On the Estimation of an Almost Ideal Demand System with Autoregressive Errors

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A new GLS estimation procedure is employed in the estimation of a dynamic version of Almost Ideal Demand System with vector autoregressive errors. The procedure enables us to have more natural form of autoregressive parameters. Also, a new habit formation specification obeying the adding up restriction is proposed. In an application for four categories of food in the U.S., test results show that the error process is a first order vector autoregressive process and that the demand system is characterized by habit formation. (*JEL* Classifications: C51, D12)

I. Introduction

Since Deaton and Muellbauer (1980a) first proposed and estimated the Almost Ideal Demand System (AIDS), there have been some efforts to extend AIDS to incorporate dynamic element. Such dynamic generalization has been approached in two ways. The first approach uses some hypotheses about the agent's behavior. The habit formation hypothesis proposed by Pollak (1970) was introduced in the linear form into AIDS by Blanciforti and Green (hereafter, BG) (1983) and Ray (1986), and a forward-looking intertemporal utility maximization hypothesis was introduced into the AIDS model by Parshardes (1986). The second approach, used mostly by Anderson and Blundell (1982, 1983, 1984), tried to accommodate any dynamic behavior by giving a

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general lag structure to both the dependent and the independent variables.

This paper belongs to the first approach and aims to extend BG (1983) by modifying their model. They proposed a dynamic version of AIDS by assuming that each expenditure share equation in AIDS is linearly related to the past consumption of each good. They applied this model to find out the habit effect on the consumption pattern of postwar U.S.. However, their specification turned out to violate the adding-up restriction implied by the demand theory. In view of the restriction, the correct specification should be that each equation is linearly related to the past consumption of *all the goods* considered. About the stochastic specification, we assume that errors follow a first order vector autoregressive process. This is a more general assumption than that of the previous authors.

On the estimation side, the AIDS is an expenditure share equation system, where the sum of dependent variables results in one of the independent variable. Therefore, the error terms show linear dependency, and the covariance matrix becomes singular. The estimation procedure of the singular equation system has been dealt with by many authors, notably by Barten (1969), Theil (1971), Deaton (1975) and recently by Dhrymes and Schwarz (1987a, 1987b) in the case of standard non-autocorrelated errors, and by Berndt and Savin (hereafter, BS) (1975) and recently by Dhrymes (1989) in the case of autocorrelated errors.

In the demand analysis, the conventional practice to deal with the singularity with autocorrelation has been to adopt BS's Maximum Likelihood (ML) estimation procedure. It suggests to delete one equation and also modify the autocorrelation coefficient matrix (say, H) to form a likelihood function.

The GLS estimation procedure proposed by Dhrymes, however, requires much less restriction on H than those of BS. Also the hypothesis test on H is not limited. Moreover, it enables us to estimate all the parameters of all the equation at the same time. So, we purpose to estimate dynamic AIDS of four categories of food in the U.S. to find out the effect of habit persistence on food consumption patterns by employing the GLS estimation procedure, and to test hypotheses, including hypothesis of habit formation.

The plan of the paper is as follows. Section II introduces Dynamic Linear Approximate AIDS (DLA/AIDS) which is consistent with the adding up condition. Section III presents the GLS estimation procedure and test statistics for various hypotheses under plausible assumption about the error term. Empirical results are presented in section IV, and section V concludes.

II. The Model

A. The Almost Ideal Demand System with Habit Formation

In the studies of household budgets and Engel curves, Working (1943) and Leser (1963, 1976) suggested that budget shares are best explained by a linear function of the log of total expenditure, $w_i = \alpha_i + \delta_i \ln x$. To make this compatible with time series analysis and demand theory, Deaton (1978) suggested to include the log of prices using the duality approach, and found that the cost function behind it falls into Muellbauer's (1975, 1976) Price Independent Generalized Log-Linear (PIGLOG) form, which allows perfect aggregation. And Deaton and Muellbauer (1980a) have proposed an AIDS which now allows for interaction between prices. The advantages of this system are as follows:[1] "(1) it gives an arbitrary first order approximation to any demand system; (2) it satisfies the axioms of choice exactly; (3) it aggregates perfectly over consumers; (4) it has a functional form which is consistent with previous household-budget data; (5) it is simple to estimate, largely avoiding the need for non-linear estimation; (6) it can be used to test for homogeneity and symmetry."

The static AIDS proposed by Deaton and Muellbauer (1980a) is a demand system from a preference represented by a static cost function of the following log form,

$$\ln c(u, p_t) = (1 - u) \ln a(p_t) + u \ln b(p_t),$$

with utility u obeying $0 < u < 1$, and with linear homogeneous functions $a(p_t)$ and $b(p_t)$ with price vector p_t at time t. They take specific functional forms for $\ln a(p_t)$ and $\ln b(p_t)$ such that

$$\ln a(p_t) = \alpha_0 + \sum_i \alpha_i \ln p_{ti} + \frac{1}{2} \sum_i \sum_j \gamma_{ij}^* \ln p_{ti} \ln p_{tj}, \text{ and}$$

$$\ln b(p_t) = \ln a(p_t) + \delta_0 \prod_i p_{ti}^{\delta_i}.$$

So the static AIDS cost function is given by

[1]Deaton and Muellbauer (1980a), p.312.

$$\ln C(u, p_t) = \alpha_0 + \sum_i \alpha_i \ln p_{ti} + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \ln p_{tk} \ln p_{tj} + u\delta_0 \prod_k p_{tk}^{\delta_k},$$

where α_0, α_i, γ^*_{ij}, δ_0, δ_i are parameters.

