A Hierarchy of Human Capital and Economic Growth

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This paper focuses on the relationship between economic growth and the hierarchical structure of human capital of different levels of abstraction. The módel is the usual endogenous growth model with the inclusion of Beckmann's "span of control" technology. Major findings are: (1) the optimal structure of human capital is pyramidal, and (2) when there exist externalities of technology from the most developed foreign countries to the catching-up, the catching-up country shows more rapid economic growth and relatively more investment on the lower level human capital than on the higher. The comparison between the U.S.A. and Japanese or Korean data provides the empirical support for these implications. (*JEL* E13, J24)

I. Introduction

"What is the engine of growth?" has reappeared in the last several years as one of the most important topics among economists. Most economists think of "human capital" as one of the most promising answers to this question.[1] Further, by introducing the endogenous growth mechanism into the model, many researches are found to be fruitful, with very interesting results being derived from them.

Nevertheless, they do not break down human capital into different categories, even though it can be classified into various levels of knowledge. The top level of human capital or knowledge is related to the higher level of education, i.e., the more fundamental and abstract theo-

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[1]Romer (1986a, 1986b) and Lucas (1988) reignited this issue, human capital and growth, by formally introducing the endogenous growth mechanism.

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ries in physics, chemistry and other disciplines. On the other hand, the bottom level is related to the lower level of education, i.e., the more applicable skills and methods how to produce goods and services. The different levels of abstraction of knowledge are related to each other and have different effects on economic growth.[2] In the endogenous growth models, the existence of the different levels of human capital has been ignored.

Empirically speaking, Japan, Germany, Korea and other rapidly developing countries have built up the lower level of human capital more than the higher level, and show higher growth rates of income than the U.S.A..[3] At this point, more specific and interesting questions arise: "What is the optimal hierarchy of different level human capitals?" and "is there any relationship between the specific structure of human capital and economic growth?" Furthermore, what is the optimal resource allocations among different levels human capital and why do countries show different structures of human capital?

The main objective of this paper is to answer these questions. Several economic growth models with small modifications are set up and characterized, introducing the technology with which each level of knowledge is used as input to the formation of the immediately lower level of knowledge, along with the other input of physical capital good (consumption good).

To find more specific implications, CRRA utility and Cobb-Douglas functional forms are assumed for preference and technology, respectively. The main implications of the paper are as follows: (1) in an economy with the domestic externality of human capital, the social planner's allocation shows more rapid growth, higher interest rate, and more investment in the human capital than the competitive economy: and (2) the economy with exogenous positive externalities of knowledge from foreign countries shows higher growth rate of income, higher

[2]Murphy and others (1990) argued that the U.S.A. shows a lower growth rate of income because it has relatively fewer engineers and more lawyers than Japan and Germany.

[3]This empirical fact can be manifested by the number of professional publications or by the breakdown of total enrollment by the level of education. In 1986-87, the proportions of third level education (highest level education) enrollment are 9.0, 12.5 and 20.7 per cent for Japan, Germany and the U.S.A., respectively. Additionally, the percentage of the population in education out out of the total for Japan, Germany and the U.S.A. are 22.0 (0.4), 20.6 (3.2) and 24. 9 (2.2) per cent, respectively. The numbers in the parentheses stand for part-time education.

interest rate, and lower investment ratio. This economy also shows a less steep pyramidal structure of investments on the knowledge from the top to bottom levels, irrespective of which level of knowledge the exogenous externalities affect.

The organization of the paper is as follows. Section II provides the mathematical formulation of the basic model and the model with endogenous externalities. Section III gives the steady state solutions to the problems stated in section II and the proof of the existence and uniqueness of their solutions. Section IV contains the various implications of the above models, as well as the modified models. Finally, the last section presents a brief summary of this study and a prosed agenda for future research.

II. Model

In this section, descriptions of the two different models, competitive equilibrium and social planner's problem are provided. For expositional convenience, the social planner's problem is dealt with first and then the competitive equilibrium model in the economy with externalities.

The technology in this paper consists of an array of different level human capitals, which are separately located on a spectrum of the degrees of abstraction (the other extreme being application). The top level of human capital (knowledge being the abstract) will be increased by the input of physical capital. Next period, the previously produced one level higher knowledge is used as input to the formation of the next lower level knowledge along with the physical capital input. In this way, each level of knowledge is used as input to the next lower level knowledge along with the physical capital investment. To make the model simpler, the time allocation between human capital (knowledge) investment and leisure is not considered. The bottom level of knowledge is used to produce the final output (consumption good, physical capital good) along with physical capital input. The knowledge of h level at time t ($Y_h(t)$) is accumulated using two kinds of input, one level higher knowledge produced at time $t-1$ ($Y_{h+1}(t-1)$) and physical capital invested one period before ($X_h(t)$) as:

$$Y_h(t) = F_h(X_h(t)), Y_{h+1}(t-1)), \text{ for all } h \neq H, \tag{1}$$

where F_h is a concave function.

