Income Mobility, Ratchet Effect, and Optimal Level of Redistribution

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This paper hypothesizes that individuals are more sensitive to how their relative position within the various income classes change over time than to their relative position within a class at any one point in time. If the poor perceives no foreseeable escape from poverty, a strong sense of inequity is likely to develop. This creates several important socio-economic problems, including increased crime and social instability. Other problems arise, however, as macro-economic activity is affected by changing individual behavior. Little is known about the relationship between the degree of income mobility and individuals' behavior. The purpose of this paper is to give some insight into how individual behavior changes when degree of income mobility changes. The results of my analysis suggest that policy which increases the degree of income mobility can promote the equity in anticipated utility while reducing the level of redistribution. When income is redistributed through the social security system, this policy promotes capital accumulation and economic growth. (*JEL* Classification : D31, I32, J62)

I. Introduction

Most of the conventional analyses on income distribution have been focused on the static aspects of income distribution. An equity criterion behind of their arguments is the "equity of outcome." One of the rules for distributing income derived from this criterion is "distributing income according to a degree of contribution." "Distributing income according to a degree of necessity," or "distributing income according to a degree of effort" are also candidates for the rules (see Ishikawa 1991). As is well known, these rules of distributing income based on the "equity of outcome" contain the important drawbacks. When

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income is distributed according to a degree of contribution, initial differences in abilities or talents may be ignored. Distributing income according to a degree of necessity brings about dis-incentive effect on labor supply behavior.

Contrary to this static criterion, there exists a dynamic criterion such as the "equality of opportunity." This sense of equity is violated in a society where a son of the rich is always rich and a son of the poor is always poor. Thus, we more or less perceive the degree of "equality of opportunity" by the degree of income mobility across generations. It is conceivable that some individuals are more sensitive to how their relative position in income classes changes over generations than to their relative position at any one point of time. This is because an intergenerational transmission of inequality and an income mobility is uncontrollable to each individual in many cases.

Many researchers have noticed the importance of income mobility in the issues of income distribution. Shorrocks (1976) scrutinized the assumptions used in Markov model, and explored the property of the transition probability matrix which expresses a state of income mobility. Tachibanaki and Atoda (1982) tested the adequacy of these assumptions by using Michigan Panel Data. Shorrocks (1978) proposed the measure of income mobility. Atkinson (1980) estimated the degree of income mobility in the U.K., and Atkinson (1983) analyzed a change in the degree of income mobility by using the *stochastic dominance* conditions. Hart (1976a, 1976b) estimated the degree of income mobility empirically by estimating the serial correlation coefficient.

Recently, the literature on this subject is growing. Studies on a relation between social policy and income mobility is followed by Markandya (1982, 1984). Kanbur and Stiglitz (1986) derived conditions for the transition matrix which leads to the higher equilibrium Lorenz curve for lifetime income. Conlisk (1989) presented a sufficient condition for the theorem given by Kanbur and Stiglitz. Following the approaches of Markandya and Kanbur and Stiglitz, Dardanoni (1990) considered welfare aspects of state of income mobility by comparing streams of income distribution that are generated under different mobility structures. Peters (1992) characterized patterns of the intergenerational mobility in the U.S. and identified an influence of family background characteristics on the mobility. Solon (1992) measured the intergenerational correlation in long-run income by using the U.S. Panel Study of Income Dynamics.

Contrary to the existing researches, this paper examines a relationship between an optimal level of redistribution of income and a degree of income mobility by incorporating the "ratchet effect" into the model. For this purpose, I introduce the modified version of the overlapping generation model, and consider the redistribution of income through the social security system.

This paper is composed of four sections. In section II, the model is presented. The optimal conditions for redistribution of income in a mobile society is presented in section III. In section IV, a numerical example is presented and an impact of a change in income mobility on the optimal tax rate and capital accumulation is examined.

II. Model

A. Ratchet Effect

To introduce an inter-generational relation, a modified version of the Samuelson (1958)-Diamond (1965) overlapping generations model is used. Each individual's life consists of two periods. In the first period, he works, while in the second period he is in retirement. At the beginning of the first period, he determines his lifetime consumption plan.

In this model, we assume that a level of income during the working period is not certain for each individual, although an expected income is certain. When he starts his own life, he inherits the consumption level of his parents, which is a function of parents' lifetime income. In this model, we assume that the standard of living inherited at the beginning of the first period affects the level of utility during the working and the retirement period. This effect is called as "ratchet effect".[1]

When the "ratchet effect" exists, the utility from one unit of consumption during the working and the retirement period is larger when the standard of living inherited from parents is less. Thus, it is assumed that the utility function involves a parameter ε which expresses the "ratchet effect" in consumption.

