Capital Structure and Production Decisions under the Differentiated Duopoly: Price and Quantity

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It is shown that managers who act in the interests of corporate insiders behave more (less) aggressively if the firm has more debt, when firms compete with quantities (prices) in the product market. The equilibrium Quantities are larger than in the traditional Cournot duopoly if the products are (demand) substitutes and strategic substitutes. The equilibrium prices are higher than in the traditional Bertrand duopoly if they are (demand) substitutes and strategic complements. These relationships between production and financial decisions explain why firms have relatively low leverage ratios irrespective of tax advantages of debt financing. Increases in debt payments have three effects on the firm's net present value: the negative own effect, the positive strategic effect and the tax advantage. The negative own effect increases as firm has more debt payment, while the positive strategic effect is irrelevant with debt payment. Therefore, according to the basic conditions of firms and their market structure, industry can have different capital structures and firms which are apparently similar in the same industry can have different capital structures. (*JEL* Classification: L11, D21)

I. Introduction

Modigliani and Miller (1958) showed that the value of the firm is independent of financial decisions under the perfect capital market assumption, and that investment and financial decisions are completely separable. This implies that financial products merely serve as a mechanism to allocate real products, and hence the firm value is completely determined by its real decisions (investment and production) in the product market. After this irrelevance theorem, the literature on

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capital structure and on industrial organization has placed relatively little emphasis on the strategic relationships between financial decisions and product market decisions.

According to the Modigliani-Miller arbitrage theorem(1963), firms should only be financed by debt because of the corporate tax advantage of debt financing. Empirical regularities, however, show that most firms have relatively low debt ratios (20-30%) irrespective of the corporate tax advantage (Gordon and Malkiel 1981; Taggart 1985) and that apparently similar firms often have very different capital structures (Remmers et al. 1974). In order to explain these empirical regularities, many literature on capital structure have explained how firms make their financial decisions based on the personal tax advantage of equity, bankruptcy costs, agency costs, asymmetric information, and so on.

According to the "basic conditions—market structure—conduct—performance" paradigm, conceived by Mason and extended by Bain, the industrial organization analysis has been primarily concerned with relationships or tendencies inducing a causal flow from firm's basic conditions and/or market structure to firm's conduct and market performance. Financial structure does not have any role in the industrial organization analysis. Firms are assumed to be completely equity financed, so that both the possibility of financial distress and the strategic interaction between equity and debt holders are usually ignored. Each firm is assumed to maximize its expected profit, that is, the net present value. Investment costs and production costs are considered as a flow concept so that financing problem does not need to be considered.[1]

If the firm decides to invest physical capital in order to either enter a new industry, develop a new product, install a new capacity or reduce its marginal cost, then the future returns from the investment will be affected by the output decisions (quantity and price) in the product market. These output decisions depend on the firm's basic conditions such as the extent of uncertainty of demand and cost, the firms' competing strategy and the firms' managerial incentive, and on the market structure such as the number of firms, the product differentiation (substitutes and complements) and barriers to entry.

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[1]The concept of opportunity (user) cost of capital is used and thus the cost for capital investment charges as recurring cost flows rather than one lump-sum charge incurred at the time capital is purchased. Therefore, the financing problem does not occur.

Jensen and Meckling (1976) pointed out the agency problems in managerial behavior, that is, shareholder-manager conflicts and stockholder-bondholder conflicts.[2] They showed that managers, acting in the interests of stockholders, have an incentive to undertake risky projects when the projects are unobservable.[3] Myers (1977) showed that if the firm already has a risky debt it may fail to undertake investments which have a positive net present value. Grossman and Hart (1982) showed that managers can use debt to precommit not to consume too many perquisites in a large corporation where manager are not shareholders. Friend and Hausbrouk (1986) pointed out that it is difficult to explain relatively low use of debt in the U.S. on the basis of stockholder optimization and that the corporate insiders (officers, directors and principal shareholders)[4] have a much greater incentive to maintain a low debt ratio than other stockholders do.

Recently, some literature has analyzed the strategic relationship between financial decisions and output decisions. Brandner and Lewis (1986) showed that the liability provisions of debt financing imply that changes in financial structure alter the distribution of returns between debt and stockholders. Therefore, the output strategy favored by stockholders is done. As the firm has more debt, managers act more aggressively, producing more quantity, in the product market. Allen (1986) showed that bankruptcy is likely to cause liquidation, which is due to the imperfection of the product market and the effect of bankruptcy to delay investment in the next period. Although bankruptcy cost itself is very small relative to tax benefit, bankruptcy is likely to cause liquidation. Thus,firms tend to have a relatively low debt ratio irrespective of

[2]The shareholder-manager agency problem (principal-agent problem) is that manager has wrong incentives when effort (investment) is unobservable since he no longer receives his full marginal product of effort. This agency problem between managers and shareholders is not considered in this paper, because it can be solved by monitoring of the principal shareholders or by reputation effect in multi-period. If the salary of the managers is assumed to depend on the expected profit only if firm is not bankrupt, then the managers will behave in the interests of the initial shareholders.

[3]If the investment is unobservable, the entrepreneurs tend to invest more risky and inferior investment when firm uses debt. So the rational investors expect this adverse selection and pay the debt value less than the equity value. Therefore, the entrepreneurs will not use debt.

[4]According to SEC disclosure rules, an investor who holds more than 5% of an equity issue is classified as a principal shareholders. The corporation whose insiders all combined hold more than 15% of the common stock outstanding is classified as closely held corporation (CHC).

tax advantages.

The empirical results such that there are systematic differences in capital structure between different industries as well as some similarities within industries (Kester 1986) and that apparently similar firms often have very different capital structure (Remmers et al. 1974) can be explained by the strategic relationship between capital structure and market structure. Thus, this paper develops the limited liability effect of Brandner and Lewis (1986) and analyzes the strategic relationship between financial decisions and output decisions under different market structures (product differentiation, price or quantity competition, managerial incentives, and market uncertainty).

If managers behave in the interests of the initial shareholders and thus maximize the equity value retained by the corporate insiders (entrepreneurial firm), then they have an incentive to pursue quantity (price) strategies that raise returns in good states and lower returns in bad states by the limited liability effect. Under quantity competition, firms behave more aggressively (increase quantity) than traditional oligopoly firms do, and hence equilibrium quantities are higher than those in the traditional Cournot model. Under price competition, however, firms behave less aggressively (increase price) than the traditional oligopoly firms do, and hence equilibrium prices are higher than those in the traditional Bertrand model. Thus, financial structure has a role for promoting or relaxing competition according to the type of competition or the characteristics of the products. Different financing mixes which pay the costs of a given investment will cause different equilibrium quantities (prices), and hence different firm values. This is inconsistent with the result of the Modigliani-Miller capital structure irrelevance theorem.

Section II presents the basic model and shows the effect of capital structure on the optimal output decisions under quantity competition. Section III also shows the effect of the managerial incentive on the optimal output decisions. Section IV shows the effect of market structure and competition on the optimal output decisions and the comparative statics. The optimal financial decisions are shown in section V. Finally, section VI contains some concluding remarks.

II. The Model

Firm i and j are rivals in an oligopolistic product market where they

produce their products by the amount of X_i and X_j respectively, place them on the product market, and then the market clearing prices P_i and P_j are determined in this product market.

The Inverse demands for the industry are stochastic, and given by

$$P_i = \alpha_i - \beta_i X_i - \gamma X_j = f_i(X_i, X_j)$$
$$P_j = \alpha_j - \beta_j X_j - \gamma X_i = f_j(X_i, X_j)$$

in the region of quantity space where prices are positive.[5] The intercepts α_i and α_j are random variables, but the slopes β_i and β_j are constant. The random variables, α_i and α_j are independently and uniformly distributed over the interval $[\alpha^L, \alpha^u]$. It is assumed that α^L is sufficiently large so that the equilibrium prices are positive for all possible realization of α. Letting $\mu = \beta_i \beta_j - \gamma^2 > 0$, $a_i = (\alpha_i \beta_j - \alpha_j \gamma)/\mu$, $a_j = (\alpha_j \beta_i - \alpha_i \gamma)/\mu$, $b_i = \beta_j/\mu$, $b_j = \beta_i/\mu$ and $c = \gamma/\mu$, the direct demand functions are written as

$$X_i = a_i - b_i P_i + c P_j = h_i(P_i, P_j)$$
$$X_j = a_j - b_j P_j + c P_i = h_j(P_i, P_j)$$

provided that quantities are positive. The random variables, a_i and a_j, are also uniformly distributed over the interval $[a^L, a^u]$. The goods are substitutes, independent, or complements according to the sign of c. The demand for goods i, X_i is always decreasing in its own price, P_i, and is increasing (decreasing) in the price of the competitor, P_j, if the goods are demand substitutes (complements).