Nonetheless, the top level knowledge is accumulated as

$$Y_H(t) = Y_H(t-1) + Y_H(t-1) \cdot g[\frac{X_H(t)}{Y_H(t-1)}], \tag{2}$$

where $g(\cdot)$ is a concave function.
Thus, we can see that the engine of growth comes along with the top level of knowledge, not from the other levels of knowledge.
The final output (consumption good) is produced through

$$Y_0(t) = F_0\,(X_0(t),\ Y_1(t-1)). \tag{3}$$

The production function of the final output can be expressed as

$$\begin{aligned} Y_0\,(t) = F_0\,(X_0(t),\ &F_1(X_1(t-1),\ F_2(X_2(t-2) \ldots F_{H-1}(X_{H-1}(t-H+1), \\ &Y_H(t-H)) \ldots))), \text{ for } t \geq H. \end{aligned} \tag{4}$$

The model here is similar to the one in Beckmann (1977) and the technology of top level knowledge and the one period time lag are distinguishing features.

To make the model easily manipulatable, we will specify the function, $F_h(\cdot)$, to be Cobb-Douglas and $g(\cdot)$ to be linear, especially for an easy characterization of the steady state, as

$$Y_h(t) = X_h(t)^{1-\gamma}\, Y_{h+1}\,(t-1)^{\gamma},^{4} \quad 0 \leq \gamma \leq 1, \text{ for } h \neq H \tag{5}$$

$$Y_H(t) = Y_H(t-1) + X_H(t). \tag{6}$$

Now, the counterpart for Equation (4) is

$$\begin{aligned} &Y_0(t) = Y_H(t-H)^{\gamma^H} \cdot \prod_{h=0}^{H-1} X_h(t-h)^{(1-\gamma)\gamma^h}, \text{ for } t \geq H, \text{ and} \\ &Y_0(t) = Y_t(0)^{\gamma^t} \cdot \prod_{h=0}^{t-1} X_h(t-h)^{(1-\gamma)\gamma^h}, \text{ for } t < H, \text{ with} \\ &\text{initial capital stocks, } Y_0(0),\ Y_1(0),\ \ldots,\ Y_H(0) \text{ given.} \end{aligned} \tag{7}$$

This technology exhibits constant returns to scale in Y_H, X_0, X_1, ..., X_{H-1}, because

$$\gamma^H + \sum_{h=0}^{H-1} (1-\gamma)\gamma^h = 1.$$

One more important aspect of the technology is that the number of technology levels is given exogenously by H.

The preference is described as follows: every consumer is identical,

[4]The elasticity of γ could be different over different levels of knowledge, but the main implications of the paper will remain intact.

therefore, this model is a representative economic agent model with the preference $\sum_{t=0}^{\infty}\beta^t U(C(t))$. For convenience, treat the utility function as CRRA, $(C(t)^{(1-\sigma)} - 1)/(1 - \sigma)$, where σ is the nonnegative risk aversion parameter and $C(t)$ is the consumption at time t.

In this case, without any externality, the competitive equilibrium model is supported by the social planner's problem. Then the social planner's is:

PS

$$\max_{\{C(t),\, X_0(t+1), X_1(t+1), \ldots, X_H(t+1):\ \text{nonnegative}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t \frac{C(t)^{1-\sigma}-1}{1-\sigma} \tag{8}$$

$$Y_0(t) = C(t) + X_0(t+1) + X_1(t+1) + \ldots + X_H(t+1) \tag{9}$$

$$Y_0(t) = Y_H(t-H)^{\gamma^H} \prod_{h=0}^{H-1} X_h(t-h)^{(1-\gamma)\gamma^h}, \text{ for } t = 0,1,2,\ldots \tag{10}$$

with Equation (6), and $Y_0(0)$, $Y_1(0)$, ..., $Y_H(0)$ given.

Let $MU(t)$ be the marginal utility of consumption at time t, $\beta^t U'(C(t))$, then we have

$$MU(t) = \frac{\partial Y_0(t+h+1)}{\partial X_0(t+1)} MU(t+h+1), \text{ for } h = 0, \ldots, H. \tag{11}$$

$$C(t)^{-\sigma} = \frac{Y_0(t+h+1)}{X_h(t+1)}(1-\gamma)\gamma^h C(t+h+1)^{-\sigma}\beta^{h+1}, \tag{12}$$

for $h = 0, \ldots, H-1$.

$$C(t)^{-\sigma} \geq \sum_{i=0}^{\infty}\beta^{H+1+i} C(t+H+1+i)^{-\sigma}\gamma^H \frac{Y_0(t+H+1+i)}{Y_H(t+1+i)}.^{5} \tag{13}$$

