Interactions between Nominal and Real Variables with Observable Nominal Aggregates

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This paper shows that serially uncorrelated monetary noise can affect output, even though every agent observes the price level and the money stock, if government responds to real shocks imperfectly and agents are asymmetrically informed. Furthermore, procyclical money can be associated with countercyclical price. (*JEL* Classification: E32)

I. Introduction

In this paper, serially uncorrelated monetary noise can affect output even though each agent observes the money stock and the price level. One condition for such monetary nonneutrality is that some (but not all) agents mistake monetary noise for persistent real shocks. Thus, the model in this paper is related to the island type macro models such as Lucas (1972), Barro (1976), Weiss (1980) and King (1982). Yet, unlike them, this paper is not subject to the criticism that monetary aggregates are promptly available in practice (see, e.g., McCallum 1982; Gordon 1990). Furthermore, this paper is consistent with the fact that the most of the information about inflation is known as it occurs (see Huberman and Schwert 1985).

In addition to the prompt availability of nominal aggregates, a realistic model of the business cycle should incorporate the fact that government can observe current real shock (if not perfectly) and conducts monetary policy in response (see Friedman and Schwartz 1963; Romer and Romer 1989). In this paper, money growth depends partly on real

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shocks; the part of money growth independent of real shocks is called a monetary noise. Thus, at least partly, this paper treats money as endogenous, similarly to King and Plosser (1984).

Also, in actuality, information on current changes in tastes or production technology is difficult to acquire, especially when such information helps to predict future changes in tastes or technology. That is, acquiring information on real shocks—in particular, persistent real shocks—is costly. This implies that an asymmetric information structure is more realistic than a symmetric one, since given that information acquisition is costly, if everyone were to have identical information regardless of his or her information acquisition activity, then such information is of no value in equilibrium and nobody would spend resources to acquire such information (see Grossman and Stiglitz 1980). In this paper, I assume that some are better informed on persistent real shocks than others.

Monetary noise affects aggregate output for two reasons. (i) The uninformed mistake it for the innovation to persistent real shocks. (ii) As the responses of the uninformed affect the return to labor, the informed also respond to monetary noise even though they know the nature of shocks. This is similar to a result in De Long, Shleifer, Summers, and Waldman (1990) that irrational agents' behavior may affect rational agents' decision making. This paper shows that even without any irrationality, rational informed agents' behavior may depend on the confused behavior of the uninformed when responding to monetary shocks. This seemingly irrational behavior results from uninformed agents' optimizing behavior. Namely, to the uninformed, the knowledge of temporary monetary shocks is worth much less than that of persistent real shocks. Thus, in an attempt to figure out persistent real shocks, uninformed agents over-react to monetary shocks. Such over-reaction of the uninformed distorts the return to labor of the informed. Thus, the informed respond to temporary monetary shocks even though they know the nature of shocks. Furthermore, since it is the uninformed who distort the return to labor of the informed, the output response of the uninformed to the monetary noise is greater than that of the informed. This paper also shows that if agents have the symmetric information set—either all the agents observe the shocks or none of the agents observe the shocks, then the monetary noise does not affect the output. Since even if all are uninformed, they can infer the monetary noise perfectly from observing the money stock and the price level. Finally, this paper shows that procyclical money may be associated with countercyclical price, which seems consistent with recent findings by Kydland and Prescott (1990).

II. Structure of the Economy

Consider an infinitely lived economy, populated by many two-period lived agents. For simplicity, agents work only when young; consume only when old. Population changes stochastically over time; θ_t denotes the current ratio of the number of the young to that of the old. Each young agent wants to maximize the following expected utility function:

$$v(n_t, c_{t+1}) = -\frac{1}{1+\alpha} n_t^{1+\alpha} + \frac{1}{1-\beta} E\{c_{t+1}^{1-\beta} | \Omega_t\}, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$.[1] n_t is the amount of labor, which yields the same amount of the non-storable consumption good. c_{t+1} is an agent's old age consumption. β is the Arrow-Pratt measure of relative risk aversion, which also governs intertemporal substitution. Ω_t is an agent's information set.

Government issues fiat money; new money is injected by means of a beginning-of-period transfer, in a quantity proportional to initial money holdings. Let m_t denote the (gross) money growth rate. The money balances of a young agent equal the market value of his labor-output:

$$M_t = p_t n_t, \tag{2}$$

where p_t is the price level in period t.

After a proportional transfer, the agent, now old, spends all his money to consume goods:

$$M_{t+1} = M_t m_{t+1}, \tag{3}$$

$$M_{t+1} = P_{t+1} c_{t+1}. \tag{4}$$

The population growth rate θ_t consists of a temporary shock ε_t and a persistent shock z_t, the log of which follows an AR (1) process:

$$\theta_t = z_t \varepsilon_t \text{ and } z_t = z_{t-1}^{\rho} \eta_t, \tag{5}$$

where $\rho \in (-1,1)$ and η_t is the innovation to persistent shocks.

