Capital and Unemployment in a Two-Class Growth Model

Wei-Bin Zhang*

The paper proposes a simple dynamic one-sector and two-class growth model with endogenous unemployment and government intervention. The model explains dynamics of capital owned by the two classes and the market structure with labor unemployment. The dynamic properties of the model and effects of changes in some parameters are investigated. (*JEL* Classification: E24)

I. Introduction

Four schools or modeling frameworks in theoretical (mathematical) economics have been proposed to explain various aspects of the complexity of economic systems. The first is the Arrow-Debreu equilibrium theory which mainly deals with equilibrium of demand and supply under perfectly competitive environment (e.g., Arrow and Hahn 1971). In this approach, monetary variables such as prices and wages are determined by interdependence of demand and supply. The second theory which is called disequilibrium or Keynesian economics accept prices and wages not equal to their perfectly competitive marginal values (e.g., Leijonhufvud 1968; Malinvaud 1977; Hahn 1978; Green and Laffont 1981; Negishi 1985). When an economic activity is attempted at the sticky price, it is the adjustment of the quantity that leads to a temporary equilibrium. The works in the field of disequilibrium analysis have increased our understanding of the functioning of the economic system when transactions take place at certain fixed prices. This approach may explain the existence of unemployment, for instance, by taking account of fixed wage rates due to the government, the labor

*Institute for Futures Studies, Hagagatan 23B, 3TR., Box 6799 113 85 Stockholm, Sweden. The author is grateful to Prof. Keehyun Hong and two anonymous referees for important comments.
[**Seoul Journal of Economics** 1993, Vol. 6, No. 4]

union or other economic or institutional factors.

The third theory is the so-called neoclassical growth theory (e.g., Burmeister and Dobell 1970). The standard model is the Solow-Swan one-sector growth model. This approach emphasizes long-run aspects of economic dynamics. How capital is accumulated over time and what kinds of effects capital stocks may have on economic structure are two of the main concerns in this approach. Monetary variables are treated as fast variables in comparison to real variables in the dynamic system. At each point of time, monetary variables are determined at their marginal values and real variables evolve over time. That is, in modern nonlinear dynamic terms, monetary variables are "enslaved" by dynamics of real variables such as population and capital dynamics in this approach. The fourth approach is initiated by Kaldor and Passinetti on distribution of wealth between the workers and capitalists, explaining Marx's economic system (e.g., Pasinetti 1974; Kaldor 1966; Sato 1966; Marglin 1984). In this approach, behavior of individuals is conditioned by their class. It is the relationship among long-run growth, profit and the savings propensities of the two social classes that this approach is mainly concerned with.

Indeed, the boundaries among these schools are not clearly cut as we just described. Ideas of each school have affected the development of the other schools in some way or another. There are a large amount of published papers in the literature of each of these schools. Although the study will not examine similarity and difference among these schools in details, this study attempts to propose a dynamic model on the basis of the main ideas in those schools. This paper attempts to propose a dynamic one-sector growth model with capital accumulation, wealth and income between two classes and possible unemployment of one class on the basis of these traditional approaches. The paper is concerned with an one-sector and two-class economy with possible unemployment of one class.

The remainder of the study is organized as follows. Section II defines the model. Section III represents the economic structure and dynamics in terms of two-dimensional differential equations of the two classes' capital accumulation. Section IV examines the dynamic properties of the economic system when the two classes have an identical preference structure. Section V analyzes effects of changes in some parameters on the long-run behavior of the system. Section VI concludes the study. Appendix A proves some of the results in section IV. Appendix B discusses the dynamics of the system in the case that the two classes have different preferences.

II. The Model

This study is concerned with an economic system similar to the Solow-Swan one-sector growth model (e.g., Burmeister and Dobell 1970). Only one commodity is produced and the commodity is composed of homogeneous qualities and is either invested or consumed. Similarly to the multi-group models by, for instance, Pasinetti (1974), Sato (1966), the population is classified into two classes, working class and knowledge class. The classification is similar to that of skilled and unskilled labor force in the literature (e.g., Bhagwati and Hamada 1974; Rodriguez 1975; Miyagiwa 1989). Let the population of the working class and knowledge class be denoted by L and N, respectively, where L and N are fixed. The population and the population structure are assumed to be invariant.

Each member of the two classes may be employed by the industrial sector. Various reasons have been suggested to explain the existence of unemployment with fixed monetary variables such as commodity prices and wage rates in the literature of neo-Keynesian economics (e. g., Balsko 1982; Barro and Grossman 1971; Bohm 1978; Silvestre 1982). This study follows the neo-Keynesian tradition. It is assumed that each member of the working class has identical knowledge utilization efficiency, irrespective of whether he is employed or not. The total qualified labor force, Q^*, and the total capital stock, $K(t)$, of the system are defined by

$$Q^* = \check{L} + \check{N}, \quad K = K_1 + K_2, \tag{1}$$

where $\check{L}$ and $\check{N}$ are the qualified labor force of the working class and knowledge class, respectively, and K_j is capital stocks owned by class j, where $j = 1$ and $j = 2$, respectively, represent the working class and knowledge class.

