The New Product Choice of an Innovating Country

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We analyze a North-South product-cycle model where the innovating North can choose both the rate of product innovation and the allocation of new products between two multi-commodity sectors, one in which it has a comparative advantage and the other in which the South has a comparative advantage. The relative wage of the North is an increasing function of its penetration in the Southern sector. If the degree of comparative advantage is below a critical point, the North maximizes its welfare by invading the Southern sector, resulting in global inefficiency. However, with a licensing agreement, Pareto improvement can be made through bargaining between the two countries.(*JEL* Classification. F13, O32.)

I. Introduction

Krugman's formulation (1979) of the product-cycle model gave rise to a literature analyzing the effects of product innovation and technology transfer on the pattern of trade and the world distribution of income: Dollar (1986, 1987), Jensen and Thursby (1986, 1987), Flam and Helpman (1987), Grossman and Helpman (1989), Segerstrom, Anant, and Dinopolous (1990), Stokey (1991). One issue which has been sidestepped by this literature is the choice of the new product mix by an innovating country. We will treat this problem systematically in the present paper. The lack of previous such research is notable in view of the recent debates on "targeting" and "industrial policy".[1]

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[1]See, for example, Krugman and Obstfeld (1991).

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Since innovation generates externalities, both across individual economic units and across time, the competitive market mechanism can not be relied upon to generate an efficient outcome. To mitigate this problem, a modern government typically intervenes in the market process both by setting up a patent system and by directly supporting research and development. The literature on product innovation has mostly focused on private initiatives in innovation. We will deal with public initiatives in the present paper. Our approach provides a theoretical benchmark in understanding models of privately financed innovation, since an optimum can be achieved through publicly financed innovation. Moreover, the approach also has some practical importance in that a substantial fraction of the world's R&D activities are in fact financed publicly. In the context of a closed economy, Shell (1967) studied a growth model with the productive and inventive sectors. The inventive sector produces technical knowledge which enters the production function of the productive sector. In his model, the revenue from an excise tax is used to pay for factors employed in the inventive sector. In the present paper, we will consider a two-country model where the government of an innovating country finances innovation activities by an income tax.

A key concept we use in analyzing the product-choice issue is the *external bias* of the new product menu. It is defined as the extent to which an innovating country's new demand is drawn from foreign rather than domestic products, and can be measured in principle by a comparative statics ratio,

$$\frac{\text{(decrease in share of foreign products in world expenditure)}}{\text{(share of new products in world expenditure)}}.$$

External bias can be an important factor shaping the impact of product innovation on trade and the distribution of world income. In particular, it would seem that the more a country's new product menu succeeds in raising the demand for its products, the more favorable will be the resulting factoral terms of trade effects.

Furthermore, new products undoubtedly *differ* in their external bias characteristics. For example, nylon, which was introduced by DuPont in the 1930s, was heavily biased against foreign fibers, mainly Asian silk. Black and white television, on the other hand, appears to have gained its market in the late 1940s and 1950s largely at the expense of radio and the movies, primarily domestic products for the U.S.. Everything else being equal, it would be to a country's advantage to introduce commodities more strongly biased against foreign-produced goods. And even though a country suffers some long-run comparative disadvantage in producing a new commodity, the terms of trade gains from external bias may more than compensate for the higher production costs.

To introduce *differential* external bias we need to specify both demand and production asymmetries in the product-cycle model. Each asymmetry complicates the analysis considerably. To keep things tractable, we work within the simplest North-South framework, Krugman's (1979) Ricardian-technology product-cycle model, with the following modifications. First, for the utility functions we assume two commodity groups, both represented by identical symmetric CES utility functions with substitution elasticities greater than one.[2] These, in turn, are nested in a symmetric Cobb-Douglas utility function, In this specification, a new product is a closer substitute for commodities in the group to which it is added than for those of the other.

Second, it seems natural to make comparative advantage, oddly excluded from previous Ricardian product cycle models, the source of production asymmetries. We assume that each country enjoys a comparative advantage in one of the two CES commodity groups. Thus, without innovation, each country specializes according to comparative advantage.[3] With innovation, the innovating country has the option of introducing commodities in which it has a comparative advantage. However, by choosing commodities from the other group instead, it will deflect demand disproportionately from products in which the other country is specialized, thereby enhancing the innovator's terms of trade.

Third, to bring out the potential for national gain, we assume that the Northern government can choose both the level of innovation activity and the composition of new products so as to maximize the welfare of their households. In the absence of the coordinating government, profit maximizing firms would allocate new products differently between the two groups and fail to reap the full terms of trade gains.[4]

Fourth, in one version of the model we allow the immediate licensing

[2]This is not done in the spirit of product differentiation and varieties approach to economies of scale. Rather, it is used only as a convenient if oversimplified way of introducing demand asymmetries.

[3]In Krugman-type models the absence of new products removes the only production differences between the two countries.

[4]Note that higher wages represent higher costs, not higher profits for such firms.

of new products in the context of a bargaining game between the two governments. Endogenizing technology transfer in this manner permits the North to exploit the differential external bias without having actually to produce commodities in which it has a comparative disadvantage. The North's credible threat that, without agreement, it would indeed produce such commodities induces the South to pay royalties so that it might produce them instead. Rodríguez (1975) and Feenstra and Judd (1982) have analyzed in rather different contexts how the innovating country could use instruments such as import and export duties and/or taxes on technology transfer to extract more surplus from technological monopoly. They agree that licencing combined with sufficiently large lump-sum royalty payments would be the most effective option. Both, however, leave open the question of how such a policy could be implemented. More generally, their treatments of the various instruments assume a basic asymmetry: the non-innovating country remains passive throughout. In our bargaining approach, each country is active and has a fair chance of suggesting the terms of the final agreement.

