Wages, Service Prices and Rent: Urban Division of Labor and Amenities

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This study proposes an equilibrium model to explain differences in living costs between two cities within a perfectly competitive framework. The economic system consists of two production centers. Each production center specifies in supplying one commodity which is demanded by the residents in the two cities. Services are city-specified in the sense that services are consumed only by the households who work in the city. We assume that all the households have an identical preference and that professional changes and movement between the two cities are costless and the two cities have different levels of amenity and technology. It is proved that the system of the economic geography has equilibria at which the land rent, service prices and wage rates are different between the two cities. We also examine the impact of changes in amenity upon the economic geography. (*JEL* Classification: D33)

I. Introduction

Time and space are the two most essential factors for explaining economic reality. Any economic activity takes place at certain place and at certain time. Although economic dynamics has caused much attention from theoretical economists (e.g. Zhang 1991), the complexity of economic geography has largely been ignored in the mainstream of theoretical economics (e.g. Krugman 1991). It has now become clear that there are a number of potentially important spatial influences, such as public goods, amenities, different externalities, transportation costs, that may challenge the validity of competitive equilibrium theory for explaining a regionally heterogeneous economy. For instance, one of

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these factors is the so-called capitalization, which implies that the price of land is interdependent with local amenities, economic agents' densities, transportation costs and other local variables or parameters (Scotchmer and Thisse 1992). But most of urban models are not suitable for examining the issue, as residential location and production location — the two main topics of spatial economics — are separated in the main stream of urban and regional economic literature (e.g. Beckmann 1968; Fujita 1989; Greenhut, Norman and Hung 1987; Miyao 1981, 1987). Utilizing some of the main ideas in these two approaches, we attempt to suggest an equilibrium framework of economic geography with capitalization. We examine the issue within the linear-two-city framework geographically similarly to that suggested in Suh (1988).

The seminar paper on compensating regional variation in wages and rents by Roback (1982) has caused a wide interest among regional and urban economists to theoretically investigate how the value of location attributes is capitalized into wages and services. Since the publication of Roback's model, many empirical and theoretical studies have also shown that across urban areas wages may capitalize differences in amenity levels or living costs (e.g. Blomquist, Berger and Hoehn 1988; Simon and Love 1990; Bell 1991; Voith 1991). But as far as I know, only a few urban models with endogenous residential structure in the literature both take division of labor into account and endogenously determine incomes of households within a perfectly competitive framework (e.g. Suh 1991; Zhang 1993).

In this study, we classify production into different sectors. In particular, we emphasize the geographical character of services in our modeling. Services are consumed simultaneously as they are produced and thus cannot be transported like commodities. Accordingly, when explicitly modeling economic geography, we have to take account of this special character of services. Many services such as schools, hospitals and restaurants have to be consumed where they are supplied. Accordingly, services have special location property in comparison to commodity production.

When dealing with economic geography, we have to explain how spatial parameters or slow changing variables, such as infrastructures, city culture and climates, may affect attractiveness of the location under consideration. For simplicity of discussion, we measure these various factors, in an aggregated term, by a single variable of urban amenity (e.g. Diamond and Tolley 1981). It is obvious that some location amenities such as pollution level, residential density and transportation congestion are dependent upon economic agents' activities (e. g. Kanemoto 1980; Miyao 1981; Suh 1988), while other amenities such as climates, transportation structure, and historical buildings, may not be strongly affected by economic agents' activities, at least, within a short-run period of time. Accordingly, in a strict sense, it is necessary to classify amenities into endogenous and exogenous amenities when modeling economic geography. Which kinds of amenities should be classified as endogenous or exogenous also depend upon time scale of the analysis and the economic system under consideration (e.g. Zhang 1991).

The paper is organized as follows. Section II defines the two-city and three-sector equilibrium model with endogenous residential structure. Section III provides conditions for the existence of economic equilibria. Section IV examines the impact of changes in amenity of city 1 upon the economic geography. Section V concludes the study. The Appendix solves all the variables in the system in terms of the parameters.