In order to describe the relationship between the number and qualified labor force of each class, the concept of human capital difference, z, is introduced to distinguish difference in productivity of the two classes. With the working class' human capital as basis of measurement, one introduces the following relationship

$$\check{L} = L, \quad \check{N} = zN. \tag{2}$$

It is assumed $z > 1$. This simply implies that the knowledge class accumulates more human capital than the working class (in the term of productivity). Here, possible effects of education, training and other costly learning efforts on $\check{L}$ and $\check{N}$ are neglected. It should be remarked that it is possible to introduce endogenous human capital accumulation into our model (e.g., Romer 1986, 1990; Lucas 1988; Zhang 1991).

Let $L_i(t)$ and $L_u(t)$ denote, respectively, the labor force of the working class employed by the industrial sector and unemployed. One has

$$L_i(t) + L_u(t) = L. \tag{3}$$

The total qualified labor force, $Q(t)$, employed by the industrial sector is defined by

$$Q = L_i + zN. \tag{4}$$

The production function of the economy is specified as follows

$$F(t) = K^{\alpha}Q^{\beta},\ \alpha + \beta = 1,\ \alpha,\ \beta > 0, \tag{5}$$

where $F(t)$ is the total output at time t. The commodity is selected to serve as numeraire. The marginal conditions are given by

$$r = \alpha TF/K,\ w = \beta TF/Q,\ W = \beta zTF/Q, \tag{6}$$

where r is interest rate, w and W are wage rates of the working and knowledge classes, respectively, and T is defined by $T \equiv 1$ – tax rate. By (6), the ratio of the wage rates is given by: $W/w = z$. The ratio of the wage rates is determined by the difference in the human capital between the two classes.

To guarantee the existence of unemployment of the working class, it is assumed that the wage rate of the working class be exogenously fixed at w (see, e.g., Eckalbar 1981; Picard 1983). How the wage rate of the working class is determined may depend on many factors such as power of labor union and actual economic conditions of the working class. It should be remarked that in this study the knowledge class is assumed to be fully employed. In certain circumstances of economic reality, the actual labor market may different from this study in that the knowledge class may be paid at a fixed wage rate.

Let each unemployed people be paid w_u amount of money by the government. It is assumed that w_u is at least not higher than the wage rate of the employed people from the same class, i.e., $0 < w_u < w$. The existence of unemployment makes it difficult to discuss the issue of wealth distribution and capital accumulation. For convenience of analysis, it is assumed that any member of working class has identical income from wage and interest rate, irrespective of whether they are employed or unemployed. To satisfy this requirement, it is assumed that the time of unemployment is equally "shared" among all the members of the working class at any point of time. This implies that wage income of each member of the working class is actually equal to $(wL_t + w_uL_u)/L$. This is obviously a strict assumption, but it makes it possible to write down the capital accumulation equation of the working class. According to the definitions, the net incomes of the two classes at any point of time are given by

$$Y_1 = wL_t + w_uL_u + rK_1, \; Y_2 = WN + rK_2. \tag{7}$$

Let $S_j(t)$ denote the savings made by class j at time t. It is assumed that utility level of class j is dependent on the temporary consumption level, $C_j(t)$ and the total wealth at time t, $K_j(t) + S_j(t) - \delta_0 K_j$, where δ_0 is the fixed depreciation rate of capital, in the following way

$$U_j(t) = C_j^{\zeta_j}(K_j - \delta_0 K_j + S_j)^{\lambda_j}, \; \zeta_j, \; \lambda_j > 0, \; j = 1,2. \tag{8}$$

Maximizing U_j subject to the budget constraint, $C_j + S_j = Y_j$, yields

$$S_j = \{\lambda_j Y_j - (1 - \delta_0)\zeta_j K_j\}/(\zeta_j + \lambda_j),$$
$$C_j = \{Y_j + (1 - \delta_0)K_j\}\zeta_j/(\zeta_j + \lambda_j). \tag{9}$$

The equality of savings and investment yields the following capital accumulation of class j, $dK_j/dt = S_j - \delta_0 K_j$. Substituting S_j in (9) into the two equations yields:

$$dK_j/dt = s_j Y_j - \delta_j K_j, \; j = 1, 2, \tag{10}$$

where $s_j \equiv \lambda_j/(\zeta_j + \lambda_j)$ and $\delta_j \equiv (1 - \delta_0)\zeta_j/(\zeta_j + \lambda_j) + \delta_0$.

It is assumed that the government has only one income source, taxing the industrial sector. The government's budget constraint is given by

$$w_uL_u = (1 - T)F. \tag{11}$$

Adding Y_1 and Y_2 in (7) yields: $Y_1 + Y_2 = wL_t + w_uL_u + WN + rK$. Substituting (11) and (6) into the above equation yields

$$F = Y_1 + Y_2 = wL_t + w_uL_u + WN + rK. \tag{12}$$

The total income of the two classes is equal to the total product of the society at any point of time.