[1] If $\beta = 1$, then from (2) through (6), (10) and (11), the equilibrium price under perfect foresight is: $p_t = M_t/\theta_t$. That is, even though each agent has complete information on the future states, each has no use of such information. In the text, I only consider the case of $\beta \neq 1$.

Government conducts monetary policy in response to real shocks; yet, it responds imperfectly. The current money growth rate m_t consists of current real shocks and a monetary noise x_t:

$$m_t = x_t \theta_t^{\delta} \equiv x_t (z_{t-1}^{\rho} \eta_t \varepsilon_t)^{\delta}. \tag{6}$$

If $\delta > 0$, the government accommodates real shocks; if $\delta < 0$, it counteracts real shocks.

As for the information structure, the uninformed observe the price level, the money stock, and all the shocks realized through period t-1. The informed observe η_t, the innovation to persistent real shocks, in addition to the information that the uninformed have. That is,

$$\begin{aligned} \Omega_t^I &= \{\eta_t, M_t, p_t, \hat{\Omega}_{t-1}\} \; ; \; \Omega_t^U = \{M_t, p_t, \hat{\Omega}_{t-1}\} \\ \hat{\Omega}_{t-1} &= \{x_{t-1}, x_{t-2}, \ldots; \eta_{t-1}, \eta_{t-2}, \ldots; \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}, \end{aligned} \tag{7}$$

where the superscripts I and U refer to informed and uninformed, respectively.

Substituting (2), (3) and (4) into (1), then, differentiating it with respect to n_t, one gets the amount of the consumption good produced by an agent j for $j = I, U$:

$$n_t^j = [E\{(p_t m_{t+1} / p_{t+1})^{1-\beta} | \Omega_t^j\}]^{\frac{1}{\alpha+\beta}}. \tag{8}$$

Note that $p_t m_{t+1}/p_{t+1}$ is the return to labor. If an agent works and produces one unit of the consumption good, the agent gets p_t dollars from the old. After a transfer from government, the agent has $p_t m_{t+1}$ dollars. Thus, in exchange for a unit of current good, an agent gets $p_t m_{t+1}/p_{t+1}$ units of future good. According to (8), production varies with the return to labor if $\beta \in (0,1)$; it varies inversely with the return if $\beta > 1$. This results from the fact that substitution effects dominate wealth effects if $\beta \in (0,1)$; wealth effects dominate substitution effects if $\beta > 1$.

For simplicity, geometric averaging is used to aggregate agents' output. Thus for $\lambda \in [0,1]$,

$$\log \bar{n}_t = \lambda \log n_t^I + (1-\lambda) \log n_t^U, \tag{9}$$

where $\bar{n}_t$ is the average output per young agent, and λ is the share of the informed.[2]

[2]Following Grossman and Stiglitz (1980), one could extend the model in this paper to endogenize the fraction of the informed using a non-cooperative Nash equilibrium concept. Such an extension would be interesting since it would

Let the size of the period t old be normalized to 1. Then, θ_t is the size of the period t young, and the aggregate production in period t, $\bar{y}_t$, becomes:

$$\bar{y}_t = \theta_t \bar{n}_t. \tag{10}$$

Note that θ_t can be interpreted as a productivity shock in a constant population environment.

In equilibrium, the real balances of the old equal the consumption good produced by the young:

$$\frac{M_t}{P_t} = \bar{y}_t. \tag{11}$$

Finally, I assume that $(x_t, \varepsilon_t, \eta_t)$ is independent and has a stationary joint log-normal distribution. More specifically,

$$\begin{bmatrix} \log x_t \\ \log \eta_t \\ \log \varepsilon_t \end{bmatrix} \sim N(0, \Sigma), \text{ where } \Sigma = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{bmatrix}. \tag{12}$$

III. Characteristics of Equilibrium

From (7), (8), (9) and (10), the equilibrium condition (11) can be rewritten as: for any $\lambda \in [0,1]$,

$$\frac{M_t}{P_t} = \theta_t [E\{(p_t m_{t+1} / P_{t+1})^{1-\beta} | \Omega_t^I\}]^{\frac{\lambda}{\alpha+\beta}} [E\{(p_t m_{t+1} / P_{t+1})^{1-\beta} | \Omega_t^U\}]^{\frac{1-\lambda}{\alpha+\beta}}. \tag{13}$$

In the appendix, I show that the following price equation satisfies (13) for any $\lambda \in (0,1]$,[3]

$$P_{\lambda,t} = k_\lambda M_{t-1} z_{t-1}^{\gamma_4} (x_t \eta_t^{\gamma_2} \varepsilon_t^{\gamma_3})^{\gamma_1}, \tag{14}$$

where

show how the fraction of the informed would respond to changes in the variances of the shocks. In this paper, however, I do not consider such an extension, since it would complicate the analysis without altering the main results.