From the Shephard's lemma and after substitution of u, the AIDS demand functions in share form is written as

$$w_{ti} = \alpha_i + \sum_j \gamma_{ij} \ln p_{tj} + \delta_i \ln\left(\frac{\mu_t}{P_t}\right), \quad (1)$$

$$\text{where } \ln P_t = \alpha_0 + \sum_k \alpha_k \ln p_{tk} + \frac{1}{2}\sum_j \sum_k \gamma_{kj} \ln p_{tk} \ln p_{tj}, \quad (2)$$

and $\gamma_{ij} = 1/2\ (\gamma^*_{ij} + \gamma^*_{ji})$. The restrictions implied upon the parameters are

$$\text{Adding-up: } \Sigma_i \alpha_i = 1,\ \Sigma_i \gamma_{ij} = 0,\ \Sigma_i \delta_i = 0,$$
$$\text{Homogeneity: } \Sigma_j \gamma_{ij} = 0,$$
$$\text{Symmetry: } \gamma_{ij} = \gamma_{ji}.$$

The δ_i's indicate whether goods are luxuries ($\delta_i > 0$) or necessities ($\delta_i < 0$). The γ_{ij}'s measure the effect on the ith budget share of a one proportional change in p_{tj}, $(dw_{ti}/(dp_{tj}/p_{tj}))$, with (μ_t/P_t) held constant.

The substitution of (2) in (1) gives a non-linear system of equations,

$$w_{ti} = \alpha_i - \delta_i\alpha_0 + \sum_j \gamma_{ij} \ln p_{tj} + \delta_i\left\{\ln \mu_t - \sum_k \alpha_k \ln p_{tk} + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \ln p_{tk} \ln p_{tj}\right\}.$$

However, they also suggest to approximate P_t as proportional to some known index, such as Stone's index, $\ln P^*_t = \Sigma_i w_{ti} \ln p_{ti}$, where $P \simeq \phi P^*$.

So,

$$w_{ti} = \alpha^*_i + \sum_j \gamma_{ij} \ln p_{tj} + \delta_i \ln\left(\frac{\mu_t}{P^*_t}\right), \quad (3)$$

where $\alpha^*_i = \alpha_i - \delta_i \ln\phi$.

And they employ equation (3) to find out the consumption pattern of postwar Britain and to see whether homogeneity and symmetry hold, using annual British data from 1954 to 1974 on eight nondurable groups of consumer's expenditure. The system is estimated for each category separately by OLS using P^*_t, and homogeneity is tested. Homogeneity failed for four commodity groups, and when homogeneity restriction was imposed, serial correlation was introduced for those groups. As one way to remedy this, they suggested to incorporate dynamic effects in the α's

In the demand literature, one of the popular ways to incorporate dynamic element in the static model is to hypothesize that current consumption is influenced by past behavior. Pollak (1970) introduced the habit formation scheme in the Stone-Geary utility function and thus derived a Linear Expenditure System with habit formation. And this approach has been widely applied.

BG (1983) extended the static AIDS by specifying the α_i in (3) to be linearly related to previous consumption levels to reflect persistence in consumption patterns. That is,

$$\alpha_i = \alpha_i^* + \alpha_i^{**} q_{t-1i}.$$

So the dynamic linear approximate AIDS (DLA/AIDS) becomes,

$$w_{ti} = \alpha_i^* + \alpha_i^{**} q_{t-1i} + \sum{}_j \gamma_{ij} \ln p_{tj} + \delta_i \ln\left(\frac{\mu_t}{P_t^*}\right). \tag{4}$$

And the model was applied to annual U.S. time series for eleven aggregate commodities and for four food groups. However, a significant error in this formulation was that *for the equation (4) to satisfy the adding-up constraint, α_i^{**} should be equal to zero for each good, since q_{t-1i} appears only once in each equation.* Therefore, their system unfortunately violates the requirement for theoretically correct demand system.

Therefore to incorporate habit effect, we should assume α_i to be linearly related to previous consumption levels of all the goods. That is,

$$\alpha_i = \alpha_i^* + \Sigma_j \alpha_{ij}\, q_{t-1j}.$$

Therefore, DLA/AIDS should be written as,

$$w_{ti} = \alpha_i^* + \sum{}_j \alpha_{ij} q_{t-1j} + \sum{}_j \gamma_{ij} \ln p_{tj} + \delta_i \ln\left(\frac{\mu_t}{P_t^*}\right), \tag{5}$$

and the adding-up restriction should be,

$$\text{Adding-up: } \Sigma_i \alpha_i = 1,\ \Sigma_i \alpha_{ij} = 0,\ \Sigma_i \gamma_{ij} = 0,\ \Sigma_i \delta_i = 0.$$

In approximating P_t, one thing worth mentioning is that the use of Stone's index $\ln P_t^* = \Sigma_i w_{ti} \ln p_{ti}$, where $P \simeq \phi P^*$, may incur simultaneity bias, since the dependent variable w_{ti} appears on the right hand side in its multiplicative form with $\ln p_{ti}$. Besides, the index fails to remain unchanged when all individual price series don't change.[2] Therefore, we decided to use $\ln P_t^* = \Sigma_i \bar{w}_i \ln p_{ti}$, where $\bar{w}_i = \Sigma_t w_{ti}/T$, where T is the

[2]Theil et al. (1989), p.185.

sample period.

In the calculation of elasticities, we consider the effect of our group price index, $\ln P^*_t$, following the suggestion by Green and Alston (1990). In our case total food expenditure elasticity, η_i, is $1 + \delta_i/w_i$. Ordinary price elasticity is $\varepsilon_{ij} = -\delta_{ij} + (\gamma_{ij} - \delta_i \bar{w}_j)/w_i$, where $\delta_{ij} = 1$, when $i = j$, and $\delta_{ij} = 0$, otherwise.