As this study is mainly concerned with possible impact of the government intervention in an economy with unemployment, it is necessary to fix the wage rates such that unemployment exists. Let $w_1(t)$ denote the wage rate of the working class at which the class is fully employed for a given value of K. To guarantee the existence of unemployment, it is necessary to require $w > w_1(t)$; otherwise all the members of the working class will be employed by the industrial sector. It can be seen that $w_1(t)$ is given by

$$w_1(t) = \beta F/Q, \quad Q = L + zN, \tag{13}$$

with $T = 1$ and $L_u = 0$.

On the other hand, if the wage rate, w, is fixed too high, the industrial sector will employ no one from the working class. For instance, if w is determined at a level higher than the marginal value of the working class when no member of the working class is employed, the industrial sector prefers paying tax to employing any people from the working class. For any given level of $K(t)$, when no member of the working class is employed by the industrial sector, the marginal value of the working class is determined by

$$w_2(t) = \beta T_1 F/Q, \quad Q = zN, \quad (1 - T_1)F = w_u L, \tag{14}$$

where $1 - T_1$ is the tax rate at which all the members of the working class are paid by the government at the fixed rate, w_u.

***Assumption* 1**

For any given level of $K(t)$ at any point of time, it is assumed that the wage rate, w, and unemployment payment rate, w_u, of the working class are fixed by the government in the following way: $w_2(t) > w > w_1(t)$, $w > w_u$.

The assumption means that the government fixes the wage rate of the working class in a level that is not lower than the market wage rate at which the working class is fully employed by the industrial sector and not higher than the wage rate at which no member of the working class will be employed by the industrial sector.

It should be noted that it is not difficult to introduce certain time-dependent policy, $w(t)$, such that the above assumption is satisfied. For instance, for any given $K(t)$, one may specify $w(t)$ by $w(t) = hw_1(t) + (1 - h)w_2(t)$, where h is a constant, $0 < h < 1$. The value of the parameter, h, is determined, for instance, by the power of the labor union. This wage policy satisfies the above assumption.

The assumption about the wage policy can be relaxed in different ways. For instance, there are a large amount of the literature in labor economics about how the wage rates of various groups of labor are endogenously determined under various policies (e.g., Stockey 1980; Spence 1976; Samuelson 1985; Bulow and Summers 1985; Pissarides 1986). It is conceptually not difficult to further extend our approach on the basis of the literature. As shown below, even our simple model is analytically very complicated, we deal with the simple case at this initial stage.

The model has been constructed. The system has 17 endogenous variables, K_j, Y_j, C_j, S_j, U_j (j = 1, 2), K, Q, F, L_t, L_u, T, and r. It also contains the same number of independent equations.

III. The Dynamics in Terms of K_1 and K_2

The dynamics consist of two dimensional differential equations for K_1 and K_2. In order to analyze the dynamic properties, it is necessary to express the dynamics in terms of the two variables. From (10), one sees that it is sufficient to express Y_1 and Y_2 in terms of K_1 and K_2 at any point of time.

From $w = \beta TF/Q$, one has: $TF = wQ/\beta$. Substituting this equation and $L_u = L - L_t$ into the government's budget constraint yields

$$G(L_t) \equiv F - wQ/\beta - w_u L + w_u L_t = 0, \tag{15}$$

where $Q = L_t + zN$. It is direct to check that at $L_t = L$, i.e., $L_u = 0$, one has

$$G(L) = (\beta F/Q - w)Q/\beta = (w_1 - w)Q/\beta < 0, \tag{16}$$

where Assumption 1 is used. At $L_t = 0$, one gets

$$\begin{aligned} G(0) &= F - wQ/\beta - w_u L = T_1 F - wQ/\beta + (1 - T_1)F - w_u L \\ &= (w_2 - w)Q/\beta > 0, \end{aligned} \tag{17}$$

where $(1 - T_1)F = w_u L$ and Assumption 1 are used. As $G(0) > 0$ and $G(L) < 0$, the equation, $G(L_t) = 0$, $0 < L_t < L$, has at least one solution. It is direct to check that the second derivative of G with respect to L_t is negative, i.e., $d^2G/d^2L_t < 0$. As $G(0)$ and $G(L)$ have the opposite signs and $d^2G/d^2L_t < 0$ holds for L_t for any $0 < L_t < L$, $G(L) = 0$ has a unique solution in the interval $(0, L)$. Let the unique relationship between L_t and K be denoted by: $L_t(t) = \Lambda(K(t))$, where the functional form of Λ is determined by (15). From (15), one obtains

$$(w/\beta - \beta F/Q - w_u)d\Lambda/dK = \alpha F/K. \tag{18}$$

As $(w/\beta - \beta F/Q - w_u)$ may be either positive or negative, an increase in the total capital may either increase or decrease the unemployment rate, $1 - \Lambda/L$, at any point of time. From $L_i(t) = \Lambda(K(t))$, all the other variables are determined as functions of $K_1(t)$ and $K_2(t)$ at any point of time in the following way:

$L_u = L - \Lambda \rightarrow Q^*$ by (1) and Q by (4) $\rightarrow F$ by (5) $\rightarrow T$ by (11) $\rightarrow r$ by (6) $\rightarrow Y_j, j = 1, 2$, by (7) $\rightarrow S_j$ and C_j by (9) $\rightarrow U_j$ by (8).