In this model, the following form of lifetime utility function is considered.

$$U = U(c_1, c_2; \varepsilon i) \quad U_i' > 0, \; U_i'' < 0, \tag{1}$$

[1]The term "ratchet effect" is used differently from the original meanings of "ratchet effect in consumption." There, however, are some similarity between them in the sense that consumption habits acquired in the previous period affect utilities from the consumption in the succeeding periods.

where c_1 and c_2 are consumption during the working and retirement periods, respectively.[2]

B. Income Mobility and Individual Behavior

In this paper, we consider that there exists earnings income mobility from generation to generation.[3] Assuming that there exists a finite number of income classes n, we define a transition probability p_{ij}, which expresses a probability of moving from i-th income class in the father's generation to j-th income class in son's generation. Only the transition probability matrix P, which expresses the degree of income mobility, is a prevailing information about earnings income during the working period.

I define the degree of income mobility by the trace of transition matrix P (see Shorrocks 1978). A decrease in tr P reflects a decreasing proportion of individuals who stay in the same income class as their parents. In this way, an increase in the degree of income mobility can be expressed by a decrease in tr$P$. Although I admit that this measure is the simplest one, this measure has an advantage in a sense that the degree of income mobility is expressed by a single parameter.

It should be noted that an impact of income mobility expressed by the transition probability matrix is considered to be different from an impact of uncertainty about future income level. I will avoid the problems relating to uncertainty for three reasons. First, the impact of changes in income mobility differs among income classes, while the analysis of uncertainty has less obvious relations with an asymmetry among income classes. An increase in income mobility implies an increase in anticipated future income for the poor, but it implies a decrease in anticipated future income for the rich. Second, since the impact of future income uncertainty on savings behavior has been examined by many researchers such as Abel (1985) and Hubbard and

[2]The formulation of the "ratchet effect" could be more specific. For example, one may use $u = u(c_1/c_0, c_2/c_1)$, where c_0 is a standard of living inherited from parents. The referee suggests the it is possible to incorporate the "ratchet effect" into the budget constraint of the "family" more explicitly by using this formulation. At this moment, I prefer to formulate the "ratchet effect" as general as possible.

[3]In this model, we do not consider the bequest motive explicitly regardless of its importance in analyzing the intergenerational transmission of inequality. However, the strength of bequest motive and institutional factors such as an inheritance tax system are included in the transition matrix implicitly.

Judd (1987), it is better to concentrate on the impact of changes in the degree of income mobility via the changes in expected earnings income during the working period. Third, the average income of the individuals whose parents belong to the same income class is certain once the transition probability matrix is given, although the earnings income during the working period is not certain for each individual.

C. Individual Behavior

Each individual determines his optimal savings so as to maximize lifetime utility (1) subject to his budget constraint

$$(1-t)y_1 + \frac{a}{1+r} = c_1 + \frac{c_2}{1+r}, \tag{2}$$

where r is an interest rate, t is a tax rate, a is social security benefits, and y is an earnings income.

Given the level of tax rate, social security benefits, and earnings income during the working period, each individual determines his optimal consumption plan. In this model, earnings income during the working period is linearly related to earnings income of parents via transition probability matrix P. Thus, utility levels are functions of t, a, y_0, r, P, and ε. That is, we arrive at the following indirect utility function:

$$V = V(a, t; r, y_0, P, \varepsilon). \tag{3}$$

III. Optimal Level of Redistribution

In this section, we examine an effect of a change in the degree of income mobility on an optimal level of redistribution of income. Since this model includes an intergenerational linkage of welfare that is expressed by the ratchet effect, the determination of the optimal level of redistribution is not a trivial problem.

In this model, the optimal level of redistribution is determined once an optimal tax rate is determined. The following comparative static analysis gives insight into not only pressures for redistribution in a mobile society, but also the growth capacity in a mobile society, because individual savings behavior is affected by corresponding changes in the optimal tax rate.

A. Conditions for Optimality

Initially, I postulate government behavior. In this paper, we assume an additively separable social welfare function $\mathcal{W}$, which is defined by

$$\mathcal{W} = \sum_{i=1}^{n} V(a,t;r,y_0,P,\varepsilon) f(y_0^i), \tag{4}$$

where V is an indirect utility function and $f(y_0^i)$ is a proportion of individuals whose parents belong to the i-th income class.

In this paper, we assume "Pay-As-You-Go" type social security system. Then, the budget constraint which the government faces is

$$\sum_{i=1}^{n} a^0 f(y_0^i) = \sum_{j=1}^{n} t^1 y_1 f^1(y_1^i), \tag{5}$$

where $f^1(y_1^i)$ is a proportion of individuals whose expected income belong to the j-th income class, a^0 is an annuity benefit received by parents' generation and t^1 is a tax rate levied on son's generation. Since the annuity benefit a^0 is the same for all individuals, (5) is rewritten as

$$a^0 = t^1 \bar{y}_1,$$

where $\bar{y}_1$ is the average income. In the following discussion, the superscripts which express generations will be omitted unless an explicit expression is necessary.