B. Stochastic Specification of DLA/AIDS

As to the error structure of the AIDS, Deaton and Muellbauer (1980a) assumed that errors are not autocorrelated. However, they report that serial correlation was introduced when homogeneity restriction was imposed. Also, BG (1983) report that they couldn't reject the hypothesis of no autocorrelation, even after introducing lagged quantity of each good in both the case of dynamic AIDS and DLA/AIDS when they applied their model to food data. Therefore, we presume that besides habit effects, there may be other factors to influence consumers expenditure pattern. BG assumed that the autocorrelation coefficient matrix, H, is diagonal. As will be shown later, in the singular system such as AIDS, this implies that each element of H is the same, which appears somewhat restrictive. Instead, we propose that the error terms exhibit first order vector autocorrelation. That is,

$$u_t = u_{t-1.}H + \varepsilon_{t.},\ t = 2, \ldots, T. \tag{6}$$

$\{u_{t.}' : t = 2, \ldots, T\}$ is a sequence of random vectors following the above first order vector autoregressive process; H is a $m \times m$ matrix of autoregressive parameters and $\{\varepsilon_{t.}' : t = 2, \ldots, T\}$ is a sequence of i.i.d. random vectors, where $E(\varepsilon_{t.}') = 0$, $Cov(\varepsilon_{t.}') = \Sigma$.

III. Econometric Issues

The singular equation system without autocorrelation in the demand literature has a covariance matrix with rank $m-1$, which is not generally invertible. To cope with the difficulty posed by this, Barten (1969) has proved that ML estimates of the m-equation system can be derived from those of $m-1$ equations and the latter are invariant with respect to the equation deleted. Theil (1971) proposed a Generalized Least Squares (GLS) estimation using generalized inverse (g-inverse) of the singular covariance. Recently, Dhrymes and Schwarz (1987a) has clarified the Barten's estimator and compared the efficiency of Barten's ML estimator and Theil's GLS estimator. Also, Dhrymes and Schwarz (1987b) provided the condition that Theil's estimator can exist.

Usually, the linear singular equation system without autocorrelation often doesn't present problem in terms of estimation. That is because demand theory requires all price and total expenditure appear in all equations, and in this case, OLS estimator will be the same as GLS estimator. Static AIDS or dynamic AIDS without autocorrelation belong to this case.

However, when the error vector is specified to follow a first order autoregressive process in the linear singular system, generally, we cannot employ OLS procedure. Parks (1967) first specified a first order autoregressive error. BS (1975) extended Barten's ML estimator in the autoregressive error case and found that the singularity imposes restrictions on the parameters of the autoregressive process. Also, they provided a ML estimator of the coefficients of the system with a equation deleted, and thus extended Barten's result in the autocorrelated case. Their proposal is to delete one equation and also modify the autocorrelation coefficient matrix (say, H) to form a likelihood function. However, unless the same number of zero restrictions as the number of equations (say, m) are imposed, some of the elements of H are not identified. Also, the hypothesis testing by likelihood ratio test based on modified H matrix often becomes limited .

Recently, Dhrymes (1989) formulated the problem as a generalization of GLS procedure employing the g-inverse in the GLS minimand. Thus, he provided a method by which all the parameters of all the equations can be estimated at the same time. The contributions of his work are seen as follows:[3] "(1) the estimation and distribution results are asymptotically simple generalizations of the restricted linear least squares (RLLS) in the context of general linear model (GLM); (2) The formulation is perfectly symmetrical in that we can estimate all the parameters at the same time, so, it eliminate the ambiguities relating to the deletion of an equation; (3) The formulation doesn't rely on the "invariance property" of MLE where a likelihood function may not exist; (4) the specification of the autoregressive errors turned out less restrictive than that of BS (1975); (5) the proposed Cochrane-Orcutt like estimation procedure is likely to reduce computational cost; (6) it allows for maintained hypotheses regarding the exclusion of explanatory variables." The procedure of Dhrymes was compared with that of BS and

[3]Dhrymes (1989), pp. 2-3 and pp. 24-45.

employed in the estimation of static AIDS by Bush (1989). In the following subsection, we outline the GLS estimation procedure and the distribution of the estimators by Dhrymes (1989), which will be employed in our estimation.

A. GLS Estimator of Singular System with Autoregressive Error and its distribution by Dhrymes(1989)

The following system of equations with first order vector autoregressive errors is considered:

$$y_{t} = x_{t}B + u_{t.}, \text{ where } u_{t.} = u_{t-1}H + \varepsilon_{t}, \; t = 2,\ldots, T. \tag{7}$$

where $y_{t.}$ is an m-element row vector of dependent variables; $x_{t.}$ is a G-element row vector of independent variables with unity as the first element; B is a $G \times m$ matrix of parameters; $u_{t.}$ is an m-element row vector containing the structural errors; H is a $m \times m$ matrix of autoregressive parameters. $\{\varepsilon_{t}' : t = 1,\ldots, T\}$ is a sequence of i.i.d. random vectors, where $E(\varepsilon_{t.}') = 0$, $\mathrm{Cov}(\varepsilon_{t.}') = \Sigma$.

In a share equation system, $y_{t.}$ satisfies the adding up condition

$$y_{t.}e = x_{t1}, \tag{8}$$

where e is an $m \times 1$ vector of unity and x_{t1} is first element of $x_{t.}$, which is unity. From (7) and (8),

$$Be = e_{.1} \text{ and } u_{t.}e = 0, \tag{9}$$

where e_{1} is an G-element column vector of zeros except the first element of unity. BS (1975) proved that $u_{t.}e = 0$ implies the row sums of H are all equal, i.e., $He = ce$. Furthermore, Σ is singular. $He = ce$ means

$$I^{*}_{m-1}He = 0, \tag{10}$$

where $I^{*}_{m-1} = [I_{m-1}, -e_{m-1}]$ and I_{m-1} is an identity matrix of order $m-1$ and e_{m-1} is an $(m-1)$- element column vector of unities.