[3]With certain restrictions on α and β, one can show that (14) is unique among the class of uniformly bounded, forward looking, and market fundamental solutions.

$$\gamma_1 = 1 + \delta \frac{(\alpha+\beta)(1-\lambda)}{(1+\alpha)\lambda} \sigma_\eta^2 \sigma_\varepsilon^2 / D;$$

$$\gamma_1\gamma_2 = \delta - \frac{\alpha+\beta}{1+\alpha} - (\delta\rho - \gamma_4)\frac{1-\beta}{1+\alpha} + \frac{1}{\delta\rho-\gamma_4}\frac{(\alpha+\beta)^2(1-\lambda)}{(1+\alpha)(1-\beta)\lambda^2}\sigma_x^2\sigma_\varepsilon^2 / D;$$

$$\gamma_1\gamma_3 = \delta - \frac{\alpha+\beta}{1+\alpha} - \frac{(\alpha+\beta)(1-\lambda)}{(1+\alpha)\lambda}\left(1 + \frac{1}{\delta\rho-\gamma_4}\frac{\alpha+\beta}{(1-\beta)\lambda}\right)\sigma_x^2\sigma_\eta^2 / D;$$

$$\gamma_4 = \delta\rho - \rho \Big/ \left\{1 + (1-\rho)\frac{1-\beta}{\alpha+\beta}\right\};$$

$$k_\lambda^{-1} = \exp\left[\frac{1}{2}\frac{(1-\beta)^2}{\alpha+\beta}\{(1-\gamma_1)^2\sigma_x^2 + (\delta-\gamma_1\gamma_2)^2\sigma_\eta^2 + (\delta-\gamma_1\gamma_3)^2\sigma_\varepsilon^2\right.$$

$$\left. + (1-\lambda)\left(\frac{\alpha+\beta}{(1-\beta)\lambda}\right)^2 \sigma_x^2\sigma_\eta^2\sigma_\varepsilon^2 / D\}\right];$$

$$D = \sigma_x^2\left\{\left(1 + \frac{1}{\delta\rho-\gamma_4}\frac{\alpha+\beta}{(1-\beta)\lambda}\right)^2\sigma_\eta^2 + \left(\frac{1}{\delta\rho-\gamma_4}\frac{\alpha+\beta}{(1-\beta)\lambda}\right)^2\sigma_\varepsilon^2\right\} + \delta^2\sigma_\eta^2\sigma_\varepsilon^2.$$

According to (6), (10), (11) and (14), the log of aggregate output becomes:

$$\log \bar{y}_t = \log k_\lambda^{-1} + (\delta\rho - \gamma_4)\log z_{t-1} + (1-\gamma_1)\log x_t + (\delta - \gamma_1\gamma_2)\log \eta_t + (\delta - \gamma_1\gamma_3)\log \varepsilon_t. \tag{15}$$

Some of the relevant characteristics of the equilibrium solution (14) and (15) are as follows. First, as long as both the informed and the uninformed coexist, $\lambda \in (0,1)$, a monetary shock x_t (an unexpected increase in money growth to the uninformed) affects aggregate output.[4]

[4]King and Trehan (1984) have also considered endogenous and observable money growth. They claimed that even without the asymmetric information structure—that is, even in case of $\lambda = 0$ or $\lambda = 1$, serially uncorrelated monetary noise can be non-neutral. According to the discussion below, this is in contrast to the results in this paper. King and Trehan's result, however, follows from their restriction that agents can not use all the available information. More specifically, according to their money growth equation (equation (5a), p. 388), money growth is a known linear combination of aggregate real shocks and the monetary noise. According to their output equation (equation (9), p. 391), output is a linear combination of money growth and the aggregate real shock. Note that since agents are identical in their setup, each knows the equilibrium level of output. Since agents know both money growth and output, they can infer real shocks. Using the money growth equation (equation (5a), p. 388), agents now can infer monetary noise perfectly. Consequently, monetary noise can not affect output in their setup.

Since $\gamma_1 < 1$ iff $\delta < 0$, a positive monetary shock increases output iff $\delta < 0$. That is, as long as agents are asymmetrically informed, the condition under which monetary noise increases output depends on how government conducts monetary policy. Unlike Lucas (1972), the condition does not depend on the degree of relative risk aversion. Also, unlike Lucas (1972), γ_1 can be negative—a positive monetary shock can even decrease the price level, while it increases output.[5] Second, if all the agents have the same information set—all are either informed ($\lambda \to 1$) or uninformed ($\lambda \to 0$), then $\gamma_1 \to 1$. That is, even though all become uninformed, a monetary shock does not affect aggregate output. This results from the fact that an agent, even if uninformed, can infer the current monetary shock x_t from the price level and the money stock since all the agents behave identically.[6] Finally as $\sigma_x^2 \to \infty$, $\gamma_1 \to 1$. Thus, as expected, if the variance of monetary shocks increases, a monetary shock tends to affect aggregate output less in magnitude.