The government determines the optimal tax rate so as to maximize (4) subject to (5). Form the Lagrangean

$$\mathcal{L} = \mathcal{W} + \lambda(a - t\bar{y}_1), \tag{6}$$

then the first-order conditions for the problem are

$$\mathcal{L}_1 = \frac{\partial \mathcal{L}}{\partial a} = \sum_{i=1}^{n} \frac{\partial V}{\partial a} f(y_0^i) + \lambda = 0, \tag{7}$$

$$\mathcal{L}_2 = \frac{\partial \mathcal{L}}{\partial t} = \sum_{i=1}^{n} \frac{\partial V}{\partial t} f(y_0^i) - \lambda \bar{y}_1 = 0, \tag{8}$$

$$\mathcal{L}_3 = \frac{\partial \mathcal{L}}{\partial \lambda} = a - t\bar{y}_1 = 0. \tag{9}$$

Let us denote by Vr'_a and Vr'_t the effect of changes in a and t, respectively, on the indirect utility through the "ratchet effect" Then, (7) and (8) are rewritten as

$$\mathcal{L}_1 = \sum_{i=1}^{n}(\frac{\mu}{1+r} + Vr_a')f(y_0^i) + \lambda = 0, \tag{10}$$

and

$$\mathcal{L}_2 = \sum_{i=1}^{n}(-\mu y_1 + Vr_t')f(y_0^i) - \lambda\bar{y}_1 = 0, \tag{11}$$

where μ is a marginal utility of income. From (7) to (9), we get the following conditions for optimal redistribution of income through the annuity system.

$$\sigma_{W,a} = \sigma_{W,t}, \tag{12}$$

where

$$\sigma_{W,a} = \frac{a}{W}\frac{\partial W}{\partial a},$$

and

$$\sigma_{W,t} = \frac{t}{W}\frac{\partial W}{\partial t}.$$

The L.H.S. of (12) is an elasticity of social welfare with respect to an annuity. The R.H.S. of (12) is an elasticity of social welfare with respect to a tax rate. Thus, the optimality conditions imply that the optimal tax rate is determined at the point where these two elasticities are equal.

B. Formulation of Changes in Income Mobility

I denote the degree of changes in income mobility by ζ. The change in the income mobility is formulated as

$$p_{ij}^1 = p_{ij}^0 - \zeta \quad \text{for } i = j, \tag{13}$$

and

$$p_{ij}^1 = p_{ij}^0 + g_{ij}(\zeta) \quad \text{for } i \neq j,$$

where $g_{ij}'(\zeta) \geq 0$ for all $i \neq j$. Assume that the form of $g_{ij}'(\zeta)$ and the range of ζ is selected so that $p_{ij} \geq 0$ for all i and j. Since the degree of income mobility is defined by the value of tr$P$, an increase in ζ implies an increase in income mobility. On the contrary, a decrease in ζ implies a decrease in income mobility. It is necessary to restrict the form of function $g_{ij}(\zeta)$ so as to maintain the order in sizes of the off-diagonal elements for each row. This excludes the case where an individual with a low income parent has a larger expected income than an individual with a high income parent.

C. The Optimal Tax Rate and the Degree of Mobility

In this paper, a change in income mobility affects the individual's behavior only through a change in expected income. The change in income mobility, however, affects the optimal tax rate not only through a change in individuals' expected income, but also through a change in average income of the economy. To make the discussion clear, we shall consider the case where a change in income mobility keeps an average income constant in the first place.

A) Constant Average Income Case

The effect of a change in the degree of income mobility on the optimal tax rate is examined using the following comparative statics analysis.

$$\frac{dt}{d\zeta}=\sum_{i=1}^{n}\left[\left(\frac{\bar{y}}{1+r}-y_1\right)\mu'_{y1}-\mu+\bar{y}(Vr'_a)'_{y1}+(Vr'_t)'_{y1}\right]\frac{dy_1}{d\zeta}f(y_0^i)/|A|, \tag{14}$$

where

$$\begin{aligned}|A|&=-\bar{y}^2\mathcal{L}_{11}-\bar{y}\mathcal{L}_{21}-\bar{y}\mathcal{L}_{12}-\mathcal{L}_{22}.\\ &=\sum_{1}^{n}\left[\bar{y}\mu'_a\left(y_1-\frac{\bar{y}}{1+r}\right)+\mu'_t\left(y_1-\frac{\bar{y}}{1+r}\right)-\bar{y}^2(Vr'_a)'_a\right.\\ &\quad\left.-\bar{y}(Vr'_t)'_a-\bar{y}(Vr'_a)'_t-(Vr'_t)'_t\right]f(y_0^i).\end{aligned} \tag{15}$$

The determinant of the Hessian matrix ($|A|$) is assumed to be positive so that the solution is maximum.

