A Reexamination on the Theory of Labor Supply with Wage Rate Uncertainty

Il Tae Kim*

This paper presents a unified reinterpretation of conditions preference derived by a decomposition of the effect of an increase in wage rate risk on labor supply. The compensated effect of risk that keeps expected utility constant is analyzed. The result of this analysis shows that preference conditions sufficient to determine the comparative static effects of risk on labor supply can be expressed in terms of the measure of endogenous partial risk aversion and the wage elasticities for labor supply. (*JEL* Classifications: D81, J22)

I. Introduction

The effect of wage rate uncertainty on labor supply has been studied by Block and Heineke (1973), Eaton and Rosen (1980), and Tressler and Menezes (1980) using models of labor supply. Their conclusion shows that conditions derived to determine the comparative static effects of risk depend on the sign of the third order derivatives of utility and that conditions on preference are insufficient to know the sign of these effects of increased wage risk. More recently, Dionne and Eeckhoudt (1987) have investigated that the uncompensated effect of increased wage rate risk on labor supply depends on the monotonicity of proportional risk aversion and the sign of supply curve of labor. However, there has been little analysis of the substitution effect with a

*Assistant Professor of Economics, Chonnam National University, Kwangju 500-757, Korea. An earlier version of this paper was presented at the 28th Annual Conference of Missouri Valley Economic Association, Saint Louis, Missouri, U.S.A., Feb. 27-29, 1992. The author gratefully acknowledges helpful comments and suggestions of an anonymous referee of this journal and Donald L. Johnson on an earlier draft of this paper.

mean utility preserving increase in risk as defined by Diamond and Stiglitz (1974). This paper extends the analysis to the effect of an increase in risk that keeps expected utility constant.

This paper decomposes the uncompensated effect of a mean preserving spread of the wage rate distribution into the substitution effect and the income effect. The analysis uses a condition on the index of endogenous partial risk aversion, denoted $P^*(Y)$, where Y is a random consumption. Thus, conditions derived previously to determine the uncompensated effect of risk are shown to be alternative statements of this decomposition. Moreover, the index $P^*(Y)$ and the income effect and substitution effects of an increase in wage rate risk can be stated in terms of the compensated and uncompensated wage rate elasticities for labor supply. This paper presents reinterpretation of the preference conditions derived previously to determine the effects of uncertainty about wage rate on labor supply by generalizing the index of partial (relative) risk aversion to the index of endogenous partial risk aversion which incorporates a direct utility interdependence between labor supply and consumption.

This paper is organized as follows. In the next section we set out the labor supply model with wage rate uncertainty and the $P^*(Y)$ is defined. In section III the comparative static effects of risk are presented. Section IV provides conclusions.

II. The Theoretical Model

Following the pure labor supply model of Dionne and Eeckhoudt without the taxation on wage,[1] the expected value of a Neumann-Morgenstern utility $U(c, L)$ is maximized by the choice of labor supply L, given that consumption c is the random variable

$$c = m + wL, \tag{1}$$

where m is exogenous, non-random and represents non-labor income, and w is the positive random wage rate, Using E to denote the expectation operator, the first- and second-order conditions for a maximum of

$$V = EU(m + wL, L), \tag{2}$$

[1]Extension of the analysis to two-periods would be an integrated treatment of consumption-saving and labor supply-leisure decisions. Such an extension, however, is a subject for future research.

by choice of L are

$$V_L = E(wU_1 + U_2) = 0, \tag{3}$$

and

$$V_{LL} = E(w^2U_{11} + 2wU_{12} + U_{22}) < 0, \tag{4}$$

where subscripts here and later denote partial derivatives. It is assumed that U is increasing in consumption c and decreasing in hours worked L, strictly concave (implying risk averse), and at least three times continuously differentiable and that c and L are positive.

