The Dynamics of Price of Equity, Capital Accumulation and Current Account in an One-Good Optimizing Model

Kiyoung Lee*

This paper analyzes the effects of changes in fiscal policies or world interests on the dynamics of the price of equity, the level of capital accumulation and the current account for a small open economy in which the process of capital accumulation entails adjustment costs. It emphasizes the role of production technology as a determinant of the dynamics of the current account. Also, the interactions among the price of equity, capital accumulation, and the current account are studied by using the fact that in the one-good model with adjustment costs, the shadow price of installed capital can be interpreted as the price of equity with some innocuous assumptions. (*JEL* Classification: E62, H87)

I. Introduction

The purpose of this paper is to analyze the effects of changes in fiscal policies or world interest rates on the dynamics of the price of equity, the level of capital accumulation and the current account for a small open economy. The model employed is a one-good, infinite-horizon optimizing model in which the process of capital accumulation entails adjustment costs.[1] Emphasis will be given to the interactions among

*Korea Tax Institute, 60 Yang Jae-Dong, Seocho-Ku, Seoul 137-130, Korea. I am grateful to Costas Azariadis, William B. English and Andrew Foster for helpful discussions and comments. I also thank to two anonymous referees. Any remaining errors are mine.

[1]The standard model of optimal growth, interpreted as a model of a market economy with infinitely long-lived agents does not allow separation of the savings decisions of agents from the investment decision of firms. Investment is essentially passive: the "one good" assumption leads to a perfectly elastic supply; the absence of installation costs for investment leads to a perfectly elas-

the price of equity, capital accumulation, and the current account.

Models of the current account based on intertemporal optimization have become standard for the analysis of disturbances in an open economy. An advantage of the explicit optimization framework is the light it throws on the interaction between private economic decisions and the balance sheets of the government. By emphasizing the intertemporal aspects of decisions to save and invest, the current account can be viewed as a mechanism by which countries can transfer consumption across time. As a result, current account imbalances generally represent the optimal level of external borrowing or lending. Analyses of current account determination in optimizing models can be divided into four categories depending on the time horizon and the structure of the goods markets in the models. One group of models uses a two-period setting while the other uses an infinite-horizon setting. The two-period setting is useful for the analysis of current account in the medium to long run while the infinite-horizon setting is useful for studying the dynamic adjustment of the current account. Models of the current account also differ in their inclusion or exclusion of a non-traded sector of the economy. Excluding the non-traded sector focuses on the determination of the current account as the difference between aggregate income and expenditure while including the non-traded sector emphasizes the relative price of non-traded goods (i.e., the real exchange rate) in the behavior of the current account. Thus the one-good, infinite-horizon model adopted in this paper is appropriate for the study of the dynamics of the current account viewed as the difference between income and absorption.

The one-good, infinite-horizon model with investment expenditure is best represented by Frenkel and Rodriquez (1975) and Blanchard (1983), who model investment by using the microeconomic theory of the firm subject to increasing adjustment costs; by Persson and Svensson (1985), who use an overlapping generations model to determine investment; and by Obstfeld (1986), who employs a time-to-build restrictions to determine investment.[2]

tic investment demand. By relaxing one of the two assumptions which imply passive investment behavior, we can study the movement of the temporary equilibrium over time in response to a particular shock or policy. This can be done either by introducing a two sector technology which generates a well-defined investment supply function (Uzawa 1964), or it can be done by introducing installation or adjustment costs which generate a well-defined investment demand function (Abel and Blanchard 1983; Hyashi 1982).

Unlike these earlier papers, this paper emphasizes the role of production technology as a determinant of the dynamics of the current account. Also, the interactions among the price of equity, capital accumulation, and the current account are studied by using the fact that in the one-good model with adjustment costs, the shadow price of installed capital can be interpreted as the price of equity with some innocuous assumptions. Thus, by adopting a one-good optimizing model in which there is capital accumulation, this paper interprets large swings in equity prices observed in reality as reflecting the outcome of optimal decisions by households and firms in response to changes in fiscal policies or world interest rates.3

There are four major findings. In an economy with low values of the world interest rate (r), the tax rate (τ), and the second derivative of the production function (f''), it is more likely that a decreasing price of equity is associated with a current account deficit along the equilibrium path. Moreover, the price of equity is more volatile in such an economy. Second, a permanent tax cut causes the price of equity to jump upwards and then decrease to the original equilibrium level. During the process a current account deficit is more likely to occur in an economy with low values of τ, r, and f''. At the new steady state equilibrium, the capital stock will be increased. In contrast, a temporary tax reduction causes the price of equity to jump upwards (but less than in the case of a permanent change) and to converge to the original level after fluctuating around it during the adjustment process. Meanwhile, alternations of current account deficit and surplus will occur. The steady state equilibrium point doesn't change. Third, an increase in government spending will have no effect on the price of equity and the capital stock. While a permanent change will not affect the current account, a temporary one will cause a current account deficit. Fourth, the effects of a decrease in the world interest rate are qualitatively similar to those of a permanent reduction in the tax rate except for its effect on the

[2]Sachs (1981) is a good example of the one-good two-period models of the current account that include investment. Bruno (1982) and Murphy (1986) are examples of optimizing two-period models of the current account that include investment and a non-traded sector. Brock (1988) is the example of optimizing infinite-horizon model of the current account that include investment and a non-traded sector.

[3]Frenkel and Razin (1986) and Edwards (1987) present related views. Another interpretation for the volatility of stock prices is that it is an evidence of equity prices diverging from market fundamentals. Shiller (1984), for instance, describes stock prices as being determined in part by "fads".

level of consumption.

Thus, the analysis demonstrates how changes in fiscal policies or world interest rates can generate sustained movements in equity prices simply because investment requires scarce resources. It also suggests important differences in the effects of a change in the government budget deficit depending on the manner in which the change is carried out. Also, by characterizing the conditions under which the optimally determined current account and the price of equity have specific relationships, we can learn a small policy lesson. i.e., In an economy with low values of the world interest rate (r), the tax rate (τ), and the curvature of the production function (f''), the price of equity is more volatile. Thus to avoid unnecessary volatility of equity prices, more conservative fiscal policies (e.g. high tax-rate policies) are required.