The sign of (14) is indeterminate a priori, because the behavior of an individual is different depending upon his parents' income class. To understand the implications of this result, it is necessary to examine the individual's behavior for each parents' income class. At the beginning, we consider the behavior of an individual whose parents are poor. In the case where the diagonal element of the transition matrix dominates the off-diagonal elements, the expected income of the individual whose parents are poor is less than the average. Thus, the first bracket in R.H.S. of (14) is positive. Recalling that the marginal utility of income is a decreasing function of income level, μ'_{y1} is negative. Thus, the first term is negative for the individual whose parents are poor. Contrariwise, the first term in R.H.S. of (14) is positive for the individual whose parents are rich. Since μ is larger for the individual whose parents are poor, the absolute size of μ'_{y1} is larger for the poor. As I explained in section II, an increase in income mobility implies an increase in expected income of the individual whose parents are poor, and vice versa for the individual whose parents are rich. That is, $dy_1/d\zeta$ is positive for the individual whose parents are poor, and negative for the individual whose parents are rich. In the absence of the "ratchet effect", the third and the fourth terms in L.H.S. of (14) disappear. Thus, we can conclude that the optimal tax rate decreases as the degree of income mobility increases when the "ratchet effect" is absent, and the shape of income distribution is not skewed strongly to the rich. That is, $dt^*/d\zeta < 0$.

When the "ratchet effect" exists, the story becomes more complex. Concerning the sign of $(Vr_a')_{y_1}'$, I give the following interpretation. The "ratchet effect" means that the utility level of an individual becomes lower as the standard of living inherited from his parents increases. As the annuity benefits increase, the parents' lifetime income increases. Therefore, utility changes through the "ratchet effect" are negative when the annuity benefits increase. This decline of utility decreases as the income in his working period increases. Thus, $(Vr_a')_{y_1}'$ has a positive sign. Contrary to this, Vr_t' takes a positive sign, because an increase in tax rate decreases lifetime income of the parents. This increase in utility is relatively small if earnings income during the working period is large. From this reasoning, $(Vr_t')_{y_1}'$ takes a negative sign. The relative size between $(Vr_a')_{y_1}'$ and $(Vr_t')_{y_1}'$, however, is not determined a priori. In case where the individual is more sensitive to the changes in the expected income through the tax than that through the annuity benefits, it is possible that the "ratchet effect" makes the poor decrease the optimal tax rate when the degree of income mobility increases, while causing the rich to increase it, because $dy_1/d\zeta > 0$ for the poor and $dy_1/d\zeta < 0$ for the rich. The change in the optimal tax rate is determined by the weighted sum of changes in individual's preference in each income class for the redistribution. Thus, whether the optimal tax rate increases or decreases depends on the relative sizes of the rich and the poor classes. When the weight of the poor class is sufficiently larger than the rich, an increase in income mobility (i.e. an increase in ζ) decreases the optimal tax rate ($dt^*/d\zeta < 0$).

This result can be interpreted as follows. When income mobility increases, an increase in the expected earnings income of individuals whose parents are poor works to decrease the tax rate, because their incentive for redistribution decreases, reflecting an increase in their anticipated lifetime income. This is shown in the first two terms in R.H.S. of (14).

When the "ratchet effect" exists, a decline in standard of living decreases utility, and an improvement in standard of living increases utility. For the person whose parents are rich, an increase in income mobility implies an increase in the probability of decline in standard of living. In this case, an increase in redistribution of income lessens the decline in standard of living. Therefore, individuals whose parents are rich desire an increase in the optimal tax rate as the degree income mobility increases. On the other hand, an increase in income mobility implies an increase in expected earnings income of the individual whose parents are poor. In other words, the probability of improvement in the standard of living of the individual whose parents are poor increases. The decrease in the level of redistribution increases the gap of this improvement. For this reason, the individual whose parents are poor desires a decrease in the optimal tax rate as the degree of income mobility increases.

B) No Constant Average Income Case

In this subsection, I examine the case where the changes in the degree of income mobility changes the average earnings income of the economy. When the changes in income mobility affect the average earnings income, we must modify (14) as follows:

$$\begin{aligned}\frac{dt}{d\zeta} = \Bigg(& \sum_{i=1}^{n}\left[\left[\frac{\bar{y}}{1+r} - y_1\right]\mu'_{y1} - \mu + \bar{y}(Vr'_a)'_{y1} + (Vr'_t)'_{y1} \right]\frac{dy_1}{d\zeta} f(y_0^i) \\ & + \bar{y}'_\zeta \sum_{i=1}^{n}\left[-\lambda + t\left[\left(\frac{\bar{y}}{1+r} - y_1\right)\mu'_a - (Vr'_a)'_a + (Vr'_t)'_a\right] \right] f(y_0^i) \Bigg) / |A|, \end{aligned} \tag{16}$$

where λ is marginal social welfare of annuity benefits. The first summation of R.H.S. in (16) is the same as that of (14).