From this procedure, one can analyze effects of changes in K_1 and K_2 at any point of time.

Substituting $r = \alpha wQ/\beta K$ and $W = zw$ into Y_2 in (7) and using $Y_1 = F - Y_2$, the dynamics (10) can be rewritten as follows:

$$\begin{aligned} dK_1/dt &= s_1(F - zwN - \alpha wQK_2/\beta K) - \delta_1 K_1, \\ dK_2/dt &= ws_2(zN + \alpha QK_2/\beta K) - \delta_2 K_2, \end{aligned} \tag{19}$$

where $Q = \Lambda(K) + zN$. At each point of time, the equations, (19), determine the capital stocks owned by the two classes. All the other variables at any point of time are given by the procedure just described.

The remainder of this study examines the behavior of the dynamic system. The next two sections examine the behavior of the system when the two classes have an identical preference structure. As it is not easy to get explicit conclusions when the two classes have different preferences, the analytical results are provided in the appendix.

IV. The Dynamics with the Identical Preference

This section examines dynamic behavior of the system when the two classes have an identical preference structure, i.e., $\zeta_1 = \zeta_2$ and $\lambda_1 = \lambda_2$ (and thus $s_1 = s_2$ and $\delta_1 = \delta_2$). The case that the two classes have different preferences is analyzed in the appendix.

Adding the two equations (19) yields

$$dK/dt = sF(K) - \delta K. \tag{20}$$

The two-dimensional dynamics are thus reduced to a one-dimensional system. An equilibrium is given by

$$F = K^\alpha Q^\beta = \theta K, \tag{21}$$

where $\theta \equiv \delta/s$. As (15) is also held, one has $F = wQ/\beta + w_u L_u = w_0 Q + w_u Q^*$, where $w_0 \equiv w/\beta - w_u > 0$. From this equation and (21), one has

$$Q = (\theta K - w_u Q^*)/w_0. \tag{22}$$

From (21), (22) and $Q = zN + L_i$, one gets

$$K = w_u Q^*/(\theta - w_0 \theta^{1/\beta}), \;\; L_i = \theta^{1/\beta} K - zN. \tag{23}$$

As this paper is mainly concerned with the case that the system has an equilibrium with a positive unemployment rate, it is necessary to require $K > 0$ and $L > L_i > 0$. From (23), the requirements are satisfied if

$$\begin{gathered} K > 0 \text{ if } \theta^{-\alpha/\beta} - w_0 > 0; \\ L > L_i \text{ if } \theta^{-\alpha/\beta} - w/\beta = \theta^{-\alpha/\beta} - w_0 - w_u > 0, \\ L_i > 0 \text{ if } w_u/(\theta^{-\alpha/\beta} - w_0) > zN/Q^*, \end{gathered} \tag{24}$$

are held. One sees that $L > L_i$ implies $K > 0$. If the wage rate of the working class is not fixed higher than $\beta\theta^{-\alpha/\beta}$, and if the ratio of the knowledge class' qualified labor force to the total qualified labor force is not very high, the problem has a unique equilibrium with a positive unemployment rate. For instance, in the case of $\theta = 1/3$, $\alpha = \beta = 1/2$, $w = 1/7$, $w_u = 1/20$ and $zN/Q^* = 1/2$, the system has a unique meaningful equilibrium.

The stability is given by taking the derivative of $sF(K) - \delta K$ with respect to K evaluated at the equilibrium. Using $F = K^\alpha Q^\beta$, $Q = zN + L_i$, $Q^* = Q + L_u$, (21) and (18), the derivative is given as follows:

$$\begin{aligned} \frac{d(sF - \delta K)}{dK} &= (F - \frac{wQ}{\beta} + w_u Q)\frac{s\beta F}{K}(\frac{wQ}{\beta} - \beta F - w_u Q) \\ &= \frac{s\beta w_u Q^* F}{KQ}(\frac{\beta w_u L_u}{Q} - \frac{\alpha w}{\beta} + w_u). \end{aligned} \tag{25}$$