The plan of the paper is as follows. Section II develops the basic model. Section III provides a graphical solution and a characterization of the saddle path and the conditions under which a decreasing price of equity is more likely to be associated with a current account deficit along the equilibrium path. Section IV presents the results of how the economy responds to the changes in fiscal policy and the world interest rate. The paper concludes in section V with a summary of its findings.

II. The Model

Consider a small economy that takes as given the foreign currency prices of all goods. These commodities can be regarded as a single composite good whose price is determined in the world markets. Since the economy is small relative to the rest of world, residents take the price of the good as given.

A. Value Maximization by Firms

Output of the good in the country is produced by perfectly competitive firms which use the services of labor and installed capital with constant returns to scale technologies. For simplicity, we assume that one unit of labor is supplied inelastically at all times by the households and the number of firms is equal to the number of agents so that the same symbol denotes the ratio of a variable per capita or per firm. As labor is in fixed supply, this normalization implies that there will be one worker per firm.

The are two perfectly substitutable assets in the economy; an internationally traded bond paying the fixed world interest rate (r) and a domestically traded equity claim on firms. Arbitrage in the domestic asset markets equates the expected yields on claims to installed capital with the interest rate on bonds:

$$\frac{\dot{V}+D}{V}=r \tag{1}$$

The expected yield on an equity claim to installed capital is composed of the expected capital gain on equity ($\dot{V}/V$) plus the dividend return (D/V). Residents of the economy are assumed to have perfect foresight so that the expected capital gain equals the actual capital gain.

In order to undertake gross investment of J units of capital, J units of output must be set aside to be installed as capital, together with $Jh(J/K)$ units which are used during installation.[4] Thus, gross investment at rate J has an opportunity cost of $J(1 + h(J/K))$ units of output. Assuming that the installation cost is an increasing function of the amount of capital formation relative to the existing capital stock, total investment expenditure is given by the following:

$$I=J[1+h(\frac{J}{K})], \text{ where } h(0)=0,\ h'>0,\ h''>0. \tag{2}$$

The assumption that the marginal cost of installation is nondecreasing in (J/K) ensures the determinacy of the capital stock despite constant-return-to-scale and perfect competition.

The representative firm is assumed to choose investment so as to maximize its market value. By assuming that the firm finances investment by retained earnings[5] and that firms have issued no previous debt, the number of equity claims outstanding will be constant and the market value of an individual firm will be proportional to the value of an equity claim. Then, integration of (1), ruling out speculative bubbles by assumption, gives the firm's market value:

$$V(t)=\int_t^{\infty} D(s)\exp^{-r(s-t)}ds, \tag{3}$$

where the number of equity shares has been normalized to unity for

[4]This formalization of adjustment costs is based on Eisner and Strotz (1963) and Lucas (1967).

[5]See Abel and Blanchard (1983) for the case where firms finance themselves both by retaining earnings and by issuing new bonds. Also see Abel (1982) for the case where the firm is financed entirely by equity.

convenience. Equation (3) defines the market value of a firm as equal to the present discounted value of its dividend stream. Dividends are equal to after-tax revenue from selling output less its investment and wage payments:

$$D = (1-\tau)[f(K) - W] - J[1 + h(\frac{J}{K})] \tag{4}$$

where τ is the tax rate on profit.

The decision problem of the representative firm is to choose a time path of investment which maximizes its market value subject to the accumulation constraint for the capital stock:

$$\begin{aligned} &\max \int_0^\infty \{(1-\tau)[f(K) - W] - J[1 + h(\frac{J}{K})]\} \exp^{-rt} dt \\ &\text{s.t. } \dot{K} = J - \delta K \end{aligned} \tag{5}$$

where δ is a constant rate of depreciation.

The present-value Hamiltonian for this problem is:

$$H = \{(1-\tau)[f(K) - W] - J[1 + h(\frac{J}{K})] + q[J - \delta K]\} \exp^{-rt}$$

where q is the shadow price of an additional unit of installed capital.

The optimality conditions are:[6]

$$q = 1 + h(\frac{J}{K}) + (\frac{J}{K})h'(\frac{J}{K}) \equiv G(\frac{J}{K}) \tag{6}$$

$$\dot{q} = (r+\delta)q - [(1-\tau)f'(K) + (\frac{J}{K})^2 h'(\frac{J}{K})] \tag{7}$$

$$\lim_{t\to\infty} \exp^{-rt} qK = 0. \tag{8}$$

Consider (6) first. $G(J/K)$ is the marginal opportunity cost of investment. Thus the condition states that investment takes place until marginal cost of investing equals the shadow value of the installed capital. Since $G'(J/K) > 0$, the equation can be inverted to give (J/K), the rate of capital formation, as an increasing function of q in a manner similar to Tobin's q theory of investment:

$$\frac{J}{K} = x(q), \quad x'(q) = \frac{1}{2h' + xh''} > 0. \tag{9}$$

[6]This problem satisfies the conditions of Weitzman's theorem [10], so that these conditions are necessary and sufficient.

Consider (7). $(J/K)^2h'(J/K)$ is the reduction in the opportunity cost of installation $Jh(J/K)$ made possible by an additional unit of capital. Therefore $(1 - \tau)f'(K) + (J/K)^2h'(J/K)$ is the total marginal product of capital. Equation (7) can be viewed as an arbitrage condition. If investors can earn a rate of return r on international bonds, then in the absence of uncertainty, arbitrage would force the price of capital q to be such that the rate of return on capital, net of depreciation, $[\{(1 - \tau)f'(K) + (J/K)^2h'(J/K)\}/q + (\dot{q}/q) - \delta]$, is equal to r. Equation (7) also describes the evolution of the marginal value of installed capital. Condition (7) can be solved subject to the transversality condition (8) to give:

$$q_t = \int_t^{\infty}[(1-\tau)f'(K)+(\frac{J}{K})^2 h'(\frac{J}{K})]\exp^{-(r+\delta)(s-t)}\,ds. \tag{10}$$

The shadow price q is the present value of marginal products.

