# Duration Dependence in Korean Business Cycles: Evidence and Its Implication Based on Gibbs Sampling Approach to Regime-Switching Model

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The hypothesis of business cycle duration dependence is tested by estimating the Hamilton regime-switching model with duration dependence using the Gibbs sampler. Data are two versions of the index of coincident indicators; (linear) dynamic factor index of Stock and Watson (1989) and (nonlinear) dynamic Markov switching factor index of Diebold and Rudebusch (1994). When the Gibbs sampler is applied to the duration dependent regime-switching model using quarterly Korean business cycle indices for the 1977:1-1994:4 period, this paper finds that the probability of a transition into an recession increases as the expansion ages, and somewhat weaker evidence for the reverse. Example of out-of-sample forecast for business cycle turning points is also provided. (*JEL* Classsifications: C11, C51, E32)

## I. Introduction

Whether the probability of a transition into an expansion increases as the recession ages, or vice versa, has been an interesting empirical question in economics. This problem has been referred to as 'duration dependence' in the literature. For example, Diebold and Rudebusch (1990) employed nonparametric hazard function and found only weak evidence of positive duration dependence in postwar U.S. recessions

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but none for postwar expansion. To the contrary, they found strong evidence of duration dependence in prewar expansions. Based on a parsimonious and flexible exponential-quadratic hazard model, Diebold, Rudebusch and Sichel (1993) found strong evidence of positive duration dependence associated with postwar recessions.

In Korea, the National Statistical Office (NSO) publishes the official business cycle dates. According to the NSO, there have been four episodes of recessions since 1976. However, the previous literature lacks the formal test for duration dependence for the Korean business cycle data.

It is well-known that a discrete and yet parsimonious two-state regime-switching model seems to well describe the business cycle data. Hamilton (1989), for example, finds that two-state Markov switching model can capture asymmetry in the U.S. GNP growth rates and produces recessionary and expansionary periods which are remarkably consistent with the NBER chronology of the business cycles.

In the context of test for duration dependence, Durland and McCurdy (1994) extend the constant transition probability Hamilton model to estimate jointly the phases and the parameters of the data generation. These authors assume that state transition probabilities are functions of the number of periods the process has been in that state. They develop the quasi-maximum likelihood estimation (QMLE) for the problem. Kim and Kim (1995) examine the same problem, but estimate the model using the gibbs sampler. The Gibbs sampler treats the Markov-switching state variables as parameters to be estimated and is suitable in assessing the uncertainty of the parameters. When these authors applied their methods to postwar quarterly U.S. GNP data, they found the evidence of positive duration dependence in contractions but none for expansion periods.

The purpose of the paper is to test formally duration dependence in Korean business cycles. This task is done in two steps: since the individual data are often noisy and seasonal, this paper derives two measures for business cycle in the first step. One version is derived from the probabilistic model of coincident indicators of Stock and Watson (1989, 1991) and the other is from the dynamic Markov switching factor (MSF) model of Diebold and Rudebusch (1994), the estimation of which is made possible later by Kim and Yoo (1995). The second step is to estimate the duration dependent regime-switching model using the derived series. Note that the Stock-Watson index is essentially linear, whereas the MSF index is nonlinear.

This paper is organized as follows. Section II briefly discusses the duration dependent regime-switching model. Section III describes the implementation of the Gibbs sampler that estimates the model and the statistics that can assess convergence. Section IV discusses the derivation of two versions of Korean business cycle data and presents empirical results. Section V examines the issue of out-of-sample forecast. Section VI provides a summary and conclusions.

## II. Regime-Switching Model with Duration Dependence

In modeling intrinsic macroeconomic shifts between periods of recession and expansion, Hamilton (1989) considered the following Markov-Switching model of business cycle:

$$\phi(L)(y_t - \mu_{S_t}) = e_t, \tag{1}$$

$$e_t \sim \text{i.i.d. } N(0, \sigma_e^2), \tag{2}$$

$$\mu_{S_t} = \mu_0 + \mu_1 S_t, \tag{3}$$

where $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L_p$ is a $p$-th order polynomial in the lag operator with its characteristic roots lying outside the complex unit circle. A variable $y_t$ is assumed to have state-dependent mean $\mu_{S_t}$, where a discrete-valued state variable $S_t$ evolves according to a first order Markov-switching process with transition probabilities given by

$$\begin{aligned} q &= Pr(S_t = 0 \mid S_{t-1} = 0), \\ p &= Pr(S_t = 1 \mid S_{t-1} = 1). \end{aligned} \tag{4}$$

$\mu_0$ and $\mu_1$ are parameters.

The extended Hamilton model that is considered in this paper allows the state transitions to be duration dependent following Durland and McCurdy (1994) and Kim and Kim (1995). Specifically, assuming that the evolution of $S_t$ can be described by a simple profit model like Filardo and Gordon (1993), the transition probabilities can be specified as follows:

$$Pr(S_t = 1) = Pr(S_t^* \geq 0), \tag{5}$$

where $S_t^*$ is a latent variable defined by

$$\begin{aligned} S_t^* = g_0 + g_1 S_{t-1} + g_2(1 - S_{t-1})\, D_{0,\, t-1} + g_3\, S_{t-1} D_{1,\, t-1} + u_t, \\ u_t \sim \text{i.i.d. } N(0,\, 1), \end{aligned} \tag{6}$$

where $g_0, \ldots, g_3$ are parameters, $D_{1,\, t-1} = \{j\!: S_{t-j-2} = 0,\ S_{t-j-1} = \ldots = S_{t-2} =$

