Economic Growth, Engel's Law, and Structural Transformation

Do-Chull Shin*

This paper presents a model of structural transformation in the course of economic development and contrasts simulation results with actual data. Important features of the United States historical data such as the diminishing importance of agriculture in terms of labor force or GDP fraction can be generated by the model. It is shown that the structural dynamics bringing about diminution of agriculture calls for a nonhomothetic preference specification. (*JEL* Classifications: O14, R23)

I. Introduction

Fundamentally, economic development involves structural transformation. This can be observed from historical experiences and cross-sectional data of different countries.

Table 1 shows that all income groups of countries have experienced diminution of agriculture in terms of both labor force and GDP fractions from 1965 to 1980. For the cross-sectional data of countries, the general pattern is that countries with higher income have a smaller fraction of the labor force devoted to the agricultural sector and a smaller proportion of the GDP originating in farming.

In the case of the United States, the fraction of the labor force devoted to the agricultural sector changed from 78.8% in 1820 to 20.1% in 1940 and 3.4% in 1980, and the proportion of the farm sector GDP to

*Sookmyung Women's University, 53-12 Chungpa-dong 2-ka, Yongsan-ku, Seoul 140-742, Korea. This paper is based on work from my dissertation (Shin 1990). I wish to thank Professors Paul M. Romer, Yair Mundlak, and especially Robert E. Lucas, Jr. (the chairman of my thesis committee) for their helpful comments. I have also benefited from suggestions by an anonymous referee. Any remaining shortcomings are, of course, my own. Financial support from Sookmyung Women's University is gratefully acknowledged.
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TABLE 1
SHARES OF AGRICULTURE IN LABOR FORCE AND GDP FOR DIFFERENT INCOME GROUPS OF COUNTRIES

	GNP Per Capita (Dollars, 1980)	Share in Labor Force (%, 1965)	(%, 1980)	Share in GDP (%, 1965)	(%, 1985)
Developing Countries	670	70	62	30	19
Low-Income	270	77	72	41	34
Middle-Income	1,510	56	43	22	14
Industrial Countries	10,760	14	7	5	3

Source: *World Development Report* (1987), Tables A.2 (p.187), A.5 (p.189) and 31 (pp.282-3).

the total business GDP decreased from 7.2% in 1940 to 2.8% in 1980.[1]

For the problem of structural transformation in the course of economic development, this paper tries to construct, as Lucas (1988) puts it, "a mechanical, artificial world ... that is capable of exhibiting behavior the gross features of which resemble those of the actual world..." This paper presents a simple but fully specified competitive equilibrium model of structural transformation.

The model incorporates an important finding forwarded by the 'development economics' or previous studies of structural transformation.[2] That is, to represent the nonproportionate expansion of demands with income growth, or more specifically, to incorporate the idea that income elasticity of demand for agricultural (nonagricultural) good is smaller (larger) than unity, the model introduces a nonhomothetic utility function. For technology, a simple linear production function is introduced, where (labor) productivity grows continually, but not necessarily at a same rate across sectors. The model, however, is so specified that there is neither duality between agricultural and nonagricultural sectors nor unemployment in any sector, in contrast with the traditional 'development economics' literature.

The model also utilizes the tools developed by growth theorists for analyzing dynamic general equilibrium models. Specifically utilized is the finding that, in many cases, a competitive equilibrium can be characterized by a maximization problem.[3]

[1] See Lebergott (1966) and *Economic Report of the President* (various years).

[2] For a survey of the literature on structural transformation, see Pasinetti and Scazzieri (1987) or Chenery (1988).

[3] The Kuhn-Tucker theorem offers a general procedure for reducing the prob-

The paper is organized as follows. Section II presents the model of structural transformation. Preferences and technology are specified and the resulting equilibrium paths of various variables are characterized. In section III, we simulate the equilibrium paths by providing specific values for parameters and initial conditions and compare those with the actual historical data of the United States. It is shown that the observed structural transformation calls for nonproportionate expansion of demands for agricultural and nonagricultural goods. In section IV, the model is extended to incorporate one more dimension of reality and concluding remarks are provided.

II. Model of Structural Transformation

In this section, after specifying preferences and technology for a closed economy with identical agents and competitive markets, we will get some characteristics of competitive equilibrium paths in connection with the problem of structural transformation.

A. Preferences

At each date, there are two kinds of consumption goods, c_1 and c_2. If we let c_1 and c_2 represent agricultural and nonagricultural good, respectively, then it is mostly likely that c_1 turns out to be a necessary good and c_2 a luxury good.

Preferences over (per capita) consumption streams $\{c_1(t), c_2(t);\ t \geq 0\}$ are assumed to be given by:

$$\int_0^\infty e^{-\rho t} U(c_1(t), c_2(t))dt, \tag{1}$$

where $\rho > 0$ is the discount rate.

We will consider a specific form of period utility function:

$$U(c_1, c_2) = \frac{1}{\gamma}\left\{\left(\frac{c_1^{\alpha_1} - 1}{\alpha_1} + a\frac{c_2^{\alpha_2} - 1}{\alpha_2}\right)^\gamma - 1\right\}, \quad 0 < a < \infty,\ -\infty < \alpha_1 \leq \alpha_2 \leq 1,\ \alpha_1 \neq 1,\ -\infty < \gamma \leq 1. \tag{2}$$

Here we exclude the case $\alpha_1 = \alpha_2 = 1$ to get some curvature at the indifference curves. Note that the limit of $(1/\alpha)[c^\alpha - 1]$ as $\alpha \to 0$ is $\ln(c)$. The

lem of calculating a competitive equilibrium to the problem of solving a maximization problem. See Romer (1989).

values of c_1 and c_2 are assumed to be such that $U(c_1, c_2)$ is well defined. If $\gamma = 1$, $U(c_1, c_2)$ is well defined for $c_i > 0$, $i = 1, 2$.[4] Another sufficient condition for $U(c_1, c_2)$ being well defined is $c_i > 1$, $i = 1, 2$. Since the preference ordering implied by (2) is in general not independent of the measuring unit,[5] we should choose an appropriate measuring unit to make $U(c_1, c_2)$ in (2) represent the agents' preferences. Thus, confining the values of c_1 and c_2 so that $U(c_1, c_2)$ is well defined is not so restrictive as it might seem.