For the remainder of this section, I discuss how a positive monetary shock affects aggregate output, provided that agents are asymmetrically informed and the government accommodates real shocks. To do that I need to discuss the interactions between the responses of the informed and that of the uninformed to various shocks. First, as indicated in (7), the informed know the current innovation to persistent real shocks. To the uninformed, the innovation to persistent real shock η_t is the most important among the current shocks, η_t, ε_t, and x_t (see (A3) in the appendix). Thus, not surprisingly, the market mechanism transmits, at least partly, the information of the informed on persistent real shocks to the uninformed. That is, according to (14) and (15) (see also the appendix), the uninformed learn imperfectly the information on persistent real shocks that the informed have from the price level and the money stock. Furthermore, as the share of the informed λ increases, the market mechanism reveal more about the innovation to persistent real shocks under normal circumstances—I consider circumstances are normal if persistent real shocks are positively serially correlated ($\rho > 0$) and substitution effects dominate wealth effects ($\beta < 1$).[7]

[5] γ_1 becomes negative if σ_x^2 is sufficiently small and $0 < -\delta < \frac{\alpha+\beta}{1+\alpha}\frac{1-\lambda}{\lambda}$.

[6] The case of $\lambda = 0$ is essentially the case considered by King and Trehan (1985).

[7] According to (A2) in the appendix, the market mechanism reveals $\eta_t^{\gamma_2-\delta}\varepsilon_t^{\gamma_3-\delta}$. From (A10), $\frac{\gamma_2-\delta}{\gamma_3-\delta} = 1 + \frac{(1-\beta)\lambda}{\alpha+\beta}(\delta\rho - \gamma_4)$. Thus, as λ increases, $\frac{\gamma_2-\delta}{\gamma_3-\delta}$ increases if $0 < \rho$ and $\beta < 1$.

This is consistent with a conjecture in Grossman and Stiglitz (1980, p. 394, Conjecture 1). Thus, under normal circumstances, the behavior of the uninformed depends on the way the informed respond to the real shocks.

As for the monetary shocks, however, the opposite results: the confused behavior of the uninformed affects the way the informed respond. This results from the fact that the persistent real shocks are most relevant for the agents in predicting the future state of the economy (see (A3) in the appendix). Thus, to the uninformed, the knowledge of persistent real shocks is much more important than that of temporary monetary shocks. In an attempt to figure out persistent real shocks, uninformed agents over-react to monetary shocks—this is a consequence of uninformed agents' optimizing behavior. Such over-reaction of the uninformed distorts the return to labor of the informed. Consequently, even though they know the nature of shocks, the informed respond to temporary monetary shocks. Note that this seemingly irrational behavior does not occur when agents respond to persistent real shocks.

More specifically, the coefficient of log x_t in (15) can be written as:

$$\left\{\delta \frac{1-\beta}{1+\alpha}\frac{1-\lambda}{\lambda}\sigma_\eta^2\sigma_\varepsilon^2 / D\right\}\lambda + \left\{\left(\delta \frac{1-\beta}{1+\alpha}\frac{1-\lambda}{\lambda} - \delta\frac{1}{\lambda}\right)\sigma_\eta^2\sigma_\varepsilon^2 / D\right\}(1-\lambda). \quad (16)$$

(To derive (16), first, compare (13) with (A3) in the appendix, then, substitute γ_1 in (14) into (A3).) The first term in (16) is the output response of the informed to monetary noise. Suppose $\delta > 0$ and only monetary noise buffets the economy currently: $x_t \neq 1$ and $z_{t-1} = \eta_t = \varepsilon_t = 1$. Then, from (6), $m_t = x_t$. Since $\delta > 0$, γ_1 is greater than 1 due to the confusion of the uninformed (see (14)). Thus, x_t changes p_t more than proportionally. The informed know that p_{t+1} will change proportionally to x_t, since when the next period comes, the shocks realized in period t are known to all, and that x_t does not affect m_{t+1}. Thus, x_t increases the return to labor $p_t m_{t+1}/p_{t+1}$. Therefore, even though the informed know the nature of shocks, they respond to monetary noise in a non-neutral way.