The paper proceeds as follows: In the next section we set up a dynamic North-South model and study its temporal equilibrium. We then turn to two dynamic scenarios regarding technology transfer in sections III and IV, and analyze their steady state equilibria. The first excludes product licensing, and the second permits the immediate licensing of new products to the South. Some concluding remarks are given in the last section.

II. The North-South Model

In this section, we will set up a North-South product-cycle model where the innovating North can choose both the rate of product innovation and the allocation of new products between two multi-commodity sectors, one in which it has a comparative advantage and the other in which the South has a comparative advantage. We will also solve the model for its temporal equilibrium in this section in preparation for analyzing the dynamic equilibria in the next two sections.

A. The Specification of the Model

There are two countries, the North and South, denoted by N and S, respectively. At each time point $t \in [0, \infty)$, the population of each country consisits of a continuum of homogeneous households and is normalized to be of measure 1. At each t, each living household is endowed with one unit of labor, which is the only productiove input. There are two commodity groups I, J. The comparative advantages of countries N and S lie in groups I and J, respectively. Each country requires 1 unit of labor to produce 1 unit of a commodity in its comparative advantage group and $a(> 1)$ units to produce 1 unit of a good in the other group.

All commodities are produced under conditions of perfect competition and free trade. Perfect competition, combined with the constant returns to scale technology, dictates zero profits in equilibrium. Letting the numeraire be the South's labor, $w(t)$ the North's current wage, and $p(t)$ the current commodity-price vector, one has $p_i(t) = w(t)$ (a, respectively) if commodity $i \in I$ is produced by the North (the South, respectively), and $p_j(t) = 1$ ($aw(t)$, respectively) if commodity $j \in J$ is produced by the South (the North, respectively).

Households of both countries have the same instantaneous utility function. Denoting the current consumption vectors of the North and South by $x(t)$ and $x^*(t)$, the Northern and Southern instantaneous utility functions are, respectively,

$$u(x(t)) = [\{\sum_{i \in I} x_i(t)^\rho\}^{\frac{1}{\rho}}]^{\frac{1}{2}} [\{\sum_{j \in J} x_j(t)^\rho\}^{\frac{1}{\rho}}]^{\frac{1}{2}},$$

$$u(x^*(t)) = [\{\sum_{i \in I} x_i^*(t)^\rho\}^{\frac{1}{\rho}}]^{\frac{1}{2}} [\{\sum_{j \in J} x_j^*(t)^\rho\}^{\frac{1}{\rho}}]^{\frac{1}{2}},$$

where $0 < \rho < 1$.

We assume that there are no savings instruments, financial or physical, so that households spend all of their disposable incomes to maximize their current utilities at each t. A Northern household is subject to an income tax of rate $v(t)$ so that it maximizes its current utility subject to the budget constraint $p(t)x(t) \leq \{1 - v(t)\}w(t)$. A Southern household is not subject to any taxes so that it maximizes its current utility subject to the budget constraint $p(t)x^*(t) \leq 1$.

The North introduces new commodities at each t. The Northern government finances research efforts for product innovation by income taxes in the North. The technical knowledge for developing the products are then made freely available to Northern firms. Thus the economy can be divided into the productive sector and the inventive sector. And the amounts of labor used for the production and innovation activities are, respectively, $1 - v(t)$ and $v(t)$. Let $n(t)$ be the total number of commodities at time point t. Its growth rate is determined by the amount of labor put into R&D. For simplicity of analysis, we assume $\dot{n}(t) = \{\alpha v(t) + \beta\}n(t)$ where $\alpha > 0$, $\beta \geq 0$.[5] According to this specification, labor used for R&D is more productive as α is greater. The presence of β indicates that there may be other sources of growth than the government financed R&D. When new commodities are introduced, they are immediately producible at minimum cost by any firm in the North.[6] The government of the North also chooses the fractions of the new commodities in the two commodity groups. The objective of the Northern government is to maximize the intertemporal utility function.

$$\int_0^\infty e^{-\delta t}u(x(t))dt.$$

For this integral to be well defined, it is assumed that $\delta > (\alpha + \beta)/(\sigma - 1)$ where $\sigma = 1/(1 - \rho)$.

The South does not introduce new commodities. Its firms can, however, transfer $(1/2)\dot{n}(t)$ additional commodities from the North at each t. Here the automatic transfer rate 1/2 is not *ad hoc* but deliberately chosen to highlight the role differential external bias plays in the North's new-product policy. Note that 1/2 is the expenditure share parameter for the Cobb-Douglas utility function over the two commodity groups. Thus, in order to maximize the symmetric utility function, the North will try to balance commodities in both groups by maintaining one half of total products in group J, quite apart from its motive to exploit differential external bias. Thus, with the automatic transfer rate of 1/2, a spread between 1/2 and the fraction of commodities in either group is attributable to the North's exploitation of differential external bias.

Let $I(t)$ be the number of commodities in commodity group I at time t, and let $J_N(t)$ and $J_S(t)$ be the numbers of commodities in group J produced by the North and South, respectively. Then, obviously, $I(t) + J_N(t) + J_S(t) = n(t)$. Assume $J_N(0) = 0$ and $I(0) = J_S(0)$. That is, each country initially specializes according to comparative advantage, and the two commodity groups have equal numbers of commodities. The utility maximizing behavior of the Northern government implies $I(t) \leq J_N(t) +$

[5]Note that the number of commodities, $n(t)$, may not be an integer. Technically, we may assume that the set of commodities is a continuum. We chose not to assume this in the beginning in an effort to save readers unnecessary technical distractions.