Let us investigate some characteristics of our period utility function given in (2). Consider the problem of maximizing the period utility (2) subject to the period budget constraint, $p_1c_1 + p_2c_2 = e$, where p_1 and p_2 are the prices of c_1 and c_2 in terms of some numeraire, and e is the total expenditure measured in unit of the numeraire. The (uncompensated) demands for c_1 and c_2, which solves the maximization problem, are functions of p_1, p_2, and e.

It is not difficult to derive explicitly the income elasticities, the own and cross price elasticities, and the elasticity of substitution implied by the period utility function (2).[6] The income elasticity of demand for good 1 turns out to be:

$$\varepsilon_1 \equiv \frac{\partial \ln(c_1)}{\partial \ln(e)} = \frac{1-\alpha_2}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}, \tag{3}$$

where $R_i = p_ic_i/e$, $i = 1, 2$ are expenditure fractions. (The income elasticity of demand for good 2 is obvious from the symmetry of the formula.) Notice that $\alpha_1 = \alpha_2$ implies unit income elasticities. If $\alpha_1 < \alpha_2$, c_1 is a necessary good with income elasticity smaller than one and c_2 a luxury good with income elasticity larger than one.

The own price elasticity of good 1 is obtained as:

$$\varepsilon_{11} \equiv \frac{\partial \ln(c_1)}{\partial \ln(p_1)} = \frac{(1-\alpha_2)R_1 + R_2}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}. \tag{4}$$

The cross price elasticity of c_1 with respect to p_2 is:

$$\varepsilon_{12} \equiv \frac{\partial \ln(c_1)}{\partial \ln(p_2)} = \frac{\alpha_2 R_2}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}. \tag{5}$$

[4]The period utility function (2) with $\gamma = 1$ was named "direct addilog" by Houthakker (1960).

[5]For example, $U(3, 2) > U(2, 3)$ does not necessarily imply $U(30, 20) > U(20, 30)$.

[6]For derivation of these elasticities, see appendix A of Shin (1990).

Finally, the elasticity of substitution between the two goods turns out to depend upon which price to take as varying. If p_1 varies while p_2 (and e) is fixed, for example, we get:

$$\sigma_1 \equiv \frac{\partial \ln(c_2 / c_1)}{\partial \ln(p_1)} = \varepsilon_{21} - \varepsilon_{11} = \frac{(\alpha_1 - \alpha_2)R_1 + 1}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}. \tag{6}$$

On the other hand, we can consider the dual problem of minimizing the expenditure $p_1c_1 + p_2c_2$ subject to a given utility level u being secured. The resulting (compensated) demands for c_1 and c_2 are functions of p_1, p_2, and u.

The compensated (own and cross) price elasticities are easily obtained by utilizing the established relationship $\varepsilon_{ij}^* \ (\equiv \partial \ln(c_i(p_1, p_2, u))/\partial \ln(p_j)) = \varepsilon_{ij} + \varepsilon_i R_j$, $i, j = 1, 2$ (see, for example, Deaton and Muellbauer 1984):

$$\varepsilon_{11}^* = \varepsilon_{11} + \varepsilon_1 R_1 = -\frac{R_2}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}, \tag{7}$$

$$\varepsilon_{12}^* = \varepsilon_{12} + \varepsilon_1 R_2 = \frac{R_2}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}. \tag{8}$$

The elasticity of substitution between compensated demands, differently from the uncompensated case, does not depend upon which price to take as varying and thus well defined as $\sigma^* \equiv - \partial \ln(c_1/c_2)/\partial \ln(p_1/p_2)$, since the solution for the expenditure minimization problem depends only upon the level of utility and *relative* price. The (compensated) elasticity of substitution between the two goods turns out to be:

$$\sigma^* \equiv -\frac{\partial \ln(c_1 / c_2)}{\partial \ln(p_1 / p_2)} = \frac{1}{(1-\alpha_2)R_1 + (1-\alpha_1)R_2}. \tag{9}$$

It is interesting to note that all these elasticities depend only upon α_1, α_2 and the expenditure fractions of goods, R_1 and R_2 ($R_1 + R_2 = 1$).

Next, when we consider $(1/\alpha_1)(c_1^{\alpha_1} - 1) + a(1/\alpha_2)(c_2^{\alpha_2} - 1)$ as a composite good, $1/(1 - \gamma)$ may be regarded as the elasticity of substitution between the composite goods at different dates. Thus it may be said that α_1 and α_2 determine intra-temporal substitution and γ represents inter-temporal substitution.[7]

[7]More rigorously, it might be said that γ together with α_1 and α_2 determines inter-temporal substitution between goods at different dates in time. When $\gamma = 1$ ($1/(1 - \gamma) = \infty$), for example, the elasticity of substitution between $c_i(s)$ and $c_i(t)$, $s \neq t$, is $1/(1 - \alpha_i)$ which is not infinite for $\alpha_i \neq 1$.

B. Technology

The good i ($i = 1, 2$) is produced with the Ricardian technology:

$$c_i(t) = h_i(t)u_i(t), \ i = 1, 2, \tag{10}$$

where $h_i(t)$ is the (labor) productivity for the production of good i and $u_i(t)$ is the fraction of the labor force devoted to producing good i. The productivity is assumed to grow exogenously at a constant rate:

$$\dot{h}_i(t) = \delta_i h_i(t), \ \ \delta_i > 0, \ \ i = 1, 2. \tag{11}$$

Since no deliberate effort is made to enhance the productivity, all labor forces are directed to goods production; hence, $u_1 + u_2 = 1$.