There is a simple relationship between the value of the owners' equity, the shadow value q, and the capital stock:

$$qK = V. \tag{11}$$

It says that "average q" and "marginal q" are equal.[7] In other words, the market value of an equity claim on a unit of installed capital equals the marginal value of an additional unit of installed capital. Accordingly, we can identify the variable q as the price of equity measured in terms of goods.

By using (9) to substitute for the rate of capital formation in equations (2), (5) and (7), we obtain relationships describing investment expenditure, the dynamics of the price equity, and the dynamics of the capital stock as functions of q and K:

$$I = x(q)K[1 + h(x(q))] \equiv I(q, K) \tag{12}$$

$$\dot{q} = (r + \delta)q - [(1 - \tau)f'(K) + x(q)^2h'(x(q))] \equiv \phi(q, K) \tag{13}$$

$$\dot{K} = [x(q) - \delta]K \equiv \Theta(q, K) \tag{14}$$

where the partial derivatives of the functions are given as:

$$I_q = qKx' > 0,\ I_K = x(1 + h) > 0,$$

[7]This equality depends on the production and investment installation functions being linearly homogeneous in labor, capital and investment. See Hayashi (1982), Abel and Blanchard (1983). The proof is a special case of Hayashi (1982) and is given in Appendix.

$$\phi_q = r + \delta - x,\ \phi_K = -(1-\tau)f''(k) > 0,$$
$$\Theta_q = x'K > 0,\ \Theta_K = x - \delta.$$

The two derivatives with ambiguous signs (ϕ_q and θ_K) will both be determined at the steady-state equilibrium for the model where the rate of capital accumulation equals the rate of depreciation ($x = \delta$). For a given path of exogenous variables, these equations determine how investment, the capital stock, and the price of equity evolve through time.

B. Utility Maximization by Consumers

Each consumer supplies one unit of labor inelastically and receives a wage w. His only decision problem is to choose a sequence of consumption which maximizes the present value of utility:

$$U = \int_t^{\infty} u(c_s)\exp^{-\beta(s-t)}\,ds, \tag{15}$$

where β is the constant rate of time preference.[8] Equation (15) is maximized subject to an accumulation constraint that sets the change in household bond holdings (B_h) equal to the excess of income over consumption expenditure:

$$\dot{B}_h = rB_h + D + (1-\tau)w + R - C, \tag{16}$$

where income is the sum of after-tax wage ($(1-\tau)w$), interest on bond holdings (rB_h), dividend (D) and lump-sum transfer payments (R). It must be allocated either to consumption or to savings in the form of bonds. The present value Hamiltonian for this problem is the following:

$$H = \{u(c) + \lambda[rB_h + D + (1-\tau)w + R - C]\}\exp^{-\beta t}, \tag{17}$$

where λ is the multiplier on household's accumulation constraint.

The optimality conditions are:

$$u'(c) = \lambda \tag{18}$$

$$\dot{\lambda} = (\beta - r)\lambda \tag{19}$$

$$\lim_{t\to\infty} \exp^{-\beta t}\lambda B_h = 0 \tag{20}$$

With a constant world interest rate and constant rate of time prefer-

[8]See Uzawa (1968) and Obstfeld (1981a, b) for the models with a time-varying rate of time preference.

ence, we must impose the condition $r = \beta$ to ensure the existence of a steady state with a non-zero but finite level of consumption. Therefore, from (18) and (19), consumers will smooth consumption for any given path of life-time income. Using (4), (16) and (20), we can express the consumption as the annuity value of life-time income.

$$c = rB_h + r\int_t^{\infty}[(1-\tau)f(K)+R-I]\exp^{-r(s-t)}ds \tag{21}$$

C. The Government

The government is assumed to finance transfer payments and its purchase of consumption good through a tax on incomes from labor and firm's profit and by borrowing on international markets. We assume that the government obeys its intertemporal budget constraint so that the present value of future receipts minus the present value of future expenditures exactly offsets the initial stock of government debt. The accumulation constraint for government debt is given as:

$$\dot{B}_g = rB_g + g + R - \tau f(K) \tag{22}$$

where B_g is the stock of government debt. With a no-Ponzi-game condition,[9] $\lim_{t\to\infty} \exp^{-rt}B_g = 0$, residents in this economy fully anticipate that the government budget will balance intertemporally:

$$B_g = \int_t^{\infty}[\tau f(K) - g - R]\exp^{-r(s-t)}ds, \tag{23}$$

where the government is assumed to adjust transfers (R) whenever a change in spending or tax revenue occurs so as to ensure that equation (23) holds.

D. The Current Account

Substitution of the government flow budget constraint (22) into the consumers budget constraint (16) gives the following:

$$\dot{B} = f(K) + rB - I(q, K) - c - g, \text{ where } B \equiv B_h - B_g. \tag{24}$$

It demonstrates the equality of the capital account deficit and current account surplus. In other words, it equates the home country's foreign lending to the excess of domestic income over absorption by the domestic private and public sectors. It describes the equilibrium path

[9]See O'Connel and Zeldes (1988) for a rational Ponzi game.

of foreign bond holdings also. From (21) and (23), consumption can be written as:

$$c = rB + r\int_t^{\infty}[f(K) - I - g]\exp^{-r(s-t)}ds. \tag{25}$$

By employing (25) to substitute for consumption in (24), the current account surplus can be expressed as:

$$\dot{B} = f(K) - I(q,K) - g - r\int_t^{\infty}[f(K) - I(q,K) - g]\exp^{-r(s-t)}ds. \tag{26}$$

This equation states that the current account will be in surplus or deficit, respectively, depending on whether $[f(K) - I(q, K) - g]$ is falling or rising over time. For a given level of government spending, the current account will be in deficit during a phase of capital accumulation and in surplus during a phase of capital decumulation under some conditions.[10]