$S_{t-1} = 1\}$ is the duration of state one, say, boom, up to time $t-1$ after it started at time $t-j-1$ and $D_{0,\,t-1} = \{j: S_{t-j-2} = 1,\ S_{t-j-1} = \ldots = S_{t-2} = S_{t-1} = 0\}$ is the duration of state zero, say, recession, up to time $t-1$ after it started at time $t-j-1$. They represent how long a recession or a boom has lasted up to time $t-1$. Hence, the transition probabilities can be written

$$p_t = Pr(S_t = 1 \mid S_{t-1} = 1,\ D_{1,\,t-1}) = Pr[u_t \geq -(g_0 + g_1 + g_3\, D_{1,\,t-1})], \quad (7)$$

$$q_t = Pr(S_t = 0 \mid S_{t-1} = 0,\ D_{0,\,t-1}) = Pr[u_t < -(g_0 + g_2\, D_{0,\,t-1})]. \quad (8)$$

Durland and McCurdy (1994) consider the QMLE of the above model with the transition probabilities $p_t$ and $q_t$ of (7) and (8) replaced by the logistic functions of $D_{1,\,t-1}$ and $D_{0,\,t-1}$. To make the QMLE feasible, maximum durations of recessions or booms, denoted $\bar{D}_0$ and $\bar{D}_1$ respectively, need to be specified. Durland and McCurdy (1994), for example, set $\bar{D}_0 = \bar{D}_1 = 9$ quarters based on a grid search with the likelihood value as a criterion. This paper adopts the Gibbs sampler approach used in Kim and Kim (1995), which can address the uncertainty of the parameters, and thus one can estimate the proper empirical distribution suitable for statistical inferences. In addition, duration variables need not be truncated and the procedure for computing the posterior distribution of the out-of-sample forecast is straightforward.

## III. Estimation of Duration Dependent Regime-Switching Model Using Gibbs Sampler

### A. Background

Let the history of the sequence $\{y_t\}$ at time $t$ be given by $Y_t = \{y_s\}$ ($s = 1, \ldots, t$). A model refers to a sequence of probability density functions (p.d.f's)

$$f_t\,(y_t \mid Y_{t-1},\ \theta),$$

in which $\theta$ is a $(k \times 1)$ vector of unknown parameters. The p.d.f. of $Y_T$, conditional on the model and the parameter vector $\theta$, is

$$p(Y_T \mid \theta) \prod_{t=1}^{T} f_t(y_t \mid Y_{t-1}, \theta).$$

The likelihood function is any function $L(\theta;\ Y_T) \propto p(Y_T \mid \theta)$.

The objective of Bayesian inference can in general, by Bayes theorem, be expressed

$$E(g(\theta) \mid Y_T) = \int_\theta g(\theta)p(\theta \mid Y_T)d\theta = \frac{\int_\theta g(\theta)L(\theta; Y_T)p(\theta)d\theta}{\int_\theta L(\theta; Y_T)p(\theta)d\theta},$$

in which $g(\theta)$ is a function of interest, $p(\theta \mid Y_T)$ is the posterior density of $\theta$, and $p(\theta)$ is the prior density of $\theta$.

Bayesian methods are operational to the extent that posterior moments can actually be computed. If the posterior distribution is sufficiently simple, the posterior moment may be obtained analytically. If the required integration takes place, however, in more than, say, five dimensions then classical quadrature methods are not often practical. As such, a class of posterior simulators have been suggested in recent years and have become known as 'Markov chain Monte Carlo.' The idea is to construct a Markov chain with state space $\Theta$ ($\theta \in \Theta$) and invariant distribution with p.d.f $p(\theta \mid Y_T)$. Following an initial transient phase, simulated values from the chain form a basis for approximating $E(g(\theta) \mid Y_T)$. What is required is to construct an appropriate algorithm, like the Gibbs sampler and the Metropolis-Hastings algorithm, and establish that its invariant distribution is unique, with p.d.f. $p(\theta \mid Y_T)$. The following section briefly summarizes the Gibbs sampler (see, Geweke (1995) and Tanner (1993) for detailed discussions for the Metropolis-Hastings algorithm).

*B. The Gibbs Sampler*

The Gibbs sampler provides a method for sampling from the posterior density $p(\theta \mid Y_T)$. As before, the $k$-element vector $\theta$ contains quantities of interest and $Y_T$ is the vector of observed data. Given the starting point $(\theta_1^0, \theta_2^0, \ldots, \theta_k^0)$, this algorithm draws a sample from the conditional distributions by iterating the following loop:

- sample $\theta_1^{(i+1)}$ from the conditional distribution $p(\theta_1 \mid \theta_2^{(i)}, \ldots, \theta_k^{(i)}, Y_T)$;
- sample $\theta_2^{(i+1)}$ from $p(\theta_2 \mid \theta_1^{(i+1)}, \theta_3^{(i)}, \ldots, \theta_k^{(i)}, Y_T)$;
- $\vdots$
- $\theta_k^{(i+1)}$ from $p(\theta_k \mid \theta_1^{(i+1)}, \ldots, \theta_{k-1}^{(i+1)}, Y_T)$.

Each of the $k$ conditional sampling performed per iteration $(i + 1)$ may be referred to as a *step*. The completion of the first $k$ steps, resulting in the vector $\theta^{(i+1)}$, may be referred to as the $(i + 1)$st *pass* through the vector $\theta$.