Let us construct the problem with externalities in the technology of knowledge formation, such that the social planner's problem of this model should be *PS*. Instead of Equation (5), we have

$$Y_h(t) = \tilde{Y}_{h+1}(t-1)^{\gamma-\alpha} \cdot Y_{h+1}(t-1)^{\alpha} \cdot X_h(t)^{1-\gamma}, \text{ for } h \neq H, \tag{14}$$

where $\tilde{Y}_h(t)$, equal to $Y_h(t)$, denotes the external effects from the aggregate level of knowledge, thus being treated exogenously, and $\gamma - \alpha$ cap-

[5]The first order condition Equation (12) holds with equality because the utility function and production function satisfy the Inada conditions. However, for Equation (13) to hold with equality, we need an additional restriction on $Y_H(0)$ at the initial point. If the initial $Y_H(0)$ is too high compared with the other initial values, then there will be an incentive to have a negative $X_H(1)$ to make more investments on other knowledge, but the nonnegativity constraint of $X_H(1)$ will lead to an inequality in Equation (13).

tures the degree of external effects on h level knowledge with $\gamma > \alpha \geq 0$. For the top level of knowledge, we impose an externality such as

$$Y_H(t) = (1 - z)\tilde{Y}_H(t-1) + zY_H(t-1) + X_H(t), \tag{15}$$

where $0 \leq z \leq 1$, $1 - z$ captures the degree of externality, and $\tilde{Y}_H(\cdot)$ equals $Y_H(\cdot)$ and is treated exogenously given.

Successive iteration gives us

$$Y_0(t) = \left[\prod_{i=0}^{H} \tilde{Y}_i(t-i)^{\gamma\alpha^{i-1}}\right] \cdot Y_H(t-H)^{\alpha^H} \cdot \prod_{h=0}^{H-1} X_h(t-h)^{(1-\gamma)\alpha^i} \tag{16}$$

Let us call this problem with externalities *PE*.

PE

$$\max_{\{C(t),\ X_0(t+1), X_1(t+1), \ldots, X_H(t+1):\text{ nonnegative}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C(t)^{1-\sigma} - 1}{1-\sigma} \tag{8}$$

$$Y_0(t) = C(t) + X_0(t+1) + X_1(t+1) + \ldots + X_H(t+1) \tag{9}$$

$$Y_0(t) = \left[\prod_{i=0}^{H} \tilde{Y}_i(t-i)^{\gamma\alpha^{i-1}}\right] \cdot Y_H(t-H)^{\alpha^H} \cdot \prod_{h=0}^{H-1} X_h(t-h)^{(1-\gamma)\alpha}, \text{ for } t \geq H,$$

and (16)

$$Y_0(t) = \left[\prod_{i=0}^{t} \tilde{Y}_i(t-i)^{\gamma\alpha^{i-1}}\right] \cdot Y_t(0)^{\alpha^t} \cdot \prod_{h=0}^{t-1} X_h(t-h)^{(1-\gamma)\alpha^h}, \text{ for } t < H.$$

$$Y_H(t) = (1 - z)\tilde{Y}_H(t-1) + zY_H(t-1) + X_H(t), \tag{15}$$

$$\tilde{Y}_i(t) = Y_i(t), \text{ for } t = 0, 1, \ldots, \text{ and for } i = 1, 2, \ldots, H, \text{ and} \tag{17}$$
$$Y_0(0), Y_1(0), \ldots, Y_H(0) \text{ given.}$$

As Romer (1986a) proved, *PE* solves the competitive equilibrium with externalities, as in Equation (15) and (16), exogenously given. The first best solution of social planner's to *PE*, is *PS*, and *PE* has the fixed point argument imbedded as we can see in Equation (17).

The analytic solutions of these two dynamic models are rather complicated, therefore we will study the steady state behaviors of these two models. One final comment is that, because of the continuity and strict concavity of the preference and technology, Equation (7) and (16), the existence and uniqueness of the solutions to *PS* and *PE* is guaranteed, following Romer (1986a).

III. Steady State Equilibrium

In order to pursue further implications, the economic behaviors of the steady state equilibrium (balanced growth path equilibrium) must be studied. We initially calculate the balanced growth path equilibrium for *PS* and later for *PE*. The strategy is that after assuming a constant growth rate (g) of consumption and investments, and a constant gross interest rate (R), we calculate the savings and consumption rates out of total income and prove the uniqueness of g and R in the balanced growth path equilibrium. By this process, even though there exists an uncountable number of balanced growth path equilibria, we can characterize the equilibrium behavior.