III. The Equilibrium Dynamics

The dynamics of the equilibrium in this economy can be described as following three equations system already derived in (13), (14) and (24).

$$\dot{g} = (r + \delta)q - [(1 - \tau)f'(K) + x(q)^2h'(x(q))] \equiv \phi(q, K) \tag{13}$$

$$\dot{k} = [x(q) - \delta]K \equiv \Theta(q, K) \tag{14}$$

$$\dot{B} = f(K) + rB - I(q, K) - c(q, K, B) - g \tag{24}$$

All the optimality conditions of households and firms [(6), (7), (8), (18), (19) and (20)], the budget constraints of households, firms and government [(5), (16) and (22)] and the budget constraint of the economy as a whole (24) are captured in these three equations. This system is recursive one because the equilibrium path of q and K can be determined by first two equations independent of the third one, and the evolution of the equilibrium foreign bond holdings can be described with the help of the equilibrium path of q and K determined in (13) and (14), given the initial value of B. Thus, we can concentrate on the first two equations.

To solve the model, we linearize equations (13) and (14) around the steady state, i.e., $\dot{q} = \dot{K} = 0$.

[10]The conditions will be characterized in section III.

$$\begin{bmatrix} \dot{q} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} \bar{\phi}_r & \bar{\phi}_L \\ \bar{\Theta}_r & \bar{\Theta}_L \end{bmatrix} \begin{bmatrix} q-\bar{q} \\ K-\bar{K} \end{bmatrix} = \begin{bmatrix} r & -(1-\tau)f'' \\ x'K & 0 \end{bmatrix} \begin{bmatrix} q-\bar{q} \\ K-\bar{K} \end{bmatrix}, \tag{27}$$

where bar(–) denotes the stationary value. The trace of the system's matrix is r (>0) and the system's determinant is $(1 - \tau)f''x'K(<0)$. Since this trace is equal to the sum of the system's characteristic roots and the determinant is equal to the product of the system's characteristic roots, this proves that in the neighborhood of long-run equilibrium where $q = \dot{K} = 0$ is a saddle point and the system has a unique convergence path. For constant levels of exogenous variables, the solution for q and K along the convergence path is given in general form as:

$$\begin{aligned} q - \bar{q} &= \omega_2 z \exp^{\mu u} \\ K - \bar{K} &= \omega_3 z \exp^{\mu u}, \end{aligned} \tag{28}$$

where z is an arbitrary constant determined by the initial value of K, μ is the negative characteristic root of the transition matrix and ω_i are the elements of the characteristic vector associated with μ. By using equations (28), the relationship between q and K along the stable path can be expressed as:

$$q-\bar{q} = \left(\frac{\bar{\omega}_2}{\bar{\omega}_3}\right)(K-\bar{K}), \text{ where } \frac{\omega_2}{\omega_3} = \frac{-\bar{\phi}_L}{\bar{\phi}_r-\mu} = \frac{(1-\tau)f''}{r-\mu} < 0. \tag{29}$$

As a result, the price of equity and the capital stock will move in opposite directions along the stable path.

A graphical solution of the model is presented in Figure 1 where the steady-state values of q and K are given by the intersection of the $\dot{q} = 0$ schedule with $\dot{K} = 0$ schedule. $\dot{q} = 0$ schedule is negatively sloped while $K = 0$ schedule is horizontal as we can verify with the following calculations:

$$\left.\frac{\partial q}{\partial K}\right|_{\dot{q}=0} = -\frac{-(1-\tau)f''}{r} < 0, \quad \left.\frac{\partial q}{\partial K}\right|_{\dot{K}=0} = -\frac{x-\delta}{x'K} = 0.$$

The $q = 0$ schedule is downward sloping because an increase in K lowers the return to capital (and thus dividends), thereby requiring a fall in the price of equity so that the yield on equity with zero expected capital gains equals the world interest rate. $\dot{K} = 0$ schedule is horizontal because the rate of capital formation (x) equals the rate of depreciation (δ).

As shown in Figure 1, the stable saddle path (ss), described by (29),

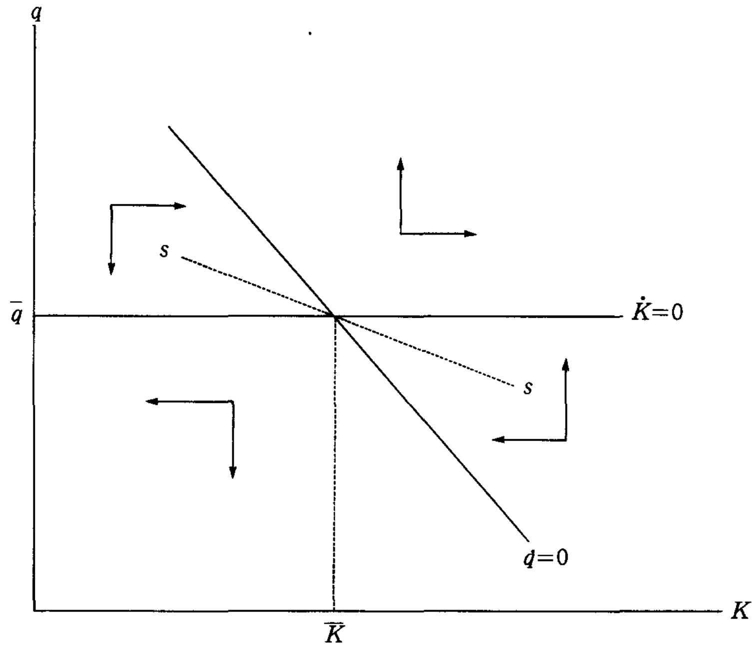

FIGURE 1
THE DYNAMICS OF THE ECONOMY

is downward sloping and follows from the pattern of adjustment for points off of the $\dot{q} = 0$ and $\dot{K} = 0$ schedules.