If $d(sF(K) - \delta K)/dK$ is negative (positive), the system is stable (unstable). By (25), the sign of $d(sF(K) - \delta K)/dK$ is the same as that of the term, $\beta w_u L_u/Q - \alpha w/\beta + w_u$. It can be seen that the system may be either stable or unstable. The system is unstable, for instance, in the case of $\alpha/\beta = 0.5$ and $w = 1.9\, w_u > 0$. The system is stable, for instance, in the case of $L/zN < 1$ and $w/w_u > \max\{1, (1 + \beta)\beta/\alpha\}$. To further explain the stability condition, by (23), $L_u = Q^* - Q$, $-\alpha w/\beta + w_u = w - w_0$, and $w_0 = w/\beta - w_u$, one obtains

$$\frac{\beta w_u L_u}{Q} - \frac{\alpha w}{\beta} + w_u = \frac{w_u Q^*(\beta\theta^{-\alpha/\beta} - w_0)}{Q(\theta^{-\alpha/\beta} - w_0)}, \tag{26}$$

where $\theta^{-\alpha/\beta} - w_0 > 0$. The sign of $d(sF(K) - \delta K)/dK$ is the same as that of the term, $\beta\theta^{-\alpha/\beta} - w_0$.

Summarizing the above discussion, one has the following proposition.

***Proposition* 1**

It is assumed that the government fixes the wage rate of the working class and unemployment payment rate in the following way: $\theta^{-\alpha/\beta} - w_0 > w_u$ and $w_u/(\theta^{-\alpha/\beta} - w_0) > zN/Q^*$. Then, the system has a unique equilibrium with a positive unemployment rate. If $\beta\theta^{-\alpha/\beta} - w_0 < 0$ (> 0), the system is stable (unstable).

V. Parameters and the Long-Run Behavior

This section examines the effects of changes in some parameters on the long-run behavior of the system. It is assumed that the conditions that the system has a unique equilibrium with positive unemployment rate are satisfied.

As $\theta = \delta/s = \zeta/\lambda + \delta_0$ where ζ and λ are, respectively, marginal utility values of consumption and wealth, one may interpret an increase in θ as a decrease in the propensity to accumulate wealth. Taking derivatives of K and L_t in (23) with respect to θ yields

$$\frac{dK}{d\theta} = -\frac{K(\beta - \theta^{\alpha/\beta} w_0)}{\theta\beta(1-\theta^{\alpha/\beta} w_0)},$$
$$\frac{dL_t}{d\theta} = \frac{\alpha K \theta^{1/\beta}}{\beta(1-\theta^{\alpha/\beta} w_0)} > 0. \tag{27}$$

An increase in the propensity to accumulate wealth reduces the employment rate. If the system is stable (unstable), i.e., $\beta - \theta^{-\alpha/\beta} w_0 < (> 0)$, an increase in the propensity reduces (increases) the total capital stocks. As a change in the preference affects the consumption components, the employment structure and production, the conclusion is not surprising. It should be noted that to explain how the new equilibrium is achieved one has to examine how all the equations in the system are affected through a shift in the parameter.

The impact on F is given by

$$\frac{dF}{d\theta} = \frac{\alpha\theta^{\alpha/\beta} w_0 K}{\beta(1-\theta^{\alpha/\beta} w_0)} > 0. \tag{28}$$

A reduction in the propensity to accumulate wealth increases output. From (11), one gets the impact on the tax rate as follows:

$$\frac{F}{w_u}\frac{dT}{d\theta}=\frac{dL_t}{d\theta}+\frac{L_u}{F}\frac{dF}{d\theta}>0. \tag{29}$$

As tax rate is equal to $1-T$, a decrease in the propensity to accumulate wealth reduces the tax rate. From (6) and (21), one has $r = \alpha TF/K = \alpha\theta T$. Taking derivatives of this equation with respect to θ yields

$$dr/d\theta = \alpha T + \alpha\theta dT/d\theta > 0. \tag{30}$$

An increase in θ increases the interest rate. From (7) and $Y_j = \theta K_j$, one has:

$$RK_1 = (w - w_u)L_t + w_u L,\ RK_2 = WN, \tag{31}$$

where $R \equiv \theta - r > 0$. One thus has

$$\begin{aligned} RdK_1/d\theta &= (w - w_u)dL_1/d\theta - (1 - dr/d\theta)K_1, \\ RdK_2/d\theta &= -(1 - dr/d\theta)K_2. \end{aligned} \tag{32}$$

If $1 < dr/d\theta$, the two classes' capital stocks are increased. But if $dr/d\theta > 1$, the knowledge class' capital is reduced. The working class may have more or less capital, depending on how the employment rate is affected. From $dY_j/d\theta = K_j + \theta dK_j/d\theta$, one sees that each class' capital stocks tends is increased if the interest rate is not strongly affected by changes in the propensity. The effects on the ratios of incomes and capital stocks of the two classes are given by

$$d(Y_1/Y_2)/d\theta = d(K_1/K_2)/d\theta = \{(w - w_u)/WN\}dL_t/d\theta > 0. \tag{33}$$

A decrease in the propensity increases the working class' wealth and income in comparison to the knowledge class' ones. From (9), the effects on consumption are given as follows

$$\begin{aligned} (\zeta + \lambda)dC_j/d\theta &= C_j\lambda^2/\zeta + \zeta K_j + \zeta(\theta + 1 - \delta_0)dK_j/d\theta\,, \\ d(C_1/C_2)/d\theta &= d[\{Y_1 + (1 - \delta_0)K_j\}/\{Y_2 + (1 - \delta_0)K_2\}]/d\theta > 0. \end{aligned} \tag{34}$$

The effects on C_j may be positive or negative, but C_1/C_2 is certainly increased.