A. PS Problem

From Equation (12), we can calculate the "span of control", i.e., the ratio of the physical capital input of $h - 1$ level of knowledge with respect to that of h level of knowledge.

$$S_h = \frac{X_{h-1}(t+1)}{X_h(t+1)} = \frac{1}{\gamma\beta(1+g)^{1-\sigma}} = \frac{1}{e}, \text{ for } h \neq H.^6 \tag{18}$$

The intuition behind Equation (18) is: the higher γ is, the less the span of control is, because the higher level knowledge has a less dissipating effect on the creation of the next level of knowledge. Additionally, $\beta(1 + g)^{-\sigma}$ captures the disfavorable effect of the one period time lag between the creation and usage of the knowledge. The last multiplicative term in the denominator, $(1 + g)$, captures the growth effect at the steady state.

From Equation (18), we know that

$$X_h(t+1) = \frac{1}{e^{H-1-h}} X_{H-1}(t+1), \text{ for } 0 \leq h \leq H-1. \tag{19}$$

And from equation (13), which is assumed to hold with equality, we can easily derive

[6]To have the finite summation of utility in the steady state, we derive that $\beta(1 + g)^{1-\sigma} < 1$. Therefore $1/e > 1/\gamma$, where $1/\gamma$ is the span of control when there is no time lag between creation and usage as an input of knowledge.

$$\frac{Y_H(t+1)}{Y_0(t)}=\frac{(1+g)^{(1-\sigma)H+1}}{(1+g)^{\sigma}-\beta}\gamma^H\beta^{H+1}. \tag{20}$$

With the constant growth rate g and Equation (6), assuming g is positive, we can express $Y_H(t)$ in terms of $X_H(t)$ as

$$\begin{aligned} Y_H(t) &= Y_H(t-1)+X_H(t) \\ &= \frac{1}{1+g}Y_H(t)+X_H(t) \\ &= \frac{1+g}{g}X_H(t), \end{aligned} \tag{21}$$

where $g > 0$.

Therefore, from the above two equations, we get

$$\frac{X_H(t+1)}{Y_0(t)}=\frac{g(1+g)^{(1-\sigma)H}\gamma^H\beta^{H+1}}{(1+g)^{\sigma}-\beta}, \tag{22}$$

and Equation (12) gives us

$$\frac{X_H(t+1)}{Y_0(t)}=(1+g)^{(1-\sigma)(h+1)}(1-\gamma)\gamma^h\cdot\beta^{h+1}, \tag{23}$$

where $0 \le h \le H-1$.

Equation (22) and (23) give us

$$X_H(t+1)=\left[\frac{\beta g}{(1+g)^{\sigma}-\beta}\cdot\frac{\gamma}{1-\gamma}\right]X_{H-1}(t+1)\equiv kX_{H-1}(t+1). \tag{24}$$

At this point, we are in a position to prove the following proposition.

***Proposition* 1**

Equation (13) holds with equality, in the steady state.

Proof: When nonnegativity constraint on $X_H(t+1)$ binds, $X_H(t+1)=0$. But, in the steady state with $g \neq 0$, whose existence is proved in Proposition 2, it is impossible to have $X_H(t+1)=0$.

Q.E.D.

Equation (22) and (23) give the savings rate as

$$\begin{aligned} \frac{I(t+1)}{Y_0(t)} &= \left[\frac{1-(1+g)^{(1-\sigma)H}\beta^H\gamma^H}{1-(1+g)^{1-\sigma}\beta\gamma}\right](1+g)^{1-\sigma}\beta(1-\gamma) \\ &\quad + \frac{g(1+g)^{(1-\sigma)H}\gamma^H\beta^{H+1}}{(1+g)^{\sigma}-\beta}\equiv d, \end{aligned} \tag{25}$$

where $I(t + 1)$ denotes the total investment at time t. Because of Proposition 1 and Inada conditions imposed on utility and production functions, d is greater than zero and less than one. Similarly, the consumption ratio out of income is

$$\frac{C(t)}{Y_0(t)} = 1 - d. \tag{26}$$

Equation (7), (19) and (24) enable us to express $Y_0(t)$ in terms of $X_0(t)$ as

$$Y_0(t) = \left[\frac{\beta}{(1+g)^{\sigma} - \beta} \cdot \frac{\gamma}{1-\gamma}\right]^{\gamma^H} [\gamma\beta(1+g)^{-\sigma}]^{\gamma^{H+1}+\gamma^H} X_0(t) \tag{27}$$

After defining an interest rate (R) as the marginal productivity of one unit of physical capital, one plus interest rate in equilibrium, we have, from Equation (12) and (27),

$$R = \left[\frac{\beta}{(1+g)^{\sigma} - \beta} \cdot \frac{\gamma}{1-\gamma}\right]^{\gamma^H} [\gamma\beta(1+g)^{-\sigma}]^{\gamma^{H-1}+\gamma^H} (1-\gamma). \tag{28}$$

We have another relationship between g and R, using Equation (12) as representing the condition of the intertemporal substitution of consumption.

$$R = \frac{1}{\beta}(1+g)^{\sigma} \tag{29}$$

This equation predicts that, to induce the given change of consumption over time, the bigger interest change is necessary, the more risk averse consumers are.