The passes, $\theta^{(0)}, \theta^{(1)}, \ldots, \theta^{(i)}, \theta^{(i+1)}, \ldots$, are a realization of a Markov chain, with transition probability from $\theta^{(m)}$ to $\theta^{(n)}$,

$$T(\theta^{(m)}, \theta^{(n)}) = p(\theta_1^{(n)} \mid \theta_2^{(m)}, \ldots, \theta_k^{(m)}, Y_T) \times P(\theta_2^{(n)} \mid \theta_1^{(n)}, \theta_3^{(m)}, \ldots, \theta_k^{(m)}, Y_T) \times \ldots \times p(\theta_k^{(n)} \mid \theta_1^{(n)}, \theta_2^{(n)}, \ldots, \theta_{k-1}^{(n)}, Y_T).$$

It has been shown that, under weak conditions, the joint distribution of $(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)})$ converges geometrically to $p(\theta_1, \theta_2, \ldots, \theta_k \mid Y_T)$ as $l$ tends to infinity (Geman and Geman 1984; Gelfand and Smith 1990; Tierney 1991; Chan 1993). Further,

$$\frac{1}{M}\sum_{j=1}^{M} g(\theta^{(j)}) \stackrel{a.s.}{\rightarrow} \int g(\theta)p(\theta \mid Y_T)dv(\theta), \quad \text{as} \quad M \rightarrow \infty.$$

In words, given independent realizations of $\theta^{(j)}$, $E(g(\theta) \mid Y_T)$ would be approximated using the sample averages by invoking the strong law of large numbers. It is noted, however, that when $k$ steps are simulated each pass, convergence of the Gibbs sampler can be quite slow if highly correlated individual $k$ components of the parameter $\theta = (\theta_1, \ldots, \theta_k)$ are treated individually (see Geweke 1992, 1995 and Tanner 1993 for more details). Hence, for the Gibbs sampler to be practical, it is essential that the blocking be chosen in such a way that one can make drawings from the posterior distributions in an efficient manner.

The Gibbs sampler is an attractive solution of the Bayesian multiple integration problem when the conditional densities are simple and easy to obtain. Conditional densities of the block of parameters for the problem at hand, i.e., $\tilde{\mu} = (\mu_0, \mu_1)'$, $\tilde{\phi} = (\phi_1, \phi_2, \ldots, \phi_p)'$, $\tilde{g} = (g_0, g_1, g_2, g_3)'$, $\sigma_e^2$ and for latent variables $\{S_t, S_t^*\}$ $(t = 1, 2, \ldots, T)$, are summarized in the appendix 1 following Kim and Kim (1995) (see, also, Albert and Chib 1993 and Filardo and Gordon 1993).

### C. Convergence Diagnostic

The formulation of the convergence diagnostic (CD) is suggested by Geweke (1992). The idea is that, given the sequence $\{\theta^{(j)}\}$, comparison of values early in the sequence with those late in the sequence is likely to reveal failure of convergence. Let

$$m_A = M_A^{-1}\sum_{j=1}^{M_A}\theta^{(j)}, m_B = (M - M_B + 1)^{-1}\sum_{j=M_B}^{M}\theta^{(j)}, (j = 1, \ldots, M_A, \ldots, M_B, \ldots, M)$$

are means of the subsample of the Gibbs sequences and let $SD_A$ and $SD_B$ denote the corresponding consistent estimates for their standard deviations. If the ratios $M_A/M$ and $M_B/M$ are fixed, with $(M_A + M_B)/M < 1$, then as $M \rightarrow \infty$,

$$\frac{m_A - m_B}{SD_A + SD_B} \Rightarrow N(0,1) \tag{9}$$

if the sequence $\{\theta^{(j)}\}$ is stationary. This paper uses the Newey-West (1987) consistent estimator to compute *SD*'s with lags equal to 100 and $M_A = 0.1M$ and $M_B = 0.5M$.[1]

## IV. Empirical Results

### *A. Data*

This paper considers two measures for Korean business cycles. These are two quarterly indices of coincident indicators from 1977:1 to 1994:4 estimated from using the dynamic factor index model of Stock and Watson (1989, 1991) (Stock-Watson index hereafter) and the dynamic Markov switching factor index model of Diebold and Rudebusch (1994), later successfully estimated by Kim and Yoo (1995) (MSF index hereafter).[2] This paper chose these derived series because the published raw data are quite noisy and often highly seasonal, making it difficult to identify the business cycle turning points. Two models mentioned above produce business cycle dates that reasonably approximate the ones published by the National Statistical Office (NSO) in Korea, who adopts more or less the NBER method. Next section briefly describes methods that estimate two versions of quarterly business cycle indices.

### *B. Estimation of Two Versions of Quarterly Korean Business Cycle Indices*

Following Kim and Yoo (1995), the dynamic Markov switching factor model that derives quarterly business cycle index can be written as follows:

[1]Note that *SD*'s can be computed from the spectral densities of $\theta^{(j)}$'s, evaluated at frequency zero as suggested by Geweke (1992).

[2]The raw data that are used to construct quarterly indices consist of: (1) total index of industrial production (IP) (monthly, SA, NSO), (2) utilization rate in manufacturing (UTIL) (monthly, SA, NSO) (3) shipment in manufacturing (SHIP) (monthly, SA, NSO) (4) gross domestic product (GDP) (quarterly, SA, Bank of Korea). Monthly data begin from January of 1976. Quarterly series are computed as averages of monthly observations.

$$\boldsymbol{y_t} = \gamma(L)\, n_t + \boldsymbol{\zeta_t}, \tag{10}$$

$$\phi(L)\, n_t = \beta_S + \eta_t,\ \eta_t \sim \text{ i.i.d. } N(0,\ 1), \tag{11}$$

where $\phi(L) = (1 - \phi_1 L - \ldots - \phi_s{}^{t} L^s)$, and the boldface letter $\boldsymbol{y_t}$ denotes $(N \times 1)$ vector of the growth rates of the coincident indicators. It is expressed as deviation from its mean, divided by its standard deviation. $\gamma(L)$ is a vector polynomial. An $(N \times 1)$ vector $\boldsymbol{\zeta_t}$ denotes the idiosyncratic component of the coincident indicators, which is unrelated to the state of the economy. That is,

$$\boldsymbol{A(L)}\ \boldsymbol{\zeta_t} = \boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t \sim MVN(\boldsymbol{O}, \Sigma),$$

where $\boldsymbol{A(L)} = (\boldsymbol{I}_{(N)} - \boldsymbol{A}_1\boldsymbol{L} - \ldots - \boldsymbol{A}_r\boldsymbol{L}_r)$ and $I_{(i)}$ denotes an $i$ dimensional identity matrix. It is assumed, for identification of the model, that the covariance matrix $\Sigma$ and $(N \times N)$ coefficient matrices $\boldsymbol{A}_i$, $i = 1, 2, \ldots, r$, are diagonal. The variable representing the state of the economy $(n_t)$ is assumed to be common to $N$ coincident indicators. $n_t$ follows a first-order Markov process with transition probabilities given previously by equation (4).