Firms have constant marginal costs, m_i and m_j. From now on, price intercepts net of marginal cost, $z_i = \alpha_i - m_i$ and $z_j = \alpha_j - m_j$, are considered. The random variables z_i and z_j reflect the effects of the uncertain environment (market demands and costs) on the profits of firm i and j, respectively. The variables z_i and z_j are assumed to be distributed uniformly over the interval $[z^L, z^u]$. The operating profit for firm i, $\pi_i(X_i, X_j; z_i)$ is defined to be the revenue net of variable cost of firm i.

There are two periods, $t = 1, 2$. The timing of the firm's decisions is as

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[5]The above linear demand structure is derived from the quadratic and strictly concave utility function. $U(X_i, X_j) = \alpha_i X_i + \alpha_j X_j - (\beta_i X_i^2 + 2\gamma X_i X_j + \beta_j X_j^2)/2$, where α_i, α_j, β_i and β_j are positive. This inverse demand function has the symmetric cross effects. The sign of γ depends on the sign of $\partial_{ij} U$, the partial derivative of marginal utility of X_i with respect of X_j. When $\alpha_i = \alpha_j$ and $\beta_i = \beta_j = \gamma$, the goods are perfects substitutes. When $\alpha_i = \alpha$, $\gamma^2/\beta_i \beta_j$ expresses the degree of product differentiation, ranging from zero when the goods are independent to one when the goods are perfect substitutes.

follows: At $t = 1$, firms make financial decisions and issue the securities (bonds and equities). The proceeds are used to pay the costs of a given optimal investment decision, which is already made. Managers decide the face value of bond, D_i, and the amount of debt financing, B_i, is set in the perfectly competitive capital market.[6] The remainder required to finance the investment, $I_t - B_i$, should be raised by issuing equity. The fraction of equity claims required by the outside investors, $1 - \theta_i$,[7] is determined in the capital market. At $t = 1 + \varepsilon$, firms make quantity (price) decisions and produce output before the uncertain environment is realized. At $t = 2$, firms' quantities (prices) determine the market prices (quantities) in the product market after the market uncertainty is realized. Net operating profits are used to pay the corporate income tax and to make payments on the securities issued at $t = 1$.

A rational expectations subgame perfect equilibrium concept is used. The equilibrium is solved backwards by first solving for the output decisions in stage two, and then solving for the financial decisions in stage one given that firms play their equilibrium strategies in the second stage subgame. Since all the investors are risk neutral and have rational expectation, they can predict the firm's managerial incentives and output decisions and hence calculate the market value of securities, V_t^E and V_t^D. Thus, securities sell for their discounted expected value and hence B_i and θ_i are determined in the capital market.

At the second stage, corporate insiders, that is, initial shareholders will maximize the value of the equities they hold. They maximize net returns, after corporate taxes and payments to bondholders and new equityholders are made. These payments depend on the financing mix, D_i and θ_i, decided in the first stage. Thus, the second-stage objective function of corporate insiders is as follows:

$$\delta\theta_i \int_{z_i^l}^{z_i^u} [(1-t)\pi_i(X_i, X_j; z_i) - (1-tr)D_i] f(z_i) dz_i \tag{1}$$

where z_i^* is defined by

[6]Pure-discount (zero-coupon) bonds are issued. Firms usually use sinking funds in order to retire long-term liabilities, preferred stock, and purchase long-term assets. Thus, the type of bonds does not matter if the economy is stationary.

[7]Principal shareholders should sell $1 - \theta_i$ of shares they hold or issue new stocks by θ_i' of shares they hold $(\theta_i' / (1 + \theta_i') = 1 - \theta_i)$. Thus, principal shareholders only receive θ_i of net operating income and outside shareholders receive $(1 - \theta_i)$. These stocks can be purchased by the principal shareholders if firms have inside financing.

$$(1 - t)\pi_i(X_i(X_j), X_j; z_i^*) + trD_i = D_i \qquad (2)$$

assuming $z_i^l < z_i^* < z_i^u$. When $z_i = z_i^*$, firm i can just meet its obligations and has nothing left over. The expression in (1) represents the present value of expected profit retained by initial shareholders, net of corporate income taxes and debt obligations in good $(z_i \geq z_i^*)$ states of the uncertainty. At $t = 2$, the firm must pay the face value of the bond D_i, or it goes bankrupt and then bondholders acquire it. Initial shareholders receive the part, θ_i, of the profits remaining at the end of period after taxes and debt payments. Thus managers select quantities (prices) in order to maximize the expected equity value, given D_i and θ_i. For the sake of simplicity, there are no personal taxes on equity and debt income. The only difference between equity and debt financing is the tax deductibility of interest payments from debt financing. Thus, the interest payment, rD_i, should be exempted from the corporate taxes $(T = t\pi_i + trD_i)$.[8] The discount factor δ is $1/(1 + \rho)$, where ρ is the required rate of return for the relevant risk class.[9] The best response curve of firm i, $X_i^E(X_j)$ can be obtained from the derivative of the equation (1) with respective to X_i.

$$\partial_i V_i^E(X_i^E(X_j), X_j)^{10} = \int_{z_i^l}^{z_i^u} \partial_i \pi_i(X_i^E(X_j), X_j; z_i) f(z_i) dz_i = 0 \quad \text{for} \quad \forall X_j \qquad (3)$$

$$-dz_i^* / dX_i \{(1-t)\pi_i(X_i^E(X_j), X_j; z_i^*) - (1-tr)D_i)\} = 0$$

where z_i^* is solved from the bankruptcy condition (2).

$$z_i^*(X_i(X_j); X_j, D_i, D_j) = \frac{(1-tr)}{(1-t)} \frac{D_i}{X_i(X_j)} + \beta_i X_i(X_j) + \gamma X_j \quad \text{for} \quad \forall X_j, D_i, D_j \qquad (4)$$

Thus z_i^* is affected by the output strategies, X_i and X_j, given financial decisions from the first stage, D_i and D_j . Firm i decides upon its opti-

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[8]The variable r corresponds to the rate of discount, that is, the ratio of interest earned during the period to the amount of debt payments. The following relationships between the rate of discount and the rate of interest are satisfied: $r = \rho/(1 + \rho)$ and $\delta = 1/(1 + \rho) = (1 - r)$. Without default risk, interest payment, rD_i is equal to the face value of pure discount bond net of the current market price, $D_i - B_i = \rho B_i$.

[9]If firms issue perpetuity bonds instead of pure-discount bonds, tax saving stream trD is constant and perpetual. The discount factor δ becomes $1/\rho$, where ρ is the capitalization rate for the relevant risk class.

[10]$\partial_i V_i^E$ represents the partial derivative of V_i^E with respect to X_i and $\partial_{ij} V_i^E$ is $\partial^2 V_i^E / \partial X_i \partial X_j$.

mal quantity, $X_i(X_j)$, expecting that firm j will not change its quantity and considering that firm i's quantity decision will affect the level of z_i^* and thus will affect its expected profit. Thus, firm j's quantity will affect the level of z_i^* both directly and via firm i's best response, $X_i(X_j)$. Firm i's debt payment, D_i will also affect the level of z_i^* both directly and via firm i's best response. Firm j's debt payments, D_j, will affect the level of z_i^* only through firm j's and firm i's best response. The following relationships are derived from the equation (2) and (4).

$$\partial z_i^* / \partial X_i = -\partial_i \pi_i(X_i(X_j), X_j; z_i^*) / \partial_{zi} \pi_i(X_i(X_j), X_j; z_i^*)$$
$$= \beta_i - \frac{(1-tr)}{(1-t)} \frac{D_i}{X_i^2} > 0 \tag{5}$$

$$\partial z_i^* / \partial X_j = -\partial_j \pi_i(X_i(X_j), X_j; z_i^*) / \partial_{zi} \pi_i(X_i(X_j), X_j; z_i^*) = \gamma \tag{6}$$

$$\partial z_i^* / \partial D_i = (1-tr)/(1-t)\partial_{zi}\pi_i(X_i(X_j), X_j; z_i^*) = \frac{(1-tr)}{(1-t)} \frac{1}{X_i} > 0 \tag{7}$$

$$\partial z_i^* / \partial D_j = 0 \tag{8}$$

$\partial z_i^* / \partial X_i$ is positive because $\partial_i \pi_i(X_i^E(X_j), X_j; z_i^*)$ must be negative to satisfy the first order condition (3) and $\partial_{zi}\pi_i(X_i^E(X_j), X_j; z_i^*)$ is always positive.[11] Thus z_i^* increases as firm i increases its quantity, and hence firm i is more likely to be bankrupt. The sign of $\partial z_i^* / \partial X_j$ is determined by the characteristics of the products: it is positive if the products are substitutes and negative if the products are complements. The critical state of bankruptcy, z_i^*, increases as firm i has more debt payments D_i and does not depend on the level of firm j's debt payments, D_j. As firm i's debt payments increase, it is more likely to be bankrupt, but firm j's debt payments can affect the possibility of bankruptcy of firm i only via firm j's output strategy.