***Proposition* 2**

There exists a unique pair of constant growth rate and interest rate in the balanced growth path equilibrium.

Proof: Equation (29) has a strictly positive slope and Equation (28) has a strictly negative slope in the (g, R) plane. Moreover, given σ, β, γ and H, as g approaches infinity, R in Equation (29) blows up to infinity, whereas R in Equation (28) approaches zero. And, as g approaches $\beta^{1/\sigma} - 1$, R in Equation (28) goes to infinity, whereas R in Equation (29) has a finite value. Therefore, there exists only one cross section of these two curves, as in Figure 1.

Q.E.D.

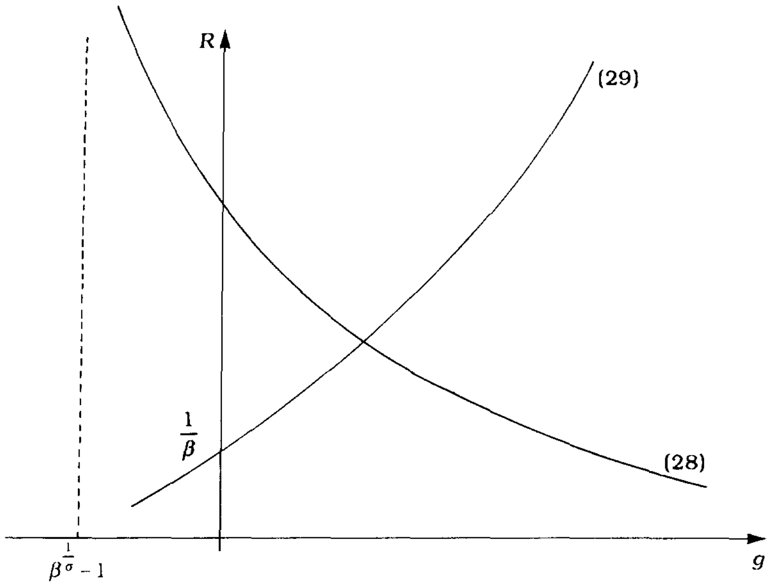

Figure 1

Proposition 3
If $[\beta/(1-\beta)]^{\gamma^H} \cdot [\gamma/(1-\gamma)]^{\gamma^H} \cdot (\gamma\beta)^{\gamma^{H-1}+\gamma^H} \cdot (1-\gamma)$ is greater than $1/\beta$, then there exists a unique positive pair of growth rate and interest rate in the balanced growth path equilibrium.

Proof: As g approaches zero, with this condition, R in Equation (28) has a greater value than R in Equation (29). With the proof in the previous proposition, the proof is completed. If β is sufficiently close to one, then the above inequality holds.

Q.E.D.

With this constant growth rate, it is easy to see that Equation (9), (10), (12), and (13) hold continuously over time. Now, the following theorem can be constructed.

Theorem 1
Given an array of initial human capital stocks, $\{Y_h(0)\}_{h=0}$, with the proper ratio between them, and with the positive growth restriction in Proposition 3, the balanced growth path equilibrium can be characterized by Equation (19), (24), (26), (27), (28) and (29). These equations

will give a unique path of $\{C(t), X_0(t+1), X_1(t+1), \ldots, X_H(t+1), Y_0(t), R, g\}_{t=0}^{\infty}$, even though there exist uncountably many balanced growth path equilibria.

Proof: Because the utility and production functions satisfy the Inada condition and Proposition 1, the first order conditions (12) and (13) hold with equality. The result is immediate with this and the above set of equations.

Q.E.D.

B. PE Problem

By an analogy with the previous subsection, the characterization of the balanced growth path equilibrium can be easily performed. The counterparts for Equation (12) and (13) are, respectively,

$$C(t)^{-\sigma} = \frac{Y_0(t+h+1)}{X_h(t+1)} \cdot (1-\gamma)\alpha^h \cdot C(t+h+1)^{-\sigma} \cdot \beta^{h+1}, \tag{30}$$

for all $h \neq H$.

$$C(t)^{-\sigma} \geq \sum_{i=0}^{\infty} \frac{Y_0(t+H+1+i)}{Y_H(t+1+i)} \alpha^H Z^i \beta^{H+1+i} C(t+H+1+i)^{-\sigma}.^7 \tag{31}$$

From Equation (30), we have

$$X_h(t+1) = \frac{1}{f^{H-1-h}} X_{H-1}(t+1), \text{ for } 1 \leq h \leq H-1, \tag{32}$$

where $f = \alpha\beta(1+g)^{1-\sigma}$.
Therefore, the span of control is $1/f$.

Proposition 4
The shape of the structure of physical capital investments for different knowledge formations, from next-to top to bottom, is pyramidal.

Proof: Equation (18) and (32) give the desired result of the proposition.

Q.E.D.