The specification of technology given above is simplistic. The only factor of production is labor, and (labor) productivity grows spontaneously without any human effort. Admittedly, these assumptions are extreme. First, introducing (physical) capital as another factor of production may help to explain the data better, since the exogeneity assumption of labor productivity growth in (11) does not seem to properly capture the effects on labor productivity of capital accumulation, which basically hinges on the saving and investment decisions of economic agents. Second, if we let the sustained long-run growth be driven not by exogenous technological change as in (11) but by endogenous technological change that arises from intentional actions taken by profit-maximizing agents, we may get different policy implications. Thus, a more adequate specification for technology remains an important topic for future research.

For the construction of a model of structural transformation in the course of economic development, however, the simplistic specification of technology given by (10) and (11) has its own merits: not only is the analysis made easier, but also the effect of preference structure on structural dynamics is apprehended more clearly.

C. Competitive Equilibrium Paths

A competitive equilibrium for this economy can be stated in terms of the problems of a representative firm and a representative consumer. As Romer (1989) made it clear, however, for the particular specification here of the preferences and technology with no external effects or distortions of any kind, the unique competitive equilibrium can be characterized by the following maximization problem together with the law of motion for $h_i(t)$:

$$\max \int_0^\infty e^{-\rho t} \frac{1}{\gamma}\left[\left\{\frac{c_1(t)^{\alpha_1}-1}{\alpha_1} + a\frac{c_2(t)^{\alpha_2}-1}{\alpha_2}\right\}^{\gamma} - 1\right] dt$$

$$\begin{aligned} \text{s.t.}\quad & c_1(t) = h_1(t)u_1(t) \\ & c_2(t) = h_2(t)u_2(t) \\ & 0 \le u_i(t) \le 1,\ \ i = 1, 2 \\ & u_1(t) + u_2(t) = 1. \end{aligned}$$

The Lagrangian for this problem, assuming that the constraints $0 \le u_i \le 1$, $i = 1,2$ are not binding, is:

$$\begin{aligned} & L(c_1, c_2, u_1, u_2, \xi_1, \xi_2, \mu;\ h_1, h_2) = \\ & \int_0^\infty e^{-\rho t}[\frac{1}{\gamma}\{(\frac{c_1^{\alpha_1}-1}{\alpha_1} + a\frac{c_2^{\alpha_2}-1}{\alpha_2})^{\gamma} - 1\} \\ & + \xi_1(h_1u_1 - c_1) + \xi_2(h_2u_2 - c_2) + \mu(1 - u_1 - u_2)]dt. \end{aligned} \tag{12}$$

The first order conditions are written down in full for subsequent convenience:

$$\left(\frac{c_1^{\alpha_1}-1}{\alpha_1} + a\frac{c_2^{\alpha_2}-1}{\alpha_2}\right)^{\gamma-1} c_1^{\alpha_1-1} = \xi_1 \tag{13}$$

$$\left(\frac{c_1^{\alpha_1}-1}{\alpha_1} + a\frac{c_2^{\alpha_2}-1}{\alpha_2}\right)^{\gamma-1} a c_2^{\alpha_2-1} = \xi_2 \tag{14}$$

$$\xi_1 h_1 = \mu \tag{15}$$

$$\xi_2 h_2 = \mu \tag{16}$$

$$c_1 = h_1 u_1 \tag{17}$$

$$c_2 = h_2 u_2 \tag{18}$$

$$u_1 + u_2 = 1. \tag{19}$$

Restate the law of motion for $h_i(t)$:

$$\dot{h}_1 = \delta_1 h_1 \tag{20}$$

$$\dot{h}_2 = \delta_2 h_2. \tag{21}$$

Here $\xi_i(t)$ is the (current) price of good i in terms of util and $\mu(t)$ is the (current) wage or income rate in terms of util. Hence $q \equiv \xi_2/\xi_1$ is the price of good 2 in terms of good 1 and $w \equiv \mu/\xi_1 = h_1$ is the wage or income rate in terms of good 1. Note that the wage or income rates in sector 1 and sector 2 are equalized because of the free mobility of labor force between sectors. Equality of wage or income rates across sectors, on the other hand, implies that the relative price is equal to the (labor) productivity ratio (see (15) and (16)):

$$q\left(=\frac{\xi_2}{\xi_1}\right)=\frac{h_1}{h_2}, \tag{22}$$

which in turn implies that the sectoral fractions of labor force and GDP (or GNP) are equalized:

$$\frac{c_1}{c_1+qc_2}=u_1;\quad \frac{qc_2}{c_1+qc_2}=u_2. \tag{23}$$

Now consider the equilibrium paths of various variables. From (13), (14), and (22), we get:

$$\frac{h_1}{h_2}=\frac{ac_2^{\alpha_2-1}}{c_1^{\alpha_1-1}}. \tag{24}$$

In view of (17)-(19), equation (24) can be rewritten as:

$$h_1^{\alpha_1}h_2^{-\alpha_2}=\frac{au_1^{1-\alpha_1}}{(1-u_1)^{1-\alpha_2}}. \tag{25}$$

The value of $u_1(t)$ is determined from (25) given the values of $h_1(t)$ and $h_2(t)$, the evolution of which is in turn governed by (20) and (21). Thus, the paths of various variables such as $h_1(t)$, $h_2(t)$, $u_1(t)$, $u_2(t)=1-u_1(t)$, $q(t)=h_1(t)/h_2(t)$, $c_1(t)=h_1(t)u_1(t)$, $c_2(t)=h_2(t)u_2(t)$ can be derived once the initial values h_{i0} of $h_i(t)$, $i=1, 2$ are provided. One comment is in order: since economic agents can make decisions on $c_i(t)$ and $u_i(t)$, $i=1, 2$ in a time separable manner (see the Lagrangian given in (12)), the time preference parameter ρ and the inter-temporal substitution parameter γ have no influence on the evolution of various variables.