The Cournot-Nash equilibrium output vector, $X^E = [X_i^E, X_j^E]$, satisfying that

$$\theta_i V_i^E(X_i^E(D_i, D_j), X_j^E(D_i, D_j)) \geq \theta_i V_i^E(X_i(D_i, D_j), X_j^E(D_i, D_j)) \text{ and}$$
$$\theta_j V_j^E(X_i^E(D_i, D_j), X_j^E(D_i, D_j)) \geq \theta_j V_j^E(X_i^E(D_i, D_j), X_j(D_i, D_j)) \text{ for } \forall X_i \text{ and } X_j,$$

has a unique stable equilibrium output vector, since $\partial_{ii} V_i^E(X_i^E, X_j^E) < 0$ and $\partial_{jj} V_j^E(X_i^E, X_j^E) < 0$ and the Jacobian matrix, $[\partial_{ij} V_i^E]_{j \neq i}$ has a domi-

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[11]Since z_i represents the price net of marginal cost, $\partial_{zi}\pi_i$ is always positive. If z_i represents the stochastic marginal cost only, $\partial_{zi}\pi_i$ will be negative.

nant negative diagonal, that is,

$$\partial_{ii}V_i^E + \sum_{j \ne i}\left|\partial_{ij}V_i^E\right| < 0 \quad \text{for} \quad \forall X_i. \quad \text{Thus} \quad \partial_{ii}V_i^E \cdot \partial_{jj}V_j^E > \partial_{ij}V_i^E \cdot \partial_{ji}V_j^E.$$

Once managers decide the face value of bond, B_i and θ_i are determined in the capital market by the following equations.

$$E_i = (1 - \theta_i)V_i^E(X_i(D_i, D_j), X_j(D_i, D_j)) \tag{9}$$

where $V_i^E(X_i(D_i, D_j), X_j(D_i, D_j))$
$$= \delta\int_{z_i}^{z_i^u} [(1 - t)\pi_i(X_i(D_i, D_j), X_j(D_i, D_j); z_i) - (1 - tr)D_i]f(z_i)dz_i$$

$$B_i = V_i^D(X_i(D_i, D_j), X_j(D_i, D_j)) \tag{10}$$

where $V_i^D(X_i(D_i, D_j), X_j(D_i, D_j))$

$$= \delta\int_{z_i}^{z_i^u} D_i f(z_i)dz_i + \delta\int_{z_i^*}^{z_i^u}[(1-t)\pi_i(X_i(D_i,D_j),X_j(D_i,D_j);z_i)+trD_i]f(z_i)dz_i.$$

By putting equations (9) and (10) into (1), equilibrium payoff to the initial shareholders is the following;

$$\theta_i V_i^E(X_i^E, X_j^E)$$
$$= NPV_i(X_i^E, X_j^E) = \delta\int_{z_i^l}^{z_i^u}(1-t)\pi_i(X_i^E, X_j^E; z_i)f(z_i)dz_i + \delta trD_i - I_i \tag{11}$$

This net present value, $NPV_i(X_i^E, X_j^E)$[12] is different from the net present value when the firm maximizes its expected profit, $NPV_i(X_i^F, X_j^F)$, since the equilibrium quantity vector $X^E = [X_i^E(D_i, D_j), X_j^E(D_i, D_j)]$ is different from $X^F = [X_i^F(D_i, D_j), X_i^F(D_i, D_j)]$ which is independent of the financing mix.

$$NPV(X_i^F, X_j^F) = \delta\int_{z_i^l}^{z_i^u}(1-t)\pi_i(X_i^F, X_j^F; z_i)f(z_i)dz_i + \delta trD_i - I_i \tag{12}$$

where $X_i^F = \arg\max_{X_i}\int_{z_i^l}^{z_i^u}\pi_i(X_i, X_j^F; z_i)f(z_i)dz_i$.

The net present value of a given investment can be different depending on the firm's managerial incentives as well as on the firm's capital structure. As the firm has more debt payments, it will behave more aggressively and hence the reaction curve will shift out because bad

[12]Equation (1) is the objective function in stage two where managers decide quantity given the firm's capital structure. Since securities are priced for their present value in the capital market, $\theta_i V_i^E(D_i; I_i)$ is equal to $NPV_i(D_i; I_i)$. Thus, at the first stage where managers decide the face value of bond, the objective function will be equation (11).

states of uncertainty are no longer relevant to equity holders.

Lemma 1

The reaction function of firm i, $X_i^E(X_j)$ is downward sloping if $\partial_y V_i^E(X_i^E(X_j), X_j) < 0$ and upward sloping if $\partial_y V_i^E(X_i^E(X_j), X_j) > 0$.

Proof: See the Appendix.

Lemma 2

The reaction function of firm i shifts out when firm i has more debt payments, D_i.

Proof: See the Appendix.

If managers behave in the interests of the corporate insiders, they will behave more aggressively as the firm has more debt payments. Thus the equilibrium quantity of firm i, X_i^E always increases, and the equilibrium quantity of firm j, X_j^E decreases (increases) as firm i uses more debt to finance investment if the reaction functions of firm i and j are downward (upward) sloping.

According to Bulow et al. (1985), the product of firm j is defined as a strategic substitutes for the goods of firm i if $\partial^2 \pi_i / \partial S_i \partial S_j < 0$, and as a strategic complements if $\partial^2 \pi_i / \partial S_i \partial S_j > 0$, where S_i and S_j are the strategies of firm i and j. Thus, with a strategic substitutes, firm i's optimal response to more aggressive play by firm j (increase in X_j or decrease in P_j) is to be less aggressive (firm i decreases X_i or increases P_i). With a strategic complement, firm i responds to more aggressive play with more aggressive play (firm i increases X_i or decreases P_i). If the products are demand substitutes, then $\partial \pi_i / \partial X_j < 0$, and if they are demand complements, then $\partial \pi_i / \partial X_j > 0$, since $\partial_j \pi_i = X_i \partial_j f_i = -\gamma X_i$. When the products are demand substitutes firm i earns less profit if firm j adopts a more aggressive strategy, while firm i earns more profit by firm j's more aggressive behavior when the products are demand complements. With linear demand functions, however, $\partial_y f_i = \partial_{ji} f_j = 0$ and thus $\partial_y \pi_i = \partial_j f_i + X_i \partial_y f_i = \partial_j f_i = -\gamma$. Therefore, the products are also strategic substitutes or strategic complements according to whether γ is positive or negative. Under price competition, however, the products are strategic complements or strategic substitutes according to whether c is positive or negative, that is, whether they are demand substitutes or demand complements.

Lemma 3

If the products are strategic substitutes, then $\partial_{ij}V_i^E < 0$ and $\partial_{ji}V_i^E < 0$. If the products are strategic complements, the $\partial_{ij}V_i^E > 0$ and $\partial_{ji}V_i^E > 0$.

Proof: See the Appendix.

Corollary

With nonlinear demand functions, if the products are both demand substitutes and strategic substitutes, then $\partial_{ij}V_i^E(X_i^E, X_j^E) < 0$. If the products are both demand complements and strategic complements, then $\partial_{ij}V_i^E(X_i^E, X_j^E) > 0$.