II. The Model

We consider an economic system consisting of two cities, indexed by 1 and 2, respectively. We assume that the spatial pattern of each city is similar to that of the standard urban economic model suggested by Alonso (1964). The basic issue of the Alonso model is to determine urban equilibrium pattern with the assumption that all the households maximize utility levels subject to the exogenously given incomes. Utility levels are dependent upon consumption levels of a composite good and land size for housing. Urban economists have made many efforts to generalize and extend the Alonso model (e.g. Straszheim 1987; Fujita 1989). Unfortunately, most of those models treat incomes as exogenously given parameters.

Similarly to the Alonso model, we assume that each city consists of two parts — the CBD and a residential area; and the locations of the CBDs are pre-specified points and all product activities are concentrated at the CBDs. We feature a linear two-city system on a homogeneous plain whose width is a unit distance (Figure 1). The geographical structure of our model is similar to that proposed by Suh (1988). But as we assume an identical population rather than heterogeneous residents as in the Suh model, we may directly omit the issue of intercity commuting examined by Suh. Also due to the assumption of identical popula-

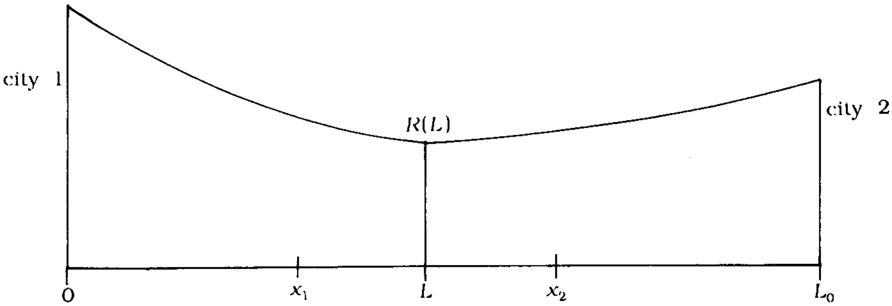

FIGURE 1
THE GEOGRAPHY OF THE TWO-CITY SYSTEM

tion in our study, the geographical structure in our study is simpler than that in the Suh model. Although the fundamental aspects of our study are dealt also by Suh (1988, 1991), from the discussion below it can be seen that we propose a simpler compact framework in which both the division of labor between the two cities and the residential location can be simultaneously and endogenously determined.

In economic geography, different spatial patterns have been discussed (e.g. Beckmann and Puu 1985; Greenhut, Norman, and Hung 1987; Rauch 1991). It has become clear that geographical patterns may make economic analysis extremely complicated. To get some explicit conclusions from our framework with endogenous residential location and incomes, we accept this almost simplest economic geographical arrangement with multiple city centers. It should also be mentioned that a similar spatial structure is suggested by Sivitanidou and Wheaton (1992) and Suh (1988), even though the production aspects of the model are different from the model in this study.

We assume that the system produces two kinds of consumption commodities, indexed by 1 and 2, respectively. We assume that production is characterized by urban specification and that commodity i is produced at CBD i. We neglect transportation costs of commodities. The CBDs are not only centers of economic production but also places for services such as school, hospitals, hotels and restaurants. Services are city-specified in the sense that services are consumed only by the households of the city which supply the services. Here, we neglect the possibility that services supplied in one city are consumed by the households from the other (such as tourism). Accordingly, each city has two production sectors — commodity and service. As we neglect transportation cost of goods, any commodity is sold at the same price in the two cities. But services may have different prices in the two cities as they cannot be transported between the cities.

We assume that labor markets are characterized by perfect competition and that people are freely mobile. For simplicity of analysis, we assume that commodities are produced by only one input factor — labor. We neglect other inputs such as capital and land. We assume a homogeneous labor force and select commodity 1 to serve as numeraire, with all the other prices being measured relative to its price. Although they may be different between the two cities, the wage rates for different sectors in the same city are identical. The assumption of the homogeneous labor force and labor mobility results in identical utility level in the entire system. We introduce

L_0: fixed distance between the CBDs;
L: distance from the CBD 1 to the boundary of the two residential areas;
x_i: dwelling location of city i's residents, $0 \leq x_1 \leq L$ and $x_2 \leq L_0 - L$;
$R(x_i)$: land rent at location x_i, $i = 1, 2$;
N_{ih} and N_{is}: labor force employed by industrial sector and service sector in city i, respectively;
N_i: city i's employment;
N: the total labor force of the system, $N = N_1 + N_2$;
F_{ih} and F_{is}: product of the industrial sector and service sector of the ith city, respectively;
p: price of commodity 2;
p_{is}: prices of services in the ith city;
w_i: wage rate in city i.