***Proposition* 2** (The impact of changes in ζ/λ)
Let the conditions that the system has a unique equilibrium with a positive unemployment rate be satisfied and the two classes have an identical preference structure. Then, a decrease in the propensity to

accumulate wealth, ζ/λ, has the following impact on the system:

1) if the system is stable (unstable), the total capital is increased (decreased);
2) the total output and the interest rate are increased, and the tax rate and the unemployment rate are reduced;
3) the income and consumption level of each class and the capital stocks owned by each class may be either increased or decreased; and
4) the ratios of the incomes, capitals stocks and consumption between the working class and the knowledge classes are increased.

Similarly to Proposition 2, one may directly get the effects of changes in other parameters. The wage rate of the working class, w, the unemployment payment rate, w_u, and the human capital of the knowledge class, z, on the long-run values of the variables. The following propositions summarize the effects of changes in the wage rate, w, of the working class and human capital, z, of the knowledge class.

***Proposition* 3** (the impact of changes in w)
An increase in w has the following impact on the system:

1) the total capital is increased;
2) the total output and the interest rate are increased, and the tax rate and the unemployment rate are reduced;
3) the income and consumption level of each class and the capital stocks owned by each class are increased; and
4) the ratios of the incomes, capital stocks and consumption between the working class and the knowledge classes may be either increased or reduced.

***Proposition* 4** (the impact of changes in z)
An increase in z has the following effects:

1) the total capital and output are increased;
2) the interest rate and tax rate are not affected, and the unemployment rate is increased;
3) the income and consumption level of and the capital stocks owned by the knowledge (working) class are increased (decreased); and
4) the living conditions of the working class in comparison to the knowledge class decline.

The above two propositions are checked in the appendix. It should be remarked that all the conclusions are held under the requirement that the system has a unique equilibrium with a positive unemployment rate.

VI. Concluding Remarks

This study proposed a dynamic one-sector and two-class growth model with endogenous unemployment and government intervention on the basis of equilibrium theory, neo-classical growth theory, the Kaldor-Passinetti two-class growth model, and Keynesian economics. The model explains the dynamics of capital owned by the two classes, income distribution and market structure with unemployment. The dynamic properties of the model were examined. The effects of changes in some parameters were also provided.

It should be remarked that although the model is claimed to be much inspirited by the four schools mentioned above, this does not mean that the model can explain all important aspects of those schools. For instance, in order to explain Keynesian features (about inflation, unemployment and relations of savings and investment) within the framework proposed in this study, it is necessary to explicitly introduce dynamics of money and relax the "neoclassical" assumption of savings being automatically equal to investment. This will cause further complications of modeling (e.g., Picard 1983; Zhang 1991).

The study may be extended in various ways. For instance, it is important but perhaps analytically difficult to introduce processes of class transformation. The population may be further classified into more classes. It is significant to extend the one-sector framework to a multi-sector one within which dynamic interdependence among production, employment and knowledge structures may be analyzed in a more satisfactory way.

Appendix A: Proving Propositions 3-4

To check Propositions 3 and 4, the following relationships will be used:

$$K = w_u Q^* / (\theta - w_0 \theta^{1/\beta}),\ L_i = \theta^{1/\beta} K - zN,\ F = \theta K,$$
$$w_u L_u = (1 - T)F,\ r = \alpha TF/K = \alpha\theta T,\ Y_j = \theta K_j,$$
$$RK_1 = (w - w_u)L_i + w_u L,\ RK_2 = WN,$$
$$Y_1/Y_2 = K_1/K_2 = \{(w - w_u)L_i + w_u L\}/WN.$$

Each proposition can be checked merely by taking derivatives of the above equations with respect to the parameter under consideration.

