A Generalization of Modeling for Markets of Substitutive Raw Materials: Effects of Introducing New Good on the Volatility of Price and Trade Volume of the Gross Substitutes

John-ren Chen*

In this paper we generalize a commodity market model to n gross substitutes. We analyse the specification errors if some gross substitutes are neglected. We proved the following main proposition that the volatility of market price and trade volume for each of the n gross substitutes will be underspecified in a model in which at least one of the gross substitutes is not considered, and the degree of the specification error for the price volatility and the trade volume is more serious the higher the number of gross substitutes not considered in a model. (*JEL* Classification: D50, C62)

I. Introduction

The fluctuation of the world market prices for raw materials are of major concern for almost all developing countries, since raw materials belong to the most important export goods of these countries. The problem of stabilization of world market prices for raw materials is therefore a much discussed theses for the development policy. A lot of theoretical as well as commodity models were applied for policy consideration.

Up to now there are almost only "isolative" commodity models propounded, in which the related goods are not suitably considered, i.e. the related raw materials are either considered as exogenous or

*Institut für Wirtschaftstheorie und Wirtschaftspolitik der Universität Innsbruck, Innrain 52, 6020 Innsbruck. I want to thank a referee for his useful comments.

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ignored. Needless to say that these models used as theoretical foundations for problems of world markets for raw materials suffer from a serious information loss, if the raw material considered is related with other goods. The high positive correlation coefficients between the world market prices of some raw materials seem to point to relativeness of these raw materials (see Chen 1990). In an earlier paper we studied the implications of an "isolative model" in which a raw material is modeled isolatively without suitable consideration of the related goods. We studied five kinds of relationship applying two goods partial equilibrium commodity market models. (See Chen 1990)

In section II we are going to generalize the "connective model" for gross substitutes to any number of raw materials.

A gross substitute for a raw material can either be another raw material (e.g. wool for cotton, coffee for tea, rice for wheat, etc.) or an industrial product (e.g. man made fibre for cotton). What are the implications in order of stabilizing raw material price, if the gross substitute is an industrial product instead of another raw material.

In section III we will analyze this problem. The measure of support price is performed in many countries. What is the effect of such a measure on volatility of price and trade volume of gross substitutes. This question will be answered in section IV.

In section V we will show that the results of the analysis in the section II to IV can be applied as a corollary about the effect of introducing a new good on the volatility of the price and the trade volume of gross substitutes.

In this paper we will only consider the nonsystematic risk in the sense of Newberry and Stiglitz (see Newberry and Stiglitz 1985, p. 48) and treat the exogenous variables besides the additive random disturbances in the demand and supply functions as non random variables. This assumption does not influence our main proposition the Le Chatelier principle in this paper. But the variances and the covariances of the prices will be underspecified, if in reality these exogenous variables are random. (See Chen 1990)

II. Generalization of the Connective Competitive Markets Model for Any Number of Gross Substitutes: The Le Chatelier Principle.

In this section we will generalize the "connective model" for any number of gross substitutes. The different kind of gross substitutes is not considered here.

$$Q = A_1P + A_2Y + U \tag{1}$$
$$Q = B_1P + B_2X + V \tag{2}$$

where Q is a $n \times 1$ vector of trade volumes

P is a $n \times 1$ vector of prices

Y is a $n \times 1$ vector of exogenous variables on the demand side

U is a $n \times 1$ vector of random variables on the demand side

A_1 is a $n \times n$ matrix with negative diagonal elements and positive off-diagonal elements.

A_2 is a $n \times m$ matrix without restriction of generalization all elements are specified to be non negative.

B_1 is a $n \times n$ diagonal matrix with positive elements on the diagonal.

B_2 is a $n \times 1$ matrix, as A_2 all elements of B_2 are specified to be non negative.

X is a $n \times 1$ vector of exogenous variables on the supply side.

V is a $n \times 1$ vector of random variables.

The equations (1) are demand equations for n gross substitutes. The property of gross substitutes (strong gross substitutes) is specified by positive off-diagonal elements of the matrix A_1. We assume that all goods (raw materials) considered in this model are non Giffen goods, i.e. all diagonal elements of the matrix A_1 are negative.

Since We consider only the strong gross substitutes on the demand side the matrix B_1 is assumed to be a diagonal matrix with positive elements.