The MSF model given by equations (10)-(12) and (4) becomes the Stock-Watson model if one assumes a single state. Test for adequacy of the MSF model against the Stock-Watson model is a nonstandard problem as the key parameters of the MSF model are not identified under the null hypothesis (Hansen 1992, 1993). Furthermore, it is computationally demanding in the multivariate model like the MSF model. Therefore, This paper estimates both models without a formal test for two states. The MSF model and the Stock-Watson model, can be written in state-space form and be estimated by using the maximum likelihood estimation via the Kalman filter (see Kim and Yoo 1995 for detailed discussions of MLE).[3]

Table 1 reports the ML estimates for the Stock-Watson model and the MSF model. Experiment with various AR lags for $s$ and $r$ suggests that three lags for both are reasonable, yielding white noise innovations. The log likelihood of the MSF model, with four additional parameters, is -263.49 which is greater than the one of the Stock-Watson model.

[3]Kim and Nelson (1995) also estimate the similar model for the U.S. data using the multi-move Gibbs sampler proposed by Shephard (1994). Instead of equation (11), they assume that $\phi(L)(n_t - \beta_{St}) = \eta_t$. When the multi-move Gibbs sampler and the approximate MLE proposed by Kim and Yoo (1995) for these specifications are applied to four U.S. coincident indicators, these research produces essentially the same results. This paper adopts the Kim and Yoo method.

**TABLE 1**

ML ESTIMATION OF STOCK-WATSON MODEL AND DYNAMIC MARKOV SWITCHING FACTOR MODEL OF KOREAN BUSINESS CYCLE INDICATORS

| Parameter | Stock-Watson Model | | MSF Model | |
|---|---|---|---|---|
| $p$ | - | | 0.8532 | (11.793) |
| $q$ | - | | 0.6181 | (5.056) |
| $\beta_0$ | - | | -1.6227 | (-4.106) |
| $\beta 1$ | - | | 0.7060 | (2.888) |
| $a_{1,1}$ | -0.0573 | (-0.283) | -0.1288 | (-0.374) |
| $a_{1,2}$ | -0.2024 | (-1.078) | 0.2975 | (2.479) |
| $a_{1,3}$ | -0.2569 | (-0.751) | -0.1654 | (-1.044) |
| $a_{1,4}$ | -0.3458 | (-2.811) | -0.3665 | (-3.041) |
| $a_{2,1}$ | 0.2646 | (2.147) | -0.1886 | (-1.052) |
| $a_{2,2}$ | 0.0593 | (0.487) | 0.0365 | (0.308) |
| $a_{2,3}$ | 0.2765 | (2.285) | -0.3289 | (-3.255) |
| $a_{2,4}$ | -0.0574 | (-0.448) | -0.0744 | (-0.574) |
| $a_{3,1}$ | -0.1844 | (-1.196) | -0.1573 | (-0.681) |
| $a_{3,2}$ | -0.3637 | (-3.237) | 0.2340 | (1.987) |
| $a_{3,3}$ | 0.0185 | (0.167) | 0.0359 | (0.344) |
| $a_{3,4}$ | -0.0352 | (-0.298) | -0.0398 | (-0.329) |
| $\sigma_{\varepsilon,1}$ | 0.1680 | (1.954) | 0.1533 | (1.819) |
| $\sigma_{\varepsilon,2}$ | 0.5411 | (10.727) | 0.5525 | (10.604) |
| $\sigma_{\varepsilon,3}$ | 0.3093 | (6.740) | 0.3258 | (7.372) |
| $\sigma_{\varepsilon,4}$ | 0.8052 | (11.550) | 0.7996 | (11.680) |
| $\gamma_1$ | 0.8325 | (11.198) | 0.5770 | (7.828) |
| $\gamma_2$ | 0.7280 | (8.073) | 0.4979 | (6.357) |
| $\gamma_3$ | 0.7817 | (10.812) | 0.5364 | (7.643) |
| $\gamma_4$ | 0.4426 | (5.412) | 0.3071 | (4.956) |
| $\phi_1$ | 0.4173 | (3.497) | 0.2330 | (1.942) |
| $\phi_2$ | -0.0205 | (-0.164) | -0.0767 | (-0.590) |
| $\phi_3$ | 0.1951 | (1.636) | 0.2628 | (2.517) |
| Log likelihood | -266.11 | | -263.49 | |

Note: Data are standardized by subtracting the sample mean from each of four growth rates of the coincident indicators (in order, IP, UTIL, SHIP and GDP) and dividing by its standard deviation. $\beta_{S_t} = \beta_0\ (1 - S_t) + \beta_1\ S_t$ for the MSF model. Data cover the 1976:2-1994.IV period. Approximate $t$-values are in parentheses.

But these numbers are not directly comparable for the reason mentioned above. Estimates for $p$, $q$, and $\beta$ are all significant.