As pointed out at the end of section II, the current account will be in surplus or deficit, respectively, depending on whether $[f(K) - I(q, K) - g]$ is falling or rising through time (see (26)). And also we know from (29) that the price of equity and the capital stock move in opposite directions along the stable saddle path. For a given level of government spending, the current account will be in deficit during a phase of capital accumulation and in surplus during a capital decumulation with some assumptions. By specifying these assumptions, we can characterize the conditions under which the decreasing price of equity is associated with either the current account deficit or the surplus respectively. Whether $[f(K) - I(q, K) - g]$ is falling or rising through time with capital accumulation depends on entirely on movements of $I(q, K)$ because $f'(K) > 0$. If the investment expenditure $I(q, K)$ either decreases or increases but less than the amount of increase in the output, then $[f(K) - I(q, K) - g]$ will be rising through time and thus the current account will be in deficit. Thus the conditions which guarantee the decrease in the investment expenditure with capital accumulation will constitute the sufficient conditions under which the decreasing price of equity is associated with current account deficit. Because the price of equity is a decreasing function of capital stock along the equilibrium path from (29), we can write the investment expenditure function in (12) in the following way.

$$I(K) = x(q(K))K[1 + h(x(q(K))].$$

Differentiating this with respect to K gives the following:

$$\frac{dI(K)}{dK} = x(q(K))[1+h(x(q(K))]+x'\frac{\partial q}{\partial K}K[1+h(x(q(x)))+xh']. \tag{30}$$

Since μ is the negative eigenvalue of the system's matrix in (27), we can calculate the value of μ:

$$\mu = \frac{r-\sqrt{r^2-4(1-\tau)x'Kf''}}{2}. \tag{31}$$

Combining (9), (29) and (31) gives

$$\frac{\partial q}{\partial K} = \frac{2(1-\tau)f''}{r+\sqrt{r^2-\dfrac{4(1-\tau)Kf''}{2h'+xh''}}}(<0). \tag{32}$$

From (30), (31) and (32), it is not difficult to show that the smaller the value of r, τ, and f'', the more likely $dI(K)/dK$ is negative. It means that when the values of r, τ, and f'' are small, other things being equal, it is more likely that decreasing the price of equity will be associated with a current account deficit rather than a current account surplus. (See (26) and discussion below (26)) In other words, the economy facing these characteristics in the tax rate, world interest rate, and the production technology has more chances that the decreasing price of equity will be associated with the current account deficit. These results are quite intuitive. For example, let's compare the two economies which are the same except for the values of f''. The economy with a higher value of $|f''|$ has a steeper saddle path (see (29)). In Figure 2, s_1s_1 represents the saddle path for this economy. (Note that the steady state value of K, K, is determined by f' independent of f''. See (13) and (14).)With a certain shock, q will adjust instantaneously because q is a jumping variable. With an adjustment of K, the rate of return on capital changes more in the economy with a higher value of $|f''|$ than in the economy with a lower value of $|f''|$. To ensure that an arbitrage condition (7)

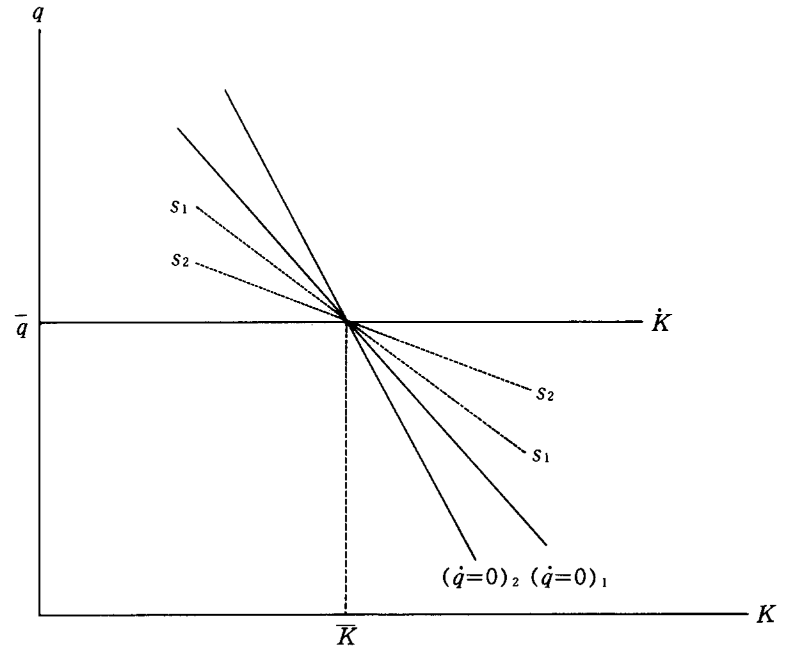

FIGURE 2

COMPARISON OF SADDLE PATHS FOR THE ECONOMIES WITH DIFFERENT CHARACTERISTICS

Note: Subscript 1 (2) stands for the economy with low (high) τ, r, and f''.

holds, q should change more in the economy with a higher value of $|f''|$. Suppose a tax rate τ is reduced. Along the saddle path, the decrease in q and thus J and I is larger in the economy with a higher value of $|f''|$. Thus during the adjustment period, investment expenditure I is more likely to decrease (see (30)) and a current account is more likely to be in deficit (see (26)).

By the same token, the economy with small values of τ and r, other things being equal, is more likely to have a current account deficit during the adjustment process in response to a tax cut. This feature will play an important role in discussing the effects of policy changes on a current account in the next section.

IV. The Effects of Policy Changes

The effects of an unanticipated reduction in the income tax rate[11] and an unanticipated increase in government spending, permanent and temporary, on the price of equity, the capital stock, and the current account will be analyzed in this section. For this purpose, the future level of lump-sum transfers to households is assumed to be the revenue variable that the government adjusts in order to ensure that the government's budget is balanced intertemporally. Because households fully anticipate this future adjustment and because transfers are non-distortionary, the exact timing of adjustment in transfers is irrelevant for the impact of the policy change on the economy.