To specify the constant term in the demand and the supply equations the first column of Y and X are all unity.

The "connective model" for n commodity markets has $2n$ endogenous variables Q and P with $2n$ structural equations.

In most commodity market models for agricultural products the supply functions are assumed to depend on the expected price of the corresponding products formed at time of production decision instead of the market price as specified in our model, because the market price for the agricultural product at harvest is rarely known at the time of decision.

At first it is important to classify between the short-run, middle-run and long-run model. For example the production of many agricultural product needs a period of several months. The supply of such goods can only be changed from period to period of production. In a short-run model the market supply is determined mainly due to the inventory decision of the market participants. Therefore the modeling of the inventory decision must be the main content of a short-run model, while the specification of the production decision is not relevant.

In a middle-run model the specification of the production decision belongs to the important content of the model. Contrary to the short-run model the specification of inventory decision is only of secondary importance. In the middle-run the capacity of raw material production is given. The change in production of raw materials is mainly due to the decision of input of variable factors and a random factor. Following points are important for specification of production decision: First, length of the time lag between the production decision and the harvest; secondly, is there a support price for the raw material producer, thirdly, is it "*production on order*" or not. If there is a support price, it must be considered as a relevant variable for the production decision. If there is no important time lag between production decision and harvest, or if production is performed in form of "production on order," the current market price is relevant for the production decision. If the time lag between the production decision and the harvest is not "neglegible," the production decision must be made according to the expected price made at the time of production decision for the harvest time. In the middle-run model the excess demand or excess supply of inventory in the sense of time aggregative excess demand or time aggregative excess supply must in general be equal to zero (ceteris paribus). In the long-run model the specification of the decision upon the change of production capacity is not important. The decision on capacity is similar to the investment decision. The specification of inventory decision and production decision is not an essential point of a long-run model.

To distinguish the short-run, middle-run and long-run models is an important point for specification of the supply function in a commodity model. This distinction is not relevant for the specification of the demand function, since demand is in general a continuous phenomenon. We believe that the substitution between a raw material and another goods is a phenomenon of the final demand (i.e. the consumer demand of raw material) mainly. For the inventory decision the problem of gross substitutes is not relevant, especially for the speculative inventory. In general the speculative inventory is not performed by the consumers, but mainly by traders who specialize in the business of special raw materials and try to gain speculation profit depending on correct expectation on the price development of the special raw material. The knowledge about the related raw materials can help to form a good expectation on the development of the raw material of special interest but it is not necessary to participate on those markets.

The distinction between models of short-, middle- and long-run is therefore only important for the specification of the supply function in a commodity model. In general the short-run model must be specified for a period of less than one year, i.e. model for daily data, weekly, monthly and quarterly data.

A middle-run model includes for the most agricultural products in general the time of one year. For several raw materials the production can be changed every day, such as the production of several mineral raw materials. In this case the distinction between the short-run and the middle-run model is neither necessary nor possible.

A long-run model is in general only relevant for more than one year. For some raw materials an increase of capacity needs many years, e.g. coffee, cocoa, tea, etc. But for some other raw materials an increase in capacity can be realized one year. In these cases a distinction between the middle-run and the long-run model is not relevant.

In this paper we are going to specify a middle-run model. In a previous paper we showed the phenomenon of high positive correlation between yearly prices of several raw materials. Our main interest in this paper is to demonstrate how these high correlations can be explained. For this reason we specify in this paper only middle-run models.

The random variables (exactly random vectors U and V) are additive in both demand and supply equations. They are assumed to be of "white noise." We assume stochastic independence between these random variables, i.e.

$$\mathrm{E}u_i v_j = \mathrm{E}u_i \mathrm{E}v_j, \;\; \mathrm{E}u_i u_j = \mathrm{E}u_i \mathrm{E}u_j, \;\; \mathrm{E}v_i v_j = \mathrm{E}v_i \mathrm{E}v_j$$
$$\text{for } i, j = 1, 2, \ldots, n \text{ and } i \quad j.$$

We call a model "connective" if the property of "gross substitutes" or "strong substitutes" is specified by the matrix A_1, such that all diagonal elements are negative and all off diagonal elements are positive. We call a model "isolative" if every raw material is specified separatively from each other without consideration of the property of gross substitutes.