Proposition 1

$dX_i^E(D_i, D_j)/dD_i$ is always positive. If the products are strategic substitutes, then $dX_j^E(D_i, D_j)/dD_i < 0$. If the products are strategic complements, then $dX_j^E(D_i, D_j)/dD_i < 0$.

Proof: Total differentiation of the first order conditions with respect to X_i, X_j and D_i yields

$$\partial_{ii}V_i^E dX_i + \partial_{ij}V_i^E dX_j + \partial_{iD_i}V_i^E dD_i = 0$$

$$\partial_{ji}V_j^E dX_i + \partial_{jj}V_j^E dX_j + \partial_{jD_i}V_j^E dD_i = 0.$$

Cramer's rule then is used to obtain the comparative static results for $dX_i^E(D_i, D_j)/dD_i$ and $dX_j^E(D_i, D_j)/dD_i$. This yields

$$dX_i^E(D_i, D_j)/dD_i = -\partial_{iD_i}V_i^E \partial_{jj}V_j^E / H$$

$$dX_j^E(D_i, D_j)/dD_i = \partial_{iD_i}V_i^E \partial_{ji}V_j^E / H$$

where $H = \partial_{ii}V_i^E \partial_{jj}V_j^E - \partial_{ij}V_i^E \partial_{ji}V_j^E > 0$.
The sign of $dX_i^E(D_i, D_j)/dD_i$ depends on the sign of $\partial_{iD_i}V_i^E$.

Since $dz_i^*/dD_i = \dfrac{(1-tr)}{(1-t)}\dfrac{1}{\partial_{zi}\pi_i(X_i^E, X_j^E; z_i^*)} > 0$,

$\partial_{iD_i}V_i^E(X_i^E, X_j^E) = -dz_i^*/dD_i \partial_i \pi_i(X_i^E, X_j^E; z_i^*)f(z_i^*) > 0$, and $\partial_i \pi_i(X_i^E, X_j^E; z_i^*) < 0$. Thus, $dX_i^E(D_i, D_j)/dD_i$ is always positive. The sign of $dX_j^E(D_i, D_j)/dD_i$ depends on the sign of $\partial_{ji}V_j^E$ and $\partial_{iD_i}V_i^E$.

$$\partial_{ji}V_j^E(X_i^E, X_j^E)$$

$$= \int_{z_j}^{z_j^u} \partial_{ji}\pi_j(X_i^E, X_j^E; z_j)f(z_j)dz_j - dz_j^*/dX_i \partial_i \pi_j(X_i^E, X_j^E; z_j^*)f(z_j^*)$$

$$\qquad\qquad (-) \qquad\qquad\qquad\qquad\qquad\qquad (+)$$

Accoding to Lemma 3,

$$\partial_{jt} V_j^E(X_i^E, X_j^E) = -\gamma \frac{1}{z_j^u - z_j^L}[(z_j^u - z_j^*) + \partial_j \pi_j(X_i^E, X_j^E; z_j^*)]$$

where $(z_j^u - z_j^*) + \partial_j \pi_j(X_i^E, X_j^E; z_j^*) > 0$.

If the products are strategic substitutes, then $\gamma > 0$ and hence $\partial_{jt} V_j^E$ $(X_i^E,$ $X_j^E) < 0$. If strategic complements, $\partial_{jt} V_j^E(X_i^E, X_j^E) > 0$. Therefore, $dX_j^E(D_i,$ $D_j)/dD_i < 0$ if the products are strategic substitutes, and $dX_j^E(D_i, D_j)/dD_i$ > 0 if strategic complements.

<div align="right">Q.E.D.</div>

The increase in debt payments will make managers behave more aggressively: they produce more quantities and hence market competition is encouraged. Thus, equilibrium quantities are larger than the traditional Cournot equilibrium quantities if firms have the same capital structure. If firm i has more debt, it affects firm i's net present value, $NPV_i(X_i^E, X_j^E)$ via its own effect on firm i's equilibrium quantity and via the strategic effect on firm j's equilibrium quantity, which depend on the characteristics of the products. From the assumption of demand functions $(\beta_i \beta_j - \gamma^2 > 0)$, the Jacobian matrix $[\partial_{ij} V_i^E]_{j \neq i}$ has a dominant negative diagonal, and thus the own effect of the increase in debt payments on its equilibrium quantity, $|dX_i^E/dD_i|$, is greater than the strategic effect on the equilibrium quantity of the other firm, $|dX_j^E/dD_i|$.

III. Control of the Firm: Managerial Incentives

Now, it will be shown that the firm's quantity strategy and the equilibrium quantities will be different according to the managerial incentive, that is, whether firms are controlled by shareholders, bondholders or both of them.

If managers behave in the interests of these equityholders, then they will make their output decision (quantity or price) in order to maximize the equity value of the firm.

$$X_i^E(X_j) = \arg\max V_i^E(X_i, X_j; I_i, D_i)$$

$$= \arg\max_{X_i} \int_{z_i}^{z_i^u} [(1-t)\pi_i(X_i, X_j; z_i) - (1-tr)D_i] f(z_i) dz_i \quad \text{for} \quad \forall X_j$$
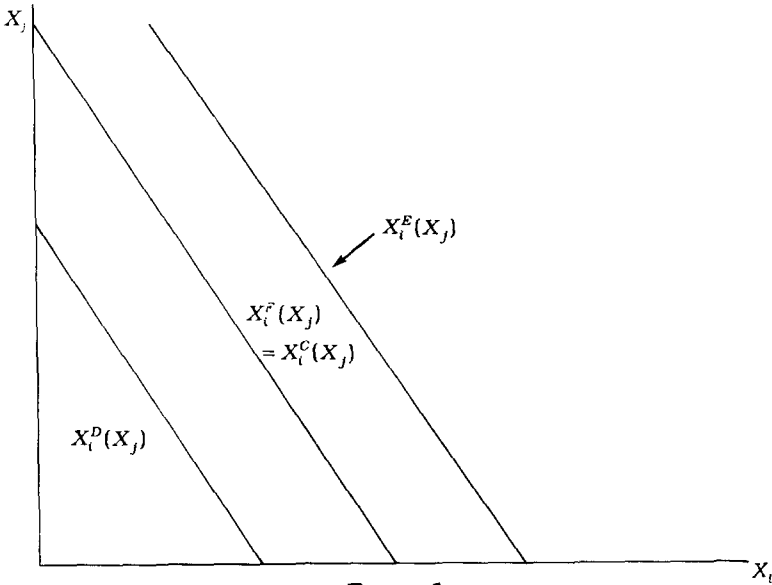
FIGURE 1

THE COMPARISON OF $X_i^E(X_j)$, $X_i^D(X_j)$ AND $X_i^F(X_j)$

If firms are controlled by bondholders and managers behave in their interests, then they maximize the debt value of the firm. The quantity strategy, $X_i^D(X_j)$, is given by the following:

$$X_i^D(X_j) = \arg\max V_i^D(X_i, X_j; I_t, D_t)$$

$$= \arg\max_{X_i} \int_{z_i^l}^{z_i^*} [(1-t)\pi_i(X_i, X_j; z_i) + trD_i] dF(z_i) + \int_{z_i^*}^{z_i^u} D_i dF(z_i) \quad \text{for } \forall X_j.$$

If managers act in the interests of both of equity and debt holders and hence maximize the market value of the firm, the output strategy $X_i^F(X_j)$ is given by the following:
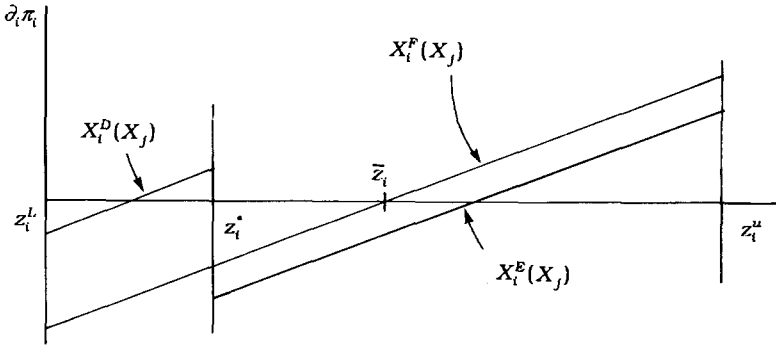
$$X_i^F(X_j) = \arg\max V_i^F(X_i, X_j; I_t, D_t)$$

$$= \arg\max_{X_i} \int_{z_i^l}^{z_i^u} (1-t)\pi_i(X_i, X_j; z_i) f(z_i) dz_i + trD_i \quad \text{for } \forall X_j.$$