We now describe the model.

Supply of Commodities and Services

We specify a linear production function of the ith industrial sector as follows

$$F_{1h} = z_1 N_{1h}, \quad F_{2h} = z_2 N_{2h}, \tag{1}$$

where z_i is city i's production efficiency parameter. As there is only one input factor in each production, we always have $w_1 N_{1h} = F_{1h}$ and $w_2 N_2 = pF_{2h}$, i.e.,

$$w_1 = z_1, \quad w_2 = pz_2. \tag{2}$$

City 1's wage rate is equal to industry 1's product value per labor input; while city 2's wage rate is equal to industry 2's product value per labor input. As the labor markets are perfectly competitive and production functions are linear with a single input, the equations in (2) provide two "accounting relations" of the industries.

Let C_{ij} denote total consumption of city i's industrial product by city j. Then, the balances of demand for and supply of the two commodities are given by

$$C_{11} + C_{21} = F_{1h}, \quad C_{12} + C_{22} = F_{2h}. \tag{3}$$

We assume that services are produced by a single input–labor. We specify production function of the service sectors as follows

$$F_{1s} = zN_{1s}, \quad F_{2s} = zN_{2s}, \tag{4}$$

where z is production efficiency parameter of the service sectors. We assume that service production has identical efficiency between the two cities. We neglect possible difference in quality of service supplies between the two cities. As become clear later on, this assumption can be easily relaxed by introducing $F_{is} = z_{is}N_{is}$ without affecting our main conclusions.

As we assume that the labor markets are perfectly competitive and labor force is homogeneous, the wage rate of the service sector in city i is identical to that of the industrial sector in the same city. As labor is the only input in service production, we have $w_iN_{is} = p_{is}F_{is}$, i.e.,

$$w_1 = zp_{1s}, \quad w_2 = zp_{2s} \tag{5}$$

As services are consumed simultaneously as they are produced we always have

$$C_{is} = F_{is}, \quad i = 1, 2 \tag{6}$$

where C_{is} is consumption of services of city i.

Demand Structure of Households and Residential Structure

We assume that the utility level of an individual from consuming commodities, services and housing can be expressed in the form of

$$U(x_i) = a_i c_1^u c_2^v c_s^\alpha c_h^\beta, \quad 1 > u, v, \alpha, \beta, > 0, \tag{7}$$

where a_i is amenity level of city i, $c_1(x_i)$, $c_2(x_i)$, $c_s(x_i)$, and $c_h(x_i)$, $i = 1, 2$, are respectively, consumption levels of commodity 1, commodity 2, serviceş and housing of a household at location x_i in city i.

As mentioned in the introduction, this study explicitly takes difference in amenity between the two cities into account. Although we admit that there are interactions among amenity, residential location and companies' location, for convenience of discussion we assume that amenity levels in each city are given in this study. It should be remarked that this assumption may be relaxed in different ways (e.g. Kanemoto 1980). The difference in amenity levels between the two cities implies that even if the components of consumption are identical among the households in the two cities, they may have different utility levels.

The consumer problem is defined by

$$\begin{aligned} &\max U(x_i) \\ &\text{s.t } c_1(x_i) + pc_2(x_i) + p_{is}c_s(x_i) + Rc_h(x_i) = y(x_i), \end{aligned} \tag{8}$$

in which $y(x_1) \equiv w_1 - \tau x_1, \; 0 \leq x_1 \leq L$
$y(x_2) \equiv w_2 - \tau(L_0 - x_2), \; L < x_2 \leq L_0.$

Here, τ is a travel cost per unity of distance and τx_i is the total travelling cost between dwelling site x_i and the CBD i.

The consumer problem has a unique optimal solution as follows

$$\begin{aligned} &c_1(x_i) = usy(x_i), \; c_2(x_i) = \frac{vsy(x_i)}{p}, \; c_s(x_i) = \frac{\alpha sy(x_i)}{p_{is}}, \\ &c_h(x_i) = \frac{\beta sy(x_i)}{R(x_i)}, \; i = 1, \; 2, \end{aligned} \tag{9}$$

in which $s \equiv 1/(u + v + \alpha + \beta)$. Here, we assume that w_i, L_0 and τ are taken on appropriate values so that any point in the interval $[0, L_0]$ is occupied by some resident.