If we evaluate (16) at $t = 0$, it becomes clear that the changes in average income affects the optimal tax rate by $-\bar{y}'_\zeta\lambda$. In other words, when an increase in income mobility (i.e. decrease in ζ) increases $\bar{y}'$, the optimal tax rate decreases, because the higher average income affords the improvement in standard of living without increasing the tax burden.

If (16) is evaluated at $t > 0$, the optimal tax rate is affected through changes in the marginal utility of income and through the "ratchet effect." Since μ'_a has a negative value, the effect of changes in the marginal utility is positive for the rich and is negative for the poor, when $\bar{y}'_\zeta > 0$.

If the "ratchet effect" is relatively small and the weight of the poor is relatively large, the effect of changes in income mobility through changes in average income on the optimal tax rate is negative, when $\bar{y}'_\zeta > 0$.

IV. Numerical Example

When the "ratchet effect" exists, the above analytical discussion suggests that the effects of changes in income mobility on the optimal tax rate is complicated. The purpose of this section is to examine the changes in optimal tax rate and corresponding individual saving's behavior numerically when both the degree of income mobility and the degree of the ratchet effect change.

A. Formulation of Changes in Income Mobility

First, we specify the transition probability matrix which expresses a state of income mobility. In specifying the mobility matrix, it is assumed that the transition probability decreases as the distance of moving increases. Thus, the transition probability from i-th income class to j-th income class, p_{ij}, is a function of $|i-j|$. Formally, the transition probability is written as

$$p_{ij} = h(|i-j|), \tag{17}$$

where $h' < 0$, $p_{ij} \geq 0$ for all i and j, and $\sum_{j=1}^{n} p_{ij} = 1$ for all i. One candidate which satisfies (17) is

$$p_{ij} = \frac{(n-1)^2-(i-j)^2}{n(n-1)^2-\sum_{j=1}^{n}(i-j)^2}. \tag{18}$$

Second, we specify the changes in income mobility. Let us denote the changes in transition probability by g_{ij}. Given the value of ζ, g_{ij} must satisfy the following conditions.

1) $\sum_{j \neq i} g_{ij} = \zeta$.
2) $g_{ij} \geq 0$ for all i and j if $\zeta > 0$, and $g_{ij} \leq 0$ for all i and j if $\zeta < 0$.
3) $|g_{ij}|$ decreases as $|i-j|$ increases.

One candidate of g_{ij} for positive ζ is

$$g_{ij} = (\alpha+\beta|i-j|)\frac{\zeta}{n-1}, \tag{19}$$

where $$\alpha = p_{in}\frac{n-1}{\zeta}-(n-1)\frac{(n-1)-(n-1)\{p_{in}(n-1)/\zeta\}}{-(n-1)^2+[(n-i)(n-i+1)+(i-1)i]/2},$$

and $$\beta = \frac{(n-1)-(n-1)[p_{in}(n-1)/\zeta]}{-(n-1)^2+[(n-i)(n-i+1)+(i-1)i]/2}.$$

For negative ζ,

$$g_{ij} = (\alpha - \beta|i-j|)\frac{\zeta}{n-1}, \tag{20}$$

where $$\alpha = \frac{(n-1)^2}{(n-1)^2-[(n-i)(n-i+1)+(i-1)i]/2},$$

and $$\beta = \frac{(n-1)}{(n-1)^2+[(n-i)(n-i+1)+(i-1)i]/2}.$$

Let us consider the case where n = 10. Figure 1 is the transition probability when ζ = –0.4. Figure 2 depicts the case when ζ = –0.1. A comparison of these two figures will help us understand how transition probability and changes in income mobility are specified.