The best response, $X_i^F(X_j)$ is equivalent to $X_i^C(X_j)$, the best response of firm i in the traditional Cournot model. From the first order conditions, the following results are obtained. (See Figure 1 and 2)

Proposition 2

The reaction functions $X_i^E(X_j)$, $X_i^D(X_j)$ and $X_i^F(X_j)$ have the following rela-

FIGURE 2

FIRM i's MARGINAL PROFIT WITH RESPECT TO X_i

tionship:

$$X_i^D(X_j) < X_i^F(X_j) < X_i^E(X_j)$$

The firm's quantity strategy is affected by its own managerial incentive as well as its own capital structure. Thus equilibrium quantities are determined by firms' managerial incentive and capital structure. The firm value depends on both the amount of debt payments and on the group who controls it. As the firm increases debt payments, the firm controlled by bondholders as well as by equityholders behaves more aggressively and hence produces more quantity. The reaction function of the firm controlled by bondholders, $X_i^D(X_j)$, approaches $X_i^F(X_j)$, as the firm increases the amount of debt, while the reaction function of the firm controlled by equityholders, $X_i^E(X_j)$, approaches $X_i^F(X_j)$, as the firm decreases the amount of debt.

IV. Market Structure and Competition: Comparative Statics

From now on, we will analyze the effect of capital structure on output decisions under different market structure. If firms compete with price, then the increase in debt payments will make the firm behave less aggressively. This relaxes competition in the product market. Thus, the equilibrium prices will be higher than the traditional Bertrand equilibrium prices if firms have symmetric capital structures.

Proposition 3

Suppose firms compete with price. $dP_i^E(D_i, D_j)/dD_i$ is always positive. If the products are strategic substitutes, $dP_j^E(D_i, D_j)/dD_i > 0$. If strategic complements, $dP_j^E(D_i, D_j)/dD_i < 0$

Under price competition, each firm regards its product as a strategic complements $(\partial_y \pi_i = \partial_j h_i + \partial_y h_i P_i > 0)$ to the products of the other firm if the products are demand substitutes ($\partial_j h_i > 0$) and $\partial_y h_i > 0$, and as a strategic substitutes $(\partial_y \pi_i = \partial_j h_i + \partial_y h_i P_i < 0)$ to the products of the other firm if the products are demand complements ($\partial_i h_j < 0$) and $\partial_{ji} h_j < 0$. With a strategic complements, debt financing increases the equilibrium prices. Therefore, debt financing has a role of relaxing price competition as capacity constraint, advertising and product differentiation relaxes price competition. (Kreps and Scheinkman 1983; Schmalensee 1983; Shaked and Sutton 1982).

Under the traditional Bertrand and Cournot models with product differentiation, Cournot equilibrium prices are not lower than Bertrand equilibrium prices. Hence Cournot competition is viewed as being more monopolistic than Bertrand competition. (Okuguchi 1985; Vives 1985) With linear demand functions,

$$\bar{P}_i^C - \bar{P}_i^B = \alpha_i \gamma^2 / (4\beta_i \beta_j - \gamma^2) > 0 \quad \text{and}$$
$$\bar{X}_i^C - \bar{X}_i^B = -a_i \gamma^2 / (4b_i b_j - c^2) < 0.^{13}$$

As firms have more debt payments, Bertrand equilibrium prices increase and expected Cournot equilibrium prices decrease. Thus expected Cournot equilibrium prices may be lower than Bertrand equilibrium prices if firms have a large amount of debt. The increase in debt payments encourages competition under Cournot competition and it relaxes competition under Bertrand competition. In an industry where firms compete with price, equilibrium prices and net present values are higher, and thus pareto superior to the traditional Bertand duopoly. In an industry where firms compete with quantity, however, equilibrium expected prices and net present values are lower, and thus pareto inferior to the traditional Cournot duopoly. Thus debt financing gives the situation like the prisoners' dilemma. (See Figure 3 and 4)

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[13] The Cournot equilibrium prices depend on the realization of future uncertainty. Thus, the expected Cournot equilibrium price is compared with the Bertrand equilibrium price.
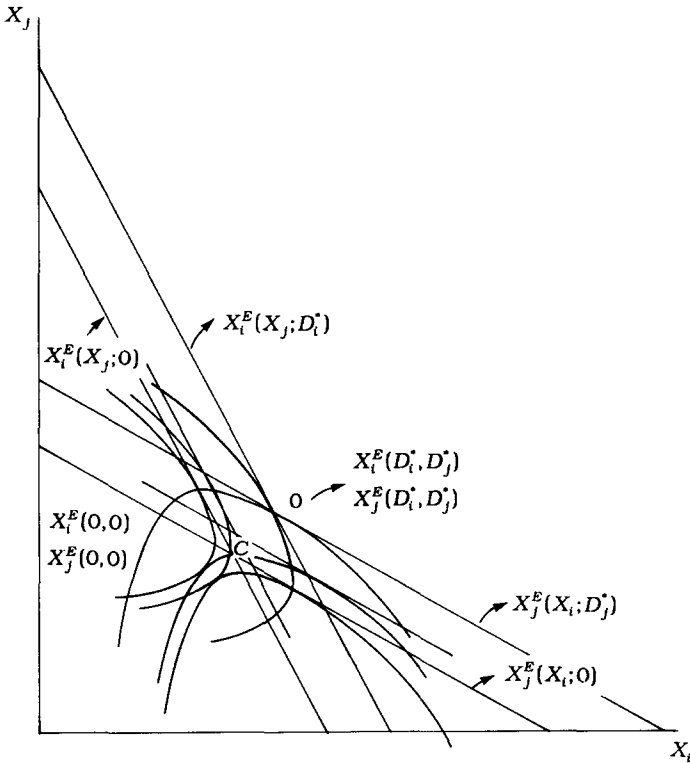
FIGURE 3
THE OPTIMAL DEBT PAYMENTS (D_i^*, D_j^*) UNDER QUANTITY COMPETITION

Proposition 4

Suppose demand systems are symmetric. Cournot equilibrium prices (quantities) are lower (more) than the Bertrand equilibrium prices (quantities) if firms have large amounts of debt.

Proof: See the Appendix.

Under either a monopoly or a perfectly competitive market, the increase in debt payments does not have any strategic effect on the equilibrium quantities of the other firms, since firms need not consider the responses of the other firms when choosing their quantities. Thus, since firms increase their quantities and hence earn less as they increase their debt payments, firms will not have any incentive to use debt financing if there is no tax advantage.
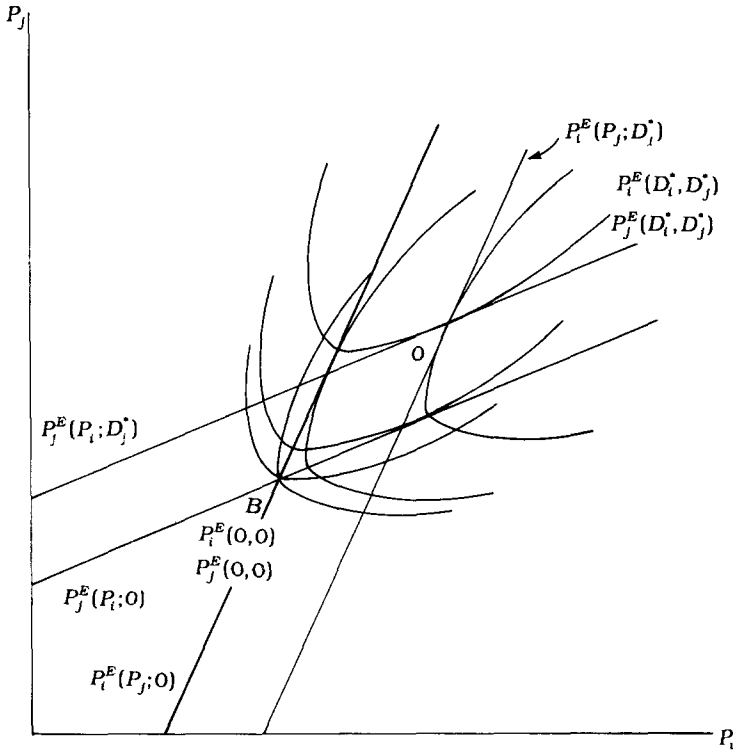
FIGURE 4
THE OPTIMAL DEBT PAYMENTS (D_i^*, D_j^*) UNDER PRICE COMPETITION

Comparative Statics

Tax Effect: $dX_i^E/dt > 0$, $dP_i^E/dt > 0$

Since the increase of marginal corporate tax rate causes firms to be default more likely $(\partial z_i^* / \partial t > 0)$, firms behave more aggressively in quantity competition or more collusive in price competition.