To develop the comparative static analysis, for a fixed value of L, the Arrow-Pratt measure of absolute risk aversion for risks in the c dimension is defined as

$$A(Y) = -U_{11}(Y, L)/U_1(Y, L). \tag{5}$$

The measure of partial (relative) risk aversion introduced by Menezes and Hanson (1970) is defined as

$$P(Y) = -YU_{11}(m + Y, L)/U_1(m + Y, L). \tag{6}$$

When m and L are constant and Y is random, the associated risk premium π is defined implicitly by the relation

$$EU(m + \alpha + \beta Y, L) = U(m + \alpha + \beta EY - \pi, L),$$

where α and β are parameters that induce, respectively, an additive and a multiplicative shift of the random variable Y with mean EY. Initially, $\alpha = 0$ and $\beta = 1$. Since L is endogenous in the theory of labor supply, extensions of the index of absolute or partial risk aversion that account for the optimal choice of L are relevant.[2] The extension of the measure of risk aversion used in this paper associates decreasing (increasing) partial risk aversion with

$$d\pi/d\beta + (d\pi/dL)(dL/d\beta)\,|_{dEU=0} < (>)\ \pi/\beta.$$

For a random consumption Y, the function (6) depends on Y and L.

[2]Sandmo (1970) has developed the extension of the absolute risk aversion measure based on the attitude toward risk that associates decreasing (increasing) absolute risk aversion with negative (positive) changes in the risk premium when the mean of the random gross return to savings increases and saving changes to maintain expected utility constant. Brown and Snow (1990) have applied to this concept to introduce the measure of endogenous absolute and partial risk aversion measure in the two-argument utility function.

Hence, as shown in the Appendix, the variation of the measure of risk aversion indicative of this attitude toward risk can be derived for simultaneous changes in c and L along the budget constraint (1) and thus is given by

$$P^*(Y) = -[LU_{12}(m+Y, L) + YU_{11}(m+Y, L)]/U_1(m+Y, L), \tag{7}$$

which can be interpreted as a measure of endogenous partial risk aversion.

III. The Comparative Static Results

In this section, to examine the effects of an increase in risk on labor supply, we replace w for the random wage rate with

$$w = \bar{w} + \gamma\varepsilon, \tag{8}$$

where $\bar{w}$ is the mean of w, γ is a positive parameter and is initially equal to unity, and ε is the random variable with mean zero. An increase in the risk parameter γ induces a mean-preserving increase in the spread of the probability distribution for w and reduces expected utility without changing the mean. *A compensated increase in spread* is defined to be an increase in the parameter γ accompanied by an increase in the mean $\bar{w}$ to maintain expected utility. The following proposition shows that labor supply increases (decreases) with a compensated increase in spread if the measure of endogenous partial risk aversion decreases (increases) with the random wage rate.

***Proposition* 1**
Labor supply L increases (decreases) with a compensated increase in spread in the distribution of w if $P^*(Y)$ is a decreasing (increasing) function of Y.

***Proof*:** This proposition follows by implicitly differentiating the first-order condition (3) with respect to γ, having replaced w with $\bar{w} + \gamma\varepsilon$, and setting $d\bar{w}/d\gamma = -E(\varepsilon U_1)/E(U_1)$ in order to preserve expected utility. Hence, the effect on labor supply is given by

$$\begin{aligned}\frac{dL^*}{d\gamma} &= -\{V_{L\gamma} + V_{L\bar{w}}(d\bar{w}/d\gamma)\}/V_{LL} \\ &= -E[(wU_{11} + U_{12})L(\varepsilon - \frac{E(\varepsilon U_1)}{E(U_1)})]/V_{LL}\end{aligned}$$

$$= E[P^*(Y)\phi U_1]/V_{LL},$$

where $\phi = \varepsilon - [E(\varepsilon U_1)/E(U_1)]$. Note that $E(\phi U_1) = 0$ and that $Y = wL$ is higher when ε is higher. Under the assumption of decreasing endogenous partial risk aversion $[P_Y^* < 0]$,