***Proposition* 3** (The impact of w)

$$dK/dw = K\theta^{1/\beta}/\beta(\theta - w_0\theta^{1/\beta}) > 0,\ dL_l/dw = \theta^{1/\beta}dK/dw > 0,$$
$$dF/dw = \theta dK/dw > 0,\ FdT/dw = (1 - T)dF/dw + w_u dL_l/dw > 0,$$
$$dr/dw = \alpha\theta dT/dw > 0,$$
$$RdK_1/dw = L_l + (w - w_u)dL_l/dw + K_1 dr/dw > 0,$$
$$RdK_2/dw = zN + K_2 dr/dw > 0,\ dY_j/dw = \theta dK_j/dw > 0,$$
$$d(Y_1/Y_2)/dw = d(K_1/K_2)/dw = (dL_l/dw - w_u L_u/w)/WN.$$

***Proposition* 4** (The impact of z)

$$dK/dz = NK/Q^* > 0,\ dF/dz = \theta dK/dz > 0,$$
$$dL_l/dz = (Q/Q^* - 1)N < 0,\ dT/dz = dr/dz = 0,$$
$$dY_1/dz = \theta dK_1/dz < 0,\ dY_2/dz = \theta dK_2/dz > 0,$$
$$RdK_1/dz = (w - w_u)dL_l/dz < 0,\ RdK_2/dz = wN > 0,$$
$$d(Y_1/Y_2)/dz = d(K_1/K_2)/dz < 0.$$

Appendix B: The Behavior When the Classes have Different Preferences

Appendix B examines the dynamic behavior of the system in the case that the two classes have different preferences.

An equilibrium of (19) is given by

$$F - zwN - \alpha wQK_2/\beta K = \delta_1 K_1/s_1,\ zwN + \alpha wQK_2/\beta K = \delta_2 K_2/s_2. \quad \text{(A1)}$$

From the above two equations, one has

$$F \doteq \delta_1 K_1/s_1 + \delta_2 K_2/s_2. \quad \text{(A2)}$$

Substituting F into (15) yields

$$Q = (\delta_1 K_1/s_1 + \delta_2 K_2/s_2 - w_u Q^*)/w_0, \quad \text{(A3)}$$

where $L_l = Q - zN$ and $w_0 = w/\beta - w_u > 0$ are used. Substituting (A2) and (A3) into (15), one gets

$$K_1 = (\lambda_2 + \lambda_3 K_2)K_2/(z\beta w_0 N + \lambda_1 K_2), \quad \text{(A4)}$$

where $\lambda_1 \equiv \alpha\delta_1/s_1 - \delta_2 w_0/ws_2,\ \lambda_2 \equiv \alpha w_u Q^* - z\beta w_0 N,$
$\lambda_3 \equiv (w_0/w - \alpha)\delta_2/s_2.$

From (A4), one gets K and F as functions of K_2 as follows:

$$K = \alpha(w_u Q^* + s^* K_2)K_2/(z\beta w_0 N + \lambda_1 K_2),$$
$$F(K_2) = \delta_1 K_1/s_1 + \delta_2 K_2/s_2 = (\delta_1 s_2 \lambda_2 + \delta_2 s_1 z\beta w_0 N + w_0 s^* s_1 \delta_2 K_2/w)K_2/s_1 s_2(z\beta w_0 N + \lambda_1 K_2), \tag{A5}$$

in which $s^* \equiv \delta_1/s_1 - \delta_2/s_2$. As

$$s^* = \delta_1/s_1 - \delta_2/s_2 = \zeta_1/\lambda_1 - \zeta_2/\lambda_2,$$

we may interpret $s^* > 0$ (< 0) as that the working class is less (more) patient than the knowledge class. In other words, $s^* > 0$ (< 0) means that the working class' savings propensity is lower (higher) than that of the knowledge class.

From (A2) and (A3), one has: $Q = (F - w_u Q^*)/w_0$. Substituting this equation and (A2) into the first equation in (A1) yields

$$(\delta_1 K_1/s_1 + \delta_2 K_2/s_2)(K - \alpha w K_2/\beta w_0) + \alpha w w_u Q^* K_2/w_0\beta = (\delta_1 K_1/s_1 + zwN)K. \tag{A6}$$

Substituting (A4) and (A5) into (A6) yields

$$\Phi(K_2) \equiv K_2\Phi_1(K_2) + \alpha s_1 s_2\{\Phi_2(K_2) - \Phi_3(K_2)\} = 0, \tag{A7}$$

where
$$\Phi_1(K_2) \equiv (\delta_1 s_2\lambda_2 + \delta_2 s_1 z\beta w_0 N + w_0 s^* s_1\delta_2 K_2/w)\{(\lambda_2 + \lambda_3 K_2) + (w - w_u)(z\beta w_0 N + \lambda_1 K_2)/w_0\},$$
$$\Phi_2(K_2) \equiv w w_u Q^*(z\beta w_0 N + \lambda_1 K_2)^2/w_0\beta,$$
$$\Phi_3(K_2) \equiv (w_u Q^* + s^* K_2)(\delta_1\lambda_2 K_2/s_1 + \delta_1\lambda_3 K_2^2/s_1 + z\beta w w_0 zN + zw\lambda_1 N K_2), \tag{A8}$$

where $1 - \alpha w/w_0\beta = (w - w_u)/w_0$ is used. An equilibrium value of K_2 is given as a solution of $\Phi(K_2) = 0$ for $K_2 > 0$. From the above equations, one sees that for any meaningful solution of $\Phi(K_2) = 0$, the other variables are uniquely determined. Unfortunately, it is difficult to generally guarantee the existence of solutions of the equation.

