Table 2. The estimated source parameters during spontaneous breathing in two normal subjects

Subject No.	Total Respiratory Resistance (cm ₂ O/1/s)	Total Dynamic Compliance (1/cmH ₂ O)	Respiratory Source Pressure (cmH ₂ O)
4	4.43±0.54	0.062±0.009	2.01±0.32
5	4.92±0.63	0.123±0.024	2.16±0.43
Mean±S.D.	4.68±0.25	0.093±0.031	2.09±0.08

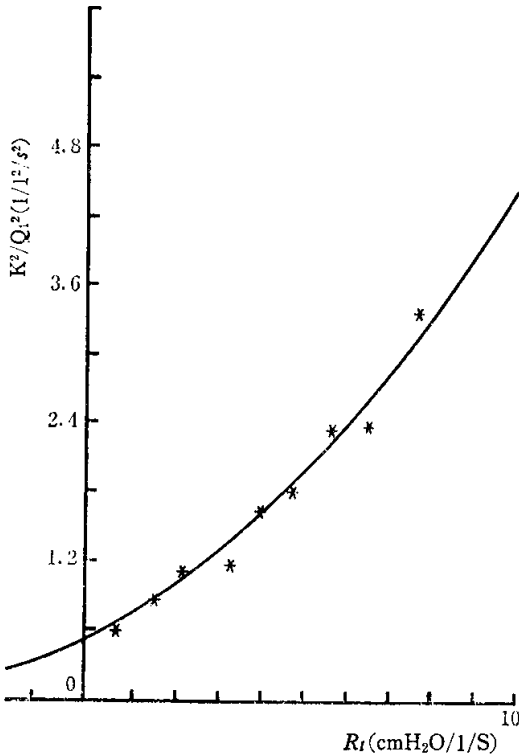


Fig. 7. The measured data(*) in relating $(k/Q_{lR})^2$ and the load resistance(R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{lR})^2 = 0.02 R_l^2 + 0.19 R_l + 0.54$ (optimally fitting the measured data) is shown by a connected line(subject 5).

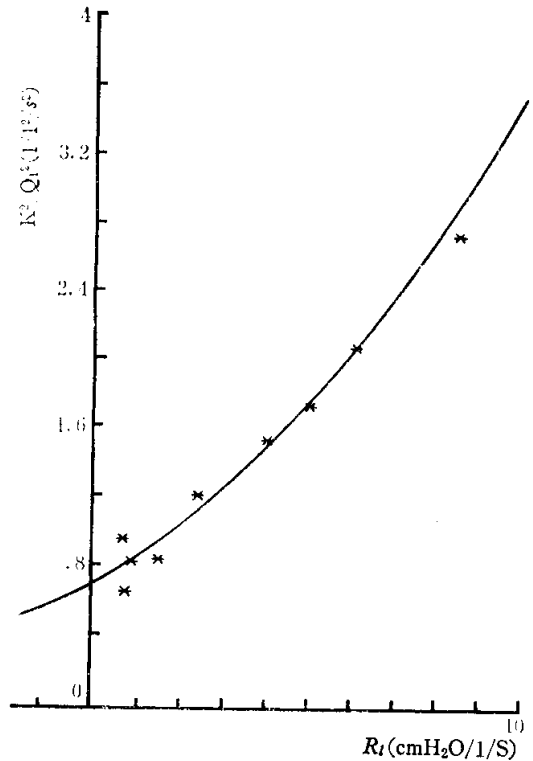


Fig. 8. The measured data(*) in relating $(k/Q_{lR})^2$ and the load resistance(R_l) for various loading conditions in maximal breathing. A parabolic function of $(k/Q_{lR})^2 = 0.01 R_l^2 + 0.14 R_l + 0.70$ (optimally fitting the measured data) is shown by a connected line(subject 6).

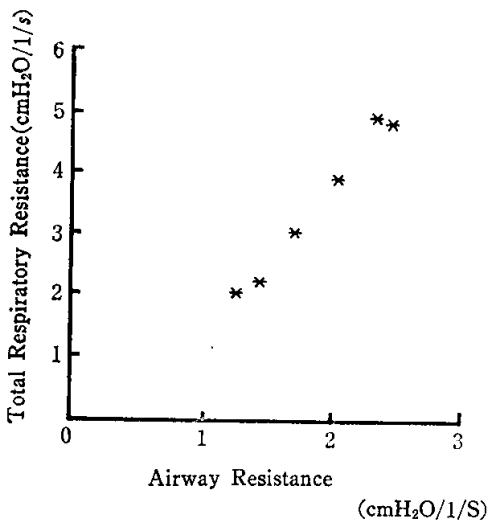


Fig. 9. The relation between the measured airway resistance using plethysmographic method and the estimated total respiratory resistance.

For these two subjects, the source pressures in maximal breathing are shown to be significantly larger than those measured during spontaneous breathing. But there are no significant changes in total respiratory resistance and the dynamic compliance measured using two breathing procedures.