A. A Permanent Reduction in the Income Tax Rate

A permanent reduction in the income tax rate (τ) that if financed by government borrowing will initially raise the after-tax return to equity and thereby increase the demand for equity claims. This increased demand for equity leads to an immediate rise in the price of equity in order to restore equilibrium in the equity market. This increase in the price of equity also represents an increase in the marginal value of installed capital. The higher price of equity induces an increase in the rate of investment by firms.

As illustrated in Figure 3, a permanent tax cut shifts the $q = 0$ schedule to the right. Since the capital is predetermined at each point in time, the immediate impact is a jump in the price of equity sufficient to place the economy on the saddle path. If the increase in the price of equity is either less than or greater than the amount needed to place the economy on the stable saddle path, then the economy will never reach the steady state equilibrium given by the intersection of the $\dot{q} = \dot{K} = 0$ schedules. The higher rate of investment then drives the dynamics as capital accumulates and the price of equity declines along the saddle path. While the capital stock increases in the new long-run equilibrium, the price of equity restores its original equilibrium value. Note that in the short-run the price of equity overshoots its long-run value and other things being equal, the price of equity is more volatile in the economy with low values of τ, r, and f''.

Whether the tax cut will increase the level of consumption and thus make a current account deficit depends on the magnitudes of τ, r, and f''. In the economy with small values of τ, r, and f'', the consumption is

[11]The effects of anticipated policy changes can be analyzed in the same spirit. Also we can analyze the effects of various forms of tax policies: a head tax or lump-sum tax; a proportional tax on gross output $f(K)$; a proportional tax on net output $f(K) - J[1 + h(x)]$.

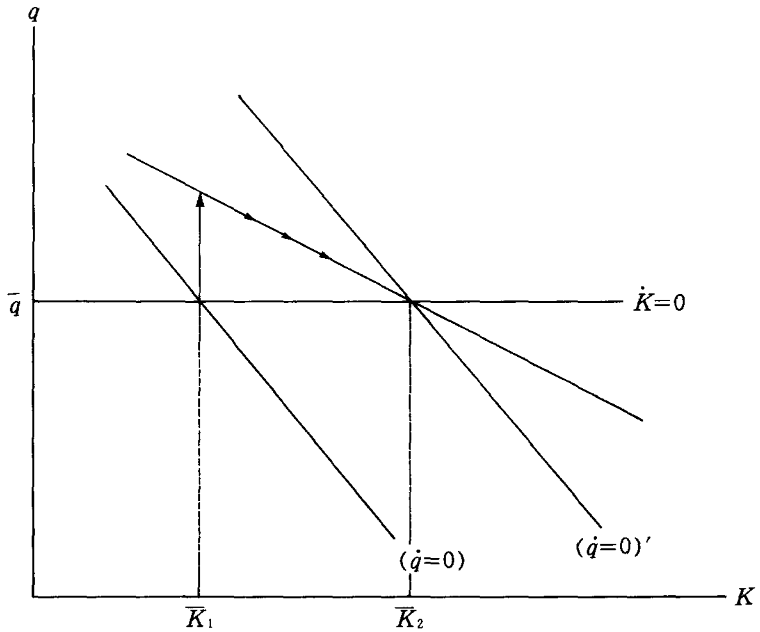

FIGURE 3
THE EFFECTS OF A PERMANENT REDUCTION IN THE INCOME TAX RATE

more likely to increase (see (25)) and thus a current account will be more likely to be in deficit (see (26)). As capital accumulates along the adjustment path, the economy runs a slowly diminishing current account deficit as time passes.

The time paths of the price of equity q, capital stock K, and current account CA are summarized in Figure 4.

B. A Temporary Reduction in the Income Tax Rate

A temporary reduction in the income tax rate is illustrated in Figure 5. The diagram for this policy change can be constructed as a combination of an unanticipated permanent decrease in the tax rate and an anticipated increase in the tax rate. As in the case of permanent reduction, the initial effect is an upward jump in the price of equity. The jump in the price of equity will be greater, the larger the period of time during which the tax cut is in effect. Also the jump in the price of equity must be smaller than for a permanent tax change so as to yield dynamics that place the economy on the original saddle path at the