Also, the issue of possible exclusion restrictions are dealt with in the model. When the model in (7) is written more compactly as

$$Y = XB + U,$$

its ith column may be represented as $y_{i} = Xb_{\iota} + u_{\iota}$. If some variables are excluded from the ith equation, it may be written as $y_{\iota} = X_{i}\beta_{\iota} + u_{\iota}$, where β_{ι} contains the elements of the ith column of B, $b_{.\iota}$, not known a priori to be zero. This is dealt with by means of selection matrix S_{i1}

which is a permutation of G_i of the columns of I_G such that $X_i = XS_{i1}$, which leads to $b_{.i} = S_{i1}\beta_{.i}$. A similar convention may be adopted for H, in $u_{t} = u_{t-1.}H + \varepsilon_{t.}$. Likewise, among the lags appearing in the equation for u_{ti}, given by $h_{.i}$, (the ith column of H), only m_i of these may appear. Then, $h_{.i} = S_{i2}\gamma_{.i}$ holds where $\gamma_{.i}$ are the elements of $h_{.i}$ not known a priori to be zero, and S_{i2} is a permutation of m_i of the columns of I_m. Therefore, $\mathrm{vec}(B) = S_1\beta$ where $S_1 = \mathrm{diag}(S_{11}, S_{21}, \ldots, S_{m1})$ and, $\beta = (\beta_{.1}', \beta_{.2}', \ldots \beta_{.m}')'$. Also, $\mathrm{vec}(H) = S_2\gamma$ where $S_2 = \mathrm{diag}(S_{12}, S_{22}, \ldots, S_{m2})$ and $\gamma = (\gamma_{.1}', \gamma_{.2}', \ldots \gamma_{.m}')'$.

(7) can be rewritten as

$$\varepsilon_{t.} = y_{t.} - w_t \quad \text{where } w_{t.} = y_{t-1.}H + x_{t.}B - x_{t-1.}BH, \ \ t = 2, \ldots, T,$$

and $\varepsilon_{t.} \sim (0, \Sigma)$ with $r(\Sigma) = m - 1$. Since the covariance matrix, Σ of $\varepsilon_{t.}$, is singular, with rank $m-1$, the inverse is its Penrose-Moore inverse, Σ_g. Now the Aitken minimand is

$$\sum_{t=1}^{T}(y_{t.} - w_{t.})\sum{}_g(y_{t.} - w_t)'$$

subject to the constraints, (9), (10) and possible exclusion restrictions.

The estimators are derived from minimizing the following Aitken Lagrangian:

$$L = (y - w)' \ (\Sigma_g \otimes I_T)(y - w) + 2\lambda_1'(R_1\beta - r_1) + 2\lambda_2'(R_2\gamma - r_2), \tag{11}$$

with respect to β, γ, λ_1 and λ_2, where $y = \mathrm{vec}(Y)$, $w = [I_m \otimes (Y_{-1} - X_{-1}B)]\ S_2\gamma + (I_m \otimes X)\ S_1\beta$, $R_1 = (e' \otimes I_G)S_1$, and $R_2 = (e' \otimes I^{*}_{m-1})S_2$. λ_1 and λ_2 are vectors of Lagrangian multipliers.

$R_1\beta = r_1$ is adding-up restriction to the structural parameters and $R_2\gamma = r_2$ is adding-up restriction to the parameters of autoregressive error.[6]

The first order conditions can be written as a matrix equation system in terms of a stacked vectors of parameters and Lagrangian multipliers, $(\beta', \gamma', \lambda_1', \lambda_2')'$, which is defined as θ,

$$\begin{bmatrix} P_{11} & P_{12} & R_1' & 0 \\ P_{21} & P_{22} & 0 & R_2' \\ R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} d_{.1} \\ d_{.2} \\ d_{.3} \\ d_{.4} \end{bmatrix} \tag{12}$$

[4]See Berndt and Savin (1975), pp. 939-40.
[5]Since we don't take any exclusion restriction about B, $S_1 = I_{mG \times mG}$
[6]Dhrymes (1989) pp. 5-10.

where

$$C = (I \otimes X) - (H' \otimes X_{-1}), \qquad D = I \otimes (Y_{-1} - X_{-1}B)$$
$$P_{11} = S_1' C' (\Sigma_g \otimes I_T) C S_1, \qquad P_{12} = S_1' C' (\Sigma_g \otimes I_T)(I \otimes Y_{-1}) S_2,$$
$$P_{21} = S_2' D' (\Sigma_g \otimes I_T)(I \otimes X) S_1, \qquad P_{22} = S_2' D' (\Sigma_g \otimes I_T) D S_2,$$
$$d_{.1} = S_1' C' (\Sigma_g \otimes I_T) y, \qquad d_{.2} = S_2' D' (\Sigma_g \otimes I_T) y,$$
$$d_{.3} = r_1 \text{ (in this case, } r_1 = e_1), \qquad d_{.4} = r_2 \text{ (in this case, } r_2 = 0).$$

The invertibility condition of the matrix in (12) is given to guarantee that the system in (12) is solvable. Under the condition, the system is amenable to an iterative procedure beginning with an initial consistent estimator of B, H and Σ. A procedure to obtain the initial estimators is proposed as follows:
first, estimate the unrestricted lease squares estimator of B,

$$\tilde{B} = (X' X)^{-1} X' Y.$$

second, regress $\tilde{U}_1$ on $\tilde{U}$ subject to the restriction $R_2\gamma = r_2$, and finally, obtain $\tilde{E} = \tilde{U} - \tilde{U}_1\tilde{H}$, and compute $\Sigma_g = ((\tilde{E}' \tilde{E})/T)_g$.

Also, an alternative computation algorithm based on Cochrane-Orcutt iteration is given. It is based on the fact that the system in (12) can be decomposed into two in terms of (β, λ_1) and (γ, λ_2).