DISCUSSION

The present theoretical analysis is based upon the following two assumptions: One assumption is that the external function of the ventilation can be modeled by a Thevenin equivalent source. The other assumption is that the magnitudes of the fundamental components of the source pressure and source impedance do not change in the first respiratory cycle after abrupt relea-

sing of the external resistive loading, when the loads are small.

Most of the models of the ventilatory function are based upon the linear system theory, and this linearity assumption has been shown to be reasonable in its physiological operation ranges. Thus, using Thévenin network theorem, the linear ventilatory function can be modeled by an equivalent pressure source without any additional assumption, even when the external loads are non-linear or time-varying components.

The second assumption on the source parameters is evaluated by comparing the measured data with the theoretical analysis which is based upon this constancy requirement in two consecutive cycles with different loads. While the measured data were closely approximated by the computed parabolic function in maximal breathing procedure, the results became unreliable in spontaneous breathing measurements. Also, when the applied load resistances were larger than $6.0(\text{cmH}_2\text{O}/1/\text{s})$, the results became unreliable.

The above differences of the reliability may suggest that the constancy requirement of the source parameters in two respiratory cycle is more acceptable when the external loads are eliminated during maximal breathing. Thus, these results suggest that the responding ventilatory changes of the source parameters against the reduction of the load may be more limited during maximal breathing as compared with the responses during spontaneous breathing procedure.

As the load resistance increases over $6.0(\text{cmH}_2\text{O}/1/\text{s})$, the tidal volume has increased in several case, as other investigators reported for the steady state measurements(Gothe et. al., 1980; Zechman et al., 1976). This may be caused by the ventilatory feedback responses with a large change in the load. Also, the source may become nonlinear in the high loading

condition. In this case, the estimation would become unreliable. For the magnitude of the source pressure, any probable changes in one loading measurement to the next one with different loading were compensated by the k factors of eq. 6. In whole six experiments k remained within the range from 0.8 to 1.3.

Constancy of the source impedance in the transient release of the load resistance can also be observed in the accuracy test of the forced oscillation method(Tsai et al., 1977). When the external load resistances were serially connected to the mouth in animal experiments, the difference of the total resistances(the sum of the source and load resistances) in pre-and post-loading has provided the resistance value of the added mechanical loads. Therefore, these results show that the source resistance did not change before and after addition of the loads.

The estimated values of the total respiratory resistance and total dynamic compliance are within the previously reported physiological ranges of the ventilatory parameters(Comroe, 1968; Franetzki et. al., 1979; Landser et. al., 1976; Landser et. al., 1976). Also, the measured airway resistances are shown to be 45~65% of the estimated total respiratory resistance, and the two resistances are related with a high correlation coefficient. Only the fundamental components of the source parameters are used for analysis to eliminate the effects of the inductive components, as the inductive components are negligible in low frequency ranges(Mead et al., 1952, 1964; Otis et al., 1950). Also, the source parameters are computed using only the magnitude components of the phasor analysis, as the reliable phase angle information is difficult to obtain.

Physiological significances of the equivalent source pressure are unclear at this time. However, the present results of the higher source pressures during maximal breathing than spon-

taneous breathing for the same subject may suggest that the source pressure magnitude provides the flow-independent ventilatory pressure, as the flow-dependent pressure component consumed in the source impedance is eliminated from the total mouth output pressure. An average value of the estimated source pressure of 6.73(cmH₂O) in maximal breathing is smaller than the reported intrapleural pressure of about 10(cmH₂O) (Guyton, 1976). This difference could be explained by the fact that the fundamental components have the magnitudes of 60~70% of the maximal values. Another cause may be due to the fact that the equivalent source pressure reflects the overall effective pressure of the various active sources in which the intrapleural pressure is one major component.

In conclusion, the present Thévenin equivalent source model of the ventilatory function may provide another way of evaluating the intrinsic mechanical characteristics of the respiration.

==국문초록==

Thevenin 등가회로를 利用한 Source-Pressure 와 呼吸임피던스 測定에 관한 연구

車殿宗* · 閔丙九* · 李忠雄** · 朴光錫* · 金建烈***

*서울대학교 병원 醫工學科

**서울대학교 공과대학 電子工學科

***서울대학교 병원 呼吸器內科

본 논문은 single noninvasive test로 active respiration source pressure와 Lung impedance를 측정하는 새로운 임상적인 방법에 관한 연구이다. 이 방법은 호흡계를 Thévenin 등가회로로 모델화한 것에 근거를 두고 있다. 이 모델을 이용하여 순수 저항성인 부하를 연결하고 측정한 호흡주기와 부하없이 측정한 연속적인 두주기의 호흡 flow와 pressure의 변화로부터 등가 source pressure와 source impedance를 컴퓨터를 사용하여 계산하였다. 6명의 정상인의 경우 maximal breathing에서는 source pressure의 값들이 근사했으며, spontaneous 방법보다 신뢰도가 높았다. Maximal breathing에서는 Total respiration resistance가 평균 3.46

cmH₂O이고, total dynamic Compliance는 0.078(1/cmH₂O)이었다. Plethysmographic 방법으로 측정된 기도저항은 본 논문에서 제시된 새로운 방법으로 계산된 총호흡저항의 45~65%의 범위내에 있었다. 이 두 저항값간의 상관계수는 0.98의 높은 상관관계를 나타내 주었다. Source Pressure의 기본파 성분의 평균값은 maximal breathing인 경우 6.73bm H₂O이고 Spontaneous breathing인 경우 2.09cm H₂O이었다.