We denote $n(x_i)$ the residential density at dwelling site x_i. According to the definitions, we have

$$\begin{aligned} &n(x_i) = \frac{1}{c_h(x_1)}, \; 0 \leq x_1 \leq L, \\ &n(x_2) = \frac{1}{c_h(x_2)}, \; L < x_2 \leq L_0. \end{aligned} \tag{10}$$

We have the following constraints upon the population distribution between the two urban areas

$$\begin{aligned} &\int_0^L n(x_1)dx_1 = N_1, \; \int_L^{L_0} n(x_2)dx_2 = N_2, \\ &N_i = N_{ih} + N_{is}, \; i = 1,2, \; N_1 + N_2 = N. \end{aligned} \tag{11}$$

The constraints (3) and (6) can be expressed in the following forms

$$\begin{aligned} &\int_0^L n(x_1)c_{11}(x_1)\ dx_1 + \int_L^{L_0} n(x_2)c_{21}(x_2)\ dx_2 = F_{1h}, \\ &\int_0^L n(x_1)c_{12}(x_1)\ dx_1 + \int_L^{L_0} n(x_2)c_{22}(x_2)\ dx_2 = F_{2h}, \\ &\int_0^L n(x_1)c_s(x_1)\ dx_1 = F_{is},\ \int_L^{L_0} n_2(x_2)c_s(x_2)\ dx_2 = F_{2s}. \end{aligned} \tag{12}$$

As we assume an identical population, we see that the equilibrium condition of the residential conditions is given by

$$U(x_i) = U(x_j),\ \text{for any } x_i,\ x_j \in [0,\ L_0] \tag{13}$$

That is, the utility level is identical, irrespective of location in the system. It is obvious that the land rent at the boundary between the two cities must be equal, i.e.,

$$R(x_1) = R(x_2) = R(L),\ \text{when } x_1 \text{ and } x_2 \to L \tag{14}$$

We have thus completed constructing the equilibrium model. The system consists of 23 variables, N_{ih}, N_{is}, N_i, F_{ih}, F_{is}, w_i, $p_{is}(i = 1,2)$, c_1, c_2, c_s, c_h, n, R, p, L, U. It also contains the same number of independent equations. We now show that these variables can be explicitly expressed as functions of the parameters, z_1, z_2, a_1, a_2, z, and L_0.

III. Existence of Equilibria

This section shows that the 23 endogenous variables can be expressed as functions of the parameters in the system. First, we note that utility level for any household must be identical in the interval $[0,\ L_0]$. Substituting (9) into $U(x_i) = U(L)$ in (13) yields

$$\frac{R(x_i)}{R(L)} = \left[\frac{y(x_i)}{y_i(L)}\right]^m > 1,\ i = 1,2, \tag{15}$$

where we use (14),

$$m \equiv \frac{1}{\beta s} > 1,\ y_1(L) = w_1 - \tau L,\ y_2(L) = w_2 - \tau(L_0 - L).$$

As $y(x_i)/y(L) > 1$, $R(x_i)/R(L) > 1$. The equations in (15) state that land rent in each urban area declines as the residential location is further away from its CBD. As the workers employed in a city have the identical wage rate and travelling cost is positively proportional to the distance, housing price will decline as the workers travel further from their dwelling sites to the working place.

***Lemma* 1**

The land rent of a urban area declines as the distance from the CBD is increased. That is, $dR/dx_1 < 0$, for $0 \leq x_1 < L$ and $-dR/dx_2 < 0$, for $L \leq x_2 \leq L_0$.

Substituting (9) into the relation of $U(x_1) = U(x_2)$ at $x_1 = x_2 = L$ yields

$$\frac{p_{1s}}{p_{2s}} = \left(\frac{a_1}{a_2}\right)^{\frac{1}{\alpha}} \left\{\frac{y_1(L)}{y_2(L)}\right\}^{\frac{1}{\alpha s}}. \tag{16}$$

Service prices are different between the two cities when urban amenities and wages are different between the two cities. In the case of $y_1(L) = y_2(L)$, if city 1's amenity is higher than that in city 2, city 1's service price is higher than that in city 2, and vice versa. From $y_1(L) = w_1 - \tau L$ and $y_2(L) = w_2 - \tau(L_0 - L)$, we note that even if city 1's amenity and wage rate are higher than those in city 2, city l's service price is not necessarily higher than city 2's service price when $L > L_0/2$ and the transportation cost per unity of distance is high.