Let us investigate how the equilibrium dynamics are determined by the values of parameters. From (25), we get:

$$\alpha_1\frac{\dot{h}_1}{h_1}-\alpha_2\frac{\dot{h}_2}{h_2}=(1-\alpha_1)\frac{\dot{u}_1}{u_1}+(1-\alpha_2)\frac{\dot{u}_1}{1-u_1}. \tag{26}$$

Combining (20), (21) and (26), we derive:

$$\dot{u}_1=\frac{u_1(1-u_1)(\alpha_1\delta_1-\alpha_2\delta_2)}{(1-\alpha_2)u_1+(1-\alpha_1)(1-u_1)}. \tag{27}$$

The law of motion for h_i specified in (20) and (21) makes the change rate of u_1 (and u_2) dependent on the change rates of h_i, but not on the levels of h_i, $i=1, 2$.

Equation (27) indicates that $u_1(t)$ approaches zero monotonically and asymptotically when and only when $\alpha_1\delta_1 - \alpha_2\delta_2 < 0$. Our model, therefore, can generate steadily decreasing u_1. This feature of the model is important considering the historical fact that almost all countries have experienced diminution of agriculture in the course of economic development.

One interesting question here is whether diminution of agriculture can take place when the demands for agricultural and nonagricultural goods increase proportionately as income grows, that is, when $\alpha_1 = \alpha_2$ ($= \alpha$) giving rise to unit income elasticities (see (3)). The answer depends on the relative speed of technological change in the two sectors and the magnitude of the elasticity of substitution between the two goods. If $\alpha > 0$ with the elasticity of substitution larger than one (see (6) or (9)), slower productivity growth in agriculture than in nonagriculture ($\delta_1 < \delta_2$) brings diminution of agriculture. On the other hand, if $\alpha < 0$ with the elasticity of substitution smaller than one, faster productivity growth in agriculture ($\delta_1 > \delta_2$) is called for to produce diminution of agriculture. The economics behind this result is not difficult to understand. Good substitutability between goods will make it more profitable to move resources away from the less to the more productive sector, whereas bad substitutability will cause resources to be moved away from the sector in which the productivity grows more quickly so as to secure sound amounts of both goods.

The historical experiences of various countries, however, seem to indicate that the relative size of δ_1 and δ_2 is not a crucial factor for the structural dynamics bringing about diminution of agriculture. Thus, it is suggested that α_1 be considerably smaller than α_2, in other words, that nonproportionate expansion of demands for agricultural and nonagricultural goods with income growth be a driving factor for the diminution of agriculture. The issue of whether the hypothesis $\alpha_1 = \alpha_2$ can be supported will be pursued in the next section, where we confront the model of structural transformation presented above with the actual historical data of the United States.

III. Empirical Investigation of the Model

In this section, we investigate whether the simple model presented above can reproduce the general features of the United States historical data on the problem of structural transformation. Since the productivity growth is assumed to be exogenous, the issue here is just what values assigned to the preference parameters, α_1 and α_2, can quantitatively explain the historical evolution of the sectoral labor force fractions, u_1 and u_2 (see (27)). It is shown that a nonhomothetic preference specification is necessary for the explanation of the data.

A. Construction of Data

Since our model tries to capture the long-run structural change in the course of economic development, the data need to cover a relatively long period of time. We construct an annual data set for important variables in our model running from 1889 to 1989 by referring to Kendrick (1961) and *Economic Report of the President* (various years), recognizing that combining data from different sources may cause problems due to differences in definitions and measuring methods.

Appendix section documents the construction of the annual data set for various variables.

In principle, u_i, i = 1, 2, is identified with the ratio of labor force employed in sector i to total labor force employed. One comment is called for here. In the model, the sectoral fractions of labor force and GDP (or GNP) are equalized because of the free mobility of labor force between sectors. Table 1, however, suggests that the actual GDP fraction has been less than the labor force fraction for the agricultural sector. This problem will be addressed in the next section where cost of migration is introduced.

We might match c_1 and c_2 in our model to per capita GNP or GDP (in constant prices) Originating in Nonfarm and Farm, respectively. This value-added concept of sectoral products fits in relatively well with the specification of production technology given in (10). Note that GNP or GDP Originating in Farm, for instance, is net of intermediate products used in the production process such as fertilizer, motor fuel, and insecticides. On the other hand, however, c_i, i = 1, 2, stands for both the amount of good i *produced* and *consumed*. In this regard, c_i might be matched more adequately with gross sectoral output which is not net of intermediate products, while the amount of agricultural good produced measured by Gross Farm Output, for instance, would be different from that consumed, since some portions of agricultural goods produced would be used as intermediate inputs to the production of agricultural and nonagricultural goods.[8]

[8]For some purposes, it would be more appropriate to allow for the fact that food contains other inputs beside agriculture such as processing, packing,

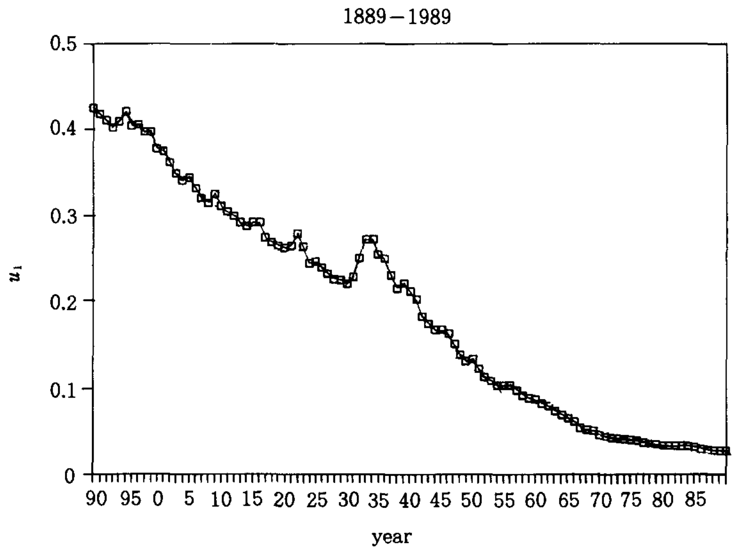

FIGURE 1
EVOLUTION OF u_1

Distinguishing between net output and gross output is particularly important for the agricultural sector because, due to a relatively large increase in purchases from the nonagricultural sector, net agricultural output has risen significantly less than gross agricultural output.[9] In this respect, the appendix section presents two kinds of c_1 series for reference. Basically, c_{1a} measures per capita GDP in constant prices originating in the farm sector (i.e., farm output net of intermediate