[6]A standard feature also of the Krugman (1979), Dollar (1986, 1987), and Flam and Helpman (1987) models.

$J_S(t)$ for any t.[7] The profit maximizing behavior of the Southern firms implies that the transferred commodities belong to group J as long as the wage rate of the North is below its relative labor productivity advantage in group I, that is, $w(t) < a$, which we will assume to be the case along the equilibrium path. Given the initial condition $J_S(0) = (1/2)n(0)$, one has $J_S(t) = (1/2)n(t)$ for any t.

Since $\dot{I}(t)/\dot{n}(t) + \dot{J}_N(t)/\dot{n}(t) + \dot{J}_S(t)/\dot{n}(t) = 1$ and $\dot{J}_S(t)/\dot{n}(t) = 1/2$, one has $\dot{J}_N(t)/\dot{n}(t) = 1/2 - \dot{I}(t)/\dot{n}(t)$. Therefore, the Northern government's choosing $\dot{I}(t)/\dot{n}(t)$, the fraction of new commodities in group I, is equivalent to choosing $\dot{J}_N(t)/\dot{n}(t)$, the fraction of new commodities which are in group J but produced by the North. Let $\mu(t) = \dot{J}_N(t)/\dot{n}(t)$. We will take $\mu(t)$ (rather than $\dot{I}(t)/\dot{n}(t)$) as the policy variable of the Northern government.

B. The Temporal Equilibrium of the Model

We will first solve the model for the equilibrium wage $w(t)$ given $J_N(t)$ and $J_S(t)$, and $v(t)$at any t. Since the intra-group utility function is a symmetric CES function,

$$\frac{x_i(t)}{x_{i'}(t)} = \frac{x_i^*(t)}{x_{i'}^*(t)} = \{\frac{P_{i'}(t)}{p_i(t)}\}^\sigma \quad \text{for any } i,\ i' \in I,$$

$$\frac{x_j(t)}{x_{j'}(t)} = \frac{x_j^*(t)}{x_{j'}^*(t)} = \{\frac{P_{j'}(t)}{p_j(t)}\}^\sigma \quad \text{for any } j,\ j' \in J.$$

Since the inter-group utility funct'on is a symmetric Cobb-Douglas function, one half of world disposable income, $(1/2)[w(t)\{1 - v(t)\} + 1]$, is spent on each commodity group. The North's revenue is

$$\frac{1}{2}[\{1-v(t)\}w(t)+1]+\frac{1}{2}[\{1-v(t)\}w(t)+1]\frac{J_N(t)aw(t)}{J_N(t)aw(t)+J_S(t)\{aw(t)\}^\sigma},$$

which should equal North's wage bill in the production sector $w(t)\{1 - v(t)\}$ in equilibrium, so that

$$\{aw(t)\}^{\sigma-1}[\{1 - v(t)\}w(t) - 1] = 2m(t), \tag{1}$$

where $m(t) = J_N(t)/J_S(t)$, the North's *invasion* in the Southern commodity group.

It is a simple matter to find a graphical solution $w(t)$ to equation (1).

[7]The North will never gain by making $I(t) > J_N(t) + J_S(t)$. It would lose from the commodity imbalance and gain nothing in its terms of trade.

Since the left hand side increases monotonically and continuously from $-a^{\sigma-1}v(t)$ to $a^{2(\sigma-1)}[a\{1-v(t)\}-1]$ as w increases from 1 to a, a unique solution $w(t)$ between 1 and a exists as long as $a^{2(\sigma-1)}[a\{1-v(t)\}-1] > 2m(t)$.

Equation (1) shows how the factoral terms of trade, $w(t)$, varies as the commodity ratio $m(t)$ varies. A higher $w(t)$ corresponds to a greater $m(t)$. That is, the greater the North's invasion in the South's group, the higher the North's relative wage. Merely adding commodities to a country's production set does not raise the relative wage, as it would in the Krugman model (1979). Rather, it is the *allocation* of commodities between the two groups that influences relative wages.

In the absence of invasion, the North's income will be the same as the South's, for one half of the world income is spent on the Northern commodity group. Even though the North expends some part of its labor resource on R&D, the favorable factoral terms of trade compensates it exactly so that the two countries have the same wage bill. Indeed, one can see easily from equation (1) that if $m(t) = 0$ then $w(t) = 1/\{1-v(t)\}$, i.e., $w(t)\{1-v(t)\} = 1$. In this sense, the research burden is "automatically" shared by the two countries in the present model.

The competitive equilibrium at t can be parametrized by $m(t)$, $n(t)$, and $v(t)$. Let us denote the instantaneous utility of a household in the North at the competitive equilibrium by $U(m(t), n(t), v(t))$ and that of a household in the South by $U^*(m(t), n(t), v(t))$. Letting $r = \sigma - 1$, one has

$$U(m(t),n(t),v(t)) = \frac{1}{2}w(t)\{1-v(t)\}[\{I(t)w(t)^{-r}\}\{J_N(t)a^{-r}w(t)^{-r}+J_S(t)\}]^{\frac{1}{2r}}$$

$$=(\frac{1}{2})^{\frac{1}{r}+1}a^{-\frac{1}{2}}\{1-v(t)\}n(t)^{\frac{1}{r}}[\{1-m(t)\}\{m(t)+a^r w(t)^r\}]^{\frac{1}{2r}},$$

$$U^*(m(t),\ n(t),\ v(t)) = [w(t)\{1-v(t)\}]^{-1}U(m(t),\ n(t),\ v(t)),$$

where $w(t)$ is the solution to equation (1), from the homotheticity of the utility function and the fact that consumers all face the same prices.