Substituting (9) into $U(0) = U(L)$ together with (16) yields

$$\frac{R(0)}{R(L_0)} = \left\{\frac{w_1 y_2(L)}{w_2 y_1(L)}\right\}^m. \tag{17}$$

The land rents between the residential areas at the CBDs will be different if $w_1 = w_2$ and $L = L_0/2$ are not held.

The following proposition which provides conditions for existence of equilibria is proved in the Appendix.

***Proposition* 1**

The equilibrium price, p, of commodity 2 is determined by

$$M(p) = 0, \ \ 0 < g(p) < L_0, \tag{18}$$

in which M and $g(p)$ are continuous functions of p with M defined in (A11) and $g(p)$ in (20). Moreover, for any positive equilibrium price, p, the other variables are uniquely determined as functions of p. In particular, the service prices and the boundary between the two cities are given by

$$p_{1s} = \frac{z_1}{z}, \ \ p_{2s} = \frac{z_2 p}{z}, \tag{19}$$

$$L = \frac{(z_1 p^{\alpha s} - a z_2 p + a\tau L_0)}{\tau(p^{\alpha s} + a)} \equiv g(p), \tag{20}$$

in which $a \equiv \{(a_2/a_1)(z_1/z_2)^{\alpha}\}^s$.

From (A11) we see that $M(p) = 0$ is a very complicated function. We discuss under what conditions the equation may have solutions in the Appendix. It should be remarked that we will not examine the conditions in detail as they are too difficult to explicitly interpret. The condition, $p_{1s} = z_1/z$, is derived from the condition of perfect competition in city 1's labor market. As labor inputs the only inputs in production and the production functions take on the linear forms, the equality of the wage rates in city 1's two sectors guarantee the condition. We see that city 1's service price is constant (in the term of the city's industrial product) as z_1 and z are fixed parameters. We can similarly interpret $p_{2s}/p = z_2/z$. The equation (20) determines the boundary between the two cities as a function of the price of city 2's industrial product. In the Appendix, we show how all the other variables can be uniquely determined as functions of p.

IV. The Impact of Amenities upon the Economic Geography

This section examines the impact of changes in city 1's amenity, a_1, upon the economic geography. First, from (19) we have

$$\frac{dp_{1s}}{da_1} = 0, \quad \frac{dp_{2s}}{da_1} = \left(\frac{z_2}{z}\right)p^*, \tag{21}$$

where $p^* \equiv dp/da_1$. Here, we can get p^* directly by taking derivative of (18) with respect to a_1. The price of services in city 1 is not affected by changes in city 1's amenity. The impact of changes in city 1's amenity upon the price of services in city 2 has the same sign as that upon the price of commodity 2. Here, we will not explicitly represent p^* as it is too complicated to explicitly interpret. It can be seen that the sign of p^* may be either positive or negative, depending upon combination of the parameter values in the entire system. As city 1 becomes more attractive, more residents tend to migrate from city 2 to city 1 (with other conditions fixed). As the number of residents is increased, city 1's services and land rent tend to be increased, which may imply a decrease in city 2's product price. On the other hand, the increased labor force in city 1 also implies a possible reduction of the wage rate in city 1 which will reduce the price of city 1's product (i.e. increasing the price of city 2's product). We see that whether p^* is positive or negative depends on various forces in the system. For convenience of discussion, we assume that $p^* < 0$ in the remainder of this section. This requirement implies that the increase of city 1's amenity will reduce the price level of city 2's product.