transportation, and restaurant service, and specify the period utility function of a representative consumer as $U(F(A, Q), N)$, where F represents food, N, nonfood, A, agricultural component of food, and Q nonagricultural component of food (see Mundlak 1988). Then, A corresponds to c_1 in our notation and $N + Q$ to c_2. The share of food in personal consumption expenditures, which corresponds to $(qA + Q)/(qA + Q + N)$ with q being the relative price, decreased from 28.5% in 1940 to 20.6% in 1980 for the U.S. (see *Economic Report of the President*, various years). Also, a cursory examination of data indicates that the share of nonagricultural component in the cost of food, which corresponds to $Q/(qA + Q)$, has been increasing over time, suggesting that quality of food has been improving.

[9]The ratio of Net to Gross Farm Output in the U.S. declined, say, from 87.2% in 1889 to 63.6% in 1957, according to Kendrick (1961, p.347). The trend continued and the ratio became 42.3% in 1980 (see *Statistical Abstract of the United States: 1987*, p.629).

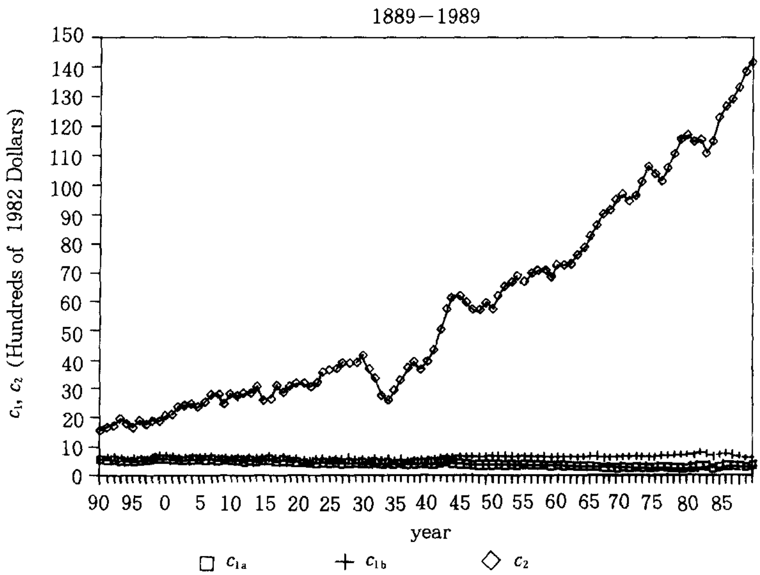

FIGURE 2
EVOLUTIONS OF c_1 AND c_2

products), whereas c_{1b} measures per capita gross farm output which is not net of intermediate products.

Measuring h_1 and h_2 also involves related problems. In view of the specification (10) of production technology, however, it seems more natural to let h_i stand for sectoral labor productivity or sectoral value-added per worker. Hence, $h_1 = c_{1a}/u_1$ in the appendix section.

Figure 1 shows that, for the United States, u_1 is steadily decreasing over time. Figure 2 indicates that c_1 stays by and large constant over time, while c_2 grows steadily. Figure 3 shows that h_1 and h_2 had grown approximately at the same rate up until the 1940s, but since then h_1 has grown more rapidly than h_2.

B. Simulation Results

Here, we investigate whether the hypothesis of proportionate expansion of demands is consistent with the data, and whether a nonhomothetic preference specification can quantitatively explain the structural transformation.

For this purpose, we first assign values of δ_1 and δ_2 which conform approximately to the United States historical data. The geometric aver-

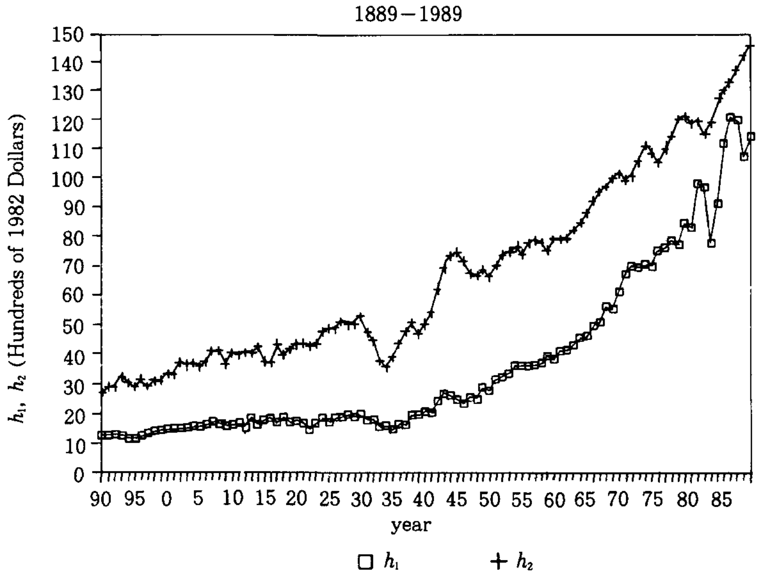

FIGURE 3
EVOLUTIONS OF h_1 AND h_2

ages of Dh_1/h_1 and Dh_2/h_2 for our sample period are 0.022 and 0.017, respectively.[10] So, we assign $\delta_1 = 0.022$ and $\delta_2 = 0.017$. Next, let us put $u_{10} = 0.42$, which corresponds to the value of u_1 around 1889.