Finally, the limiting distribution of the estimators of the parameters, β and γ, are given as follows: $\sqrt{T}(\hat{\theta} - \theta_0) \sim N(1, Q^{-1}Q^* Q^{-1})$ where $\theta = (\beta',\ \gamma',\ \lambda_1',\ \lambda_2')'$, θ_0 is a true value of θ,

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}, \quad Q^* = \begin{bmatrix} Q_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$Q_{11} = \frac{1}{T}\left[\frac{\partial w}{\partial \beta}, \ \frac{\partial w}{\partial \gamma}\right]' (\Sigma_g \otimes I_T) \left[\frac{\partial w}{\partial \beta}, \ \frac{\partial w}{\partial \gamma}\right]$$
$$= \frac{1}{T}\begin{bmatrix} S_1'C'(\Sigma_g \otimes I_T)CS_1 & S_1'C'(\Sigma_g \otimes I_T)DS_2 \\ S_2'D'(\Sigma_g \otimes I_T)CS_1 & S_2'D'(\Sigma_g \otimes I_T)DS_2 \end{bmatrix},$$

$$Q_{12} = \begin{bmatrix} R_1' & 0 \\ 0 & R_2' \end{bmatrix}, \quad Q_{21} = Q_{12}', \ \text{and} \ Q_{22} = 0.$$

B. Our Model and Testing Hypothesis

The invertibility condition of the coefficient matrix of θ requires that a restriction should be imposed on the form of H. The restriction is that "all lags should not appear in all equations." (Dhrymes 1989, pp. 11-4) Therefore, the most general formulation of H is a full matrix except only one zero element. In the estimation of our DLA/AIDS of four food, we assume that $h_{41} = 0$ among a number of possibilities. This implies that last period's error of miscellaneous foods' expenditure share doesn't affect current error of expenditure share of meats. We believe that this is an admissible assumption.

So our maintained model can be written as:

$$y_{t.} = x_{t.}B + u_t, \quad u_{t.} = u_{t-1.}H + \varepsilon_{t.}, \text{ and}$$

$$H = \begin{bmatrix} h_{11} & \cdots & h_{14} \\ \vdots & \ddots & \vdots \\ h_{41} & \cdots & h_{44} \end{bmatrix}, \text{ where } h_{41} = 0.$$

Our proposed Dynamic Linear Approximate AIDS is a share equation system with habit formation hypothesis and a first order vector autoregressive errors. Now, we need to examine the validity of our hypotheses and whether homogeneity and symmetry of our demand system can be confirmed by the data by performing a sequence of tests. Regarding tests of significance and of linear restrictions, some results are introduced in the appendix.

1) Tests on the H

We are interested in finding out whether our specification of H is justified. First, we want to test for the hypothesis that H is equal to a zero matrix. If this is the case, there is no autocorrelation. Moreover, since each equation has the same set of explanatory variables, equation by equation OLS estimator and Aitken estimator will coincide. So we perform a significance test of all of h_{ij}'s, H_0: $\gamma = 0$ against H_1: $\gamma \neq 0$. As is reproduced in the appendix, $\sqrt{T}(\hat{\gamma} - \gamma) \overset{asy}{\sim} N(0, (B_{11}^{(4)}))$, and under the null hypothesis, $T\hat{\gamma}'(B_{11}^{(4)})_g \hat{\gamma} \overset{asy}{\sim} \chi^2_{\Sigma m_i}$.

If H is turned out to be not zero, then we may think that H is a diagonal matrix. However, from the condition, $He = ce$, H should be of the form, cI. That implies we should test our null hypothesis that H is diagonal and $h_{11} = h_{22} = h_{33} = h_{44}$. This test can be carried out by a test of linear restriction. Under H_0: $R\gamma = 0$, the test statistic $T(R\hat{\gamma})'(RB_{11}^{(4)}R')_g(R\hat{\gamma}) \overset{asy}{\sim} \chi^2$ with degrees of freedom, 11.

2) Test of linear habit formation hypothesis

After testing on the specification of H, we test whether our proposed linear habit formation is justified in our model by the data. Our null hypothesis is β_i' s = 0, where β_i' s are coefficients of lagged quantity demanded. That is, in terms of coefficients of our model, β_i' s are α_{ij}' s, i, $j = 1, \ldots, 4$. In the appendix, we described a test statistic for general linear restriction in two cases. This test belongs to first case of when $r(RB_{11}^{(1)}R') < q$, because $q < r(B_{11}^{(1)})$ and $(m \times m)$ element column vector $R\hat{\beta}$, $(\hat{\alpha}_{11}, \ldots \hat{\alpha}_{11})'$, is subject to m adding up restrictions. Therefore, $RB_{11}^{(1)}R'$ is a singular matrix with rank $(m \times m) - m$. Under the null, $R\beta = 0$, $T(R\hat{\beta})'(RB_{11}^{(1)}R')_g(R\hat{\beta})$ has a chi-square distribution with 12 degrees of freedom.

3) Test of homogeneity

Once the most restrictive dynamic specification is found by a sequence of tests, we need to find out whether the model is justified in terms of an economic sense. First, we consider homogeneity of degree zero in prices and total food expenditure, $\Sigma_j \gamma_{ij} = 0$, $i = 1, \ldots, 4$.[8] This test can also be carried out in the context of testing the general linear restriction, $R\beta = r$, and also belongs to the first case of when $r(RB_{11}^{(1)}R') < q$. The reason is that $q < r(B_{11}^{(1)})$, and m element column vector $R\hat{\beta}$, $(\hat{\gamma}_{11} + \hat{\gamma}_{12} + \hat{\gamma}_{13} + \hat{\gamma}_{14}, \ldots, \hat{\gamma}_{41} + \hat{\gamma}_{42} + \hat{\gamma}_{43} + \hat{\gamma}_{44})'$, is subject to one adding up restriction. Therefore, $RB_{11}^{(1)}R'$ is also a singular matrix with rank $m - 1$. Under H_0: $R\beta = 0$, the test statistic has the following asymptotic distribution: $T(R\hat{\beta})'(RB_{11}^{(1)}R')_g(R\hat{\beta}) \overset{asy}{\sim} \chi_3^2$.