Interest Effect: $dX_i^E/dr < 0$, $dP_i^E/dr < 0$

Nominal interest rate also affects the output decisions $(\partial z_i^* / \partial r < 0)$, but the increase in the required rate of return results in less equilibrium quantities under quantity competition, or lower equilibrium prices under price competition.

Product Differentiation Effect: $dX_i^E/d\gamma < 0$, $dP_i^E/dc < 0$

If the products are substitutes($\gamma > 0$), firms reduce their quantities

or lower their prices at the products get more closely related (as γ approaches one). If the products are complements ($\gamma > 0$), firms reduce their quantities or lower their prices as the products get more unrelated (as γ approaches zero).

Uncertainty Effect: $dX_i^E/dz_i^u > 0$, $dP_i^E/dz_i^u > 0$

As the upper bound of the future uncertainty (z_i^u) increases, firms behave more aggressively under quantity competition or less aggressively under price competition. The lower bound of the future uncertainty (z_i^l) usually does not affect firm's output decisions. It only affects the possibility of bankruptcy. When z_i^l is high enough to satisfy $\pi_i(X_i^E, X_j^E; z_i^l) = (z_i^l - \beta_i X_i^E - \gamma X_j^E)X_i^E > 0$, firms produce their quantities at the Cournot equilibrium levels until firms have some amount of debt payments such that $z_i^*(X_i^E, X_j^E; D_i) \geq z_i^l$. With the same expected value of market uncertainty, the equilibrium quantities are larger or the equilibrium prices are higher as the market uncertainty gets more risky, that is, as z_i^u gets higher.

V. The Financial Decision

The optimal debt payments, maximizing the firm's net present value are solved in the first stage, given that firms will select their optimal output decisions in the second stage. In the first stage, firms choose the face value of their debt, expecting that their quantities will be determined optimally depending on their debt levels in the second stage. Therefore, (D_i^*, D_j^*) is the Nash equilibrium debt payments in the first stage, if and only if

$$NPV_i(X_i^E(D_i^*, D_j^*), X_j^E(D_i^*, D_j^*)) \geq NPV_i(X_i^E(D_i, D_j^*), X_j^E(D_i, D_j^*)) \text{ and}$$

$$NPV_j(X_i^E(D_i^*, D_j^*), X_j^E(D_i^*, D_j^*)) \geq NPV_j(X_i^E(D_i^*, D_j), X_j^E(D_i^*, D_j)) \text{ for } \forall D_i \,\&\, D_j.$$

So firm i selects its debt payments as a best response to firm j's debt payments, $D_i(D_j)$, so as to maximize the net present value of firm i.

$$\underset{D_i}{\text{Max}} \; \delta \int_{z_i^l}^{z_i^u}(1-t)\pi_i[X_i(D_i,D_j),X_j(D_i,D_j)]f(z_i)dz_i + \delta trD_i - I_i \quad \text{for} \;\; \forall D_j.$$

$$\frac{dNPV_i}{dD_i} = (1-t)\int_{z_i^l}^{z_i^u}[\frac{\partial \pi_i}{\partial X_i}\frac{dX_i^E}{dD_i} + \frac{\partial \pi_i}{\partial X_j}\frac{dX_j^E}{dD_i}]f(z_i)dz_i + tr$$

$$= (1-t)\frac{dX_i^E}{dD_i}\int_{z_i^l}^{z_i^*}\partial_i\pi_i[X_i^E(D_i(D_j),D_j),X_j^E(D_i(D_j),D_j)f(z_i)dz_i \quad (-)$$

$$+(1-t)\frac{dX_j^E}{dD_i}\int_{z_i^L}^{z_i^U}\partial_j\pi_i[X_i^E(D_i(D_j),D_j),X_j^E(D_i(D_j),D_j)]f(z_i)dz_i \quad (+)$$

$$+tr \qquad\qquad\qquad\qquad (+)$$

The first term represents the own effect of the increase in firm i's debt payments on its own net present value, which is negative. The absolute value of the own effect increases as the firm has more debt payments. The second term represents the strategic effect. It is always positive, irrespective of product differentiation. The third term represents the positive tax advantage. Thus, firms in the oligopoly product market should consider the trade-off among these three effects when they select their capital structures. If the firm has a small amount of debt payments, then z_i^* is too low, near z_i^L. Hence the own effect is too small and $dNPV/dD_i$ must be positive. The firm thus tends to increase the amount of debt. As the firm takes on more debt, however, the absolute value of the own effect gets larger. Thus $dNPV_i/dD_i$ decreases and becomes zero only if the firm has more than some amount of debt, for the absolute value of the own effect is greater than the sum of strategic effect and tax effect. Therefore, if managers act in the interests of the corporate insiders, then they will not finance investment costs by issuing bond only, but have some optimal amounts of debt payments.

Proposition 5

The own effect of debt financing is greater than the strategic effect if

$$X_i^E \geq (\bar{z}_i - \gamma X_j^E)/(2\beta_i - \gamma) \geq X_i^C(X_j^E) = (\bar{z}_i - \gamma X_j^E)/2\beta_i.$$

Proof: See the Appendix.

Figure 5 and 6 show the possible region where the own effect is absolutely greater than the strategic effect. Therefore, the equilibrium quantities should be located in the region where $X_i^C(X_j^E) < X_i^E < (\bar{z}_i - \gamma X_j^E)/(2\beta_i - \gamma)$ and $X_j^C(X_i^E) < X_j^E < (\bar{z}_j - \gamma X_i^E)/(2\beta_j - \gamma)$ when tax effect is not considered, whereas the equilibrium prices should be located in the region where $P_i^B(P_j^E) < P_i^E < (\bar{a}_i + cP_j^E)/(2b_i - c)$ and $P_j^E(P_i^E) < P_j^E < (\bar{a}_j + cP_i^E)/(2b_j - c)$.

Figure 3 and 4 illustrate the optimal debt levels (D_i^*, D_j^*) when tax effect is not considered. Firm i chooses its optimal debt payments, D_i^* (D_j^*) given the other firm's debt payments, D_j. As firm j increases its own debt payments, firm i will have a smaller amount of debt. When firm i has the optimal amount of debt, $D_i(D_j)$, firm i's iso-value curve is
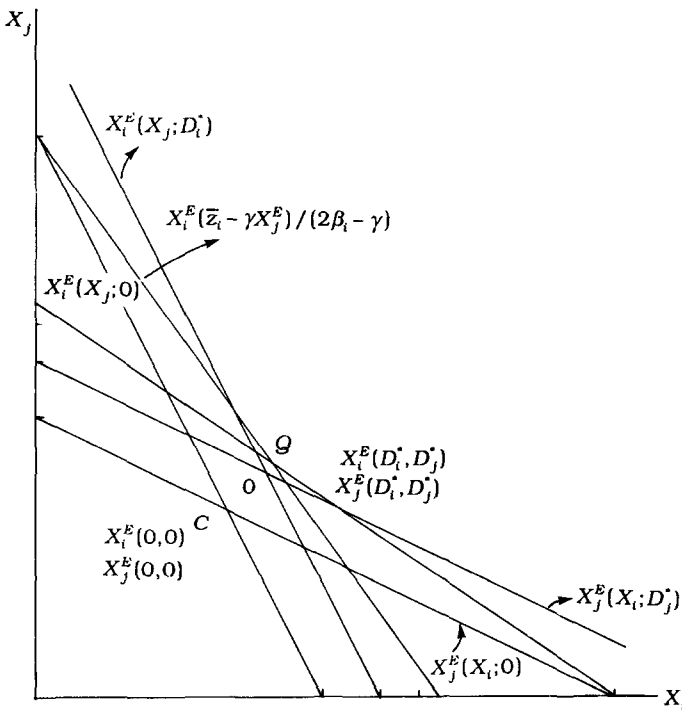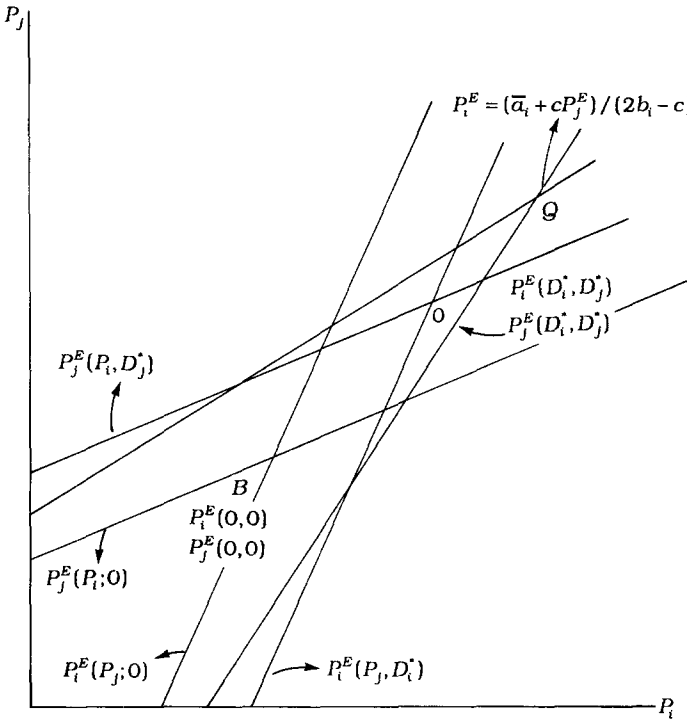
FIGURE 5
THE POSSIBLE REGION OF THE OPTIMAL DEBT PAYMENTS UNDER
QUANTITY COMPETITION