From (15) one has

$$L_i = \{F(K_2) - wzN/\beta - w_u L\}/w_0, \tag{A9}$$

where $F(K_2)$ is a given function of K_2 by (A5). The requirement, $0 < L_i < L$, is given by

$$wzN/\beta + w_u L < F(K_2) < wQ^*/\beta. \tag{A10}$$

From (A5) and (A10), one can explicitly determine the positive domain, (K', K''), to which K_2 belongs. It can be seen that within this domain, $K_1 > 0$ and $K_2 > 0$. Similarly to the case of the identical preference, one may provide conditions for the existence of solutions of $\Phi(K_2) = 0$ for $K' < K_2 < K''$. As the expressions of the conditions too complicated to be explicitly interpreted, further examination of the conditions for $\Phi(K_2) = 0$ to have meaningful solutions is omitted. It should be remarked that it is quite possible that $\Phi(K_2) = 0$ has multiple solutions when the two classes have different preferences. From (18) and (19), one may directly calculate the eigenvalues of each equilibrium and thus provide stability conditions.

References

Arrow, K.J., and Hahn, F.H. *General Competitive Analysis.* San Francisco: Holden-Day, 1971.

Balsko, Y. "Equilibria and Efficiency in the Fixprice Setting." *Journal of Economic Theory* 28 (1982): 113-27.

Barro, R., and Grossman, H. "A General Disequilibrium Model of Income and Employment." *American Economic Review* 61 (1971): 81-93.

Bhagwati, J., and Hamada, K. "The Brain Drain, International Integration of Markets for Professionals and Unemployment: A Theoretical Analysis." *Journal of Development Economics* 1 (1974): 19-42.

Bohm, V. "Disequilibrium Dynamics in a Simple Macroeconomic Model." *Journal of Economic Theory* 17 (179-99).

Bulow, J., and Summers, L. "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment." *Journal of Labor Economics* 4 (1986): 376-414.

Burmeister, E., and Dobell, A.R. *Mathematical Theories of Economic Growth,* New York: Macmillan, 1970.

Eckalbar, J.C. "Stable Quantities in Fixed Price Disequilibrium." *Journal of Economic Theory* 25 (1981): 302-13.

Green, J.R., and Laffont, J.J. "Disequilibrium Dynamics with Inventories and Anticipatory Price-Setting." *European Economic Review* 16 (1981): 199-211.

Hahn, F.H. "On Non-Walrasian Equilibria." *Review of Economic Studies* 45 (1978): 1-18.

Kaldor, N. "Marginal Productivity and the Macro-Economic Theories of Distribution." *Review of Economic Studies* 33 (No. 4 1966).

Leijonhufvud, A. *On Keynesian Economics and the Economics of Keynes.* Oxford: Oxford University Press, 1968.

Lucas, R.E. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22 (1988): 3-42.

Malinvaud, E. *The Theory of Unemployment Reconsidered.* Oxford:

Blackwell, 1977.

Marglin, S.A. *Growth, Distribution, and Prices*, Mass., Cambridge: Harvard University Press, 1984.

Miyagiwa, K. "Human Capital and Economic Growth in a Minimum-Wage Economy." *International Economic Review* 30 (1989): 187-202.

Negishi, T. *Economic Theories in a Non-Walrasian Tradition*. Cambridge: Cambridge University Press, 1985.

Pasinetti, L.L. *Growth and Income Distribution*. Cambridge: Cambridge University Press, 1974.

Picard, P. "Inflation and Growth in a Disequilibrium Macro-Economic Model." *Journal of Economic Theory* 30 (1983): 266-95.

Pissarides, C.A. "Trade Unions and the Efficiency of the Natural Rate of Unemployment." *Journal of Labor Economics* 4 (1986): 582-95.

Rodriguez, C.A. "Brain Drain and Economic Growth: A Dynamic Model." *Journal of Development Economics* 2 (1975): 223-47.

Romer, P.M. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* 94 (1986): 1002-37.

———. "Endogenous Technological Change." *Journal of Political Economy* 98 (1990): 71-102.

Samuelson, L. "Implicit Contracts and Heterogenous Labor." *Journal of Labor Economics* 3 (1985): 70-100.

Sato, K. "The Neoclassical Theorem and Distribution of Income and Wealth." *The Review of Economic Studies* 33 (1966): 331-+6.

Silvestre, J. "Fixprice Analysis in Exchange Economies." *Journal of Economic Theory* 26 (1982): 28-58.

Spence, M. "Competition in Salaries, Credentials, and Signaling Prerequisites for Jobs." *The Quarterly Journal of Economics* 91 (1976): 51-74.

Stockey, N.L. "Job Differentiation and Wages." *The Quarterly Journal of Economics* 95 (1980): 432-49.

Zhang, W.B. "Wealth Accumulation and Endogenous Technology in a Dynamic Monetary Economy." *The Indian Economic Journal* 39 (1991): 43-53.