From Equation (31) and (32), we derive

$$X_H(t) = \left[\frac{\beta g}{(1+g)^\sigma - Z\beta} \cdot \frac{\alpha}{1-\gamma}\right] X_{H-1}(t) \equiv k' X_{H-1}(t). \tag{33}$$

[7]Equation (30) holds with equality due to Inada conditions, but for Equation (31) is not as obvious as stated in the previous footnote about Equation (13). For this, Proposition 1 is necessary for the steady state.

The counterparts for Equation (23), (25), (26) and (27) are, respectively,

$$\frac{X_h(t+1)}{Y_0(t)} = (1+g)^{(1-\sigma)(h+1)}(1-\gamma)\alpha^h\beta^{h+1}, \text{ where } 0 \le h \le H-1, \tag{34}$$

$$\frac{I(t+1)}{Y_0(t)} = \left[\frac{1-(1+g)^{(1-\sigma)H}\beta^H\alpha^H}{1-(1+g)^{1-\sigma}\beta\alpha}\right]\cdot(1+g)^{1-\sigma}\cdot\beta(1-\gamma) + \frac{g(1+g)^{(1-\sigma)H+1}\alpha^H\beta^{H+1}}{(1+g)^{\sigma}-Z\beta} \equiv d', \tag{35}$$

$$\frac{C(t)}{Y_0(t)} = 1-d', \tag{36}$$

$$Y_0(t) = \left[\frac{\beta}{(1+g)^{\sigma}-Z\beta}\cdot\frac{\alpha}{1-\gamma}\right]^{\gamma^H}(\alpha\beta(1+g)^{-\sigma})^{\gamma^{H-1}+\gamma^H}\cdot X_0(t). \tag{37}$$

Then, we have

$$R = \left[\frac{\beta}{(1+g)^{\sigma}-z\beta}\cdot\frac{\alpha}{1-\gamma}\right]^{\gamma^H}(\alpha\beta(1+g)^{-\sigma})^{\gamma^{H-1}\gamma^H}\cdot(1-\gamma). \tag{38}$$

Equation (29) remains identical for this case, too. For this balanced growth path equilibrium of *PE*, Proposition 2 holds and Proposition 3 and Theorem 1 should be modified like below.

Proposition 5

If $\{[1/(1-Z)]\cdot[\alpha/(1-\gamma)]\}^{\gamma^H}\cdot(\alpha\beta)^{\gamma^{H-1}+\gamma^H}\cdot(1-\gamma)$ is greater than $1/\beta$, then there exists a unique positive pair of growth rate and interest rate in the balanced growth path equilibrium.

Theorem 2

Given an array of initial human capital stocks, with the proper ratio between them, and with the positive growth condition in Proposition 5, the balanced growth path equilibrium can be characterized by (29), (33), (34), (36), (37), and (38). These equations will give a unique path of $\{C(t), X_0(t+1), X_1(t+1), \ldots, X_H(t+1), Y_0(t), R, g\}_{t=0}^{\infty}$, even though there exist uncountably many balanced growth path of the competitive equilibria with externalities.

IV. Implications of the Model

This section provides the interesting implications from the models characterized in the previous section as well as the modified models.

A. The Previous Models

In this subsection, we compare the economic behaviors of two kinds of countries with an externality in the economy; the competitive equilibrium *PE* and the other with Pareto optimal allocation with the proper government intervention *PS*. Probably, the U.S.A. belongs to the first category, and Japan and Germany to the second.

Proposition 6
The resource allocation of *PS* is not Pareto dominated by that of *PE*, given the same set of initial stocks of knowledge.

Proof: We can prove this proposition by the following simple argument. By construction, we can easily see that if there exists a resource allocation to *PE*, then this resource allocation belongs to the opportunity set of *PS*.

Q.E.D.

With the specific problem with CRRA utility and Cobb-Douglas production function, we can prove the following proposition.

Proposition 7
With the specified assumptions above, the resource allocation of *PS* is Pareto-dominant over that of *PE*, given the identical set of initial stocks of knowledge.

Proof: If there exist solutions to *PS* and *PE*, respectively, we know that the solution to *PE* does not satisfy the first order conditions of *PS*. In other words, these two allocations are different. Moreover, a solution to *PE* belongs to the opportunity set of *PS*, but is not the only solution of *PS*. Therefore, the proof is completed.

Q.E.D.

