# A Further Note on the Theory of Labor Supply With Wage Rate Uncertainty

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It is demonstrated that Kim's (1994) results on the theory of labor supply with wage rate uncertainty, which rely on mean-preserving spread for comparative-statics analysis, hold equally well and with additional economic intuition, when the process variance is used as the measure of dispersion. (*JEL* Classifications: D81, J22)

Following in the footsteps of Block and Heineke (1973), Tressler and Menezes (1980) and Dardanoni (1988), Kim (1994) provides a most interesting exploration of the comparative-static effects of risk on labor supply with wage rate uncertainty. As has been customary for the past quarter of a century, Kim's analysis uses the concept of mean-preserving spread as a proxy for uncertainty. That concept was developed by Rothschild and Stiglitz (1970) and Sandmo (1971), who showed that relying on the variance to proxy uncertainty may be inappropriate. Specifically, Rothschild and Stiglitz showed that, contrary to what one would suppose and plausible behavioral assumptions, in *some* situations a risk-averse expected-utility-maximizing decision maker choosing between uncertain projects with equal expected means will prefer the one with the greater variance. Meyer (1987) later developed restrictions on the decision maker's choice set and preferences under which a two-moment decision model *is* consistent with expected-utility maximization.

In the latter spirit, this note shows that under the rather modest conditions placed by Kim on labor's utility function, his principal

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results hold equally well when the variance rather than mean-preserving spread is used to proxy uncertainty. Further, in this case at least, use of the variance leads to some intuitively appealing interpretations of those results. Moreover, the fact that Kim's restrictions are indeed rather modest suggests there may be many comparable problems in which *either* the variance *or* mean-preserving spread is an appropriate proxy of uncertainty.

Following Kim (1994, pp. 24-5), let $U(c, L)$ denote a von Neumann-Morgenstern preference function where: $c = m + wL$ denotes consumption or total income, which is random because the wage rate, $w$, is random; $m$ is an exogenous income component; and labor supply, $L$, is the solitary decision variable. Letting subscripts denote partial derivatives, it is assumed that $U_1 > 0$, $U_2 < 0$, and that $U$ is strictly concave in $L$ (implying $U_{11} < 0$ and $U_{22} < 0$). Labor supply is determined by choosing $L$ so as to maximize $V_L = \text{E}[U_L] = \bar{U}_L$, where E is the expectations operator and the "bar" denotes expected value with respect to $w$.

Kim notes that the optimal $L$ occurs where $V_L = \text{E}[wU_1 + U_2] = 0$, which is his equation (3). The latter may also be written

$$V_L = \bar{w}\bar{U}_1 + \sigma_0 + \bar{U}_2 = 0, \tag{1}$$

where $\sigma_0$ is the covariance between $w$ and $U_1$. But $\sigma_0$ may be written as $U_{11}(\bar{w})L\sigma^2$, where $U_{11}(\bar{w})$ denotes $U_{11}$ evaluated at the expected wage and $\sigma^2$ is the wage rate variance (Kendall and Stuart 1977, p. 247).[1] Hence, equation (1) may be written

$$V_L = \bar{w}\bar{U}_1 + \bar{U}_2 + U_{11}(\bar{w})L\sigma^2 = 0. \tag{2}$$

Taking the Taylor's series expansion about $\bar{w}$,

$$U_1 = U_1\,(\bar{w}) + (w - \bar{w})\,LU_{11}\,(\bar{w}) + (w - \bar{w})^2L^2U_{111}\,(\bar{w})/2 + \Phi\,(\bar{w}),$$

where $\Phi\,(\bar{w})$ is the remainder. Assuming $\text{E}[\Phi(\bar{w})] \approx 0$ and taking expec-

[1]Specifically, consider $g(x_1, x_2, \ldots, x_n)$ and $h\,(x_1, x_2, \ldots, x_n)$, where $x_i$ $(i = 1, \ldots n)$ is a random variable with mean $m_i$ and variance $v_i$. The covariance between $g$ and $h$ is given by:

$$\sum_{ij}\left\{\left(\frac{\partial g}{\partial m_i}\right)\left(\frac{\partial h}{\partial m_j}\right)\text{cov}(x_i, x_j)\right\},$$

where $\text{cov}(x_i, x_j)$ is the covariance between $x_i$ and $x_j$ and $\text{cov}(x_i, x_i) = v_i$. In the present context, with $w$ the only random variable, $g = w$ and $h = U_1(c = m + wL, L)$. Hence, $\partial g/\partial\bar{w} = 1$, $\partial h/\partial\bar{w} = U_{11}L$, and $v_w = \sigma^2$. The expression given for $\sigma_0$ immediately follows as the product of the latter three terms.