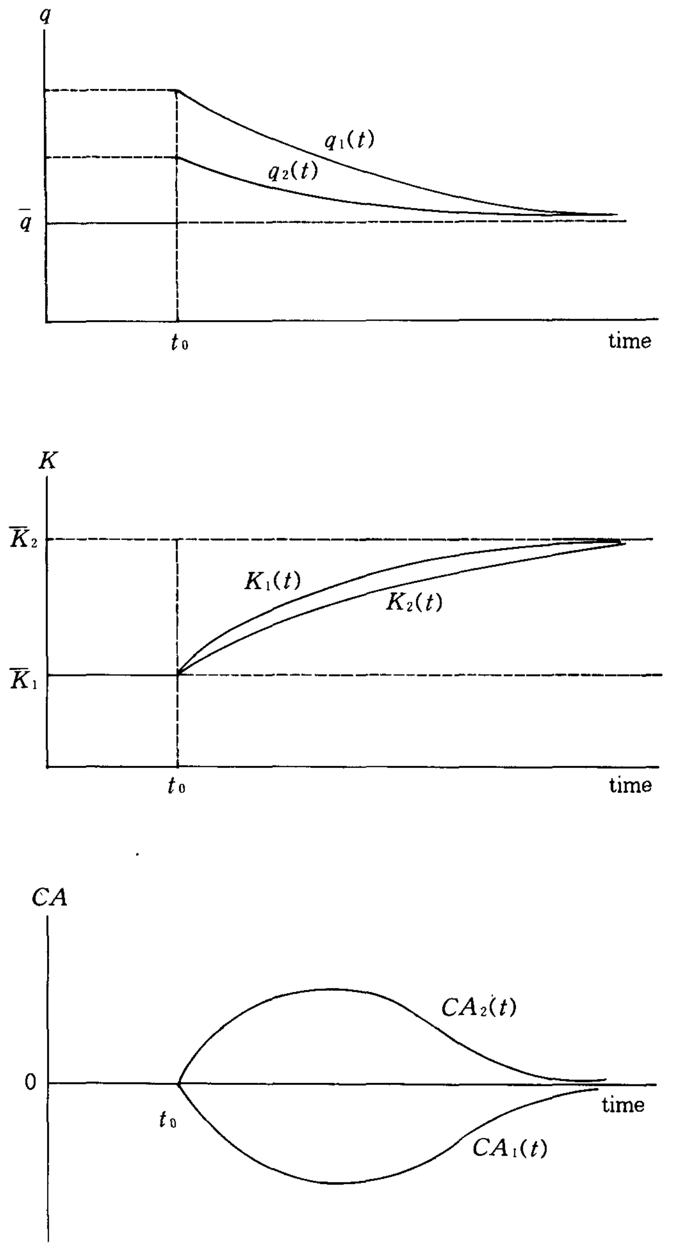

FIGURE 4

TIME PATHS OF THE PRICE OF EQUITY q, CAPITAL STOCK K, AND CURRENT ACCOUNT CA: Permanent Reduction of τ

Note: Subscript 1(2) stands for the economy with low (high) values of τ, r, and f''.

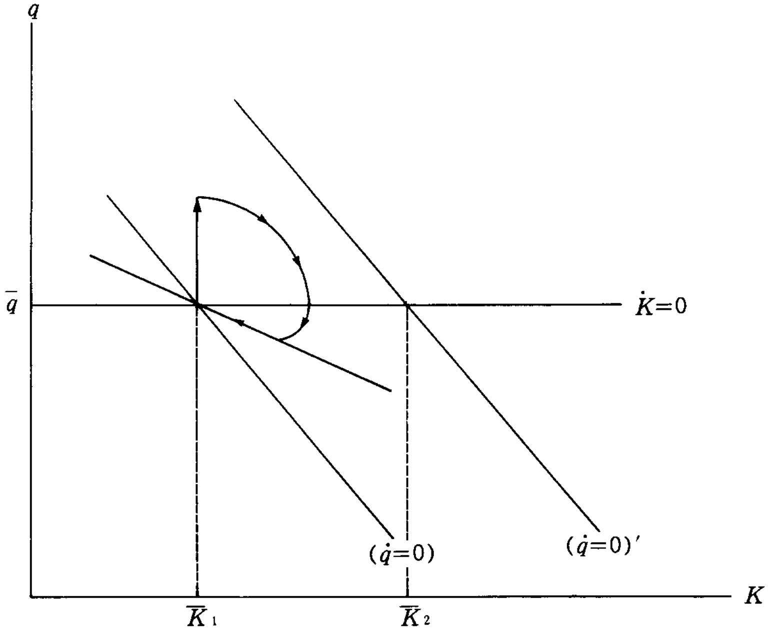

FIGURE 5

THE EFFECTS OF A TEMPORARY REDUCTION IN THE INCOME TAX RATE

time the tax cut is rescind. The price of equity declines during the period preceding the tax increase while the capital stock at first rises and then declines. After the increase in the tax rate, the economy adjusts along the original saddle path, with the price of equity rising and the capital stock falling. But the capital stock begins to decline at some point of time prior to the actual increase in the tax rate. Note that the price of equity undershoots its long-run value after the tax rate is increased (see Figure 5) and the price of equity in the economy with low values of τ, r, and f'' is more volatile (see Figure 6).

Since the temporary reduction in the tax rate has no effects on the consumption level, it will induce an immediate current account deficit due to an immediate jump in investment expenditure. But it will be followed by current account surplus depending on the fluctuation of the level of investment expenditures. Thus, contrary to the case of permanent change, the temporary decrease in the tax rate results in the alternations of current account deficit and surplus during the adjustment process.

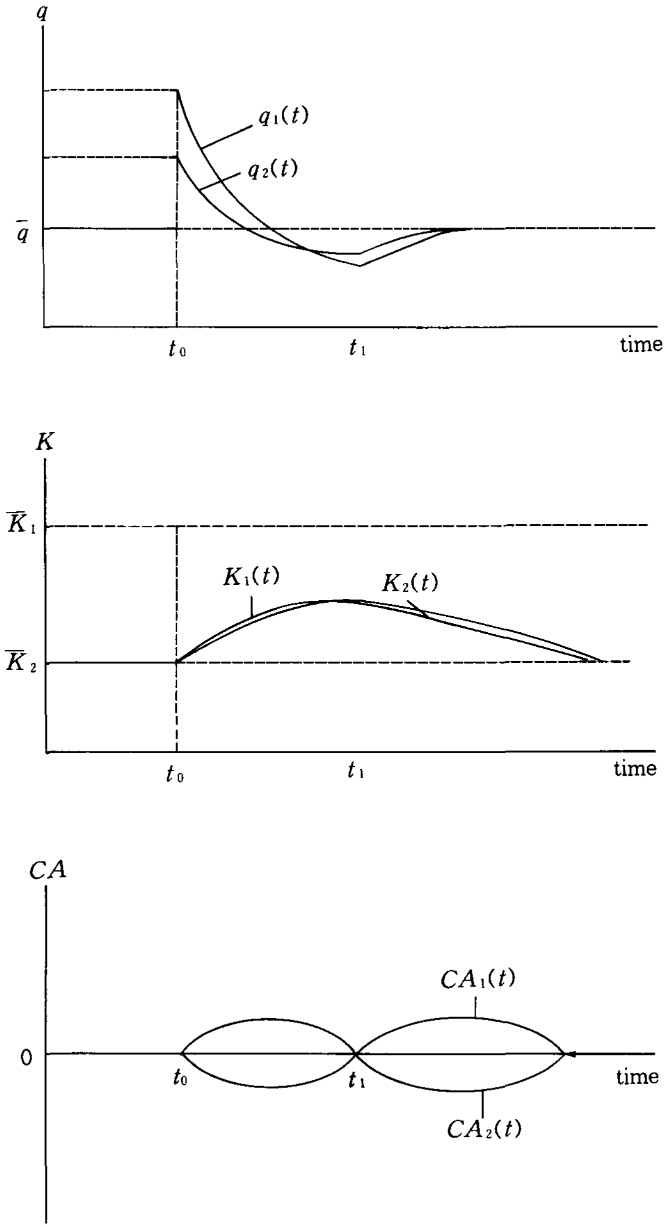

FIGURE 6

TIME PATHS OF THE PRICE OF EQUITY q, CAPITAL STOCK K, AND CURRENT ACCOUNT CA: TEMPORARY REDUCTION OF τ

Note: Subscript 1(2) stands for the economy with low (high) values of τ, r, and f''.