The equilibrium of our model is given as follows:

$$P^* = D(A_2 Y - B_2 X + U - V) \tag{3}$$
$$Q^* = B_1 D A_2 Y + (I - B_1 D) B_2 X + B_1 D U + (I - B_1 D) V \tag{4}$$

where $D = (B_1 - A_1)^{-1}$

P^*: $n \times 1$ vector of market prices

Q^*: $n \times 1$ vector of trade volumes

We assume the existence of the equilibrium e.g.

$$P^* = D(A_2Y - B_2X + U - V) > 0 \tag{3}$$

$$Q^* = B_1DA_2Y + (I - B_1D)B_2X + B_1DU + (I - B_1D)V > 0 \tag{4}$$

***Assumption* 1**

$P > 0$ for (at least one set of) parameters $(A_2Y - B_2X + U - V) > 0$.

It is well known that this implies the following properties of $(B_1 - A_1)$:

1) $(B_1 - A_1)$ is nonsingular

2) $D \geq 0$

(See Takayama, Theorem 4.D.1)

Thus the uniqueness of the equilibrium is guaranteed. It is also known that the gross substitutability assumption implies stability.

The variances of market prices and trade volumes are calculated as follows:

$$\text{Var}\,(P^*) = D\,\Omega\,D' \quad \text{with } D > 0 \tag{5}$$

The matrix of variances and covariances Var(P^*) is symmetric, where Ω is a diagonal matrix, i.e. with

$$\Omega = (\Omega_{ij}) \text{ with } \Omega_{ij} = \text{Var}(u_i) + \text{Var}(v_i) > 0, \text{ for } i = j \text{ and}$$
$$\Omega_{ij} = 0, \text{ for } i = j,\ i, j = 1, 2, \ldots, n$$

since u and v are stochastic independent random vectors.

Conveniently we write the matrix $D = (D_{ij})$ with the element D_{ij} on the i-th row and j-th column.

Similarly, the variance matrix of trade volumes can be given as:

$$\text{Var}(Q^*) = B_1D\,\text{Var}(U)\,D'B_1' + (I - B_1D)\,\text{Var}(V)(I - B_1D)' \tag{6}$$

with Var(U) as covariances matrix of U with the element Var(u_iu_j) and Var(V) as covariances matrix of V with the elements Var(v_iv_j); Var(U) and Var(V) are diagonal.

Representatively, the variance of a market price is equal to

$$\text{Var}(P_i^*) = \sum_{i=1}^{n} D_{ij}^2(\text{Var}(u_j) + \text{Var}(v_j)) \text{ for } i = 1,2,\ldots,n \tag{7}$$

A covariance between two market prices is equal to

$$\mathrm{Var}(P_i^* P_j^*) = \sum_{q=1}^{n} D_{iq} D_{jq} (\mathrm{Var}(u_q) + \mathrm{Var}(v_q)) \tag{8}$$

where D_{ij} is the element on the i-th row and j-th column of the matrix D.

Since the matrix D is positive, i.e. $D_{ij} > 0$ for all i and j, the covariance between any two market prices is therefore always positive.

Thus we have shown that "*The market prices of two gross substitutes are always positively correlative.*" Now we want to show that the variance of every market price will be misspecified (underspecified), if one of gross substitutes is not considered in the "connective" model and the degree of underspecification about the variance of an equilibrium price is the more serious, the more gross substitutes are not considered in the "connective" model. This proposition can be called as *the Le Chatelier-Samuelson-Morishima principle* (see Morishima 1964, p. 11; Samuelson 1947 and 1964, p. 36-8).

In the following discussion we number the neglected substitutes backwards from n, n-1, n-2, ...etc. If only one substitute is not considered then it is numbered as the n-th good in our model. If two goods are neglected, then they are the n-th and the n-1th good, and so on.