Note that $n_t$ in equation (11) is demeaned growth rates of the coincident index. Figure 1A plots $n_t$ estimated from both models. Two series are markedly close each other. In fact, the sample correlation was 0.999. Figure 1B plots the smoothed probability of recession computed

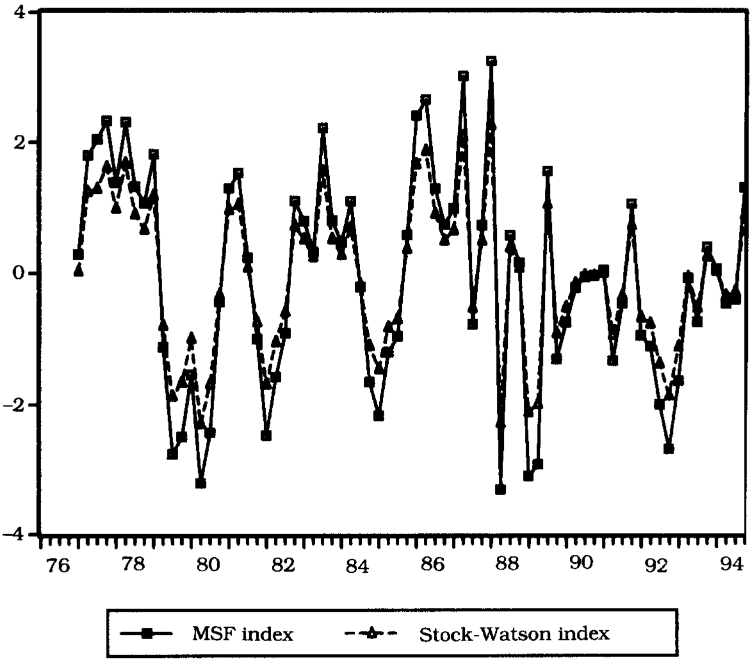



**FIGURE 1A**

QUARTERLY GROWTH RATES OF COINCIDENT INDICES FROM MSF MODEL AND STOCK-WATSON MODEL: 1977:I-1994:IV

from the MSF model (see Kim and Yoo 1995 for the expression for the full sample smoother). The model identifies that there has been five peaks (the official NSO dates are in parentheses), 79:1 (79:1), 81:3 (na), 84:2 (84:1), 88:1 (88:1), 91:4 (91:1) and five troughs, 80:3 (80:3), 82:2 (na), 85:2 (85:3), 89:2 (89:3), 93:1 (93:1). Note that the NSO did not designate the 1981-82 period as a recession. Recall, however, that during this period the Korea's major trading partners, U.S. and Japan, have experienced recessions. The call for the latest peak appears somewhat controversial. The NSO's derived series, called the cyclical component of coincident index, also suggests that a signal for recession in early 1991 was rather weak.

To see two versions of coincident indices retain the same feature of regime switches, the Hamilton model with constant transition probabilities, equations (1)-(4), is estimated for each series. ML estimates are reported in Table 2. It suggests that the two-state model is reasonable description for both series. Smoothed probabilities, depicted in Figure 2 suggest that two measures perform likewise in terms of state identifi-

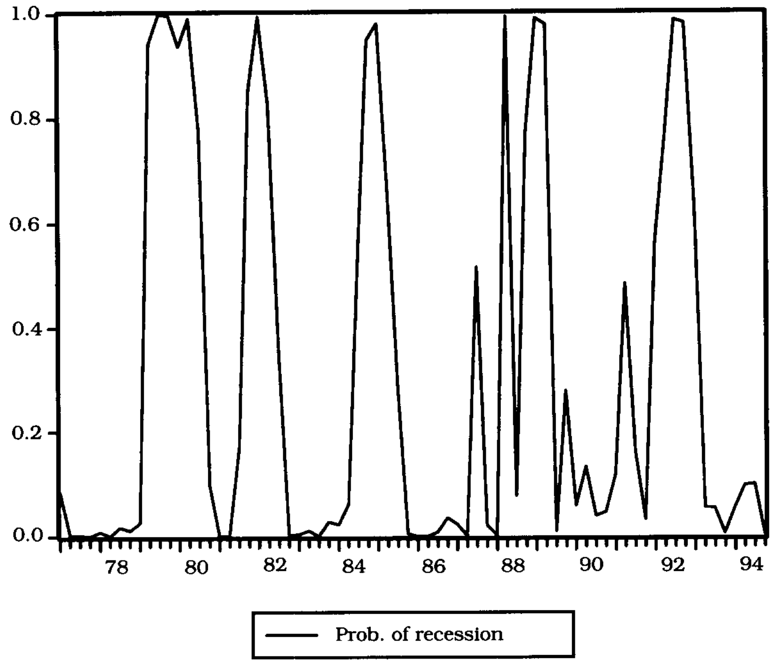



**FIGURE 1B**
SMOOTHED PROBABILITY OF RECESSION FROM MSF MODEL

**TABLE 2**
THE HAMILTON MODEL OF THE INDICES OF KOREAN BUSINESS CYCLES: ML ESTIMATION WITHOUT DURATION DEPENDENCE

| Parameter | Stock-Watson Index | | MSF Model | |
|---|---|---|---|---|
| $p$ | 0.8716 | (17.043) | 0.8749 | (17.675) |
| $q$ | 0.7103 | (7.182) | 0.7075 | (7.112) |
| $\mu_0$ | -1.3941 | (-5.812) | -2.0569 | (-5.977) |
| $\mu_1$ | 0.3478 | (1.644) | 0.5011 | (1.651) |
| $s$ | 0.6056 | (10.367) | 0.8640 | (10.577) |
| $\phi_1$ | 0.1092 | (0.929) | 0.0980 | (0.865) |
| $\phi_2$ | -0.0557 | (-0.482) | -0.0480 | (-0.432) |
| $\phi_3$ | 0.5671 | (5.024) | 0.5758 | (5.270) |
| Log likelihood | -88.49 | | -112.96 | |

Note: Data are estimated series based on estimates reported in Table 1. Effective sample covers the 1977:1-1994.IV period. Asymptotic $t$-values are in parentheses.