III. Innovation and Transfer without Licencing

The dynamic model developed in the previous section can be embedded in a variety of scenarios where the government or other decision makers of an innovating country choose strategically both the rate of innovation and the division of the new commodities between the two commodity groups. In the one that we will consider in this section agreements to license new technologies are excluded.

A. The Overall Equilibrium

Without licensing, the problem the Northern government faces is

$$\max \int_0^\infty e^{-\delta t} U(m(t),\ n(t),\ v(t))dt$$

$$\text{s.t.}\quad \dot{m}(t) = \{2\mu(t) - m(t)\}\{\alpha v(t) + \beta\}$$

$$\dot{n}(t) = \{\alpha v(t) + \beta\} n(t),$$

$$0 \le \mu(t) \le 1/2,\ 0 \le v(t) \le 1,$$

$$m(0) = 0,\ n(0) = n_0$$

$$m(t) \ge 0,$$

where $n_0(>0)$ is the initial number of commodities. The first constraint is easily derived from $m(t) = J_N(t)/J_S(t)$, $\mu(t) = \dot{J}_N(t)/\dot{n}(t)$, and $\dot{n}(t)/n(t) = \alpha v(t) + \beta$.

The above problem is a standard optimal control problem where m and n are state variables and μ and v are instrument (or control) variables. We are interested in the steady state equilibrium where m and $\dot{n}/n$ are constant. In principle, the problem can be solved using Pontryagin's maximum principle. For our problem, however, there is a more intuitive solution method which also allows us to find some economically meaningful properties of the solution.

Observe the following: From the previous section, one has

$$U(m,\ n(t),\ v) = \left(\frac{1}{2}\right)^{\frac{1}{r}+1} a^{-\frac{1}{2}} n(t)^{\frac{1}{r}} (1-v)\{(1-m)(m+a^r w^r)\}^{\frac{1}{2r}},$$

where w is implicitly given by

$$(aw)^r\{(1 - v)w - 1\} = 2m. \tag{2}$$

Notice that given some v, the value of m which maximizes the function U does not depend on $n(t)$. This implies that given a steady state value of innovation activity v, the optimal steady state invasion m does not depend on the number of commodities $n(t)$. Notice also that U does not depend on μ at all. In particular, given a steady state value of invasion m, the optimal steady state innovation activity v does not depend on the product choice μ.

The above observations regarding the function U greatly simplify the analysis and, more importantly, allow us to break down the steady state analysis into two parts: In the first part, we find the optimal invasion $m(=2\mu)$ given a steady state innovation activity v, and investigate how the optimal invasion varies as the parameters of the model vary. In the second part, we find the optimal innovation activity v given a steady state invasion $m(=2\mu)$, and study the effects of the parameters of the model on the optimal innovation. Obviously, the overall steady state equilibrium is the intersection of the solutions of the two parts.

B. Optimal Invasion

Suppose that a steady state innovation activity v is given. Maximizing $U(m, n(t), v)$ with respect to m, the North finds the optimal steady state invasion m.

Theorem 1
1) Suppose $a \geq (1-v)(2\sigma-1)^{1/(\sigma-1)}$. Then invasion does not occur, i.e., $m=0$. In particular, if $a \geq (2\sigma-1)^{1/(\sigma-1)}$, then $m=0$ no matter what v is.
2) Suppose $a < (1-v)(2\sigma-1)^{1/(\sigma-1)}$. Then invasion occurs, and one has $0 < m < 1/2$. The optimal invasion m decreases both in comparative advantage a and innovation activity v.

Proof: See the Appendix.

There are two types of steady state equilibria. In one, each country specializes according to comparative advantage, half the commodities belong to each group ($m=0$). This steady state is Pareto efficient given the level of innovation activity v. In effect, because of high labor requirements, it never pays the North to *produce* in the South's group. However, its gain from a balanced diet does lead the North to allocate exactly half of its new products directly to the South through the transfer mechanism. Thus the two countries' incomes will be equal.

In the other type of steady state, comparative advantage is sufficiently small that it pays the North to exploit external bias to improve its terms of trade. On the strength of its new-product monopoly the North produces commodities in the South's group. The rates of innovation and transfer we chose guarantee that in the steady state the *numbers* of commodities produced by the North and South will be the same. Yet, the North's wage and welfare are higher, because it takes advantage of differential external bias in its choice of new products. As a result, the world economy operates inefficiently. The efficiency may be measured by $1-m$, the steady state value of the degree of specialization.

As shown in the Appendix, in case 2) of the above theorem, the optimal invasion $m(>0)$ is the solution to the following equation:

$$\frac{1}{2}(1-v)^{\sigma-1}[\{1+(\sigma-1)\sigma(2-4m)^2\}^{\frac{1}{2}}-1+\sigma(2-4m)]$$
$$=a^{\sigma-1}\sigma^{-(\sigma-1)}[\{(\frac{1}{2-4m})^2+(\sigma-1)\sigma\}^{\frac{1}{2}}+\frac{1}{2-4m}]^{r}. \tag{3}$$

The optimal invasion m decreases in the measure of comparative advantage a. It also decreases in innovation activity v. It is non-monotonic in σ, the within group elasticity of substitution, apparently first falling and then rising as σ increases.[8]

The threshhold value of comparative advantage $a = (1 - v)(2\sigma - 1)^{1/(\sigma-1)}$ below which invasion occurs is a decreasing function of σ. (Notice that $(2\sigma - 1)^{1/(\sigma-1)} \to e^2$ as $\sigma \to 1$, and $(2\sigma - 1)^{1/(\sigma-1)} \to 1$ as $\sigma \to \infty$). The region of the parameter space on which invasion occurs is depicted in Figure 1. For a numerical example, if $\sigma = 2$ and $v = 1/3$, then invasion occurs when $a < 2$.