To compare the variances and the covariances of the equilibrium prices between the models with different numbers of neglected substitutes we use the following notations:

1) $$\mathrm{Var}(P_i^* | k) = \sum_{j=1}^{n-k} D_{ij}^2 (\mathrm{Var}(u_j) + \mathrm{Var}(v_j)) \tag{9}$$

for $i = 1, 2, \ldots, n - k$ and k is any non negative number

$k = 0, 1, \ldots, n - 1$

where $\mathrm{Var}(P^* | k)$ is the variance of the equilibrium price if k of n gross substitutes are not considered in a connective model; and

2) $$\mathrm{Var}(P_i^* P_j^* | k) = \sum_{j=1}^{n-k} D_{iq} D_{jq} (\mathrm{Var}(u_q) + \mathrm{Var}(v_q)) \tag{10}$$

for $i, j = 1, 2, \ldots, n - k$

the covariance between P_i^* and P_j^* due to the gross substitutes considered in the connective model if k of such goods are not considered.

3) $D_{ij}(k)$ is the element on the i-th row and j-th column of the inverse of $(B_1 - A_1 | k)$ where k of the n gross substitutes are not considered in a connective model. where

$$D_{ij}(k)=\frac{H_{ij}(k)}{H(k)},\ \text{with}\ H(k)=\det(B_1-A_1|k) \tag{11}$$

where $H(k) = \det(B_1 - A_1 | k)$ means determinant of the matrix $(B_1 - A_1 | k)$ and $H_{ij}(k)$ the cofactor corresponding to the element of the i-th row and the j-th column of the determinant det $(B_1 - A_1 | k)$, if k of the n gross substitutes are not considered in a connective model. Since $H(k)$ and $D_{ij}(k)$ are positive (see (L1) and (L3)), therefore

$$H_{ij}(k) > 0,\ \text{for any}\ i, j = 1, 2, \ldots, n-k$$

Now we compare

$$D_{ij}(0)\ \text{and}\ D_{ij}(k)\ \text{for all}\ i, j = 1, 2, \ldots, n-k,$$
$$\text{and}\ k = 1, 2, \ldots, n-1$$

According to (10)

$$H(0)=\begin{vmatrix} h_{11}\ldots h_{in} \\ \vdots \\ h_{n1}\ldots h_{nn}\end{vmatrix}$$

where h_{ij} is th element on the i-th raw and j-th column of $H(0)$

$$D_{ij}(0)=\frac{H_{ij}(0)}{H(0)}$$

$$H(k)=\begin{vmatrix} h_{11}\ldots & h_{in-k} \\ \vdots & \\ h_{n-k1}\ldots & h_{n-k\ k-n}\end{vmatrix}$$

$$D_{ij}(k)=\frac{H_{ij}(k)}{H(k)}=\frac{H_{ji}(H_{nn}(0))}{H_{nn}(0)}$$

Since the determinant $H(k)$ is equal to the cofactor corresponding the n-th row and n-th column of the determinant $H(0)$, i.e. $H_{nn}(0)$ where H_{ji} $(H_{nn}(0))$ is the cofactor corresponding the the i-th row and the j-th column of the cofactor of the $H_{nn}(0)$. Now we calculate the difference

$$\frac{H_{ji}(0)}{H(0)}-\frac{H_{ji}(1)}{H(1)}=\frac{H_{ji}(0)}{H(0)}-\frac{H_{n'nj'i'}(0)}{H_{nn}(0)}=$$
$$\frac{H_{ji}(0)H_{nn}(0)-H_{nnj'i}(0)H(0)}{H(0)H_{nn}(0)}= \tag{12}$$
$$\frac{(H_{ji}(0))(H_{ij}(0))}{H(0)H_{nn}(0)}>0$$

where $H_{nn'j'i}(0)$ is the cofactor of the element on the j-th row and i-th column of the cofactor of the element on the n-th row and n-th column of $H(0)$ by the Jacobi theorem on determinants. (See Samuelson 1965, p. 370)

Thus we have shown

$$D_{ij}^2(k-1) > D_{ij}^2(k) \quad \text{for } n > k \geq 0, \quad i, j = 1, 2, \ldots, n \tag{13}$$