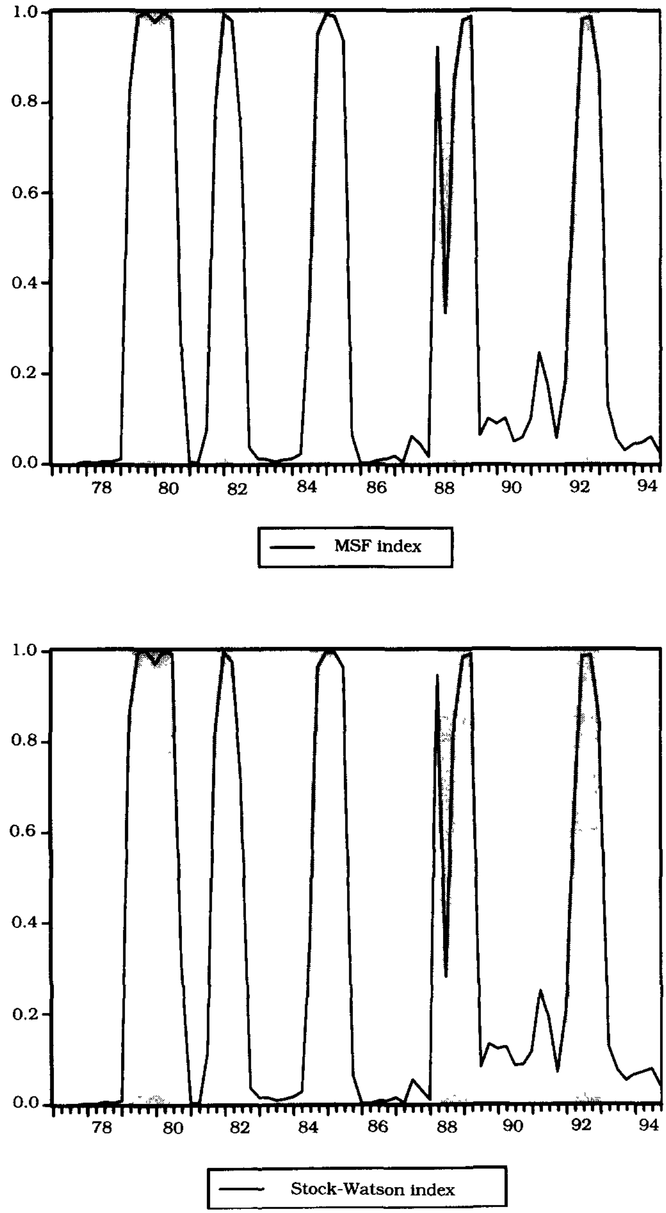



**FIGURE 2**

PROBABILITY OF RECESSION FROM UNIVARIATE HAMILTON MODEL (SHADED BARS DENOTE RECESSIONARY PERIODS IN FIGURE 1B) cation.[4] First two AR lags were insignificant for both cases, and thus constrained to zero in implementing the Gibbs sampler.

The MSF index is derived under the assumption of constant transition probabilities. Nevertheless, one can examine whether a particular chronology of Korean business cycles contains predictable content, i.e., duration dependence. Since the Stock-Watson index does not assume anything about duration dependence, next section investigates two measures for business cycle to test for duration dependence.

## *C. Test for Duration Dependence*

Prior distributions are set similar in values to Albert and Chib (1993) and Kim and Kim (1995). In particular, initial values of parameters for two duration variables are set under the null hypothesis of no duration dependence. First 5000 passes are discarded, i.e., an initial transient or burn-in phase, and then every fifth draws out of additional 10000 passes are recorded in estimating the empirical distribution of the parameters. Table 3 reports the posterior distributions as well as the 95 percent confidence intervals. Means of posterior distributions resulted in reasonable figures with appropriate signs. To see whether the proposed model is a reasonable description of the data, the Box-Pierce Q statistics with twelve lags are computed. These are, *p*-values in parentheses, 4.95 (0.960) and 5.84 (0.924) respectively for the Stock-Watson index and the MSF index. In addition, the convergence diagnostic (CD) reported in the last column suggests that the estimates were calculated using the draws from the marginal distribution.

Inferred probability of contractionary states, $Pr(S_t = 0 \mid \psi_T)$, produced from the Gibbs runs is plotted in Figure 3 (shaded bar refers to the business cycle recessions from the MSF model, which is plotted in Figure 1B). Smoothed probabilities resemble the ones of Figure 2, evidencing further the appropriateness of the current model. Note that inferred probability reflects the uncertainty of the parameters.

Primary interests of Table 3 are two coefficients, $g_2$ and $g_3$. Positive estimated posterior means were yielded for the parameter that indicates the positive duration dependence in contractionary periods, i.e.,

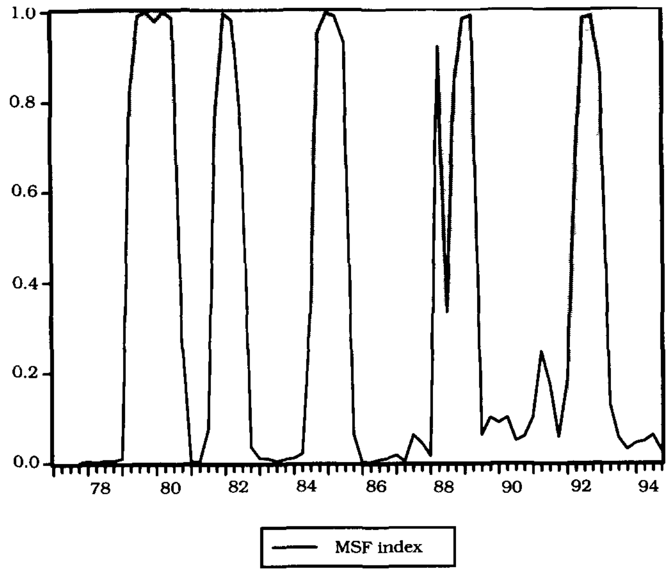
[4]In comparison with Figure 1B and Figure 2, it appears that whether one switches the unconditional mean (equation (1)) or just an intercept term (equation (11)) does not seem to matter much in practice, although there is slight disagreement of business cycle dates between Figure 1B and Figure 2 as the derived series are somewhat smoother than four component series.