The second term in (16) is the output response of the uninformed to monetary noise. The second term has two parts. In terms of the return to labor $p_t m_{t+1}/p_{t+1}$, the first part is the output response due to a change in p_t, given m_{t+1}/p_{t+1}; the second part is the response due to an expected change in m_{t+1}/p_{t+1}, given p_t. Without the second part, the output response of the uninformed would be identical to that of the informed. However, the second part dominates the first part: the uninformed respond to x_t negatively if $\delta > 0$, regardless of β. Thus, if $\beta \in (0, 1)$, the output response of the uninformed to x_t is opposite to that of the informed. Furthermore, the output response of the uninformed due to confusion dominates that of the informed when they respond to monetary noise. This result is due to the fact that it is the uninformed who distort the return to labor of the informed when responding to the monetary noise, not the other way around.

To see why an uninformed agent's output response to monetary noise is negative if $\delta > 0$, first, I re-write the information set of the uninformed in equation (A2) in the appendix as:

$$\Omega_t^U = \{\log x_t + \delta(\log \eta_t + \log \varepsilon_t),\ (\gamma_2 - \delta)\log \eta_t + (\gamma_3 - \delta)\log \varepsilon_t,\ \hat{\Omega}_{t-1}\}.$$

Since $\eta_t = \varepsilon_t = 1$, $(\gamma_2 - \delta) \log \eta_t + (\gamma_3 - \delta) \log \varepsilon_t = 0$ or $\log \varepsilon_t = -\frac{\gamma_2 - \delta}{\gamma_3 - \delta} \log \eta_t$. Thus, Ω_t^U becomes $\{\log x_t + \delta\left(\frac{\gamma_3 - \gamma_2}{\gamma_3 - \delta}\right) \log \eta_t, \hat{\Omega}_{t-1}\}$. Note that if $\rho > 0$ and $\beta \in (0, 1)$, for example, then $\frac{\gamma_2 - \delta}{\gamma_3 - \delta} > 1$ — the uninformed respond to η_t more than to ε_t. Since η_t affects $\log m_{t+1}$ by $\delta\rho\log\eta_t$ (see (6)) and $\log p_{t+1}$ by $\gamma_4 \log\eta_t$ (see (14)), it affects $\log (m_{t+1}/p_{t+1})$ by $(\delta\rho - \gamma_4) \log\eta_t$. Ignoring constant terms, η_t affects the log of an uninformed agent's production by $\frac{1-\beta}{\alpha+\beta}$ $(\delta\rho - \gamma_4) \log \eta_t$ (see (8)). Thus, an uninformed agent's output response resulting from the confusion becomes:

$$E\left\{\frac{1-\beta}{\alpha+\beta}(\delta\rho - \gamma_4)\log \eta_t \middle| \log x_t + \delta\left(\frac{\gamma_3 - \gamma_2}{\gamma_3 - \delta}\right)\log \eta_t\right\}$$
$$= -\left(\delta \frac{1}{\lambda}\sigma_\eta^2 / D'\right)\left\{\log x_t + \delta\left(\frac{\gamma_3 - \gamma_2}{\gamma_3 - \delta}\right)\log \eta_t\right\}, \tag{17}$$

where $D' = \left(\frac{1}{\delta\rho - \gamma_4}\frac{\alpha+\beta}{(1-\beta)\lambda}\right)^2 \sigma_x^2 + \delta^2\sigma_\eta^2$. Since $\eta_t = 1$, the right hand side of (17) is $-\{(\delta \frac{1}{\lambda}\sigma_\eta^2)/D'\} \log x_t$, which looks very similar to the second part of the second term in (16) (if $\sigma_\varepsilon^2 \to\infty$, they are identical).

Therefore, under normal circumstances, i.e., $\rho > 0$ and $\beta \in (0, 1)$, the uninformed would like to respond to persistent real shocks more than to temporary ones. In doing so, uninformed agents mistake a positive monetary shock for a negative persistent real shock if government increases money growth in response to positive real shocks. Such a mistake results in monetary non-neutrality. Furthermore, since it is the uninformed who distort the return to labor of the informed, the output response of the uninformed is greater than that of the informed when they respond to monetary noise.

IV. Implications for the Comovements among Money, Output, and Prices

In this section, I discuss some statistical relationships among money, output, and prices. From (6), (14) and (15), I have:

$$\mathrm{Cov}(\log M_t, \log \bar{y}_t | \hat{\Omega}_{t-1}) = \delta\left\{\frac{\alpha+\beta}{1+\alpha}(\sigma_\eta^2+\sigma_\varepsilon^2)+(\delta\rho-\gamma_4)\frac{1-\beta}{1+\alpha}\sigma_\eta^2\right\},$$

$$\mathrm{Cov}(\log M_t, \log p_t | \hat{\Omega}_{t-1}) = \sigma_x^2 + \delta\left\{\left(\delta-\frac{\alpha+\beta}{1+\alpha}\right)(\sigma_\eta^2+\sigma_\varepsilon^2)-(\delta\rho-\gamma_4)\frac{1-\beta}{1+\alpha}\sigma_\eta^2\right\},$$