tations on both sides of the equation,

$$\bar{U}_1 = U_1(\bar{w}) + U_{111}(\bar{w})L^2\sigma^2/2.$$

Similarly,

$$\bar{U}_2 = U_2(\bar{w}) + U_{211}(\bar{w})L^2\sigma^2/2.$$

Hence, equation (2) may be written

$$V_L = \bar{w}U_1(\bar{w}) + U_2(\bar{w}) + L[\bar{w}LU_{111}(\bar{w}) + 2U_{11}(\bar{w}) + LU_{211}(\bar{w})]\sigma^2/2 = 0. \quad (3)$$

Interpreting $\bar{w}$ as the certainty-equivalent wage that obtains in a certain world in which $\sigma^2 = 0$, the term $\bar{w}U_1(\bar{w}) + U_2(\bar{w})$ may be interpreted as the negatively-sloped ($V_{LL} < 0$) certainty-equivalent marginal utility of labor curve. The optimal $L$ occurs where that curve crosses the $L$ axis; that is, at the labor supply at which the marginal utility of labor equals zero.

The bracketed term is precisely the term in parentheses in Kim's (1994, p. 28) crucial equation (9). Kim shows that "a sufficient condition for labor supply, $L$, to increase (decrease) under increased wage rate risk" as proxied by mean-preserving spread (p. 27) is that this term be positive (negative). Kim then shows that the sign depends upon whether "endogenous partial risk aversion" (p. 26) is less than or greater than unity and whether it is decreasing or increasing in income (p. 28).

Suppose that the sign of the bracketed term is positive (negative). Since $\sigma^2 > 0$ in an uncertain world, the effect is to shift the certainty-equivalent marginal utility of labor curve upwards (downwards). This means that the curve will cross the $L$ axis to the right (left) of where it crossed when $\sigma^2 = 0$. This shift thus implies that wage-rate uncertainty results in an increase (decrease) in labor supply.

Totally differentiating $V_L = 0$ with respect to $\bar{w}$ ($\sigma^2$) it immediately follows from $V_{LL} < 0$ that the sign of $dL/d\bar{w}$ ($dL/d\sigma^2$) is the same as that of $\partial V_L/\partial\bar{w}$ ($\partial V_L/\partial\sigma^2$). In particular:

$$\partial V_L/\partial\bar{w} = U_1(\bar{w}) + \bar{w}LU_{11}(\bar{w}) + LU_{21}(\bar{w}) + 3L^2U_{111}\sigma^2/2, \quad (4)$$

where the higher-order terms are again assumed negligible; and

$$\partial V_L/\partial\sigma^2 = L[\bar{w}LU_{111}(\bar{w}) + 2U_{11}(\bar{w}) + LU_{211}(\bar{w})]/2. \quad (5)$$

Looking at equation (4), the only difference from the certainty-equivalent situation develops from the last term, which will be positive (negative) when $U_{111}(\bar{w}) > (<) 0$; or, when labor is decreasingly (increasingly) risk averse in income (Hanson and Menezes 1971, p. 217). Decreasing risk aversion is generally considered to be reflective of real-world behavior.

As regards the remaining terms in equation (4), these comprise the negative of the numerator of Kim's equation (11), the Slutsky equation for a change in labor supply in response to a change in the wage rate. Specifically, $\partial L/\partial w > (<) 0$ and the labor supply curve is upward (downward) sloping when the sum of the remaining terms is *positive* (negative). This being the case, equation (4) implies that the effect of uncertainty is to increase the amount of *additional* labor supplied when the expected wage increases; or, when the labor supply curve is backward bending, to ameliorate or even reverse any *decrease* in the amount of labor supplied in response to an increase in the expected or certainty-equivalent wage.

The sign of equation (5) depends solely on the bracketed term that was the focus of the previous discussion and Kim's equation (9). Thus, increases in uncertainty, proxied here by the variance in the wage rate and in Kim by mean-preserving spread, will increase (decrease) the labor supply when (a) the bracketed term is positive (negative), implying that the labor supply curve is positively (negatively) sloped, and when (b) labor evinces decreasing (increasing) risk aversion.

In the context of labor supply and wage rate uncertainty, then, under the reasonable assumptions set forth in Kim about the shape of labor's risk preference function, the process variance plays its time-honored role as a measure of dispersion in exemplary fashion, adding considerably to our economic intuition as to what, precisely, is transpiring.

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