C. Optimal Innovation

Suppose now that a steady state invasion m is given. To find the optimal steady state level of innovation activity v, the North maximizes

$$V(v)=\int_0^{\infty} e^{-\delta t}U(m,\ n(t),\ v)\ dt.$$

In the Appendix, we show that

$$\frac{V'(v)}{V(v)}=\frac{-1}{1-v}+\frac{w}{\{\sigma(1-v)w-(\sigma-1)\}\{(1-v)w+1\}}$$
$$+\frac{1}{\frac{(\sigma-1)\delta}{\alpha}-(v+\frac{\beta}{\alpha})}.$$

Each of the three terms in the last expression represents different effects of R&D effort v on the overall utility. The first term represents the opportunity cost of labor. The second term represents the favorable wage-rate effect from withdrawing labor from production for use in

[8]Due to the complexity of the implicit function defining the solution m, we did not attempt to show $d^2m/d^2\sigma < 0$ analytically. Plotting the graph of m as a function of σ for various fixed values of a and v numerically, however, indicates that this is the case.

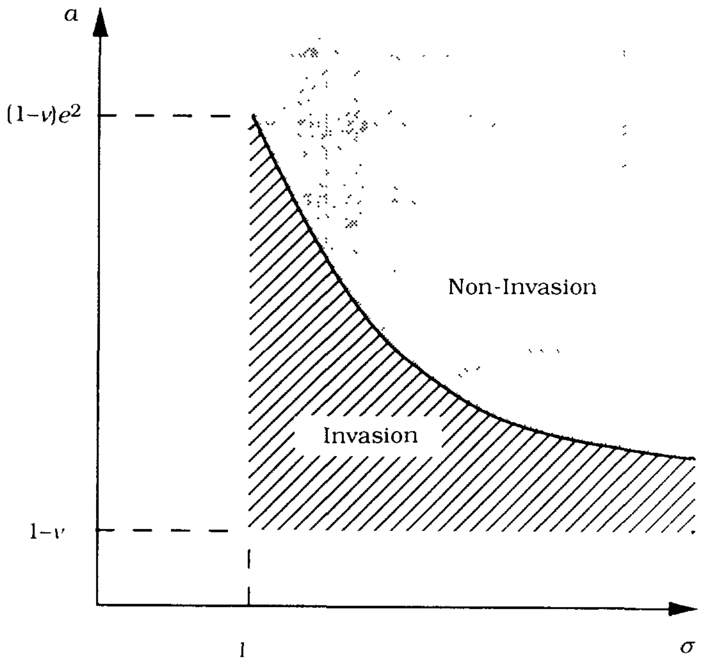

Figure 1
Invasion Parameters

R&D. The third term represents the favorable effect of the growth in the number of commodities. The optimal innovation activity will be zero only if $V'(0) \leq 0$. Otherwise, it is the solution v to $V'(v) = 0$, i.e.,

$$[\frac{1}{1-v}+\frac{-w}{\{\sigma(1-v)w-(\sigma-1)\}\{(1-v)w+1\}}]^{-1}+v=\frac{(\sigma-1)\delta-\beta}{\alpha}. \qquad (4)$$

The first term inside the bracket on the left hand side of (4) increases in v. It can be easily shown from (2) that as innovation activity v increases, the wage rate w increases but the wage bill $(1 - v)w$ in the production sector decreases. This implies that the second term inside the bracket decreases in v. Thus the effects of the parameters on the right hand side of (4) on the optimal innovation activity v are ambiguous in general. There are circumstances, however, under which we obtain definite effects.

Theorem 2

If m is is sufficiently close to zero, the optimal innovation activity v

increases in α and β and decreases in σ and δ.

***Proof*:** See the Appendix.

The results are intuitive. If the innovation activities are more productive, more resources should be put into it. A higher degree of substitutability among commodities means a lower benefit of product innovation. If the society values the future less, then it will devote less resources to innovation activities. The important point is, however, that in the international context these intuitive results obtain only in limited circumstances. For example, when there are strategic considerations such as the invasion in our model, the Northern government is in a second best world and the optimal innovation may not be monotonic in the productivity of innovation activity or the social discount factor. What the above theorem says is that we obtain the monotonic relationship when the strategic considerations are not dominant.

If m and v are both positive in the overall steady state equilibrium, they are the solution to the simultaneous system of equations (3) and (4). The steady state equilibrium wage rate is then determined by (2). Clearly, the comparative statics properties stated in Theorems 1 and 2 are valid in the overall equilibrium as well.

IV. Innovation and Transfer with Licensing

There are better ways than invasion for the North to exploit external bias in the exercise of its innovational monopoly. Perhaps the best would have the North allocating new products between the two commodities groups in their Pareto efficient proportions, immediately transferring those in the Southern group in exchange for sufficient royalty payments to compensate the North for foregoing its invasion option. If such an arrangement could be reached and furthermore, if the revenues to pay the royalties could be raised and distributed in a nondistortionary fashion (e.g., uniform income taxes and subsidies), then a Pareto improvement could be made over the steady state without such an arrangement.

In this section, we will assume that the North first determines the steady state level of innovation activity and then the two countries negotiate the licensing agreement. When the North decides on the level of innovation activity, it should take into account the negotiated licensing agreement which would follow its decision. Thus we will first study the licensing agreement for a given level of innovation activity, and then investigate the optimal innovation of the North.