If we assign values to the remaining parameters, α_1 and α_2, simulated evolutions of $u_1(t)$ can be generated from (27).[11] Now, let us investigate whether the hypothesis $\alpha_1 = \alpha_2 (= \alpha)$ can be supported by the data. Notice that, under the null hypothesis, the value assignment of δ_1 and δ_2 with $\delta_1 (= 0.022) > \delta_2 (= 0.017)$ calls for a negative value of α for u_1 to decrease over time (see (27)). Figure 4 shows three simulated evolutions of $u_1(t)$ when α is set to -1, -10, and $-1{,}000$. Note that, even if α is set to $-1{,}000$, which represents a very poor substitutability between the two goods ($\sigma \approx 0.001$; see (6) or (9)), the simulated value of u_1 decreases to reach only 0.31 in 1989 while the actual value is 0.03 in that year.

[10]Here, $Dh_i(t) \equiv h_i(t+1) - h_i(t)$, $i = 1, 2$, is the discrete time counterpart of $\dot{h}_i(t)$. Since δ_i is a rate of change, arithmetic average of Dh_i/h_i is not appropriate for estimating δ_i. Rather, we get the estimate $\hat{\delta}_i$ from the geometric relation $\prod_{t=0}^{T-1}[1 + Dh_i(t)/h_i(t)] = (1 + \hat{\delta}_i)^T$. That is, we have $\hat{\delta}_i = \exp\{(1/T)\sum_{t=0}^{T-1}\ln[1 + Dh_i(t)/h_i(t)]\} - 1$.

[11]For simulation, $\dot{u}_1(t)$ in (27) is replaced with the discrete time counterpart $Du_1(t) \equiv u_1(t+1) - u_1(t)$.

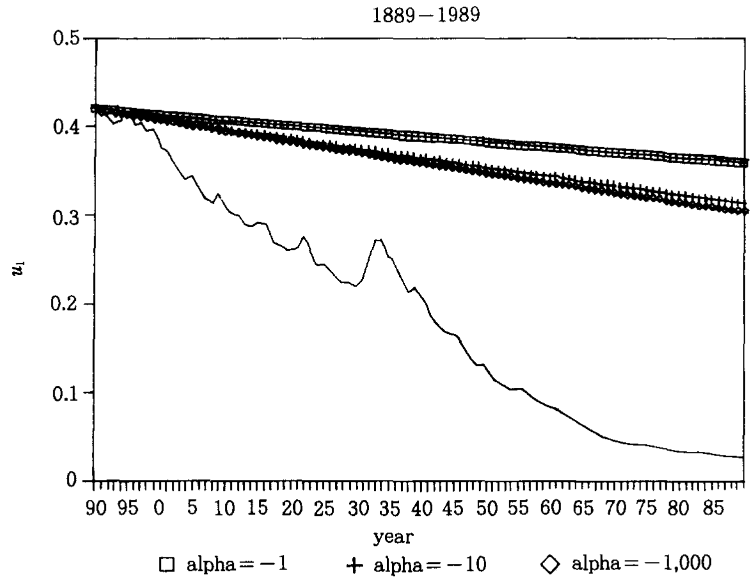

FIGURE 4
HOMOTHETIC PREFERENCES

Given the higher rate of productivity growth in agriculture than in nonagriculture, the proportionate expansion of demands for agricultural and nonagricultural goods with income growth cannot give rise to diminution of agriculture unless the substitutability between the two goods is relatively low. Moreover, however low the substitutability is set to be, the proportionate expansion of demands cannot generate the structural transformation of the magnitude observed in the United States over the last 100 years.

In fact, we observe general diminution of agriculture as well for the sample period from the 1890s to the 1940s when the average rate of productivity growth in the nonagricultural sector is about the same as or slightly higher than that in the agricultural sector. A cursory examination of historical data of other countries also indicates that the diminution of agriculture is a general phenomenon irrespective of the relative size of δ_1 and δ_2. In connection with (27), these observations suggest that the hypothesis $\alpha_1 = \alpha_2$ is inconsistent with the actual historical data, and that $\alpha_1 < \alpha_2$ (i.e., faster expansion of demand for nonagricultural good with income growth) is necessary for the structural dynamics bringing about diminution of agriculture.

Having established that the general diminution of agriculture calls

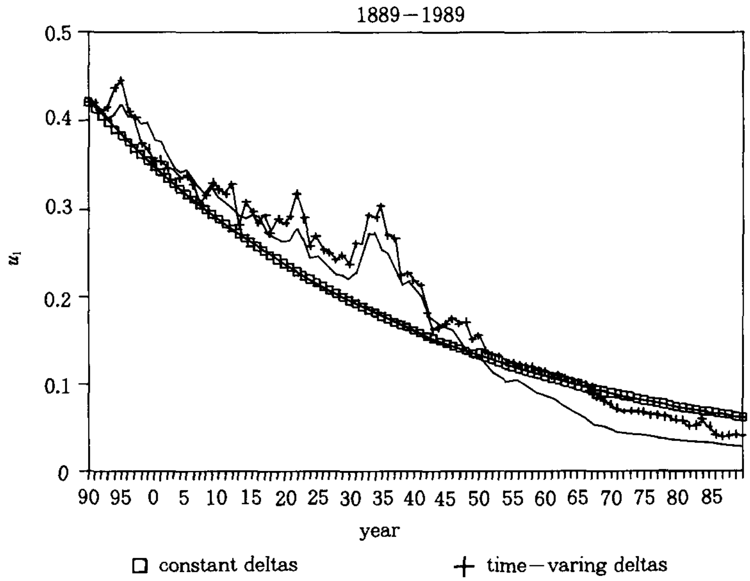

FIGURE 5
NONHOMOTHETIC PREFERENCES

for $\alpha_1 < \alpha_2$, let us investigate whether a specific value assignment to α_1 and α_2 with $\alpha_1 < \alpha_2$ can quantitatively explain the structural transformation. Figure 5 contrasts the actual United States data for u_1, and its simulated path with α_1 and α_2 set to -3.0 and 0.7, respectively.[12] Two simulation results are provided here: one with the sectoral productivity growth rates set to be constant during the sample period (δ_1 = 0.022 and δ_2 =0.017), and the other with the growth rates allowed to vary over time ($\delta_i(t) = Dh_i(t)/h_i(t)$, i = 1, 2; see (26) and (27)). The fits seem to be good even in a quantitative sense.