4) Test of symmetry

To test the symmetry restriction of the Slutsky substitution matrix, $\gamma_{ij} = \gamma_{ji}$, i, $j = 1, \ldots, 4$, we also use the test of the general linear restriction. In this case, $r(RB_{11}^{(1)}R') = q$, since $m(m-1)/2$ restriction of $R\hat{\beta}$ are not subject to adding up restrictions. Therefore, under H_0: $R\beta = 0$, the test statistic has the following asymptotic distribution: $T(R\hat{\beta})'(RB_{11}^{(1)}R')^{-1}(R\hat{\beta}) \overset{asy}{\sim} X_6^2$.

[7]Dhrymes (1989) pp. 15-8, 23-4.

[8]This homogeneity restriction is valid in the first stage of two stage budgeting, and it may not be appropriate to test this restriction in the second stage such as food expenditure allocation. However, for the sake of completeness, we perform the homogeneity test in the context of our second stage food allocation.

TABLE 1
SUMMARY OF TESTING HYPOTHESES

Model	Test	χ^2	D.F.[a]	1% C.V.[b]
1. Maintained Model				
2. No Autocorrelation	2 vs. 1	86.14	15	30.58
3. Diagonal *H* matrix	3 vs. 1	49.98	11	24.73
4. No Habit Formation	4 vs. 1	51.55	12	26.22
5. Homogeneity	5 vs. 1	22.19	3	11.34
6. Symmetry	6 vs. 1	44.17	6	16.81

Note: a. Degrees of freedom.
b. Critical values at 1% significance level.

IV. Empirical Results

The data we employ in the estimation of our proposed model and in the test of hypotheses are aggregate annual U.S. time series data on four categories of foods ranging from 1948 to 1978. The categories are 1. Meats, 2. Fruits & Vegetables, 3. Cereal & Bakery Products and 4. Miscellaneous Foods. For the purpose of comparison, we use the same data employed by BG (1983). The source of data is Blanciforti (1982, pp. 110-3).[9] Table 1 presents the results of testing on specification of the autocorrelation coefficient matrix, *H*, habit formation hypothesis, homogeneity and symmetry. Regarding the test of specification of *H*, both the hypothesis of no autocorrelation and of diagonal *H* were rejected at the 1% significance level. Therefore a first-order vector autoregressive process seems to be present. About the economic hypothesis, first of all, habit effect in food consumption seems to be present. However, homogeneity and symmetry don't seem to be supported by the data.

The rejection of homogeneity and symmetry has often been observed in the literature. Regarding this phenomena, several explanations have been given. In the following, we introduce some of them which may be pertinent to our case.

The first line of explanation is concerning the asymptotic test we employed. In light of the number of observations of thirty in our data, we may not be well justified in employing the asymptotic test and this argument can be carried over to our entire test procedure. For ex-

[9]However, our general price index to deflate total expenditure is different.

ample, Guilkey (1974) suggests "... tests for R (H in our case) equals to zero and ... diagonal can be trusted to a great degree for sample sizes equal to or greater that 50."[10] There is also some literature (See, for example, Laitinen 1978; Meisner 1979; Bera et al. 1981), which also point that especially when number of goods are large relative to number of observations, homogeneity and symmetry are often falsely rejected by asymptotic tests.

The second line of explanation is concerning the possibility of misspecification of dynamic demand functions. Our habit formation hypothesis is simple but ad-hoc in its nature. Recently, Browning (1991) showed that intertemporal nonseparability assumption can be elegantly modeled and reported that homogeneity could not be rejected.

The third line of explanation is about the use of aggregate data. According to Blundell (1988), the demand study based on micro-data shows that household characteristics have an important impact on consumer behavior. That implies the use of micro data would give us fruitful results. Even though AIDS started from a preference structure satisfying aggregation condition, we may have inevitably ignored parameters regarding taste differences between households.

With these limitations in mind, we turn to the estimation results. Although not presented here, initial OLS results without autocorrelation showed that all estimated equations explained the variation of shares well ($\bar{R}^2 \approx 0.9$) except for Cereal & Bakery Product ($\bar{R}^2 \approx 0.3$).

As shown in Table 2, about one third of variables turned out to be significant. According to the sign of δ_i, Cereal & Bakery Products and Miscellaneous Foods are necessities, and Meats and Fruits & Vegetables are luxuries. Regarding the price effect, price of Meats seems to be important for the allocation of other foods' expenditure. Habit effect seems to be dominant in the share equation for Fruits & Vegetable. Also, the error term of Fruits & Vegetables seems under influence to lagged error of other foods' share. It is not very clear why the category is peculiarly sensitive to other foods' consumption. As indicated by the test, we notice from the values of γ_{ij}'s that homogeneity and symmetry do not seem to hold.

In Table 3, the elasticities of food expenditure and prices are presented. Since the sign of coefficients of food expenditure determines the food expenditure elasticity, we notice that Meats and Fruits & Vegetables are elastic with respect to food expenditure. Ordinary own price

[10]Guilkey (1974), p.103.