Taking derivatives of (20) with respect to a_1 yields

$$\tau(p^{as}+a)\frac{dL}{da_1}=\{\alpha sp^{as}y_1(L)-aw_2\}(\frac{p^*}{p})+\frac{as(w_2-\tau L_0)}{a_1}+\frac{as\tau L}{a_1}, \tag{22}$$

in which we require $w_2 - \tau L_0 > 0$. We see that if the price of city 2's product is not much affected, i.e., $|p^*/p|$ being small, then $dL/da_1 > 0$. If $|p^*/p|$ is small, so is $|(dp_{2s}/da)/p_{2s}|$. Since the prices are little affected, we see that more people will move to city 1. Accordingly, city 1's urban area will be expanded. From (16) and (19) we have $p^{as}y_1(L) = ay_2$. The term, $\{\alpha sp^{as}y_1(L) - aw_2\}$, in (22) is equal to $a(\alpha sy_2(L) - w_2)$. As $\alpha s < 1$ and $y_2(L) < w_2$, we see that $\{\alpha sp^{as}y_1(L) - aw_2\} < 0$. We thus can conclude that under the requirement of $p^* < 0$, dL/da_1 is always positive. That is, as city 1's amenity is improved, some city 2's urban area will be "eaten" up by city 1. As city 1's improved environment increases its residents income (in comparison to city 2's), city 1's land rent tends to be increased. This further implies that the residents from city 1 will offer higher land rent than those from city 2 at boundary. As the land market is competitive. we see that the competition will move some city 2's residents away from its original area near the boundary. The process will continue until the new equilibrium is achieved.

From (A14) we directly have the following impact upon the land rent at the boundary

$$\frac{dR(L)}{da_1}=\tau N\frac{dN_0}{da_1}, \tag{23}$$

in which $$\frac{dN_0}{da_1}=-N_0^2[\{\frac{\tau mz_1^m}{y_1(L)^{m+1}}\}\frac{dL}{da_1}+\tau m\{\frac{(pz_2)}{y_2(L)}\}^m\frac{\{-\frac{Lp^*}{p}+\frac{dL}{da_1}\}}{y_2(L)}]. \tag{24}$$

We see that $dR(L)/da_1 > 0$. When the price of commodity 2 is reduced, the land rent at the original boundary is increased. As city 1 "eats up" city 2's urban area, it is expectable for the land rent at the old boundary to become higher.

From (A13), we have the following impact upon the population distribution between the two cities

$$\frac{dN_1}{da_1}=\left(\frac{N_1}{N_0}\right)\frac{dN_0}{da_1}+\tau\left\{\frac{mz_1^m N_0 N}{y_1(L)^{m+1}}\right\}\frac{dL}{da_1},$$

$$\frac{dN_2}{da_1}=-\frac{dN_1}{da_1}. \tag{25}$$

We see that city 1's employment is increased and city 2's employment is reduced. As city 1's amenity is improved, some people will migrate to city 1 from city 2 until the utility level in the two cities become identical.

From (A9), (A10), (A15), (9) and the above analytical results we can directly provide the impact of changes in a_1 upon the variables, $R(x_i)$, N_{ih}, N_{is}, $c_1(x_i)$, $c_2(x_i)$, $c_s(x_i)$ and $c_h(x_i)$, i = 1,2. We will not represent the results here.

We now compare how the consumption components of the households in the two cities are affected by changes in city 1's amenity. For simplicity, we just compare the consumption components of the households at the two CBDs. From $y(0) = w_1 = a_1$, $y(L_0) = w_2 = z_2p$ and (9) we have

$$\frac{c_1(0)}{c_2(L_0)}=\frac{z_1}{z_2p},\quad \frac{c_2(0)}{c_2(L_0)}=\frac{z_1}{z_2p},\quad \frac{c_s(0)}{c_s(L_0)}=1. \tag{26}$$

Taking derivatives of (26) with respect to a_1 yields

$$\frac{d\left\{\frac{c_1(0)}{c_2(L_0)}\right\}}{da_1}=\frac{d\left\{\frac{c_2(0)}{c_2(L_0)}\right\}}{da_1}=-\frac{z_1p^*}{z_2p^2}>0,\quad \frac{d\left\{\frac{c_s(0)}{c_s(L_0)}\right\}}{da_1}=0. \tag{27}$$

As $c_s(0)/c_s(L_0)$ is constant, we obviously have that the ratio of service consumption per household between the households at the CBD 1 and at the CBD 2 is not affected by changes in city 1's amenity. From the first equation in (27), we see that city 1's consumption levels of the two commodities are increased in comparison to these in city 2 at the CBDs. As city 1's wage rate is increased and land rent is increased (in comparison to those in city 2), it is reasonable to expect the increases in consumption of the commodities. From the condition $U(0)/U(L_0)$ = 1 and (27) we can directly have that $d\{c_h(0)/c_h(L_0)\}/da_1 > 0$. As city 1 consumes more commodities than city 2 and (relative) services consumption is not affected by improved amenity in city 1, it is reasonable to have that housing condition at city 2's CBD is improved in comparison at city 1's CBD. The population is identical in the system, improvement in some aspects will mean disadvantages in some others, otherwise free migration will result in the concentration of the population in a single city.