[12]The values set to the preference parameters are roughly in agreement with those derived from empirical analyses of consumer demand. The derivation procedure is as follows. First, calculate the income and compensated own price elasticities for food implied by empirical results. Second, set values to the corresponding elasticities for good 1 (agricultural good), taking into account the fact that the share of agricultural component in the cost of food tends to decrease as income grows. Finally, utilize the relationships $\alpha_1 = 1 + (1 - R_1\varepsilon_1)/\varepsilon_{11}^*$ and $\alpha_2 = 1 + R_2\varepsilon_1)\varepsilon_{11}^*$, where R_i is the expenditure fraction for good i, ε_1 the income elasticity of demand for good 1, and ε_{11}^* the compensated own price elasticity of good 1 (see (3) and (7)). If we set R_1 = 0.1, R_2 = 0.9, ε_1 = 0.1, and ε_{11}^* = -0.25, where the values are consistent with Deaton(1974), then we get α_1 = -2.96 and α_2 = 0.64. See Shin (1990) for more details.

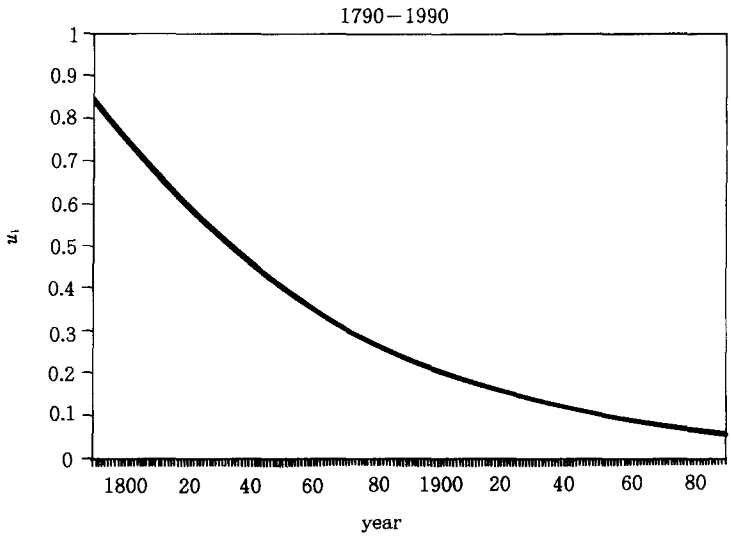

FIGURE 6
SIMULATED PATH OF u_1

For a longer-run perspective of our model, a simulated path of u_1 for a time span of 200 years is provided in Figure 6. The simulation was obtained with the same values set to the preference parameters. But, the initial value of u_1 was set to a larger number, 0.85, which roughly corresponds to the fraction of the agricultural labor force about 200 years ago in the United States. The value assignment to the sectoral productivity growth was also changed to $\delta_1 = \delta_2 = 0.015$ in view of the reported growth rate figures.[13]

If the value assignment of preference parameters, $\alpha_1 = -3$
0.7, is accepted as a reasonable description of the reality, t
not only that Engel's law is saliently among the underlying fo
structural transformation, but also that productivity growt
sector contributes positively to structural transformation (∂
$i = 1, 2$; see (27)).[14]

[13]Romer (1986) reports that the average annual compound growth
per capita GDP in the U.S. was 0.58% for the 1800-40 period, 1.44
80, 1.78% for 1880-1920, 1.68% for 1920-60, and 2.47% for 1960-7
this variation of growth rates across different periods, the simulation shown in Figure 6 could be improved in terms of goodness of fit by allowing δ_i to vary over time.

IV. Extension of the Model and Concluding Remarks

In this section, we introduce cost of migration into the model to incorporate one more dimension of reality. Concluding remarks will then follow.

A. Cost of Migration

In the model of structural transformation presented in section II, wage or income rates are equalized across sectors due to the costless mobility of labor force between sectors, so that sectoral fractions of labor force and GNP (or GDP) are equalized. But, among stylized facts seem to be that the wage or income rate in the agricultural sector is lower than that in the nonagricultural sector, and that the fraction of GNP (or GDP) originating in agriculture is less than that of labor force devoted to agriculture.

We posit here that these phenomena can be captured by introducing a restriction on the free and costless mobility of labor force between sectors. The idea is that if off-farm migration, for example, is not free and costless, then the agricultural labor force and hence agricultural production would become relatively large. This would make the price of agricultural good relative to nonagricultural good stay below the (labor) productivity ratio, and the wage or income rate in the agricultural sector less than that in the nonagricultural sector. In addition, the value share of agricultural good in GNP (or GDP) would be less than the fraction of the labor force devoted to the agricultural sector.

Let us construct a two-sector model of structural transformation which takes the cost of migration explicitly into account. Assume for simplicity that m (constant over time) units of nonagricultural good (and nothing else) are incurred per migrant as the cost of intersectoral migration. Then the equation (10) is replaced with:

$$\begin{aligned} c_1(t) &= h_1(t)u_1(t), \\ c_2(t) + m\,\dot{u}_2(t) &= h_2(t)u_2(t), \end{aligned} \qquad (10)'$$

where it is assumed that off-farm, not in-farm, migration takes place

[14]Thus, a fast growing economy with high values of δ_1 and δ_2 would exhibit relatively speedy structural transformation. In Korea, for example, it took 25 years (1963-88) for u_1 to decrease from 0.63 to 0.20, whereas it took 100 years (1840-1940) in the U.S.