A. License Fee with Bargaining

We allow the countries to bargain over a licensing contract. We assume that before the bargaining begins our two-country economy has reached a steady state equilibrium level of invasion m corresponding to some steady state innovation activity v. The licensing contract under negotiation specifies that one-half of all new products will be from the Southern group and that these will be transferred immediately to the South in exchange for a constant flow of license fees measured in terms of the South's wage. Would it be possible for the two countries to agree on the appropriate license fee, and if so, what would it be? We will provide solutions to these problems using the bargaining model of Rubinstein (1982).

The objective of the Northern government is to maximize the intertemporal utility function $\int_0^\infty e^{-\delta t}u(x(t))dt$. Similarly, suppose that the Southern government maximizes

$$\int_0^\infty e^{-\delta^* t}u(x^*(t))dt, \text{ where } \delta^* > (\alpha+\beta)/(\sigma-1).$$

The two countries bargain as follows: At time point 0, the North proposes a license fee. If the South accepts the proposal, then the agreement is made. If the South rejects the proposal, then the South becomes a proposer with a time lag of Δ. The process continues until an agreement is reached.

Let w be the steady state wage level corresponding to m and v (given by equation (2)). Let U_N and U_S be the instantaneous utilities of the North and South, respectively, at time 0 along the steady state equilibrium path in the absence of a licensing agreement, and let U_P be the utility of each country at time 0 along the steady state equilibrium path that would occur if the North does not invade the South's comparative advantage group. Then $U_N = U(m, n_0, v)$, $U_S = U(m, n_0, v)\{w(1 - v)\}^{-1}$, and $U_P = U(0, n_0, v)$. Put $\gamma = (\alpha v + \beta)/(\sigma - 1)$. Suppose that agreement on license fee $\phi(\in[0, 1))$ is reached at $t = \tau$. Then $u(x(t)) = e^{\gamma t}U_N$ for $t < \tau$ and $u(x(t)) = e^{\gamma t}(1 + \phi)U_P$ for $t \geq \tau$. Similarly, $u(x^*(t)) = e^{\gamma t}U_S$ for $t < \tau$ and $u(x(t)) = e^{\gamma t}(1 - \phi)U_P$ for $t \geq \tau$.

For this bargaining game, a perfect equilibrium constitutes a natural solution if it exists uniquely. Indeed, we can show.

Theorem 3

There exists a unique perfect equilibrium of the bargaining game. As the delay in the bargaining process Δ goes to zero, the perfect equilibrium license fee approaches

$$\phi = \frac{\delta^* - \gamma}{\delta + \delta^* - 2\gamma}(1 - \frac{U_S}{U_P}) - \frac{\delta - \gamma}{\delta + \delta^* - 2\gamma}(1 - \frac{U_N}{U_P}).$$

Proof: See the Appendix.

The license fee is greater if the North is relatively more patient. If the discount factors of both countries are the same, one has $\phi = (1/2)(U_N - U_S)/U_P$, that is, the two countries split the relative income spread. The royalty payment will be higher as the North's gain and the South's loss from the North's exploiting external bias is greater.

B. Optimal Innovation with Licensing

The North's optimal innovation without licensing was studied in section III. There the optimal innovation activity v was the maximum of $V(v)$ where

$$\begin{aligned} V(v) &= \int_0^\infty e^{-\delta t} U(m,\ n(t),\ v)\ dt \\ &= \int_0^\infty e^{-(\delta-\gamma)t} U(m,\ n_0, v)\ dt \\ &= (\delta - \frac{\alpha v + \beta}{\sigma - 1})^{-1} U_N. \end{aligned}$$

The introduction of the licensing agreement naturally affects the North's innovation activity. The North's optimal innovation activity v is now determined by maximizing $\int_0^\infty e^{-(\delta-\gamma)t}(1+\phi)U_P\ dt$. Suppose that the social discount factors of the North and South are the same, i.e., $\delta = \delta^*$. In this case, one has $\phi = (1/2)\{(U_N/U_P) - (U_S/U_P)\}$ and thus $(1 + \phi)U_P = U_P + (1/2)(U_N - U_S)$. Thus the North chooses v to maximize

$$(\delta - \frac{\alpha v + \beta}{\sigma - 1})^{-1}\{U_P + \frac{1}{2}(U_U - U_S)\}.$$

The overall effect of introducing licensing on innovation is ambiguous. On the one hand, the licensing agreement encourages the North to increase its innovation activity by compensating part of the externality from innovation. On the other hand, a greater innovation activity makes the North's threat of invasion less credible. (Recall from Theorem 1 that the optimal invasion level decreases as the innovation activity increases.) Thus a greater innovation activity may lead a lower royalty payment.

V. Concluding Remarks

In this paper, we have analyzed a Norht-South model of innovation and transfer where the innovating North can choose not only the rate of innovation but also the allocation of new products between a commodity group in which the North has a comparative advantage and another in which the South has a comparative advantage. The possibility of its invasion in the Southern group affects the North's decision on the rate of innovation in a complex way, and our results on this issue are limited.

The main focus of the paper was on the optimal choice of new products by the North. In the first dynamic scenario without the possibility of licensing, we saw that if the production inefficiencies of doing so are not too great, an innovating country will invade the other country's comparative advantage group in what amounts to a beggar-thy-neighbor attempt to improve national welfare. In the second scenario, which permits a licensing contract, we saw that the two countries will achieve a Pareto superior outcome by agreeing on a license fee which reflects their relative bargaining powers.