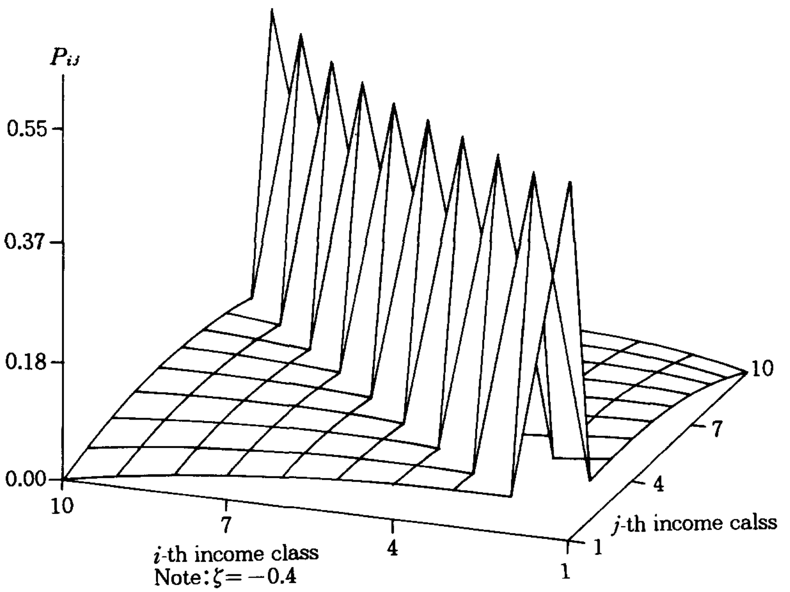

FIGURE 1
STATE OF TRANSITION PROBABILITY
Note: ζ = –0.4

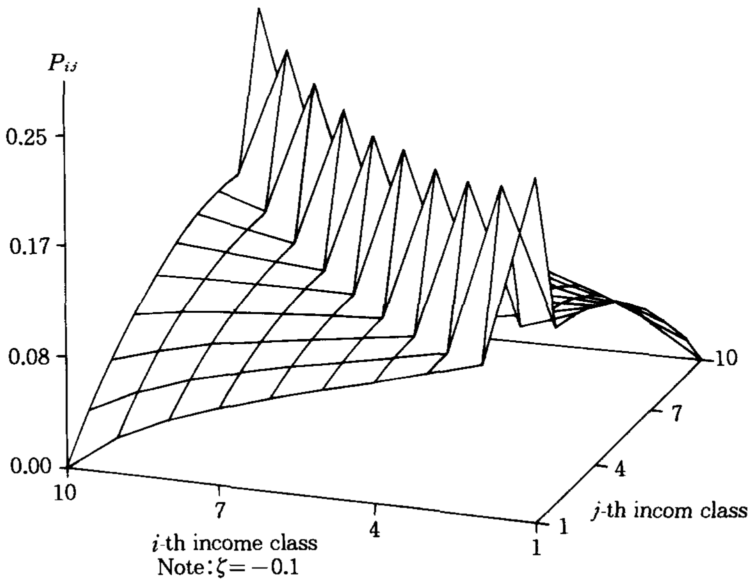

FIGURE 2
STATE OF TRANSITION PROBABILITY
Note: $\zeta = -0.1$

B. Optimal Level of Redistribution and Changes in Mobility

A) Utility Function and Social Welfare Function

We assume the following additively separable utility function.

$$U = \kappa_1 \log \frac{c_1}{1+\varepsilon(1-t)(y_0-y_1)} + \kappa_2 \log c_2, \tag{21}$$

where κ_1 and κ_2 are parameter values which express the weights of consumption during working and retirement periods, respectively. This utility function includes the "ratchet effect." The degree of this effect is expressed by ε, whose range is chosen so that the utility function has a positive value. This utility function is produced so that the utility level decreases when an individual's earnings income is less than that of his parents, and the utility level of the individual whose earnings is larger than that of his parents increases as the degree of the "ratchet effect" increases.

We modify the social welfare function with incorporating transition probabilities explicitly as follows:

$$W = \frac{1}{\xi}\sum_{i=1}^{n}\sum_{j=1}^{n}(\kappa_1 \log \frac{c_1^j}{1+\varepsilon(1-t)(y_0^i - y_1^j)} + \kappa_2 \log c_2^i)^{\xi} p_{ij} f_d(y_0^i), \tag{22}$$

where ξ is a parameter representing preference for social inequality, p_{ij} is a transition probability of moving from i-th income class in parents' generation to j-th income class in sons' or daughters' generation, and f_d (y_0^i) is a proportion of individuals who belong to the i-th income class in parents' generation. Using a Newton-Rapson method, the optimal tax rates for several parameter values are calculated. In this calculation, we set the parameter values as $r = 0.3$, $\kappa_1 = 0.5$, and $\kappa_2 = 0.5$.

I presume initial income distribution to be uniform to concentrate on the impact of changes in the degree of income mobility. Using an uniform distribution has an advantage in selecting the level of ξ, because a small value of ξ directly implies the large weight of the poor class when distribution is uniform. In addition, a constant average income for various degree of income mobility is required for the analysis of changes in savings amount. Thus, we consider the case where the average income does not change when the degree of income mobility changes as a first step of research.

B) Optimal Tax Rate and Degree of Income Mobility

Figure 3 shows the movement of the optimal tax rate corresponding to the changes in income mobility and the degree of "ratchet effect". This figure is produced for $\xi = -4.0$. In this case, large weight is given to the lower income classes. Recalling that an increase in ζ implies an increase in income mobility, the degree of income mobility is smallest at $\zeta = -0.4$ and largest at $\zeta = -0.1$ in this figure. The degree of "ratchet effect" increases as ε increases.

From this figure, one notices that the optimal tax rate decreases as the degree of income mobility increases. That is, the level of redistribution must be large in the economy where the degree of income mobility is small. This result is still valid even in the presence of the "ratchet effect", at least in the case where the weight of the poor income class is relatively large. This result provides important justification for income redistribution in an immobile society. In such a society, a larger level of redistribution is necessary to increase social welfare. On the other hand, the necessity for redistribution decreases as the degree of income mobility increases. Thus, the policy which promotes income mobility lessens the necessity for income redistribution.