$$P^*(Y) < P^*(\hat{Y}) \quad \text{for } \phi > 0$$

and

$$P^*(Y) > P^*(\hat{Y}) \quad \text{for } \phi < 0,$$

where $\hat{Y}$ is the value of Y when $\phi = 0$. Multiplying both sides of the inequality by ϕU_1 which is positive (negative) when $\phi > (<)\ 0$ yields

$$P^*(Y)\phi U_1 < P^*(\hat{Y})\phi U_1,$$

for all ϕ. Taking expectations obtains

$$E[P^*(Y)\ \phi U_1] < E[P^*(\hat{Y})\phi U_1] = P^*(\hat{Y})E(\phi U_1) = 0,$$

so that $dL^*/d\gamma$ is positive, given $V_{LL} < 0$. When $P_Y^* > 0$, the inequalities are reversed, completing the proof.

The income effect of an increase in spread which keeps its mean constant is defined to be the effect of a decrease in the mean of wage rate sufficient for a change in expectation of utility to be equivalent to the loss in utility caused by an increase in spread. Hence, it follows that the income effect of a mean-preserving increase in spread is positive (negative) if the wage elasticity of labor supply is negative (positive), i.e., $\eta_{Lw} < (>)0$, where η_{ij} denotes the elasticity of i with respect to j.

Next, using the general definition of increase in risk, termed a mean-preserving spread introduced by Rothschild and Stiglitz (1970), the uncompensated effect of a mean-preserving spread on labor supply can be derived in the context of Rothschild and Stiglitz (1971). The following proposition provides conditions on preference that determine the uncompensated effect of wage rate risk on labor supply.

Proposition 2

Labor supply L increases (decreases) with a mean-preserving spread if $P_Y^* < (>)\ 0$ and $P^*(Y) > (<)\ 1$.

Proof: Following Rothschild and Stiglitz (1971), a sufficient condition for labor supply, L to increase (decrease) under increased wage rate risk is that

$$\partial(L^2U_{11})/\partial L > (<) 0.$$

Differentiation of (L^2U_{11}) and $P^*(Y)$ shows that

$$\partial(L^2U_{11})/\partial L = L(YU_{111} + 2U_{11} + LU_{112}) \tag{9}$$

and that

$$P_Y^* = -[LU_{112} + YU_{111} + (1 + P^*)U_{11}]/U_1. \tag{10}$$

Using the expression of P_Y^*, equation (9) can be rewritten as

$$\partial(L^2U_{11})/\partial L = -LU_1[P_Y^* - (P^* - 1)A(Y)].$$

It follows that the effect on labor supply is positive (negative) if the P_Y^* is negative (positive) and $P^*(Y)$ is greater (less) than unity.

This result is related to Eaton and Rosen's (1980, pp. 367-8) analysis under the assumptions of additive separability of utility. For additive separable ($U_{12} = 0$), the measure of endogenous partial risk aversion, $P^*(Y)$ reduces to the measure of partial risk aversion, $P(Y)$ and, when no non-labor income ($m = 0$), $P(Y)$ is the measure of relative risk aversion;

$$R(Y) = -YU_{11}(Y, L)/U_1(Y, L).$$

Hence, conditions on preference sufficient for increased wage rate risk to increase (decrease), in the sense of a mean-preserving spread in w, can be expressed in terms of the monotonicity and magnitude of relative risk aversion.[3] The statement of Proposition 2 can be restated as the measure of relative risk aversion $R(Y)$ instead of $P^*(Y)$. Moreover, many empirical evidences provide relative risk aversion above two, and if the non-increasing absolute risk aversion hypothesis is accepted, as characterization of attitudes toward risk,[4] then the following inequality is satisfied $2U_{11} + YU_{111} > 0$.