A key concept we used in analyzing the product choice of an innovating country was differential external bias rooted in asymmetric preferences and comparative advantage. The empirical significance of differential external bias in this century has not, to our knowledge, been directly evaluated. We do not know whether terms of trade effects from differential external bias are potentially more important today than they were before. On the one hand, the process of technological diffusion has quickened, gradually shortening the period of time over which a country might exercise the innovator's monopoly. On the other hand, technological developments in most parts of the world have narrowed traditional comparative advantage gaps, reducing the inefficiency costs of producing products in which a country does not have a comparative advantage. With the recovery of Europe, the Japanese phenomenon, and the rise of the newly industrialized contries in Asia and Latin America, international division of labor according to comparative advantage seems to be taking a new dimension.

Appendix

***Proof of Theorem* 1**

Let $u(m) = (1 - m)(m + a^r w^r)$ for $m \in [0, 1]$, where w is implicitly given by $(aw)^r\{(1 - v)w - 1\} = 2m$. To find the optimal m, it is sufficient to maximize $u(m)$ with respect to m. Let us first show that u is strictly concave. One has

$$\frac{du}{dm} = -(m + a^r w^r) + (1-m)(1 + a^r r w^{r-1}\frac{dw}{dm}),$$

where
$$a^r w^{r-1}\{(r+1)(1-v)w - r\}\frac{dw}{dm} = 2.$$

Thus

$$\frac{du}{dm} = -(m + a^r w^r) + (1-m)\{1 + \frac{2r}{(r+1)(1-v)w - r}\}.$$

Therefore

$$\frac{d^2u}{dm^2} = -(1 + a^r r w^{r-1}\frac{dw}{dm}) - \{1 + \frac{2r}{(r+1)(1-v)w - r}\}$$
$$-(1-m)\frac{2r(r+1)(1-v)}{\{(r+1)(1-v)w - r\}^2}\frac{dw}{dm}.$$

Since $dw/dm > 0$, one has $d^2u/dm^2 < 0$. Thus u is a strictly concave function.

We will now find the maximum m^* of u. One has $m^* = 0$, if and only if $du/dm|_{m=0} \leq 0$, i.e., $a \geq (1 - v)(1 + 2r)^{1/r}$. If $a < (1 - v)(1+2r)^{1/r}$, then m^* is a solution to

$$\frac{du}{dm} = -(m + a^r w^r) + (1-m)\{1 + \frac{2r}{(r+1)(1-v)w - r}\},$$

where
$$a^r w^{r-1}\{(r+1)(1-v)w - r\}\frac{dw}{dm} = 2.$$

Put $y = (1 - v)w - 1$ (> 0). Then

$$\frac{2m}{y} = a^r(\frac{y+1}{1-v})^r \tag{A1}$$

and $$m+\frac{2m}{y}=(1-m)\{1+\frac{2r}{(r+1)y+1}\}.$$

Multiplying both sides of the latter equation by $y\{(r+1)y+1\}$, one obtains

$$m(y+2)\{(r+1)y+1\}=(1-m)y[\{(r+1)y+1\}+2r],$$

i.e., $$\frac{m}{1-m}(y+2)\{(r+1)y+1\}=y\{(r+1)y+2r+1\},$$

i.e., $$\frac{m}{1-m}\{(r+1)y^2+(2r+3)y+2\}=(r+1)y^2+(2r+1)y.$$

Notice that $m < 1/2$, for $y > 0$. Put $v = m/(1-m)$. Then

$$(1-v)(r+1)y^2+\{(1-v)(2r+1)-2v\}y-2v=0.$$

Put $z = v/(1-v)$. Then

$$(r+1)y^2+2\{(r+\frac{1}{2})-z\}y-2z=0,$$

i.e., $$y=(r+1)^{-1}[\{(z+\frac{1}{2})^2+r(r+1)\}^{\frac{1}{2}}+(z+\frac{1}{2})-(r+1)].$$

and thus $$y^{-1}=(2z)^{-1}[\{(z+\frac{1}{2})^2+r(r+1)\}^{\frac{1}{2}}-(z+\frac{1}{2})+(r+1)]$$

and $$y+1=(r+1)^{-1}[\{(z+\frac{1}{2})^2+r(r+1)\}^{\frac{1}{2}}+(z+\frac{1}{2})]$$

Since $(1-v)^r(2m/y)=a^r(y+1)^r$ from (A1) and $m = z/(2z+1)$, one gets

$$(1-v)^r(2z+1)^{-1}[\{(z+\frac{1}{2})^2+r(r+1)\}^{\frac{1}{2}}-(z+\frac{1}{2})+(r+1)]$$
$$=a^r(r+1)^{-r}[\{(z+\frac{1}{2})^2+r(r+1)\}^{\frac{1}{2}}+(z+\frac{1}{2})]^r.$$

Let $q = z+(1/2)$. Then

$$\frac{1}{2}(1-v)^r[\{1+r(r+1)(\frac{1}{q})^2\}^{\frac{1}{2}}-1+(r+1)\frac{1}{q}]$$

$$= a^r (r+1)^{-r} [\{q^2 + r(r+1)\}^{\frac{1}{2}} + q]^r.$$

Since $q = 1/(2-4m)$ and $r = \sigma - 1$, one has

$$\frac{1}{2}(1-v)^{\sigma-1}[\{1+(\sigma-1)\sigma(2-4m)^2\}^{\frac{1}{2}} - 1 + \sigma(2-4m)]$$

$$= a^{\sigma-1}\sigma^{-(\sigma-1)}[\{(\frac{1}{2-4m})^2 + (\sigma-1)\sigma\}^{\frac{1}{2}} + \frac{1}{2-4m}]^r.$$

The last equation defines m implicity. It was shown above that $m < 1/2$. Notice that the left hand side of the last equation is a decreasing function of m, while the right hand side is an increasing function of m. Thus the solution m decreases in a, and also decreases in v.

Q.E.D.