Proposition 8
PE has lower values of growth rate and interest rate (g^E, R^E) than the

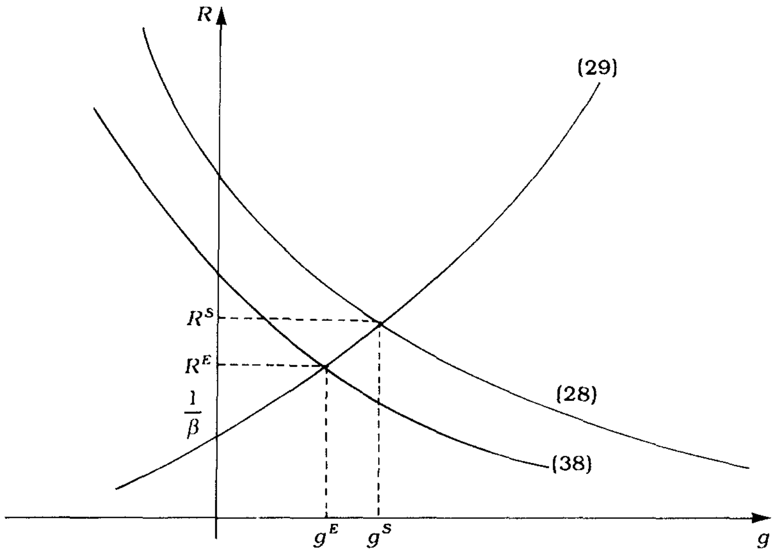

FIGURE 2

social planner's problem (g^s, E^s).

Proof: By the comparison of two Equation (28) and (38), we know that the curve representing Equation (38) lies below that of Equation (28). This is depicted in Figure 2.

Q.E.D.

***Proposition* 9**

If γ^H is sufficiently small or σ is smaller than one, then the span of control of *PS* is smaller than that of *PE*.

Proof: Under the first condition, Equation (28) and (38) do not make much difference in (g, R) plane, such that they have similar values of g and R. Then the spans of control will be different mainly due to the difference of γ and α. Under the second condition, the result is obvious from Equation (18) and (32) with g^s greater than g^E.

Q.E.D.

***Proposition* 10**

The decrease of σ with positive growth rate or increase of β induces higher investment ratio and growth rate, with interest rate unchanged. Additionally, they cause the span of control, except the top level, to be decreased, in both *PS* and *PE*.

Proof: Substituting Equation (29) into (28), then we have

$$R = \left[\frac{\gamma}{(R-1)(1-\gamma)}\right]^{\gamma^H} \cdot \left(\frac{\gamma}{R}\right)^{\gamma^{H-1}+\gamma^H} \cdot (1-\gamma).$$

The counterpart of *PE* of the above relationship is

$$R = \left[\frac{1}{R-Z} \cdot \frac{\alpha}{1-\gamma}\right]^{\gamma^H} \cdot \left(\frac{\alpha}{R}\right)^{\gamma^{H-1}+\gamma^H} \cdot (1-\gamma).$$

These equations say that the interest rate is not a function of β nor of σ in both of these problems. Furthermore, from Equation (29), g is expressed as $g = (\beta R)^{1/\sigma} - 1$, which gives the desired direction of the movement of g with respect to β and σ.

The span of control for *PS* is $1/e = R/\gamma(\beta R)^{1/\sigma}$ and that for *PE* is $1/f = R/\alpha(\beta R)^{1/\sigma}$.

Therefore, if $R\beta$ is greater than 1 (this condition is equivalent to positive growth rate), then we can achieve the desired results.

Q.E.D.

If Japan and catching-up countries are less risk averse and more patient than the U.S.A., they will show a higher growth rate and a higher investment ratio, and invest relatively more on the abstract knowledge than the U.S.A..

***Proposition* 11**

The change in σ and β does not change the span of control, except the top level, in both of *PS* and *PE*, when there is no time lag between the creation and usage as an input of knowledge.

Proof: The span of control without time lag is $1/\gamma$ for *PS* and $1/\alpha$ for *PE* except for the top level span of control.

Q.E.D.

B. Externality from Foreign Countries

From the above propositions, we can infer that if there exist externalities in the knowledge formation technology as in *PE*, the government policy should be exercised such that it can induce more investments over all levels of knowledge, resulting in a higher state of welfare, a higher growth rate and interest rate, but uncertain steepness of pyramidal structure of investments over all different knowledge. The externality comes from learning-by-doing, spillover, or imitation of technology from the domestic or international source.

For more interesting implications, consider a less-developed or developing economy which experiences externalities on a specific level (h^*) of knowledge, probably on a higher level of knowledge, by technology transfer or spill-over through international trade. This situation can be easily dealt with using the framework in the previous section if we modify it slightly.

Let us assume that this externality comes from foreign countries and is not affected by the domestic variables. Then, we have

$$\begin{aligned} &Y_h(t) = Y_{h+1}^f(t-1)Y_{h+1}(t-1)^{\gamma}X_h(t)^{1-\gamma}; \;\; h = h^*-1 \text{ and } h \neq H, \\ &Y_h(t) = Y_{h+1}(t-1)^{\gamma}X_h(t)^{1-\gamma}; \;\; h \neq h^*-1 \text{ and } h \neq H. \\ &Y_H(t) = Y_H(t-1) + X_H(t), \end{aligned} \tag{39}$$

where $Y_{h+1}^f(t-1)$ is the externality on the $h+1$ level knowledge from the foreign countries.