The measure of endogenous partial risk aversion can be expressed in

[3]For additive separable utility function, when there exists no exogenous income, the uncompensated effect of risk on labor supply can be rewritten as

$$\begin{aligned}\partial(L^2U_{11})/\partial L &= L(2U_{11} + YU_{111})\\ &= -LU_1[R_Y - (R-1)A(Y)].\end{aligned}$$

This expression implies that labor supply L increases (decreases) with a mean-preserving spread if U exhibits decreasing (increasing) absolute risk aversion and the magnitude of relative risk aversion is greater (less) than unity.

terms of the elasticities for labor supply with respect to the wage rate. When the wage rate w increases, the Slutsky equation for a change in labor supply can be written as

$$\partial L / \partial w = -U_1 / D + L(-U_{12} - wU_{11}) / D, \tag{11}$$

where $D = w^2 U_{11} + 2wU_{12} + U_{22} < 0$.
Using the definition of $P^*(Y)$ and equation (11), we find that

$$P^* - 1 = \frac{(\partial L / \partial w)}{(U_1 / D)} = \frac{\eta_{Lw}}{(-\hat{\eta}_{Lw})},$$

where $\hat{\eta}_{Lw}$ is the compensated wage elasticity of labor supply, that is,

$$\hat{\eta}_{Lw} = (w/L)(-U_1/D)$$

is positive. Hence, P^* exceeds (is less than) unity if labor supply decreases (increases) with an increase in the wage rate. Proposition 2 can be expressed in terms of the wage rate elasticities for labor supply and interpreted as the substitution and the income effect of increased wage rate risk. Therefore, we obtain the following corollary.

***Corollary* 1**
Labor supply L increases (decreases) with a mean-preserving spread if the substitution effect is positive (negative), i.e.,

$$d(\eta_{Lw}/\hat{\eta}_{Lw})/dc\,|_{dL=0} > (<)\ 0,$$

and the income effect is positive (negative), i.e.,

$$\eta_{Lw} < (>)\ 0.$$

The statement of Proposition 2 and Corollary 1 is related to Proposition 2 postulated by Dionne and Eeckhoudt (1987, pp.359-60). This corollary shows that the magnitude of the measure of endogenous partial risk aversion depends on the sign of the supply curve of labor.

As c and L simultaneously moves along the budget line given in (1), the derivative of the measure $P^*(Y)$ with respect to Y is related to the derivative of the measure $P(Y)$ with respect to both Y and L. Hence,

[4]Cohn, *et al.* (1975) have investigated empirically the effect of wealth on the proportion of individual portfolio allocated to risky assets and presented evidence of decreasing absolute risk aversion. Friend and Blume (1975) have estimated the coefficient of relative risk aversion using the market price of risk and concluded that "...the coefficient of proportional risk aversion is more likely to be in excess of two" (p.920).

totally differentiating $P(wL, L, m)$ with respect to L is equal to

$$\begin{aligned}\frac{dP}{dL} &= wP_Y + P_L \\ &= \frac{Y(-U_{112} - wU_{111}) - wU_{11}}{U_1} + \frac{YU_{11}(wU_{11} + U_{12})}{U_1^2} \\ &= wP_Y^*.\end{aligned}$$

The following corollary provides that Proposition 2 can be alternatively expressed in terms of the measure of partial risk aversion $P(Y)$ and the uncompensated wage elasticities for labor supply.

Corollary 2

Labor supply L increases (decreases) with a mean-preserving spread if $dP/dL < (>) \ 0$ and $\eta_{Lw} < (>) \ 0$.

This corollary is related to a result derived previously by Dardanoni (1988, p.442). For the economic intuitive explanation behind this corollary, Dardanoni indicates that an increase in risk about [wage rate] "is perceived as a reduction of its expected value; if the individual unambiguously increases (decreases) [labor supply, L], when [wage rate] decreases, and by doing so is faced with a subjectively less 'risky' situation, i.e., one to which he is less risk averse (as measured by $P(Y)$), then [the effect of an increase in wage rate risk on labor supply] will unambiguously be positive (negative)" (p.441).