Derivation of $V'(v)/V(v)$

Since

$$U(m,\ n(t),\ v) = (\frac{1}{2})^{\frac{1}{r}+1} a^{-\frac{1}{2}} n(t)^{\frac{1}{r}} (1-v)\{(1-m)(m+a^r w^r)\}^{\frac{1}{2r}}$$

and $n(t) = n(0)e^{(\alpha v + \beta)t}$, one has

$$V(v) = (\frac{1}{2})^{\frac{1}{r}+1} a^{-\frac{1}{2}} n(0)^{\frac{1}{r}} (1-v)\{(1-m)(m+a^r w^r)\}^{\frac{1}{2r}} \int_0^\infty e^{-\delta t} e^{\frac{\alpha v+\beta}{r}t}\ dt.$$

$$= (\frac{1}{2})^{\frac{1}{r}+1} a^{-\frac{1}{2}} n(0)^{\frac{1}{r}} (1-v)\{(1-m)(m+a^r w^r)\}^{\frac{1}{2r}} \frac{r}{r\delta - (\alpha v + \beta)},$$

where w is implicitly given by (2). Thus

$$\frac{d\ \log V(v)}{dv} = \frac{-1}{1-v} + \frac{1}{2r}\frac{a^r r w^{r-1}\frac{dw}{dv}}{m + a^r w^r} + \frac{1}{\frac{r\delta}{\alpha} - (v + \frac{\beta}{\alpha})}$$

$$= \frac{-1}{1-v} + \frac{w}{\{(r+1)(1-v)w - r\}\{(1-v)w+1\}}$$

$$+ \frac{1}{\frac{r\delta}{\alpha} - (v + \frac{\beta}{\alpha})},$$

for $dw/dv = w^2/\{(r+1)(1-v)w - r\}$ from (2). Now simply note that $r = \sigma - 1$.

Q.E.D.

Proof of Theorem 2

Denote the left hand side of (4) by $F(v, \sigma, m)$. Then $F(v, \sigma, 0) = 2 - v$. Since $\partial F(v, \sigma, m)/\partial v$ is continuous in m and $\partial F(v, \sigma, 0)/\partial v = -1$, one has $\partial F(v, \sigma, m)/\partial v < 0$ for sufficiently small m. Thus the solution v of $F(v, \sigma, m) = \{(\sigma - 1)\delta - \beta\}/\alpha$ increases in α and β and decreases in δ for sufficiently small m. In order to analyze the effect of σ on the solution v, let $G(v, \sigma, m)) = F(v, \sigma, m)) - \{(\sigma - 1)\delta - \beta\}/\alpha$ for fixed α, β, and δ. Since $\partial G(v, \sigma, m)/\partial\sigma$ is continuous in m and $\partial G(v, \sigma, 0)/\partial\sigma = -\delta/\alpha < 0$, one has $\partial G(v, \sigma, m)/\partial\sigma < 0$ for sufficiently small m. Since $\partial G(v, \sigma, m)/\partial v < 0$ for sufficiently small m, the solution v of $G(v, \sigma, m) = 0$ decreases in σ for sufficiently small m.

Q.E.D.

Proof of Theorem 3

The maximization problems of the two countries do not change if one subtract $\int_0^\infty e^{-(\delta-\gamma)t}U_N dt$ from $\int_0^\infty e^{-\delta t}u(x(t))dt$ and $\int_0^\infty e^{-(\delta*-\gamma*)t}U_S dt$ from $\int_0^\infty e^{-\delta* t}u(x(t))dt$. In other words, the North and South maximize $V(\tau) = \{(1 + \phi)U_P - U_N\} \int_\tau^\infty e^{-(\delta-\gamma)t}dt$ and $V^*(\tau) = \{(1 - \phi)U_P - U_S\} \int_\tau^\infty e^{-(\delta*-\gamma*)t}dt$, respectively. Let $A = (1 + \phi)U_P - U_N$, $A^* = (1 - \phi)U_P - U_S$, $\theta = \delta - \gamma$, and $\theta^* = \delta^* - \gamma^*$. Then $V(\tau) = (A/\theta)e^{-\theta\tau}$ and $V^*(\tau) = (A^*/\theta^*)e^{-\theta^*\tau}$.

Our model is equivalent to a bargaining model of Rubinstein (1982) with the feasible set $F = \{(A/\theta, A^*/\theta^*);\ A, A^* \geq 0,\ A + A^* \leq 2U_P - U_N - U_S\}$ and the discount factors $e^{-\theta\Delta}$ and $e^{-\theta*\Delta}$ between two adjacent bargaining rounds. Let $\pi = 2U_P - U_N - U_S$, $\delta = e^{-\theta\Delta}$ and $\delta* = e^{-\theta*\Delta}$. It can be easily shown that the scale factors $1/\theta$ and $1/\theta^*$ do not affect the result of Rubinstein (1982). Thus the bargaining game has a unique perfect equilibrium with the outcome

$$(A,\ A^*) = \pi\left(\frac{1-\delta^*}{1-\delta\delta^*}, \frac{\delta^*(1-\delta)}{1-\delta\delta^*}\right).$$

(See Rubinstein 1982, p. 108) As the delay in the bargaining process Δ goes to zero, the solution approaches $(A, A^*) = \pi(\theta^*/\theta + \theta^*), \theta/(\theta + \theta^*))$. From $A = (1 + \phi)U_P - U_N$ and $\pi = 2U_P - U_N - U_S$, one has

$$\phi = \frac{\theta^*}{\theta+\theta^*}(1-\frac{U_S}{U_P}) - \frac{\theta^*}{\theta+\theta^*}(1-\frac{U_N}{U_P}).$$

Q.E.D.

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