tangent to firm j's reaction function which is determined by firm j's debt payments, D_j. In the first stage equilibrium, each firm's iso-value curve is tangent to the other firm's reaction function at the intersection, O, so that neither firm can gain a higher net present value by altering its capital structure. Under quantity competition with substitutes, however, the optimal debt amounts cause firms to have lower firm values than the traditional Cournot duopoly, because they behave more aggressively and hence their equilibrium quantities are larger. Thus it shows the situation of prisoners' dilemma. Although they know that they may have higher firm value without debt financing, they tend to have some amounts of debt.

Additionally, under price competition with substitutes, firms have higher firm values at the optimal levels of debt, and hence this equilibrium is pareto superior to the traditional Bertand equilibrium. This occurs both because firms become less aggressive and hence their equilibrium prices are higher, and because debt financing has tax

P_j

$P_i^E = (\overline{a}_i + cP_j^E) / (2b_i - c)$

Q

$P_i^E(D_i^*, D_j^*)$

$P_j^E(D_i^*, D_j^*)$

$P_j^E(P_i, D_j^*)$

0

$P_j^E(P_i;0)$

B

$P_i^E(0,0)$
$P_j^E(0,0)$

$P_i^E(P_j;0)$

$P_i^E(P_j, D_i^*)$

P_i

FIGURE 6

THE POSSIBLE REGION OF THE OPTIMAL DEBT PAYMENTS UNDER
PRICE COMPETITION

advantage.

Under either monopoly or perfect competition, the increase in debt payments has only two different effects on the net present value of the firm. These are the negative own effect and the positive tax effect. Since monopoly or perfectly competitive firms have no strategic effects from debt increases, they simply consider the trade-off between negative own effect and positive tax benefit on the net present value. Hence they tend to have less debt than oligopoly firms do. Under the Stackelberg competition, the leader behaves like a monopolist and thus tends to have little amount of debt, whereas the follower tends to have a large amount of debt in order to avoid his disadvantage in choosing quantities.

Kester (1986) shows that regulated and mature industries have a higher debt ratio than immature and high technology industries. Regulated and mature industries generally seem to be more stable than immature and high technology industries. Therefore, these differ-

ences in capital structure between different industries can be explain-
ed by the different riskiness of market uncertainties.

Proposition 6

Suppose demand systems are symmetric. Firms have lower amount of
debt payments when market uncertainty is more risky (more variable)
than when market uncertainty is less risky (more stable).

Proof: See the Appendix.

   The industry capital structure, (D_i^*, D_j) depends on the type of compe-
tition (quantity or price) and the market structure parameters: range of
uncertainty ($[z^L, z^u]$) and the extent of product differentiation (β_i, β_j, γ).
Firms will decide on the optimal level of debt payments by comparing
the marginal loss of expected profits caused by the increase of the level
of debt above D_i^* (D_j) with the constant marginal gain from the tax
deduction of interest payments, tr, which depends both on the mar-
ginal tax rate of corporate income tax and the nominal interest rate.
Firms may have different amounts of debt (D_i^*, D_j^*) according to their
different basic conditions even if they are in the same industry.
According to the situation in which firms are located, they may have
different capital structures. Under stackelberg competition, the leader
tends to have little amount of debt and the follower to have some opti-
mal amount of debt, even if they are apparently similar. The firm which
expects more stable future uncertainty will have larger amount of debt
than the firm which expects more risky future uncertainty.

VI. Conclusion

   It is shown that the firm value is different according to its financing
mix, if managers behave in the interests of the corporate insiders. Thus
firms will not use debt financing only, irrespective of the corporate tax
advantage from debt financing. The managers who act in the interests
of the initial shareholders are likely to have incentives to use less debt
than the managers who maximize the firm value. Usually, a closely-
held corporation is likely to act in the interests of its insiders. So clo-
sely-held corporations are likely to have less debt amount than the
publicly-held corporations, which is thought to behave so as to maxi-
mize the firm value (Friend and Lang 1986). Thus, differences in the
managerial incentives could explain the fact that apparently similar

firms often have very different capital structures.

The empirical analysis by Bradley et al. (1984) and Kester (1986) show that there are systematic differences of capital structure across industries as well as similarities within industries. Regulated industries, such as utilities and railroads, have higher debt ratios than manufacturing industries. Heavy and mature industries, such as petroleum, steel, cement and chemicals, have higher debt ratios than immature and high technology industries, such electronics, computers and semiconductors. This analysis suggests that this systematic differences of capital structure across industries can, in part, be explained by differences of market structure. If market structure is different in terms of the number of firms, the extent of market uncertainty, the extent of product differentiation, the extent of riskiness and the type of competing strategy, then firms' behavior and performance like capital structure, equilibrium quantities and prices and firm value, may be different. Even in the same industry, firms may have different capital structures according to the different firm's basic conditions like the extent of demand and cost uncertainty, managerial incentives and the situation in which firms are located within the industry. Therefore, different market structure and different firm's basic conditions should be considered in the analysis of different capital structure across firms, industries and countries.

Appendix

Proof of Lemma 1
By implicitly differentiating the first order condition (3),

$$dX_i^E(X_j) / dX_j = -\partial_y V_i^E(X_i^E(X_j), X_j) / \partial_u V_i^E(X_i^E(X_j), X_j)$$

Since $\partial_u V_i^E(X_i^E(X_j), X_j)$ is negative by the second order condition, the sign of $dX_i^E(X_j)/dX_j$ depends on the sign of $\partial_y V_i^E(X_i^E(X_j), X_j)$.

$$Q.E.D.$$

Proof of Lemma 2
By totally differentiating the first order condition,

$$\partial_u V_i^E dX_i + \partial_y V_i^E dX_j + \partial_{iDi} V_i^E dD_i = 0.$$

Solving for dX_i / dD_i then yields

$$dX_i^E(X_j)/dD_i = -\partial_{iD_i}V_i^E(X_i^E(X_j),X_j)/\partial_{ii}V_i^E(X_i^E(X_j),X_j)$$

The denominator is negative by the second order condition. Thus the sign of $dX_i^E(X_j)/dD_i$ depends on the sign of $\partial_{iD_i}V_i^E$ and hence the sign of dz_i^*/dD_i.

$$\partial_{iD_i}V_i^E = -dz_i^* / dD_i \partial_i\pi_i(X_i^E(X_j),X_j;z_i^*)f(z_i^*)$$

where $dz_i^* / dD_i = \dfrac{(1-tr)}{(1-t)} - \dfrac{1}{\partial_{zi}\pi_i(X_i^E(X_j),X_j;z_i^*)} > 0.$

Since $\partial_i\pi_i(X_i^E(X_j),X_j;z_i^*) < 0$, $dz_i^*/dD_i > 0$ and hence $dX_i^E(X_j)/dD_i > 0$.