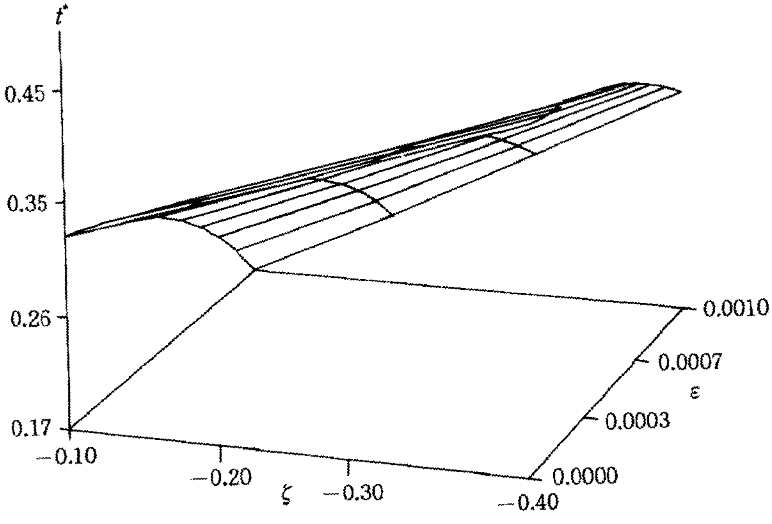

FIGURE 3
OPTIMAL TAX RATE AND INCOME MOBILITY
Note: $\xi = -4.0$

Concerning the "ratchet effect", one notices that the optimal tax rate decreases as the degree of "ratchet effect" increases when the weight of the poor is large. The individual whose parents are poor is less likely to experience a decline in standard of living. If he can raise himself to a higher income class, he gains utility from the improvement in his standard of living itself. The less the level of redistribution, the greater will be the improvement. Thus, the optimal level of redistribution decreases as the degree of "ratchet effect" increases. Since the likelihood of rise in income class increases for the poor as the degree of income mobility increases, the decline in the optimal tax rate responding to the increase in the "ratchet effect" accelerates as the degree of income mobility increases.

As is stated in the theoretical part of this paper, the response of individuals whose parents are rich to the changes in the degree of "ratchet effect" is the opposite of the response described above. They try to avoid a decline in their standard of living in more earnest, as the "ratchet effect" gets stronger. Thus, it is in their interest to increase the level of redistribution.

Finally, it should be noted that the results derived from this numeri-

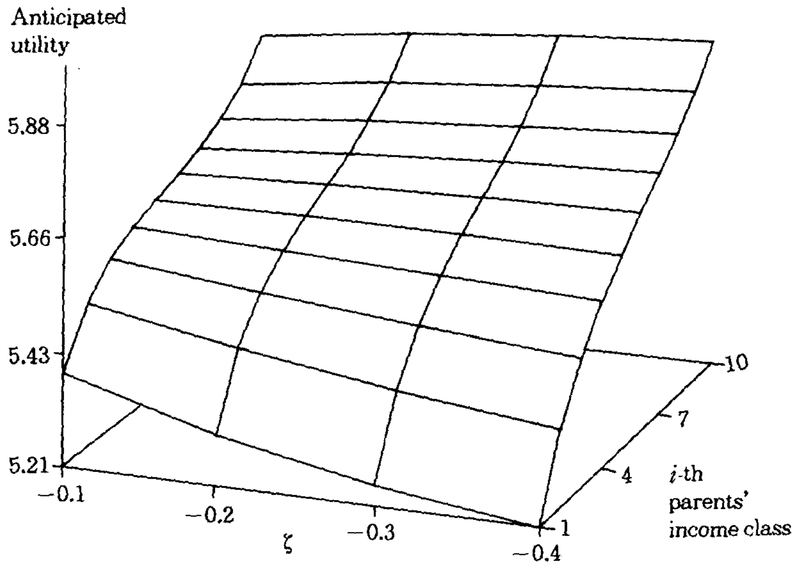

FIGURE 4
ANTICIPATED UTILITY AND INCOME MOBILITY
Note: $\varepsilon = -0.001$ and $\xi = -4.0$

cal example are consistent with the results obtained in the theoretical analyses.

C) Anticipated Utility and the Degree of Income Mobility

In Figure 4, the relation between anticipated utility and income mobility is plotted. This graph is generated when $\varepsilon = 0.001$. It seems logical that anticipated utility increases as the income level of a parent increases. In interpreting this figure, however, it should be noted that this figure is produced by changing the optimal tax rate. This figure shows that the equalization of anticipated utility is achieved with smaller redistribution of income, when the degree of income mobility increases. In other words, inequality of anticipated utility is larger in an immobile society, even though the level of redistribution is larger. This result has an important policy implication. Since an increase in tax rate is very difficult in a real world, the policy which promotes income mobility can be a more effective method for achieving equity.