$$\begin{aligned} &\mathrm{Var}(P_i^* | k-1) - \mathrm{Var}(P_i^* | k) \\ &= \sum_{j=1}^{n-k} (D_{ij}^2(k-1) - D_{ij}^2(k))(\mathrm{Var}(u_j) + \mathrm{Var}(v_j)) \\ &+ \sum_{j=n-k+1}^{n} D_{ij}^2(K-1)(\mathrm{Var}(u_{n-k+1}) + \mathrm{Var}(v_{n-k+1})) \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\mathrm{Var}(P_i^* P_j^* | k-1) - \mathrm{Var}(P_i^* P_j^* | k) \\ &= \sum_{q=1}^{n-k} (D_{iq}(k-1) - D_{jq}(k-1) - D_{iq}(k) D_{jq}(k))(\mathrm{Var}(u_k) + \mathrm{Var}(v_k)) \\ &+ \sum_{r=n-k+1}^{n} D_{ir}(k-1) D_{jr}(k-1)(\mathrm{Var}(u_r) + \mathrm{Var}(v_r)) \end{aligned} \tag{15}$$

Thus $D_{ij}(k-1) > D_{ij}(k) \quad \text{for } k = 1, 2, \ldots, n-1 \text{ and } i, j = 1, 2, \ldots, n$ (16)

is a sufficient condition for $\mathrm{Var}(P_i^* | k-1) > \mathrm{Var}(P_i^* | k)$ and $\mathrm{Var}(P_i^* P_j^* | k-1) > \mathrm{Var}(P_i^* P_j^* | k)$ where the neglected gross substitutes are numbered back from n to 1.

Thus this result can be formulated compactly as follows:

$$\mathrm{Var}(P_i^* | k-1) \geq \mathrm{Var}(P_i^* | k) \quad \text{for } i = 1, 2, \ldots, n-k$$
$$\mathrm{Var}(P_i^* P_j | k-1) \geq \mathrm{Var}(P_i^* P_j | k) \quad \text{for } i, j, = 1, 2, \ldots, n-k, \; i = j.$$

We summarize this result as follows:

"*The specification of the price volatility for each of the n gross substitutes will be biased downward in a model in which at least one of the gross substitutes is not considered. The degree of the specification bias concerning price volatility is more serious the higher the number of the gross substitutes not considered in a model.*"

As in the two-goods model the reason for the specification bias of the volatility of the price of gross substitutes in an isolative model or in a model in which at least one of the n gross substitutes is not considered is the neglection of some gross substitutes. The specification bias occurs, even if the market of the neglected substitute is deterministic, i.e. if it is not influenced by any random disturbance on the corresponding market. Due to the gross substitution a random disturbance on every one of these markets will carry over to the prices and the trade volume of the other substitutes.

Thus, the specification error of price volatility due to omission of a gross substitute in a model is not only due to the carry-over of random disturbances between the markets of gross substitutes but mainly due to their market interdependence. This can be explained by the following example: Suppose the n-th gross substitute is not influenced by any random disturbance neither on the demand nor on the supply side. In this case if we don't consider this gross substitute in our model, there will occur a specification error of the price volatility for all n gross substitutes, since

$$\begin{aligned}&\mathrm{Var}(P_i^*|0)-\mathrm{Var}(P_i^*|1)\\&=\sum_{q=1}^{n-1}(D_{iq}(0)D_{jq}(0)-D_{iq}(1)D_{jq}(1))(\mathrm{Var}(u_q)+\mathrm{Var}(v_q))>0\end{aligned} \tag{17}$$

because the n-th gross substitute is not disturbed randomly, thus both u_n and v_n disappear in the above example.

We are now going to show the specification error for the trade volume in a model due to omission of a gross substitute. For this purpose we write the variance of trade volume in a model in which k gross substitutes are not considered endogenously as follows:

$$\begin{aligned}\mathrm{Var}(Q^*|k) &= B_1(k)D(k)\,\mathrm{Var}(U|k)D'(k)\,B_1(k)\\&\quad+(I-B_1(k)D(k))\,\mathrm{Var}(V|k)\,(I-B_1(k)D(k))'.\end{aligned} \tag{18}$$

As shown above the first term on the righthand side (RS) is the lower the more the number of gross substitutes not considered in a model. To show $\mathrm{Var}(Q^*|k-1) > \mathrm{Var}(Q^*|k)$ we have to prove according to the second term on the RS of (18) that

$$\begin{aligned}&(I-B_1(k-1)\,D(k-1))\,\mathrm{Var}(V|k-1)\,(I-B_1(k-1)D(k-1))' >\\&\qquad(I-B_1(k)D(k))\,\mathrm{Var}(V|k)\,(I-B_1(k)D(k))'\end{aligned} \tag{19}$$