V. Concluding Remarks

We proposed a two-center model with endogenous incomes and output levels. We assumed that each city specifies in producing a single commodity which is consumed by the entire system. Services are city-oriented in the sense that the services supplied in one city are only consumed by the residents from the same city. The economic geography is similar to that in the Suh's two-center model. We provided the conditions for the existence of equilibria with the division of labor between the two cities under the free and costless migration institution. The model showed that wages, service prices, consumption components and land rent are different between the two cities at the perfectly competitive equilibrium. We also examined the impact of changes in city 1's amenity level upon the economic geography. It can be seen that the complicated interdependence among various forces which affect the economic geography of the two-city system makes it very difficult to explicitly judge possible effects of changes in city's amenity and technology.

We discussed the problem under some strict assumptions. For instance, the linear spatial pattern is too simple. We also omit important inputs such as capital and land in the production functions. The possible economic structures of the CBDs are more complicated than the residential areas. We may obviously extend the model by using more general utility functions. We may also extend our model into a dynamic frameworks with endogenous capital and knowledge accumulation (e.g. Zhang 1992a, 1992b).

Appendix

Proving Proposition 1

We now show how to get each equation in Proposition 1 and solve all the other variables as functions of the parameters in the system.

First, from (9) and $R(x_i) = R(L)[y(x_i)/y_i(L)]^m$, we get

$$n(x_i) = y_i(x_i)^{m-1} Y_i(L), \tag{A1}$$

where $Y_i(L) \equiv mR(L)y_i(L)^{-m}$. Substituting (9) and (A1) into (11) and (12) then integrating the formula yield

$$w_1^m - y_1(L)^m = \tau m N_1/Y_1,\ w_2^m - y_2(L)^m = \tau m N_2/Y_2, \tag{A2}$$

$$Y_1^* + Y_2^* = (m+1)F_{1h}/us,\ Y_1^* + Y_2^* = p(m+1)F_{2s}/vs, \tag{A3}$$

$$Y_1^* = p_{1s}(m+1)F_{1s}/\alpha s,\ Y_2^* = p_{2s}(m+1)F_{2s}/\alpha s, \tag{A4}$$

where $Y_1^* \equiv \{w_1^{m+1} - y_1(L)^{m+1}\}Y_1(L),\ Y_2^* \equiv \{w_2^{m+1} - y_2(L)^{m+1}\}Y_2(L)$.

From (1) and (A3) we directly have

$$p = \frac{vz_1N_{1h}}{uz_2N_{2h}}. \tag{A5}$$

From (2) and (5) we have (19). From (16) and (19) we have (20).

Substituting (A4) into $Y_1^* + Y_2^* = (m+1)F_{1h}/us$ together with (A5)

$$z_1^2N_{1s} + pz_2^2N_{2s} = \frac{\alpha z z_1 N_{1h}}{u}. \tag{A6}$$

Substituting $Y_i = mR(L)y_i(L)^{-m}$ and Y_i^* into the two equations in (A4) and then substituting $R(L)$ obtained from one equation to the other one yield

$$\frac{p^{1-\frac{\alpha}{\beta}}N_{1s}}{N_{2s}} = \left(\frac{a_1}{a_2}\right)^{\frac{1}{\beta}}\left(\frac{z_1}{z_2}\right)\frac{\{z_1^{m+1} - y_1(L)^{m+1}\}}{\{w_2^{m+1} - y_2(L)^{m+1}\}} \equiv f(p), \tag{A7}$$

in which we use (A6) and

$$\left\{\frac{y_1(L)}{y_2(L)}\right\}^m = \left(\frac{a_2}{a_1}\right)^{\frac{1}{\beta}}\left(\frac{p_{1s}}{p_{2s}}\right)^{\frac{\alpha}{\beta}},$$

which is obtained from (16). Using $L = g(p)$, we can express (A7) as a function of p. Substituting (A7) into (A6) yields

$$\frac{N_{1h}}{N_{1s}} = \frac{u\{z_1^2f + p^{2-\frac{\alpha}{\beta}}z_2^2\}}{\alpha z z_1 f}. \tag{A8}$$