$(\dot{u}_2 > 0)$.

Consider the maximization problem which characterizes the unique competitive equilibrium of an economy where the preferences and technology are given by (1), (2), (10)′, and (11). The corresponding (current value) Hamiltonian, assuming that the constraints $0 \leq u_i \leq 1$, $i = 1, 2$, are not binding, is:

$$H(c_1, c_2, u_1, u_2, \xi_1, \xi_2, \mu) =$$
$$\frac{1}{\gamma}\{(\frac{c_1^{\alpha_1}-1}{\alpha_1} + a\frac{c_2^{\alpha_2}-1}{\alpha_2})^{\gamma} - 1\} \tag{12)'$$
$$+ \xi_1(h_1u_1 - c_1) + \xi_2(h_2u_2 - c_2) + \mu(1 - u_1 - u_2).$$

For maximization, the Hamiltonian H must be maximized with respect to c_1, c_2 and u_1. Thus, the conditions (13)-(15) must hold here, too. The Maximum Principle also states that ξ_2 must satisfy the following differential equation:

$$m(\dot{\xi}_2 - \rho\xi_2) = -\frac{\partial H}{\partial u_2} = -\xi_2 h_2 + \mu, \tag{16)'$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} m\xi_2(t)u_2(t) = 0.$$

Replace (18) with:

$$c_2 + m\,\dot{u}_2 = h_2u_2.\ . \tag{18)'$$

Then, equations (13)-(21) with (16) and (18) replaced with (16)′ and (18)′ respectively characterize the competitive equilibrium.

Now, wage or income rates are no longer equalized across sectors. Since the marginal utility of good 2, ξ_2, is positive and most likely to decrease over time as income grows (see (14) noting that both c_1 and c_2 are normal goods in our specification of preferences), we conjecture that the left hand side of (16)′ has a negative value. Then, we have $\xi_1 h_1 < \xi_2 h_2$ (see (15) and (16)′) and hence $h_1 < qh_2$ which implies that wage or income rate in the agricultural sector is lower than that in the non-agricultural sector. In this situation, it is also the case that the fraction of GNP (or GDP) originating in agriculture is less than that of labor force devoted to agriculture $(c_1/(c_1 + qc_2) < u_1)$.

Having established that the introduction of migration cost would add an additional dimension of reality to our model, we leave the detailed characterization and empirical investigation of this extended model as a future research topic.

B. Concluding Remarks

This paper constructs a model that can generate some of the main features of the actual data on structural transformation, especially the feature that the labor force and GNP (or GDP) fractions of the agricultural sector have been decreasing over time. Section II presents a two-sector equilibrium model of structural transformation, where Engel's law expressed by a nonhomothetic preference specification is one of the building blocks. By way of contrasting simulation results with the actual historical data of the United States, section III establishes that a faster expansion of demand for nonagricultural good with income growth is necessary for the structural dynamics bringing about diminution of agriculture. Section IV investigates the implications of introducing cost of migration.

The model presented in this paper can be modified in various directions. Some nonhomothetic period utility function other than (2) could be tried to express Engel's law.[15] As mentioned earlier, to introduce physical capital or to endogenize technological progress might help explain the data better, and intermediate goods (or lands) might play important roles. Or, open trade could be introduced into the model in order to get an idea about the effects of trade on structural transformation.[16] In any case, however, the simple model of structural transformation presented in this paper, which makes it clear that Engel's law plays an essential role in structural dynamics, can serve as a reference model for future research.

[15]One feature to be noted of the extended addilog utility function given in (2) is that the income elasticity of the demand for a necessary good decreases as its expenditure fraction decreases (see (3)). Giving empirical support to this feature of the addilog utility, Ogaki (1992) observes that the income elasticity of demand for food is lower in the U.S. than in India.

[16]Open trade as well as the fast growth seems to have contributed to the very rapid decrease in the sectoral share of agriculture in Korea, which is now a net importer of agricultural goods with domestic supply of grains being only one-third (see footnote 14 above). Introducing open trade into the model in section II, where migration incurs no costs, would make at least one country suddenly jump into complete specialization of goods production. This suggests that open trade might be incorporated more adequately into the model above in this section where migration costs are explicitly taken into account, or into some other models.

Appendix: Construction of Data

LFAG and LFNAG — Civilian Labor Force Employed in Agricultural and Nonagricultural Sector, respectively. 1939-89: from *Economic Report of the President* (1990, p. 330). Pre-1939 period (1889-1938): from Kendrick(1961, TABLE A-VI).

u_1 = LFAG/(LFAG+LFNAG)

u_2 = LFNAG/(LFAG+LFNAG)

POP — Total U.S. Population. 1939-89: from *Economic Report of the President* (1990, p.329). Pre-1939 period: from *Historical Statistics of the United States 1789-1945* (1949, p.26).

F82 and NF82 — Business Sector GDP in 1982 dollars, Farm and Nonfarm, respectively. 1939-89: from *Economic Report of the President* (1990, p. 305). Pre-1939 period: Series (8) (Farm Gross Product) and Series (7) (Gross Private Domestic Product) minus series (8), respectively, in Kendrick (1961), TABLE A-III, rescaled to produce the same numbers for 1939 as in the 1939-89 series described above.

GFO — Gross Farm Output. 1939-89: Gross Farm Output Indexes in *Economic Report of the President* (1990, p. 404) and —— (1983, p. 270). Pre-1939 period: Gross Farm Output Indexes in Kendrick(1961), TABLE B-II. These two sets of series were linked together in 1939 and rescaled to obtain F82/GFO = 0.872 for the year 1889 (see footnote 9 in the text).

c_{1a} = F82/POP

c_{1b} = GFO/POP

c_2 = NF82/POP

$h_1 = c_{1a}/u_1$

$h_2 = c_2/u_2$

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