Then, the composite production function of this technology is

$$Y_0(t) = \left[Y_{h^*}(t-h^*)^{\gamma^{h^*-1}}\right] \cdot Y_H(t-H)^{\gamma^H} \cdot \prod_{h=0}^{H-1} X_h(t-h)^{(1-\gamma)\gamma^h} \text{[8]} \tag{40}$$

From this technology, we can prove the following propositions with ease.

***Proposition* 12**

Irrespective of which level of knowledge we have a positive externality as in equation (39), the growth rate and the interest rate are higher at the steady state where the externality remains constant over time. Moreover, if σ is greater than one, the span of control increases, except for the top level, compared to the economy without an externality. Furthermore, if σ is greater than $1 + g$, then the top level span of control increases and the investment ratio out of total income decreases.

Proof: The result can be easily obtained as in the proof of Proposition 8. For the top level span of control, we can prove the if σ is greater than $1 + g$, its span of control is $1/k$, as in Equation (24).

[8] The positive externality through trade is extensively dealt with by Grossman and Helpman (1990). The same expression holds with the externality on the top level technology.

Additionally, let $f(g) = \beta g/[(1+g)^\sigma - \beta]$ and we know that $f'(g) = \{1/[(1+g)^\sigma - \beta]^2\}\{\beta(1+g)^\sigma[(1+g) - \sigma] - \beta^2\}$. Therefore, if σ is greater than $1 + g$, then k decreases as g goes up. That is, as g goes up, the span of control increases. This proposition holds with an externality on the top level knowledge. We can easily prove the last part of the proposition, decrease of investment ratio, by Equation (22) and (23). These equations show that, as g increases, the investment ratio with respect to total income decreases. Here, investment comprises both categories of human and physical invstment.

Q.E.D.

The externality works just like a Hicksian factor neutral technology shock. The proposition above is very important in the sense that it is compatible with the empirical observation. By comparing Japan or Korea with the U.S.A., we find that Japan and Korea, the countries which have more technology import than export, show a higher growth rate and relatively more investment on the more applicable knowledge. Therefore, this mechanism can give us a partial answer to why some catching-up countries show a higher growth rate and less investment in the top level knowledge.

Another important implication is that technology spill-over on any level of knowledge, which does not have learning-by-doing by itself, affects growth rate.

The intuition of why the country with a higher growth rate has a less steep pyramidal structure of investment (a higher value of span of control) is that, as she discounts the future more, because of a higher growth rate, she will hesitate to invest on the higher level of knowledge, which takes more time to work as an input to the final output.

V. Conclusion

In this paper, several different models are formulated and their interesting implications are derived. One of the major findings is that, if there exist externalities coming from a domestic source, through knowledge spill-over or imitation, the social planner's allocation shows higher growth rate, interest rate, and steeper pyramidal structure of investments from top to bottom levels of knowledge under reasonable conditions than the competitive equilibrium solution.

Nevertheless, in the real world, the catching-up countries with more government intervention, such as Japan, Germany, up until several years ago, and NIEs, show higher growth rate and lower interest rate. They also make relatively more investments on the more applicable knowledge and have lower human capital investment ratios than the U. S.A.. Some of the empirical facts do not go along with the implications of the above model.

The second model, where externalities come from the more developed countries exogenously, shows that, irrespective of which level of knowledge the externalities affect, the country with these types of externalities shows higher growth rate, higher span of control, i.e., less steep pyramidal structure, and lower investment ratio out of GNP if economic agents are sufficiently risk averse. However, this country is supposed to show a higher interest rate, which contradicts the Japanese and German cases.[9]

Using this framework, the catching-up countries' behavior, relatively more investments on the more applicable research and less investment on the abstract knowledge can be rationalized as an optimizing behavior.

In this paper, the span of control, the structure of investments on different levels of knowledge, is endogenously determined, and, therefore, is not an exogenous engine of growth. The technology in this paper has some friction in the economy, because, as the number of levels of knowledge, H, increases, the production opportunity dwindles down. Similarly, different types of organization can be characterized by different values of H and, thus, have different exogenous engines of growth for different countries. This will be interesting. The increase of steps of knowledge formation, which can be interpreted as lack of coordination, will decrease the growth rate of income.

The time lag between the creation of knowledge and the usage as an input to the next level of knowledge, can be endogenized somehow, but this will not give us a much different implication., If the time lag can be endogenized, then this will give us the implication that a decrease of the time lag will increase the productivity of the technology in this model.

Another question is about what will happen if we have a learning-by-doing technology in the formation of other levels of knowledge besides the top level. Finally, the empirically difficult question is how to catego-

[9]Japan and Germany probably have a higher value of β, in addition to the exogenous externality.

rize the various kinds of knowledge over the hierarchy of human capital. These are on the agenda of future research.

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