Since $D(k) = [B_1(k) - A_1(k)]^{-1}$ we can show

$$(I-B_1(B_1-A_1)^{-1})\cdot(B_1-A_1) = B_1 - A_1 - B_1 = A_1$$

Therefore

$$(I - B_1(k)D(k)\,\mathrm{Var}(V|k)\,(I - B_1(k)D(k))' = A_1(k)D(k)\,\mathrm{Var}(V|k)\,D'(k)\,A_1'(k) \tag{20}$$

Substituting (20) in (19) we prove

$$A_1(k-1)\,D(k-1)\,\mathrm{Var}(V|k-1)\,D'(k-1)\,A_1'(k-1) > A_1(k)D(k)\,\mathrm{Var}(V|k)\,D'(k)\,A_1'(k) \tag{21}$$

according to (16).

Thus we proved the Le Chatelier principle for the specification error of trade volume volatility.

III. Modeling of Gross Substitutes between Raw Materials and Industrial Man-made Materials

In comparison with raw material the man-made material is characterized by its stable supply. Since man-made material is an industrial product, its production can be performed without influence of those random disturbance such as weather.

In this sense we shall specify the supply function of man-made gross substitute without additive random disturbance as those of raw materials. We assume perfect competition for the market of man-made material. To show the specification error due to omission of a man-made material we denote it as the n-th gross substitute. We assume for simplification that the random variable in the supply function of man-made material v_n is always equal to zero.

The results of the above section can be carried over easily as follows:

1) The prices of gross substitutes are positively correlated, i.e.

$$\mathrm{Var}(P_i^* P_j^*) > 0 \text{ for all } i \text{ and } j$$

2) The Le Chatelier principle

$$\mathrm{Var}(P^* | k-1) > \mathrm{Var}(P^* | k) \text{ and}$$

$\mathrm{Var}(P^* | 0) > \mathrm{Var}(P^* | 1)$, if the omitted gross substitute is the man-made material.

$$\mathrm{Var}(Q^* | k-1) > \mathrm{Var}(Q^* | k)$$

3) If one of gross substitutes is omitted, then

$$\text{Var}(P^* \mid \text{man}) > \text{Var}(P^* \mid \text{raw})$$

where $\text{Var}(P^* \mid \text{man})$ means the variance of price if a man-made material is not considered; and analogously raw means that a raw material is omitted. This proposition is due to the assumption $v_n = 0$ and means an omission of a raw material will have as a consequence higher specification error than in the case of a man-made material.

4) $\text{Var}(P^*_n \mid k-1) > \text{Var}(P^*_n \mid k)$ and $\text{Var}(Q^*_n \mid k-1) > \text{Var}(Q^*_n \mid k)$
for $n - 1 \;\; k \;\; 0$

This proposition means that the volatility of the price and the trade volume of man-made material will be the greater, the greater the number of gross substitutive raw materials.

IV. Support Price and Volatility of Price and Trade Volume

In many countries the production of raw material is influenced by support price. In this section we are going to study the effect of support price on the volatility of price and trade volume in a model for markets of gross substitutes. We now modify the supply of our model in the section II as:

$$Q = B_1 P^U + B_2 X + V \tag{22}$$

where P^U is a nx1 vector of support prices.
Our new model consists of (2a) and (1) with the following solutions:

$$P^{**} = C(A_2 Y - B_1 P^U + B_2 X + U - V) \tag{23}$$

with $C = (-A_1)^{-1}$ (see: Takayama, Theorem 4.D.1, p. 392)

$$Q^{**} = B_1 P^U + B_2 X + V \tag{24}$$

The volatility of price and trade volume in equilibrium is given as

$$\text{Var}(P^{**}) = C\,\Omega\,C' \tag{25}$$

$$\text{Var}(Q^{**}) = \text{Var}(V) \tag{26}$$

$$\text{(L4)}\ C > D \text{ i.e. } (-A_1)^{-1} > (B_1 - A_1)^{-1}$$

Proof: $-A_1$ is a matrix with positive diagonal elements and negative off-diagonal elements.

Assuming the existence of positive prices in equilibrium, i.e. $P^* > 0$, which implies (L3) with

$$A_2Y - B_1P^U + B_2X + U - V > 0$$

has the consequence that C is a matrix with all positive elements.