$$\mathrm{Cov}(\log p_t, \log \bar{y}_t | \hat{\Omega}_{t-1}) = \left(\delta-\frac{\alpha+\beta}{1+\alpha}\right)\frac{\alpha+\beta}{1+\alpha}(\sigma_\eta^2+\sigma_\varepsilon^2)$$
$$+\left(\delta-2\frac{\alpha+\beta}{1+\alpha}\right)(\delta\rho-\gamma_4)\frac{1-\beta}{1+\alpha}\sigma_\eta^2$$
$$-\left\{(\delta\rho-\gamma_4)\frac{1-\beta}{1+\alpha}\right\}^2\sigma_\eta^2+\left(\frac{\alpha+\beta}{1+\alpha}\frac{1-\lambda}{\lambda}\right)^2\frac{\sigma_x^2\sigma_\eta^2\sigma_\varepsilon^2}{D}.$$

First, the monetary policy parameter δ determines the sign of the conditional covariance between money and output, except for the case of $\beta \gg 1$ and $\rho < 0$. If for example $\delta > 0$ and $\rho > 0$, the conditional covariance between money and output is positive. λ does not affect the covariance. Thus, even in case of $\lambda = 1$, where all the agents know the nature of the shocks and, thus, monetary noise does not affect output (see (15)), money is procyclical if $\delta > 0$ and $\rho > 0$.

Second, if σ_x^2 is small and δ is positive and small, the conditional covariance between money and price can be negative, though money is procyclical. Yet, if σ_x^2 is large, the covariance is positive. Thus, even though money is procyclical, the model does not restrict price to behave in a specific way: money and price may move together or in opposite directions.

Third, the conditional covariance between price and output can be of any sign. If $\rho > 0$ and $0 < \delta \leq \min\{\frac{\alpha+\beta}{1+\alpha}, \frac{\alpha+\beta}{1+\alpha} + (\delta\rho-\gamma_4)\frac{1-\beta}{1+\alpha}\}$, the covariance is negative. Therefore, if $\rho > 0$ and δ is positive and small, procyclical money may be associated with countercyclical price. Note that such money-output-price behavior is consistent with recent findings. See, e.g., Kydland and Prescott (1990).

V. Concluding Remarks

Money can affect output for many reasons. An increase in money growth increases inflation taxes; an increase in money relaxes liquidity constraints. Even without these, this paper has shown that a serially uncorrelated monetary noise can affect aggregate output.

The novelty of this paper lies in the fact that unlike the previous models of the business cycle with incomplete information, agents are interested in knowing persistent real shocks not the monetary noise since the persistent real shocks are the most relevant for the prediction of the future state of the economy. Furthermore, under normal circumstances, the uninformed would like to respond to persistent real shocks more than to temporary ones. In doing so, uninformed agents mistake a positive monetary shock for a negative persistent real shock if government increases money growth in response to positive real shocks. Such a mistake results in monetary non-neutrality. Since it is the uninformed who distort the return to labor of the informed, the output response of the uninformed is greater than that of the informed when they respond to monetary noise. The monetary noise does not affect output if all the agents observe the shocks or none of the agents observe the shocks. In the latter case, monetary neutrality results as a consequence of the fact that even though monetary noise is not directly observable, agents can infer monetary noise from the price level and the money stock. Finally, this paper has shown that procyclical money may be associated with countercyclical price, consistent with recent empirical findings.

Appendix

In this appendix, I prove that for any $\lambda \in (0, 1]$, the price equation (14) satisfies the equilibrium condition (13). First, I conjecture that

$$p_{\lambda,t} = k_\lambda M_{t-1} z_{t-1}^{\gamma_4} (x_t \eta_t^{\gamma_2} \varepsilon_t^{\gamma_3})^{\gamma_1}. \tag{A1}$$

Then, according to (6), (7) and (A1), the information set of the uninformed can be expressed as:

$$\begin{aligned}\Omega_t^U &= \{M_t, p_t, \hat{\Omega}_{t-1}\} = \{m_t, p_t, \hat{\Omega}_{t-1}\} \\ &= \{\log m_t - E(\log m_t | \hat{\Omega}_{t-1}),\ \log p_t - E(\log p_t | \hat{\Omega}_{t-1}),\ \hat{\Omega}_{t-1}\} \\ &= \{x_t(\eta_t\varepsilon_t)^{\delta}, \eta_t^{\gamma_2-\delta}\varepsilon_t^{\gamma_3-\delta},\ \hat{\Omega}_{t-1}\}.\end{aligned} \tag{A2}$$