**TABLE 3**
THE EXTENDED HAMILTON MODEL OF THE INDICES OF KOREAN BUSINESS CYCLES: DURATION DEPENDENCE MODEL WITH THE GIBBS SAMPLER

| | Prior | | Posterior | | | |
|---|---|---|---|---|---|---|
| Parameter | Mean | SD | Mean | SD | 95 percent interval | CD |
| A. Stock-Watson Index | | | | | | |
| $\mu_0$ | -0.8 | 0.4 | -1.232 | 0.246 | (-1.688, -0.667) | 0.018 |
| $\mu_1$ | 1.5 | 0.4 | 1.674 | 0.169 | (1.315, 1.975) | -0.033 |
| $\phi_3$ | 0.0 | 0.3 | 0.441 | 0.134 | (0.153, 0.688) | -0.023 |
| $g_0$ | -1.0 | 0.4 | -0.968 | 0.286 | (-1.540, -0.407) | -0.045 |
| $g_1$ | 2.4 | 0.4 | 2.547 | 0.345 | (1.872, 3.238) | 0.010 |
| $g_2$ | 0.0 | 0.4 | 0.215 | 0.157 | (-0.074, 0.559) | -0.030 |
| $g_3$ | 0.0 | 0.4 | -0.121 | 0.073 | (-0.269, 0.017) | 0.004 |
| $\sigma^2$ | - | | 0.463 | 0.112 | (0.296, 0.740) | 0.016 |
| B. MSF Index | | | | | | |
| $\mu 0$ | -0.8 | 0.4 | -1.648 | 0.376 | (-2.263, -0.991) | -0.018 |
| $\mu_1$ | 1.5 | 0.4 | 2.568 | 0.355 | (1.672, 2.664) | 0.016 |
| $\phi_3$ | 0.0 | 0.3 | 0.436 | 0.133 | (0.165, 0.688) | 0.014 |
| $g_0$ | -1.0 | 0.4 | -0.983 | 0.281 | (-1.553, -0.452) | 0.043 |
| $g_1$ | 2.4 | 0.4 | 2.568 | 0.355 | (1.866, 3.277) | -0.010 |
| $g_2$ | 0.0 | 0.4 | 0.236 | 0.159 | (-0.060, 0.565) | 0.010 |
| $g_3$ | 0.0 | 0.4 | -0.120 | 0.072 | (-0.270, 0.016) | -0.003 |
| $\sigma^2$ | - | | 0.995 | 0.265 | (0.610, 1.652) | -0.030 |

Note: Prior distribution of $\sigma^2$ is improper. Results are based on 15,000 Gibbs sampling draws and first 5,000 burn-in draws are discarded, i.e., $l$ = 5,000, and $M$ = 10,000 (see the text for notation). To reduce the correlations between neighboring Gibbs runs, we recorded parameter estimates every fifth runs following Albert and Chib (1993). CD is the convergence diagnostic suggested by Geweke (1992). It is distributed as standard normal under the null hypothesis that the sampling technique has converged.

$g_2$ = 0.215 with the corresponding standard deviation of 0.157 for the Stock-Watson index. By conventional standards that invoke asymptotic normality for statistical inferences, there appears to be only weak evidence of increasing probability of exiting the recessionary period as it ages. Since the Gibbs sampler produces empirical distribution of parameters of interest, the exact $p$-value can be computed. The probability that $g_2$ is less than zero, one-sided $p$-value, was 0.086. However, the negative parameter representing positive duration dependence in expansionary periods ($p$-value in parenthesis) was -0.121 (0.041) with standard deviation of 0.073. Therefore, the positive duration depen-



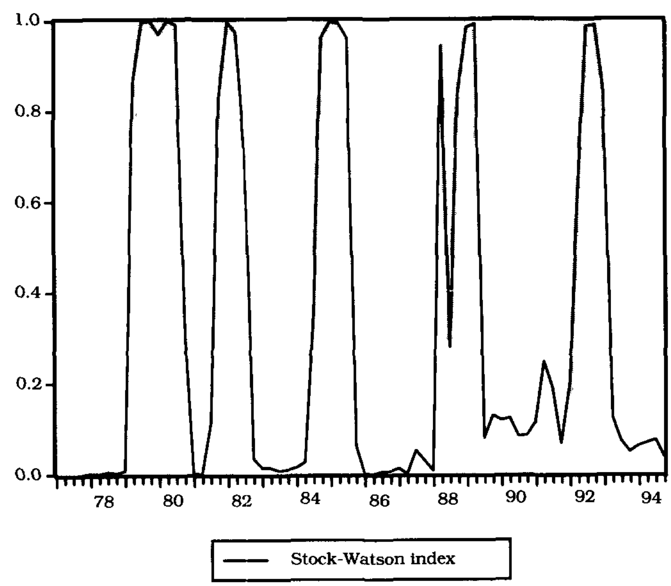




**FIGURE 3**

PROBABILITY OF RECESSION FROM DURATION DEPENDENT MODEL (SHADED BARS DENOTE RECESSIONARY PERIODS IN FIGURE 1B) dence is supported by the data.