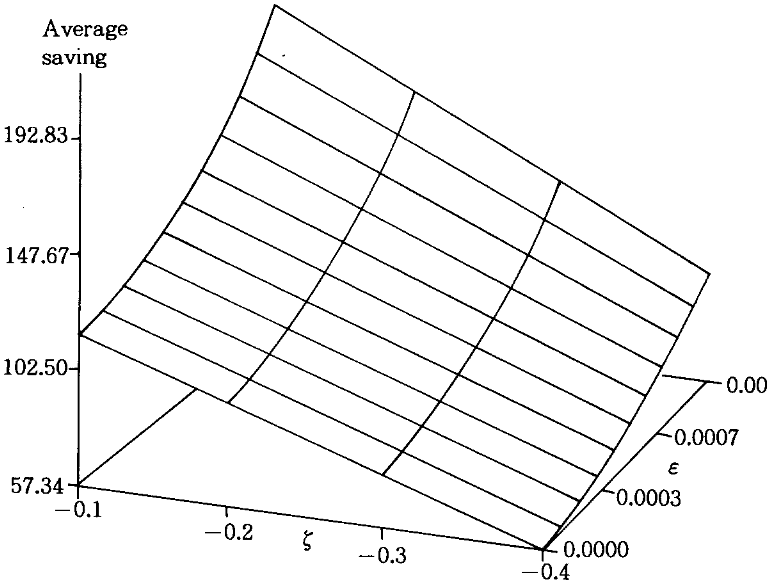

Figure 5
Average Savings and Income Mobility
Note: $\xi = -4.0$

D) Savings and the Degree of Income Mobility

The final objective of this paper is to investigate how average savings change as the degree of income mobility changes. Average savings are calculated by summing the savings of each income class with weight $fa\left|(y_i^t)\right|$. Figure 5 plots the relation between average savings and the degree of income mobility for each ε. It shows that the savings increase as the degree of income mobility increases for each level of ε. In this model, the income is redistributed through the social security system. Under the "Pay-As-You-Go" social security system, a decrease in the tax rate increases average savings, because private savings are replaced by the "social security." This analysis suggests that an increase in the degree of income mobility promotes capital accumulation with improving the equality of anticipated utility by decreasing tax rate. In other words, the policy which increases the degree of income mobility should be emphasized not only to achieve a sense of equity, but also to promote economic growth.

TABLE 1
OPTIMAL TAX RATE FOR STATIONARY DISTRIBUTION

ε	ξ	Optimal Tax Rate
0.01	−0.4	0.59893694
0.01	−0.3	0.59890717
0.01	−0.2	0.59888778
0.01	−0.1	0.59887339
0.02	−0.4	0.59886025
0.02	−0.3	0.59884777
0.02	−0.2	0.59883701
0.02	−0.1	0.59882756
0.03	−0.4	0.59881834
0.03	−0.3	0.59880923
0.03	−0.2	0.59880092
0.03	−0.1	0.59879326
0.04	−0.4	0.59878566
0.04	−0.3	0.59877807
0.04	−0.2	0.59877096
0.04	−0.1	0.59876427
0.05	−0.4	0.59875758
0.05	−0.3	0.59875087
0.05	−0.2	0.59874450
0.05	−0.1	0.59873843

Note: $\xi = -1.0$

E) Stationary Distribution and Optimal Tax Rate

In the above numerical example, we presumed that the initial distribution is uniform so that the average income is kept constant. By keeping the average income constant for various degree of income mobility, we can separate the effect of changes in the degree of income mobility from the changes in average income. This presumption, however, has drawbacks in the sense that the income distribution we considered has no relation with the transition probability matrix. To reflect the characteristics of transition probability matrix on income distribution, the stationary distribution should be used. The analysis using a constant distribution may be interpreted as the optimal redistribution problem of the first generation, where all the future generations are somehow bound to honor the policy set by their ancestor. This interpretation is not so appealing because there is no positive reason why the first generation can behave as a dictator.

To answer for this criticism, I examined how the optimal tax rate changes as the degree of income mobility changes by using the stationary income distribution generated for each degree of income mobility. Thus, the initial distribution changes as the degree of income mobility changes. The result of the calculation is shown in Table 1. Since our main concern is how the optimal tax rate changes according to the changes in the degree of income mobility, only the optimal tax rate is listed. If we get the relation between the optimal tax rate and the degree of income mobility, we can infer about the changes in the anticipated utility and savings from the above numerical example.

From Table 1, we find that the basic relationship between the degree of income mobility and the optimal tax rate is the same with that in the above numerical example where the initial distribution is kept constant. The optimal tax rate decreases as the degree of income mobility changes, although this calculation includes the changes in the initial distribution accompanied by the changes in the transition matrix. This numerical example where the stationary distribution is used strengthen the results derived in the upper part of the paper.

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