From (5), (6), (7), (A1) and (A2), the condition (13) can be re-written as: for $\lambda \in (0, 1]$,

$$\begin{aligned}k_\lambda^{-1} z_{t-1}^{\delta\rho-\gamma_4} x_t^{1-\gamma_1} \eta_t^{\delta-\gamma_1\gamma_2} \varepsilon_t^{\delta-\gamma_1\gamma_3} &= [z_{t-1}^{\rho}\eta_t\varepsilon_t][z_{t-1}^{(\gamma_4-\delta\rho)(1-\rho)} x_t^{\gamma_1-1} \eta_t^{\gamma_1\gamma_2-\delta} \varepsilon_t^{\gamma_1\gamma_3-\delta}]^{\frac{1-\beta}{\alpha+\beta}} \\ &\times \eta_t^{(\delta\rho-\gamma_4)\frac{1-\beta}{\alpha+\beta}\lambda} [E\{\eta_t^{(\delta\rho-\gamma_4)(1-\beta)} | \ x_t(\eta_t\varepsilon_t)^{\delta}, \eta_t^{\gamma_2-\delta}\varepsilon_t^{\gamma_3-\delta}\}]^{\frac{1-\lambda}{\alpha+\beta}} \\ &\times [E(x_{t+1}^{1-\gamma_1} \eta_{t+1}^{\delta-\gamma_1\gamma_2} \varepsilon_{t+1}^{\delta-\gamma_1\gamma_3})^{1-\beta}]^{\frac{1}{\alpha+\beta}}\end{aligned} \tag{A3}$$

According to Rao (1973), if $\log y \sim N(\mu, \sigma_y^2)$, then

$$E(y) = \exp\left[\mu + \frac{1}{2}\sigma_y^2\right]. \tag{A4}$$

Thus,

$$\begin{aligned}&E[(x_{t+1}^{1-\gamma_1} \eta_{t+1}^{\delta-\gamma_1\gamma_2} \varepsilon_{t+1}^{\delta-\gamma_1\gamma_3})^{1-\beta}] \\ &= \exp\left[\frac{(1-\beta)^2}{2}\{(1-\gamma_1)^2\sigma_x^2 + (\delta-\gamma_1\gamma_2)^2\sigma_\eta^2 + (\delta-\gamma_1\gamma_3)^2\sigma_\varepsilon^2\}\right].\end{aligned} \tag{A5}$$

Define Y such as

$$Y \equiv \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \equiv \begin{bmatrix} \log x_t + \delta\log\eta_t + \delta\log\varepsilon_t \\ (\gamma_2-\delta)\log\eta_t + (\gamma_3-\delta)\log\varepsilon_t \\ (1-\beta)(\delta\rho-\gamma_4)\log\eta_t \end{bmatrix}.$$

From (12), $E(Y) = 0$ and $\Sigma_y^* = E(Y'Y)$. Σ_y^*, a symmetric 3×3 matrix, can be expressed as:

$$\Sigma_y^* = \begin{bmatrix} \Sigma_{11}^* & \Sigma_{12}^* \\ \Sigma_{21}^* & \sigma_{33} \end{bmatrix},$$

where Σ_{11}^* is a 2×2 matrix, consisting of the first 2×2 elements of Σ_y^*. Σ_{11}^* is nonsingular as long as $\gamma_2 \neq \gamma_3$ and $\beta \neq 1$. I first posit $\gamma_2 \neq \gamma_3$; then verify it later. From Rao (1973, Chapter 8),

$$E(y_3|y_1,y_2) = \Sigma_{21}^{*}\Sigma_{11}^{*-1}[y_1 - E(y_1), y_2 - E(y_2)]' \qquad \text{(A6)}$$
$$Var(y_3|y_1,y_2) = \sigma_{33} - \Sigma_{21}^{*}\Sigma_{11}^{*-1}\Sigma_{12}^{*},$$

where

$$\Sigma_{11}^{*-1} = \frac{1}{D^*}\begin{bmatrix} (\gamma_2-\delta)^2\sigma_\eta^2 + (\gamma_3-\delta)^2\sigma_\varepsilon^2 & -\delta\{(\gamma_2-\delta)\sigma_\eta^2 + (\gamma_3-\delta)\sigma_\varepsilon^2\} \\ -\delta\{(\gamma_2-\delta)\sigma_\eta^2 + (\gamma_3-\delta)\sigma_\varepsilon^2\} & \sigma_x^2 + \delta^2(\sigma_\eta^2+\sigma_\varepsilon^2) \end{bmatrix};$$
$$D^* = \{(\gamma_2-\delta)^2\sigma_\eta^2 + (\gamma_3-\delta)^2\sigma_\varepsilon^2\}\sigma_x^2 + \delta^2(\gamma_2-\gamma_3)^2\sigma_\eta^2\sigma_\varepsilon^2;$$
$$\Sigma_{21}^{*} = (\Sigma_{12}^{*})' = [(1-\beta)(\delta\rho-\gamma_4)\delta\sigma_\eta^2,\ (1-\beta)(\delta\rho-\gamma_4)(\gamma_2-\delta)\sigma_\eta^2];$$
$$\sigma_{33} = (1-\beta)^2(\delta\rho-\gamma_4)^2\sigma_\eta^2.$$