REFERENCES

- Butler, J., Caro, C.G., Alcalá, R. and Dubois, A.B.: *Physiological factors affecting airway resistance in normal subjects and in patients with obstructive respiratory disease. J. Clin. Invest.*, 39: 584-591, 1960.
- Comroe, J.H.: *Physiology of respiration. Chicago. Year book medical publishers, inc. pp. 97-128, 1968.*
- Delavault, E., Saumon, G. and Georges, R.: *Characterization and validation of forced input method for respiratory impedance measurement. Respir. Physiol.*, 40:119-136, 1980.
- Desoer, C.A. and Kuh, E.S.: *Basic circuit theory. New York, McGraw-Hill Book Co., pp. 653-710, 1969.*
- Du Bois, A., Brody, A., Lewis, D. and Burgess, B. F.: *Oscillation mechanics of lungs and chest in man. J. Appl. Physiol.*, 8:587-594, 1956.
- Franetzki, M., Prestele, K. and Korn, V.: *A direct-display oscillation method for measurement of respiratory impedance. J. Appl. Physiol.*, 46(5):956-965, 1979.
- Goldman, M., Knudson, R., Mead, J., Peterson, N., Schwaber, J. and Wohl, M.E.: *A Simplified measurement of respiratory resistance by forced oscillation. J. Appl. Physiol.*, 28:113-116, 1970.
- Göthe, B., and Cherniack, N.S.: *Effects of expiratory loading on respiration in humans. J. Appl. Physiol.*, 49(4):601-608.
- Grimby, G., Takishima, T., Graham, W., Macklem, P. and Mead, J.: *Frequency dependence of flow resistance in patients with obstructive lung disease. J. Clin. Invest.*, 47:1455-1465, 1968.
- Guyton, A.C.: *Pulmonary ventilation. In: Textbook of medical physiology. Chapter 39. Philadelphia.*

- W.B. Saunders Co. pp.516-529, 1976.
- Hyatt, R., Zimmerman, I., Peters, G. and Sullivan, W.: *Direct writeout of total respiratory resistance. J. Appl. Physiol.*, 28:675-678, 1976.
- Landser, F.J., Nagels, J., Demedts, M., Billiet, L. and van de Woestijne, K.P.: *A new method to determine frequency characteristics of the respiratory system. J. Appl. Physiol.*, 41:101-106, 1976.
- Macklem, P.T.: *Obstruction in small airways A challenge to medicine. Am. J. Med.*, 52:721-724, 1972.
- Mead, J. and Whittenberger, J.L.: *Physical properties of human lungs measured during spontaneous respiration. J. Appl. Physiol.*, 5:779-796, 1952.
- Mead, J., and Milic-Emil, J.: *Theory and methodology in respiratory mechanics with a glossary of symbols. In: Handbook of Physiology. Section 3. Respiration. Vol. I, edited by W.O. Fenn and H. Rahn. Washington D.C., American physiological Society, 1964.*
- Michaelson, E.D., Grassman, E.D. and Peters, W.R.: *Pulmonary mechanics by spectral analysis of forced random noise. J. Clin. Invest.*, 56:1210-1230, 1975.
- Naimark, A., and Cherniack, R.M.: *Compliance of the respiratory system and its components in health and obesity. J. Appl. Physiol.* 15:377-382, 1960.
- Otis, A.B., Fenn, W.O. and Rahn, H.: *Mechanics of breathing in man. J. Appl. Physiol.*, 2:592-607, 1950.
- Pennock, B.E., Cox, C.P., Rogers, R.M., Cain, W.A. and Wells, J.H.: *A noninvasive technique for measurement of changes in specific airway resistance. J. Appl. Physiol.*, 46:399-406, 1979.
- Pimmel, R.L., Tsai, M.J., Winter, D.C. and Bromberg, P.A.: *Estimating central and peripheral respiratory resistance. J. Appl. Physiol.*, 45:375-380, 1978.
- Peslin, R., Papon, J., Duvivier, C. and Richalet, J.: *Frequency response of the chest: modeling and parameter estimation. J. Appl. Physiol.*, 39:523-534, 1975.
- Schwaber, J.R., and Mead, J.: *Use of a modified Thunberg barospirometer to determine airway resistance in man. J. Appl. Physiol.* 25:328-332, 1968.
- Tsai, M.J., Pimmel, R.L., Stiff, E.J., Bromberg, P.A. and Hamlin, R.L.: *Respiratory parameter estimation using forced oscillatory impedance data. J. Appl. Physiol.*, 43:322-330, 1977.
- Zechman, F.W., Franzier, D.T. and Lally, D.A.: *Respiratory volume-time relationships during resistive loading in the cat. J. Appl. Physiol.*, 40:177-183, 1976.