TABLE 2
THE PARAMETER ESTIMATES OF DLA/AIDS

No.	α_i^*	α_{i1}	α_{i2}	α_{i3}	α_{i4}	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}
1	-0.10	-0.0002	0.0002	0.0010	0.0002	0.15	-0.06	-0.10	0.04
	-0.95[a]	-1.2650	0.4504	1.3253	1.1706	8.63	-1.77	-2.11	1.55
2	0.27	00003	0.0006	-0.0017	-0.0004	-0.05	0.05	0.04	-0.06
	3.98	2.0512	1.8674	-2.5640	-2.9427	-3.54	1.78	1.23	-3.08
3	0.27	0.0002	-0.0001	-0.0006	-0.0001	-0.03	-0.02	0.06	-0.01
	3.87	1.3360	-0.4507	-1.2450	-0.6400	-2.47	-0.70	1.91	-0.76
4	0.56	-0.0003	-0.0007	0.0014	0.0003	-0.07	0.02	-0.01	0.03
	6.82	-1.1781	-1.9927	1.5337	1.5472	-3.56	0.49	-0.15	1.24

No.	δ_i	h_{i1}	h_{i2}	h_{i3}	h_{i4}
1	0.09	0.49	-5.12	1.06	3.54
	1.45	2.75	-3.81	0.82	1.79
2	0.05	0.12	-5.04	1.03	3.87
	1.02	0.32	-3.56	0.72	1.86
3	-0.06	-0.54	-4.37	2.02	2.87
	-1.50	-1.19	-3.17	1.63	1.44
4	-0.07	0.00	-4.90	1.45	3.42
	-1.05	—	-3.61	1.11	1.72

Note: 1. a: t-statistic
2. Value of objective function = 119.941
3. No.1: Meats; No.2: Fruits & Vegetables
No.3: Cereal & Bakery Products; No.4: Miscellaneous Foods

TABLE 3
FOOD EXPENDITURE AND ORDINARY PRICE ELASTICITIES

No.	η_i^a	ε_{i1}^b	ε_{i2}	ε_{i3}	ε_{i4}
1	1.26	-0.62	-0.23	-0.33	0.04
2	1.23	-0.31	-0.80	0.17	-0.38
3	0.56	-0.06	-0.03	-0.51	0.06
4	0.80	-0.13	0.10	0.01	-0.84

Note: 1. a. Food expenditure elasticities
b. Ordinary price elasticities
2. Elasticities are evaluated at the sample mean.
3. No.1: Meats; No.2: Fruits & Vegetables;
No.3: Cereal & Bakery Products; No.4: Miscellaneous Foods.

TABLE 4
COMPARISON OF ELASTICITIES

Food Expenditure Elasticity

Goods	Manser[a]	BG[b]	Ours
1. Meats	1.11	0.78	1.26
2. Fruits & Vegetables	0.29	0.67	1.23
3. Cereal & Bakery Products	0.18	0.36	0.56
4. Miscellaneous Foods	1.65	1.62	0.80

Ordinary Own Price Elasticity, ε_{ii}

Goods	Manser[a]	BG[b]	Ours
1. Meats	−0.53	−0.57	−0.62
2. Fruits & Vegetables	−0.35	−0.60	−0.80
3. Cereal & Bakery Products	−0.65	−0.55	−0.51
4. Miscellaneous Foods	−0.94	−1.01	−0.84

Note: a. Manser (1976), p. 887-8, Table V, VI.
b. Blanciforti and Green (1983), p. 515. Table 2.

elasticities seem to have right sign and their magnitude seems to be relatively reasonable.

Table 4 presents the elasticities of food expenditure and the own uncompensated price elasticities from previous works and ours for four food categories in the U.S.. First, Manser's result is based on the estimated parameters of the indirect translog system (ITL) with habit formation hypothesis using data from 1948 to 1972.[11] Also, the elasticities are evaluated at 1959 data. BG's result is based on the dynamic AIDS with their hypothesis on habit formation and diagonal H matrix.

From the comparison, first, we notice that there are some variations in elasticities among the alternative estimates. Second, the demand for Cereals is (food) expenditure inelastic. However, it may be noted that the empirical results form the three models turn out not to obey the demand theory. Manser's ITL with habit formation is reported not to satisfy the convexity, while DLA/AIDS by Blanciforti and Green not to satisfy the homogeneity. Therefore, it may be hard to depend on one particular model. However, it is often said that AIDS has more plausible characteristics than ITL. Also, our specification of the habit formation at least obeys the implication of the adding up restriction.

[11]See Manser (1976), p. 880.

V. Conclusion

In our paper, we extended a dynamic AIDS by BG (1983) by making their proposed habit formation hypothesis to obey the adding up restriction implied by the demand theory. Also we introduced a more general form of autocorrelation coefficient matrix (H) in the specification of autocorrelated error process.

Furthermore, in the estimation procedure, we employed the GLS estimator proposed by Dhrymes (1989). The GLS estimator enabled us to estimate all the parameters of all the equations at the same time. In terms of the restriction on H, the GLS estimator requires less restrictive form of H than that required by the ML estimation procedure by Berndt and Savin (1975). Moreover, although we did not explore in this work, it enables us to test for various specifications of H which the ML estimation procedure does not allow us to test.

In the application towards four categories of food in the U.S., various tests on H, on habit formation, of homogeneity and of symmetry were performed based on the asymptotic distribution of estimated parameters. Our proposed first order vector autoregressive error seemed to be present, and habit effect also appeared to exist in the consumption patterns of foods. However, characteristics of demand function such as homogeneity and symmetry don't seem to be supported by the data. Estimated parameters and elasticities seem to be reasonable.

Even though we included the lagged variable and general autoregressive error structure to capture possible dynamic behavior, our model appears to exclude some important explanatory factors.

As recent studies based on micro data show, consideration of household characteristics seems to provide a bridge between theory and data. Also, incorporating dynamic effect on consumer behavior through modeling intertemporal nonseparability seems to be an interesting area for future research.