The Proposition 2 and Corollary 1 and 2 provide the economic intuition for the response of labor supply to risk. When labor supply is upward sloping, the effect of an increase in risk reduces certainty equivalent sure wage introduced Dionne and Eeckhoudt. Hence, the budget line is flatter and supply of labor reduces. As presented in (7), the fall in consumption decreases risk aversion under endogenous partial risk aversion, and by decreasing labor supply, the individual reduces a perceived risk of the outcome. Thus, the response of labor supply to risk is negative.

Finally, consider now the compensated effect of uncertainty about wage rate on labor supply, from Diamond and Stiglitz's Theorem 2 (1974), preference conditions sufficient to determine the substitution effect of a mean-preserving spread on labor supply can be stated by applying to the fact that "it is natural to think of the mean utility preserving increase as a compensated adjustment of a mean preserving increase in risk" (p.343). Hence, we proceed to prove the following

Proposition 3.

Proposition 3

Labor supply increases (decreases) with a compensated mean-preserving spread if $P^*(Y)$ is a decreasing (increasing) function of Y.

Proof: Following Diamond and Stiglitz, the absolute value of labor supply L increases (decreases) if

$$\frac{\partial^2(\log U_w)}{\partial w \partial L} > (<) 0, \text{ that is, } \frac{\partial(LU_{11}/U_1)}{\partial L} > (<) 0.$$

Differentiating (LU_{11}/U_1) with respect to L and using the expression (10) to obtain

$$\begin{aligned}\frac{\partial(LU_{11}/U_1)}{\partial L} &= \frac{(U_{11}+YU_{111}+LU_{112})U_1 - LU_{11}(wU_{11}+U_{12})}{U_1^2} \\ &= -P_Y^*.\end{aligned}$$

It follows that the effect on labor supply is positive (negative) if P_Y^* is negative (positive).

To see the concrete form of a sufficient condition presented in Proposition 3, taking the following utility function presented by Cowell (1981) and using our notation;

$$U = -\frac{1}{\beta\rho}[(Y-K)(T-L)^{\beta}]^{-\rho},$$

where $\beta > 0$ and $\rho > -1$ are parameters, T denotes the maximum time available, and K is the subsistence minimum level of consumption. This utility function represents strictly decreasing absolute risk aversion and increasing or decreasing endogenous partial risk aversion in terms of the sign of K.

For such a utility, it is easily seen that the measure of endogenous partial risk aversion

$$P^*(Y) = \frac{(\rho+1)Y}{(Y-K)} - \frac{\beta\rho L}{(T-L)}$$

and

$$\frac{\partial(LU_{11}/U_1)}{\partial L} = \frac{(\rho+1)K}{(Y-K)^2} = -P_Y^*.$$

If K is positive (negative), $P^*(Y)$ is a decreasing (increasing) function of Y and then the effect on labor supply is positive (negative). Hence, the substitution effect with a mean utility preserving increase in risk on labor supply is determined by the monotonicity of $P^*(Y)$ depending on the sign of K. Proposition 2 and 3 show that the economic intuition given above reflects a decomposition between substitution and income effect of an increase in risk.

IV. Conclusions

This paper derives conditions sufficient to imply determinate signs for the effect of uncertainty on labor supply by using general and plausible assumptions on risk preference and the wage rate elasticities for labor supply. The finding of the paper is that conditions on preference derived previously to determine the uncompensated effect of increased wage uncertainty can be alternatively stated in terms of the index of endogenous partial risk aversion and the wage elasticities for labor supply. The uncompensated effects of a mean-preserving increase in spread consist of the substitution effect remarked in Proposition 3 and the income effect that depends on the wage elasticities for labor supply or the magnitude of endogenous partial risk aversion.

Specifically, our analysis reveals that labor supply increases (decreases) with a mean-preserving spread if utility exhibits decreasing (increasing) endogenous partial risk aversion and the magnitude of endogenous partial risk aversion is greater (less) than unity. Finally, we also find that, labor supply increases (decreases) in response to an increase in uncertainty about the wage rate that the expected utility is kept constant if the measure of endogenous partial risk aversion is monotonically decreasing (increasing) in random consumption.