C. An Increase in Government Spending

An increase in the government spending financed through borrowing will have no effect on the price of equity, the capital stock. The only effect will come as households react to the increase in the spending by smoothing the drop in lifetime income represented by the future tax-burden. When there is a permanent increase in the government spending, there will be no effect on the current account since consumption falls by the full amount of the increase in spending. For a temporary increase in spending, the current will move into deficit as consumption declines by less than the increase in spending.

D. A Reduction in the World Interest Rate

To consider a permanent reduction in the world interest rate (r), it is necessary to assume that the rate of time preference (β) also falls in order to ensure a determinate steady state with $r = \beta$. Under this assumption, a reduction in r will shift the $\dot{q} = 0$ schedule to the right. The immediate effect is an increase in the price of equity as residents attempt to shift their asset holdings from the less attractive international bonds into domestic equities. The adjustment process will be the same as in the case of a permanent reduction in the tax rate. The only difference from the case of a tax-cut concerns the level of consumption. Depending on whether the economy has a positive or negative initial net asset position, the decline in the world interest rate will either lower or raise the level of lifetime income and thus consumption. A temporary reduction in the world interest rate can be analyzed in the same way as a temporary cut in the tax rate. Again, the only important difference concerns the effect on lifetime income and consumption resulting from the initial net asset position of the economy.

A fundamental feature of the model is that a common worldwide shock, such as a change in the world interest rate, will induce similar movements in equity prices across a group of a small economies.

V. Conclusions

This paper considered the dynamics of the price of equity, the level of capital accumulation and the current for a small open economy in a one-good, infinite-horizon, optimizing model in which the process of capital accumulation entails adjustment costs. There are four major findings. First, in an economy with low values of τ, r, and f'', it is more likely that a decreasing price of equity is associated with a current account deficit along the equilibrium path. Moreover, the price of equity is more volatile in such an economy. Second, a permanent tax cut causes the price of equity to jump upwards and then decrease to the original equilibrium level. During the process a current account deficit is more likely to occur in an economy with low values of τ, r, and f''. At the new steady state equilibrium, the capital stock will be increased. In contrast, a temporary tax reduction causes the price of equity to jump upwards (but less than in the case of a permanent change) and to converge to the original level after fluctuating around it during the adjustment process. Meanwhile, alternations of current account deficit and surplus will occur. The steady state equilibrium point doesn't change. Third, an increase in government spending will have no effect on the price of equity and the capital stock. While a permanent change will not affect the current account, a temporary one will cause a current account deficit. Fourth, the effects of a decrease in the world interest rate are qualitatively similar to those of a permanent reduction in the tax rate except for its effect on the level of consumption.

The analysis emphasizes the role of tax rates, world interest rates, and production technology (f'') as determinants of the dynamics of current account. And the present paper demonstrates how changes in fiscal policies or world interest rates can generate sustained movement in equity price simply because investment requires scarce resources. Also it shows that there is a close connection between the volatility of equity price and the production technology. The results indicate that a stable and consistent set of fiscal policies can play an important role in reducing unnecessary volatility in equity prices. Especially, in an economy with low values of the world interest rate (r), tax rate (τ), and the curvature of the production function (f''), the price of equity is more volatile. Thus to avoid unnecessary volatility of equity prices. more conservative fiscal polices (e.g. high tax-rate policies) are required.

Appendix

Here, the proposition that "average q" and "marginal q" are equal in the model, i.e., $qK = V$, is proved. The value of a firm at $t = 0$ is given by the following:

$$V(0)=\int_0^\infty\{(1-\tau)[f(K)-w]-J[1+h(\frac{J}{K})]\}\exp^{-rt}dt. \tag{A1}$$

Since f is assumed to be homogeneous of degree 1 in K,

$$f(K)-w=f'(K)K. \tag{A2}$$

From the accumulation constraint for te capital stock (5) and the optimality conditions (6) and (7),

$$J=\dot{K}+\delta K \tag*{(5)$'$}$$

$$1+h(\frac{J}{K})=q-(\frac{J}{K})h'(\frac{J}{K}) \tag*{(6)$'$}$$

$$(1-\tau)f'(K)=(r+\delta)q-\dot{q}-(\frac{J}{K})^2h'(\frac{J}{K}) \tag*{(7)$'$}$$

Plugging (A2), (5)$'$, (6)$'$, and (7)$'$ into (A1) and rearranging gives the following:

$$\begin{aligned}V(0)&=\int_0^\infty(rqK-\dot{q}K-\dot{K}q)\exp^{-rt}dt\\&=\int_0^\infty[rqK-\frac{d}{dt}(qK)]\exp^{-rt}\ dt\\&=\int_0^\infty rqK\ \exp^{-rt}dt-qK\ \exp^{-rt}\Big|_0^\infty-r\int_0^\infty qK\ \exp^{-rt}dt\\&=-\lim_{t\to\infty}qK\ \exp^{-rt}dt+q(0)K(0)\\&=q(0)K(0)\end{aligned}$$

Therefore $V(0)=q(0)K(0)$.

The third equality follows from the integration by parts and the fifth equality holds from the transversality condition (8).

Since this relationship holds at any point in time along the optimal path, $V = qK$.

Q.E.D.

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