From $N_{1h} + N_{1s} = N_1$ and (A8) we have

$$N_{1h} = \frac{u\{z_1^2f + p^{2-\frac{\alpha}{\beta}}z_2^2\}N_1}{\{up^{2-\frac{\alpha}{\beta}}z_2^2 + (uz_1 + \alpha z)z_1 f\}},$$

$$N_{1s} = \frac{azz_1 N_1 f}{\{up^{2-\frac{\alpha}{\beta}} z_2^2 + (uz_1 + \alpha z) z_1 f\}}. \tag{A9}$$

From $N_{2s} + N_{2h} = N_2$, (A5) and (A9) we directly have

$$N_{2h} = \frac{\upsilon z_1 N_{1h}}{puz_2}, \quad N_{2s} = N_2 - \frac{\upsilon z_1 N_{1h}}{puz_2}. \tag{A10}$$

It is easy to check that from $w_1{}^m - y_1(L)^m = \tau m N_1/Y_1$ in (A2) and $\{w_1{}^{m+1} - y_1(L)^{m+1}\}Y_1(L) = p_{1s}(m+1)F_{1s}/\alpha s$ in (A4) we have

$$\frac{(m+1)z_1^2 N_{1s}}{\alpha sz\{w_1^{m+1} - y_1(L)^{m+1}\}} = \frac{\tau m N_1}{\{w_1^m - y_1(L)^m\}}.$$

Substituting the equation for N_{1s} in (A9) into this equation yields

$$M(p) \equiv f_1 - \{z_1{}^{m+1} - y_1(p)^{m+1}\}f_2 = 0, \tag{A11}$$

where $f_1(p) \equiv (m+1)z_1{}^3 f/\{up^{2-\alpha/\beta}z_2{}^2 + (uz_1 + \alpha z)z_1 f\}$,
$f_2(p) \equiv s\tau m/\{z_1{}^m - y_1(p)^m\}$.

As $0 < L < L_0$, we see that p has to satisfy $0 < g(p) < L_0$ where $g(p)$ is defined in (19).

Substituting $Y_i(L) = mR(L)y_i(L)^{-m}$ into (A2) yields

$$\begin{aligned} \{\frac{z_1}{y_1(p)}\}^m - 1 &= \frac{\tau N_1}{R(L)}, \\ \{\frac{pz_2}{y_2(p)}\}^m - 1 &= \frac{\tau N_2}{R(L)}. \end{aligned} \tag{A12}$$

From the two equations in (A2) and $N_1 + N_2 = N$ we obtain

$$\begin{aligned} N_1 &= [\{z_1/y_1(p)\}^m - 1]N_0(p)N, \\ N_2 &= [\{(pz_2)/y_2(p)\}^m - 1]N_0(p)N \end{aligned} \tag{A13}$$

where $N_0(p) \equiv 1/[\{z_1/y_1(p)\}^m + \{(pz_2)/y_2(p)\}^m - 2]$.

From the first equation in (A12) we directly have

$$R(L) = \tau N_0(p)N. \tag{A14}$$

By (15) we have

$$R(x_1) = [\frac{y_i(x_i)}{y_i(p)}]^m R(L), \quad i = 1, 2, \tag{A15}$$

in which y_i (x_i) and $R(L)$ are functions of p.

We now see how the 23 endogenous variables, N_{ih}, N_{is} N_i, F_{ih}, F_{is}, w_i, p_{is} $(i = 1,2)$, c_1, c_2, c_s, c_h, n, R, p, L, U, can be expressed as functions of the parameters. In Proposition 1 we solve p, p_{is} and L. The wage rates, w_i, are given by (2), the urban population by (A13), the labor division, N_{ih} and N_{is}, by (A9) and (A10), the output of the industrial sectors, F_{ih}, by (1), the output of the service sectors, F_{is}, by (4), the land rent at the boundary, $R(L)$, by (A14), the land rent distribution, $R_i(x_i)$, by (A15), the consumption components at any location, c_1, c_2, c_s, c_h, by (9), the utility level, U, by (7), the residential density at any location, n, by (10). We have thus proved Proposition 1.

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