Proof of Lemma 3

$$\partial_{ij}V_i^E(X_i^E,X_j^E) = \int_{z_i^L}^{z_i^u} \partial_{ij}\pi_i(X_i^E,X_j^E;z_i)f(z_i)dz_i - (dz_i^* / dX_j)\partial_i\pi_i(X_i^E,X_j^E;z_i^*)f(z_i^*)$$

where $dz_i^*/dX_j = -\partial_j\pi_i(X_i^E, X_j^E; z_i^*)/\partial_{zi}\pi_i(X_i^E, X_j^E; z_i^*) = \gamma.$

Since demand functions are linear and $\partial_i\pi_i(z_i)$ is strictly increasing in z_i, $\partial_{ij}\pi_i = -\gamma$ and $\partial_i\pi_i(z_i^u) = z_i^u - 2\beta_iX_i - \gamma X_j$ must be positive and $\partial_i\pi_i(z_i^L)$ $= z_i^L - 2\beta_iX_i - \gamma X_j$ must be negative for all X_i and X_j. Thus $z_i^L < 2\beta_iX_i^E + \gamma X_j^E$ $< z_i^u$ and hence $z_i^* - z_i^u < \partial_i\pi_i(X_i^E,X_j^E;z_i^*) < 0$.
Therefore,

$$\partial_{ij}V_i^E(X_i^E,X_j^E) = -\gamma \frac{1}{z_i^u - z_i^L}[(z_i^u - z_i^*) + \partial_i\pi_i(X_i^E,X_j^E;z_i^*)]$$

The sign of $\partial_{ij}V_i^E(X_i^E,X_j^E)$ and $\partial_{ji}V_i^E(X_i^E,X_j^E)$ are determined according to the sign of γ. If the products are substitutes, then $\gamma > 0$ and thus $\partial_{ij}V_i^E(X_i^E,X_j^E)$ is negative. If the products are complements, then $\gamma < 0$ and thus $\partial_{ij}V_i^E(X_i^E,X_j^E)$ is positive.

<div align="right">Q.E.D.</div>

Proof of Proposition 4
Suppose demand systems are symmetric. The Bertrand Equilibrium prices (P_i^B,P_j^B) satisfy the following first order conditions for any amount of debt:

$$\phi_i(P^B) = \int_{a_i^L}^{a_i^u} \partial_i\pi_i(P_i^B,P_j^B;a_i)f(a_i)da_i = 0 \qquad \text{for } i = 1,2.$$

The function $\phi_i(P_i, P_j)$ is downward sloping $(\partial\phi_i(P_i,P_j)/\partial P_i < 0)$ by the second order condition. At the average Cournot equilibrium prices,

$$\phi_i(\overline{P}^C) = \int_{a_i}^{a_i^u} [h_i(\overline{P}^C) + \overline{P}_i^C \partial_i h_i(\overline{P}^C)] f(a_i) da_i$$

$$= \int_{a_i}^{a_i^u} [b_i(\overline{\alpha}_i - \beta_i X_i^C - \gamma X_j^C) + (a_i - b_i \overline{P}_i^C + (c\overline{P}_j^C)] f(a_i) da_i$$

$$= \frac{1}{\mu} \int_{a_i}^{a_i^u} (a_i - a_i) f(a_i) da_i - \frac{\beta_j}{\mu} \int_{a_i}^{a_i^u} (\alpha_i - 2\beta_i X_i^C - \gamma X_j^C) f(a_i) da_i$$

$$(+) \hspace{5cm} (+)$$

$$- \frac{\gamma^2}{\mu} \int_{a_i}^{a_i^u} X_i^C f(a_i) da_i.$$

$$(-)$$

The first and second term is positive and the third term is negative. The sign of $\phi_i(\overline{P}^C)$ can be negative or positive according to the absolute values of three terms. When $D_i = 0$, $\partial_i \pi_i (X^C; \overline{\alpha}_i) = 0$ and thus $\phi_i(\overline{P}^C; D_i = 0) < 0$. Therefore, the Cournot equilibrium (expected) prices are higher than the Bertrand equilibrium prices.

The function $\phi_i(P_i, P_j)$ shifts up $(d\phi_i(P_i, P_j)/dD_i > 0)$ as firm i's debt payments increases since $dP_i^C/dD_i > 0$. There exists D_i^* such that $\phi_i(\overline{P}^C) = \phi_i(P^B) = 0$, that is, $\overline{P}_i^C = P_i^B$. If $D_i > D_i^*$, then $\phi_i(\overline{P}^C) > 0$ and $P_i^C < P_i^B$. Therefore, the Cournot equilibrium (expected) prices are not necessarily lower than the Bertrand equilibrium prices if firms have a large amount of debt.

Proof of Proposition 5
The own effect of the increase in firm i's debt payments on its own equilibrium quantity, $|dX_i^E/dD_i|$, is greater than the strategic effect on the equilibrium quantity of the other firm, $|dX_j^E/dD_i|$. Thus, the own effect of the increase in firm i's debt payments on its net present value is definitely greater than the strategic effect if,

$$\left| \partial_i \pi_i(X_i^E(D_i, D_j), X_j^E(D_i, D_j); \overline{z}_i) \right| \geq \left| \partial_j \pi_i(X_i^E(D_i, D_j), X_j^E(D_i, D_j); \overline{z}_i) \right|, \text{ that is,}$$

$$(\overline{z}_i - 2\beta_i X_i^E - \gamma X_j^E) \geq -\gamma X_i^E.$$

If $X_i^E > (\overline{z}_i - \gamma X_j^E)/(2\beta_i - \gamma) \geq X_i^C(X_j^E) = (\overline{z}_i - \gamma X_j^E)2\beta_i$, then

$$\left| \frac{dX_i}{dD_i} \int_{z_i}^{z_i^u} \partial_i \pi_i(X_i^E(D_i, D_j), X_j^E(D_i, D_j)) f(z_i(dz_i) \right|$$

$$\geq \left| \frac{dX_i}{dD_i} \int_{z_i}^{z_i^u} \partial_j \pi_i(X_i^E(D_i, D_j), X_j^E(D_i, D_j)) f(z_i(dz_i) \right|$$

So the absolute value of the own effect is greater than the absolute

value of the strategic effect if $X_i^E \geq (\bar{z}_i - \gamma X_j^E)/(2\beta_i - \gamma)$. Therefore, if the firm has a large amount of debt which satisfies the above condition, then the own effect of marginal debt increase is greater than the strategic effect.

<div align="right">

Q.E.D.

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Proof of Proposition 6

The market uncertainty of z_1 is assumed to be more variable (more risky) than the market uncertainty of z_2:

$$E_{z1} = E_{z2} \text{ and } \sigma_{z1} > \sigma_{z2}, \text{ that is, } z_1^u > z_2^u \text{ and } z_1^L < z_2^L.$$

Suppose firms have the optimal debt payments $(D_i^* = D_j^*)$ and produce outputs at the equilibrium quantity level, $X_i^E(D_i^*, D_j^*; z_2) = X_j^E(D_i^*, D_j^*; z_2)$, under the market uncertainty of z_i. Since $dX_i^E/dz_i^u > 0$, the equilibrium quantities will be larger by the amount of dX_i under the market uncertainty of z_i:

$$X_i^E(D_i^*, D_j^*; z_1) = X_i^E(D_i^*, D_j^*; z_2) + dX_i \text{ and}$$

$$X_j^E(D_i^*, D_j^*; z_1) = X_j^E(D_i^*, D_j^*; z_2) + dX_j \quad \text{where } dX_i = dX_j > 0.$$

At the optimal debt payments $(D_i^* = D_j^*)$, the first order condition of marginal debt increase will be negative:

$$dNPV(D_i^*, D_j)/dD_i$$
$$= -(1-t) \, dX_i^E/dD_i \, [2\beta_i dX_i + \gamma dX_j] + (1-t) \, dX_j^E/dD_i \, [-\gamma X_i^E(D_i^*, D_j^*)$$
$$+dX_j] < 0.$$

Therefore, firms will reduce their optimal debt payments if market uncertainty is more risky.

<div align="right">

Q.E.D.

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