Now we want to show $C > D$:

$$(-A_1)^{-1}(B_1 - A_1) = (-A_1)^{-1} B_1 + I = C \cdot B_1 + I > 0$$
since $C \cdot B_1 > 0$, and
$$(B_1 - A_1)^{-1}(B_1 - A_1) = D \cdot (B_1 - A_1) = I, \text{ therefore}$$
$$C \cdot (B_1 - A_1) - D \cdot (B_1 - A_1) = C \cdot B_1 > 0$$

Thus we proved the lemma.

We compare the variance of prices

$$\begin{aligned}\mathrm{Var}(P^{**}) - \mathrm{Var}(P^{*}) &= C\,\Omega\,C' - D\,\Omega\,D' \\ &= C\,\Omega - C^{-1}\,D\,\Omega\,D'\,C'^{-1}\,C' > 0\end{aligned} \tag{27}$$

Since the matrix in the bracket above has all positive elements, we have proved the following proposition: "*The support prices will increase the volatility (variance) of the price.*" To analyse the effect of support price on the volatility of trade volume we calculate the following difference:

$$\begin{aligned}&\mathrm{Var}(Q^{*}) - \mathrm{Var}(Q^{**}) \\ &= B_1D\,\mathrm{Var}(U)\,D'\,B_1' + (I - B_1D)\,\mathrm{Var}(V)\,(I - B_1D)' - \mathrm{Var}(V)\end{aligned} \tag{28}$$

Since the first and second term on the RS of the above equation are positive and the third term on the RS is a diagonal matrix, all off-diagonal elements of the above equation are positive. The diagonal elements of the above equation can be written out as:

$$b_{ii}^2 \sum_{j=1}^{n} d_{ij}^2 \mathrm{Var}(u_j) + (1 - b_{ii}d_{ii})^2 \mathrm{Var}(v_i)$$
$$+ b_{ii}^2 \sum_{j=i}^{n} d_{ij}^2 \mathrm{Var}(v_j) - \mathrm{Var}(v_i)$$
for $i = 1, 2, \ldots, n$

Thus we proved the following proposition: "The support price reduces the covariance between any trade volume of gross substitutes. The support price reduces in general the variance of trade volume of gross substitutes."

We remark that due to the support price measure the random disturbances in the supply and demand function of other gross substitutes cannot carry over to the trade volume of a good with support price and the random disturbance in the demand function of the good with support price has no influence on the variance of the trade volume of the good considered. But the random disturbance both in the demand and supply function of the good with support price will carry over to every other gross substitutes without support price.

V. New Gross Substitute and the Volatility of Price and Trade Volume: A Corollary

The propositions in the last sections can be applied to answer the question: "How will an introduction of a new gross substitutes influence the volatility of the price and the trade volume of the existing raw materials?"

According to the Le Chatelier principle proved in the section II an introduction of a new gross substitute will increase the volatility of the price and the trade volume of the existing goods.

The influence of a new man-made material on the volatility of the price and trade volume of the existing goods is weaker than that of a new raw material. This is a corollary of the propositions proved in the section III.

The volatility of price and trade volume of a new introduced good will be higher the greater the number of existing gross substitutes.

VI. Conclusion

In this paper we generalized a commodity market model to n gross substitutes. We analyzed the specification errors if some gross substitutes are neglected in a model. Only the case of a linear model is studies in this paper.

Applying the Le Chatelier Principle which is known in economic theory since Samuelson, we proved the following main proposition: "The volatility of market price and trade volume for each the n gross substitutes will be underspecified in a model in which at least one of the gross substitutes is not considered. The degree of the specification error for the price volatility and the trade volume is more serious the higher the number of gross substitutes not considered in a model." The Le Chatelier principle is also true even if some gross substitutes are industrial man-made materials.

The measure of support price will increase the volatility of price of every gross substitutes, but will decrease the volatility of the trade volume.

An important corollary of the Le Chatelier principle is: "An introduction of a new gross substitute will always increase the volatility of price and trade volume of the existing goods" and "an introduction of a new raw material as a gross substitute will increase the volatility of price and trade volume of the existing goods stronger than introducing a man-made material."

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