Appendix: Test of Hypotheses

In this appendix, we discuss test statistics for tests of significance and for tests of linear restriction. In Dhrymes (1989), it is shown that

$$\sqrt{T}\left(\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\end{pmatrix} - \begin{pmatrix}\beta\\ \gamma\end{pmatrix}\right) \sim N(0,\ B_{11})$$

where $B_{11} = V_{11} - V_{11}Q_{12}(Q_{21}V_{11}Q_{12})^{-1}Q_{21}V_{11}$ and $V_{11} = (Q_{11} + Q_{12}Q_{21})^{-1}$, where Q_{ij}'s are defined in section III.[12]

Note that B_{11} is a $(\Sigma G_i + \Sigma m_i) \times (\Sigma G_i + \Sigma m_i)$ singular covariance matrix with rank, $\Sigma G_i - G + \Sigma m_i - m$, due to the $(G + m)$ adding-up restrictions.

If we suppose that $B_{11} = \begin{bmatrix} B_{11}^{(1)} & B_{11}^{(2)} \\ B_{11}^{(3)} & B_{11}^{(4)} \end{bmatrix}$, then we know $\sqrt{T}(\hat{\beta} - \beta_0) \sim N(0, B_{11}^{(1)})$ and $\sqrt{T}(\hat{\gamma} - \gamma_0) \sim N(0, B_{11}^{(4)})$.

A. Testing the Significance of Sets of Coefficients

To test H_o: $\begin{pmatrix}\beta\\ \gamma\end{pmatrix} = 0$ against H_1: $\begin{pmatrix}\beta\\ \gamma\end{pmatrix} \neq 0$, a chi-square statistic is constructed by Bush (1989) as follows:[13] $B_{11} = F\Lambda_1 F'$, where Λ_1 is a $(\Sigma G_i - G + \Sigma m_i - m) \times (\Sigma G_i - G + \Sigma m_i - m)$ diagonal matrix containing the nonzero (decreasing in order of magnitude) characteristic roots of B_{11}. F is a $(\Sigma G_i + \Sigma m_i) \times (\Sigma G_i - G + \Sigma m_i - m)$ matrix containing the orthonormal characteristic vectors of the corresponding nonzero roots of B_{11}.

$$\sqrt{T}F'\left(\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\end{pmatrix} - \begin{pmatrix}\beta\\ \gamma\end{pmatrix}\right) \overset{asy}{\sim} N(0,\ F'F\Lambda_1 F'F) = N(0,\ \Lambda_1)$$

Let $\Lambda_1^{-1} = P'P$.

$$y = \sqrt{T}PF'\left(\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\end{pmatrix} - \begin{pmatrix}\beta\\ \gamma\end{pmatrix}\right) \overset{asy}{\sim} N(0,\ P\Lambda_1 P') = N(0,\ PP^{-1}P'^{-1}P') = N(0,\ I).$$

Since $FP'PF' = (B_{11})_g$,

$$y'y = T\left(\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\end{pmatrix} - \begin{pmatrix}\beta\\ \gamma\end{pmatrix}\right)'(B_{11})_g\left(\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\end{pmatrix} - \begin{pmatrix}\beta\\ \gamma\end{pmatrix}\right) \overset{asy}{\sim} \chi^2$$

with degrees of freedom, $(\Sigma G_i - G + \Sigma m_i - m)$.

In the same way, to test H_0: $\beta = 0$ and H_0: $\gamma = 0$, chi-square statistics can be formed based on $B_{11}^{(1)}$ and $B_{11}^{(4)}$ respectively.

B. Testing the General Linear Restriction, $R\beta = r$

Suppose we test H_0: $R\beta = r$ as against the alternative H_1: $R\beta \neq r$. In

[12]Dhrymes (1989), pp.23-4.
[13]Bush (1989), pp.76-7.

our model, R is $q \times \Sigma G_i$ matrix of rank q. Then, under the null, $\sqrt{T}(R\hat{\beta} - r) \overset{asy}{\sim} N(0,\ RB_{11}^{(1)}R')$. We note that either $r(RB_{11}^{(1)}R') = q$ or $r(RB_{11}^{(1)}R') < q$.

1) When $r(RB_{11}^{(1)}R') = q$.

This case arises when the two conditions that (1) $q \leq r(B_{11}^{(1)})$ and that (2) $R\hat{\beta} = r$ is not under the adding up restrictions, $\hat{B}e = 0$, are satisfied.

If we define $\Psi = RB_{11}^{(1)}R'$, Ψ becomes nonsingular. Since Ψ is symmetric and positive definite, there exists a nonsingular matrix P such that $\Psi^{-1} = P'P$. Therefore, $y = \sqrt{T}P(R\hat{\beta} - R\beta) \overset{asy}{\sim} N(0, I_q)$, and under the null hypothesis, $y'y = T(R\hat{\beta} - r)'\Psi^{-1}(R\hat{\beta} - r) \overset{asy}{\sim} \chi^2$ with q degrees of freedom.

2) When $r(RB_{11}^{(1)}R') < q$.

This case arises when either (1) $q < r(B_{11}^{(1)})$ and $R\hat{\beta} = r$ is under the adding up restriction, $\hat{B}e = 0$ or (2) $q > r(B_{11}^{(1)})$. If $q < r(B_{11}^{(1)})$ and $R\hat{\beta} = r$ is under the adding up restriction, and suppose number of adding up restriction imposed on $R\hat{\beta} = r$ is p. Then, $r(RB_{11}^{(1)}R') = q - p$.

By the same arguments as above, the test statistic, $T(R\hat{\beta} - r)'\Psi_g(R\hat{\beta} - r) \overset{asy}{\sim} \chi^2_{q-p}$.

When $q > r(B_{11}^{(1)})$, the test statistic, $T(R\hat{\beta} - r)'\Psi_g(R\hat{\beta} - r) \overset{asy}{\sim} \chi^2_{\Sigma G_i - G}$.

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