Appendix

To develop the variation of the partial risk aversion, that is, the index of endogenous partial risk aversion (7), differentiating in full the partial risk aversion function (6), we obtain

$$d\left(-\frac{YU_{11}}{U_1}\right)=Y\left[\frac{\partial}{\partial Y}\left(-\frac{U_{11}}{U_1}\right)dY+\frac{\partial}{\partial L}\left(-\frac{U_{11}}{U_1}\right)dL\right]-\frac{U_{11}}{U_1}dY.$$

Using the relationship between consumption and labor supply along the budget constraint, i.e., $dY = wdL$, it can be rewritten as

$$d\left(-\frac{YU_{11}}{U_1}\right)=\left[\frac{\partial}{\partial Y}\left(-\frac{YU_{11}}{U_1}\right)dY+\frac{\partial}{\partial L}\left(-\frac{YU_{11}}{U_1}\right)\left(\frac{1}{w}\right)dY\right]-\frac{U_{11}}{U_1}dY.$$

Since $\frac{\partial}{\partial L}\left(-\frac{L_{11}}{U_1}\right)=\left(-\frac{U_{112}U_1-U_{12}U_{11}}{U_1^2}\right)=\frac{\partial}{\partial Y}\left(-\frac{U_{12}}{U_1}\right)$, it follows that $\frac{d}{dY}\left(-\frac{YU_{11}}{U_1}\right)=\frac{\partial}{\partial Y}\left(-\frac{LU_{12}+YU_{11}}{U_1}\right)-\frac{U_{11}}{U_1}$. Hence, the index of endogenous partial risk aversion (7) is derived. *(Manuscript received September, 1993; final revision received February, 1994)*

References

Block, M.K., and Heineke, J.M. "The Allocation of Effort under Uncertainty: The Case of Risk-Averse Behavior." *Journal of Political Economy* 81 (1973): 376-85.

Brown, D.M., and Snow, A. "The Effects of Product Price Uncertainty on the Owner-Managed Firm: Theory and Evidence." *Southern Economic Journal* 56 (1990): 705-17.

Cohn, R.A., Lewellan, W.G., Lease, R.C., and Schlarbaum, G.G. "Individual Investor, Risk Aversion and Investment Portfolio Composition." *Journal of Finance* 30 (1975): 605-20.

Cowell, F.A. "Income Maintenance Schemes under Wage-Rate Uncertainty." *American Economic Review* 71 (1981): 692-703.

Dardanoni, V. "Optimal Choices under Uncertainty: The Case of Two-Argument Utility Functions." *Economic Journal* 98 (1988):429-50.

Diamond, P.A., and Stiglitz, J.E. "Increase in Risk and in Risk-Aversion." *Journal of Economic Theory* 8 (1974): 333-61.

Dionne, G., and Eeckhoudt, L. "Proportional Risk aversion, Taxation and Labor Supply Under Uncertainty." *Journal of Economics* 47 (1987): 353-66.

Eaton, J., and Rosen, H.S. "Labor Supply, Uncertainty, and Efficient Taxation." *Journal of Public Economics* 14 (1980): 365-74.

Friend, I., and Blume, M.E. "The Demand for Risky Assets." *American Economic Review* 65 (1975): 900-22.

Menezes, C.F., and Hanson, D.L. "On the Theory of Risk Aversion.' *International Economic Review* 11 (1970): 481-7.

Rothschild, M., and Stiglitz, J.E. "Increasing Risk: I.A Definition." *Journal of Economic Theory* 2 (1970): 225-43.

___________."Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 (1971): 66-84.

Sandmo, A. "The Effect of Uncertainty on Saving Decisions." *Review of Eco nomic studies* 37 (1970): 353-60.

Tressler, J.H., and Menezes, C.F. "Labor Supply and Wage Rate Uncertain ty." *Journal of Economic Theory* 23 (1980): 425-36.