Thus,

$$\begin{aligned} E(y_3|y_1,y_2) = {} & \{(1-\beta)(\delta\rho-\gamma_4)\sigma_\eta^2 / D^*\}[\delta(\gamma_3-\delta)(\gamma_3-\gamma_2)\sigma_\varepsilon^2 \log x_t \\ & + \{(\gamma_2-\delta)^2\sigma_x^2 + \delta^2(\gamma_2-\gamma_3)^2\sigma_\varepsilon^2\}\log\eta_t \\ & + (\gamma_2-\delta)(\gamma_3-\delta)\sigma_x^2 \log\varepsilon_t]; \\ Var(y_3|y_1,y_2) = {} & (1-\beta)^2(\delta\rho-\gamma_4)^2(\gamma_3-\delta)^2\sigma_x^2\sigma_\eta^2\sigma_\varepsilon^2 / D^*. \end{aligned} \qquad \text{(A7)}$$

From (A4),

$$E\{(\eta_t^{(\delta\rho-\gamma_4)(1-\beta)}\big|x_t(\eta_t\varepsilon_t)^\delta, \eta_t^{\gamma_2-\delta}\varepsilon_t^{\gamma_3-\delta}\} = \exp\left[E(y_3|y_1,y_2) + \frac{1}{2}Var(y_3|y_1,y_2)\right]. \qquad \text{(A8)}$$

Substituting (A5), (A7) and (A8) into (A3), then, matching exponents, one gets for $\lambda \in (0, 1]$,

$$\begin{aligned} \gamma_1 &= 1 + \delta\frac{(1-\beta)(1-\gamma)}{(1+\alpha)}(\delta\rho-\gamma_4)(\gamma_3-\delta)(\gamma_2-\gamma_3)\sigma_\eta^2\sigma_\varepsilon^2 / D^*; \\ \gamma_1\gamma_2 &= \delta - \frac{\alpha+\beta}{1+\alpha} - (\delta\rho-\gamma_4)\frac{(1-\beta)\lambda}{1+\alpha} - (\delta\rho-\gamma_4)\frac{(1-\beta)(1-\gamma)}{(1+\alpha)} \\ & \quad \times \{(\gamma_2-\delta)^2\sigma_x^2 + \delta^2(\gamma_2-\gamma_3)^2\sigma_\varepsilon^2\}\sigma_\eta^2 / D^*; \\ \gamma_1\gamma_3 &= \delta - \frac{\alpha+\beta}{1+\alpha} - (\delta\rho-\gamma_4)\frac{(1-\beta)(1-\lambda)}{(1+\alpha)}(\gamma_2-\delta)(\gamma_3-\delta)\sigma_x^2\sigma_\eta^2 / D^*; \\ \gamma_4 &= \delta\rho - \rho \Big/ \left\{1 + (1-\rho)\frac{1-\beta}{\alpha+\beta}\right\}; \\ k_\lambda^{-1} &= \exp[\frac{1}{2}\frac{(1-\beta)^2}{\alpha+\beta}\{(1-\gamma_1)^2\sigma_x^2 + (\delta-\gamma_1\gamma_2)^2\sigma_\eta^2 + (\delta-\gamma_1\gamma_3)^2\sigma_\varepsilon^2 \\ & \quad + (1-\lambda)(\delta\rho-\gamma_4)^2(\gamma_3-\delta)^2\sigma_x^2\sigma_\eta^2\sigma_\varepsilon^2 / D^*\}]. \end{aligned} \qquad \text{(A9)}$$

From (A9),

$$\gamma_2 - \delta = \left\{1 + \frac{\alpha+\beta}{(1-\beta)\lambda}\frac{1}{\delta\rho-\gamma_4}\right\}(\gamma_2-\gamma_3),$$
$$\gamma_3 - \delta = \frac{\alpha+\beta}{(1-\beta)\lambda}\frac{1}{\delta\rho-\gamma_4}(\gamma_2-\gamma_3). \quad \text{(A10)}$$

According to (A10), $\gamma_2 \neq \gamma_3$ as long as $\lambda > 0$ and $\beta \neq 1$; thus, Σ^*_{11} being nonsingular is verified.

Finally, using (A10) one gets (14) from (A9) and D^* in (A6).

Q.E.D.

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