The results from using the MSF index suggest likewise: there is evidence for positive duration dependence in expansions ($g_2$ = 0.236 with $p$-value = 0.048) and marginal evidence for positive duration dependence in recessions ($g_3$ = −0.120 with $p$-value = 0.059). Hence, there is the ample evidence that the Korean business cycles feature duration dependence at least in expansions, and somewhat weaker evidence for the reverse.

## V. Out-of-Sample Forecast

How useful is the finding of duration dependence in Korean business cycle data to policy makers? It is well-known that the regime switching model tends to perform poorly for the long-term forecasting. Nonetheless, the empirical evidence of duration dependence suggests that the probability of a transition from one state to another will increase as the current business cycle phase progresses. To examine this issue, this section performs exercise of computing out-of-sample forecast.

Let $S_t^k = (S_t, \ldots, S_k)$ denote states starting at time $t$ and ending at time $k$, and $\theta$ is the vector of parameters. The Bayes prediction density, $f(y_{T+1} \mid Y_T)$, is then

$$\begin{aligned} f(y_{T+1} \mid Y_T) &= \int f(y_{T+1} \mid Y_T, S_{T+1}, S_{T+1-p}^T, D_T, \theta) d(S_{T+1}, S_{T+1-p}^T, D_T, \theta \mid Y_T) \\ &= \int f(y_{T+1} \mid Y_T, S_{T+1}, S_{T+1-p}^T, D_T, \theta) d(S_{T+1} \mid S_T, D_T, \theta) \qquad (13) \\ &\quad d(S_{T+1-p}^T, D_T, \theta \mid Y_T), \end{aligned}$$

where, given $S_{T-p+1}^T$, the conditional density of $y_{T+1}$ is obtained from

$$f(y_{T+1} \mid Y_T, S_{T+1}, D_T, \theta) \propto \exp\left\{\frac{-\sigma^{-2}(y_{T+1} - \hat{y}_{T+1|T})^2}{2}\right\}, \qquad (14)$$

where $\hat{y}_{T+1|T} = (1 - \phi(L))y_{T+1} + \phi(L)\ \mu S_{T+1}$. Hence, if $S_{T+1}$ is sampled from $Pr(S_{T+1} \mid S_T, D_T, \theta)$, the future observation $y_{T+1}$ can be drawn from $N(\hat{y}_{T+1|T}, \sigma^2)$. Similarly, $\{S_{T+1}, y_{T+1}\}(l \geq 2)$ can be drawn recursively.

Figure 4A depicts the out-of-sample forecast probability of recession for the 1995:1-1996:1 period (based on 5000 burn-in phase and 10000 simulations). Since the results from using the MSF index and Stock-Watson index are very similar, this paper reports only the one based on the MSF index. The shaded area denotes the out-of-sample forecast period. The duration dependent model signals a likely switch to reces-

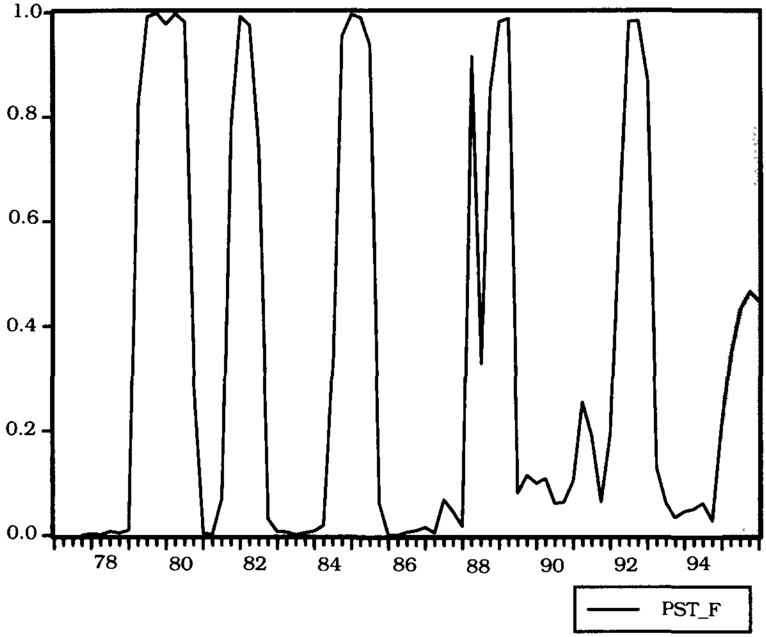



**FIGURE 4A**

OUT-OF-SAMPLE FORECAST PROBABILITY OF RECESSION FOR 1995:1-1996:1: BASED ON DURATION DEPENDENT MODEL USING MSF INDEX AND GIBBS SAMPLER

sion around the turn of the year 1995. The forecast probabilities for the 1995:4 and 1996:1, for example, were 0.467 and 0.449, respectively. On the other hand, the forecast probability based on the model without duration dependence, depicted in Figure 4B, resulted in rather weak warning: it rises only to 0.326 and 0.347 for the corresponding period.

## VI. Summary and Conclusions

This paper tests for duration dependence in the Korean business cycle data. The duration dependent regime-switching model, the MLE of which was proposed by Durland and McCurdy (1994), is estimated using the Gibbs sampler. The empirical model admits state transitions that vary endogenously as functions of the number of periods that the economy has stayed in a particular inferred state.

Data are two versions of quarterly Korean business cycle indices for the 1976 to 1994 period, constructed from the Stock-Watson (1989, 1991) dynamic factor index model and from the dynamic Markov switching factor model of Diebold and Rudebusch (1994). For the esti-

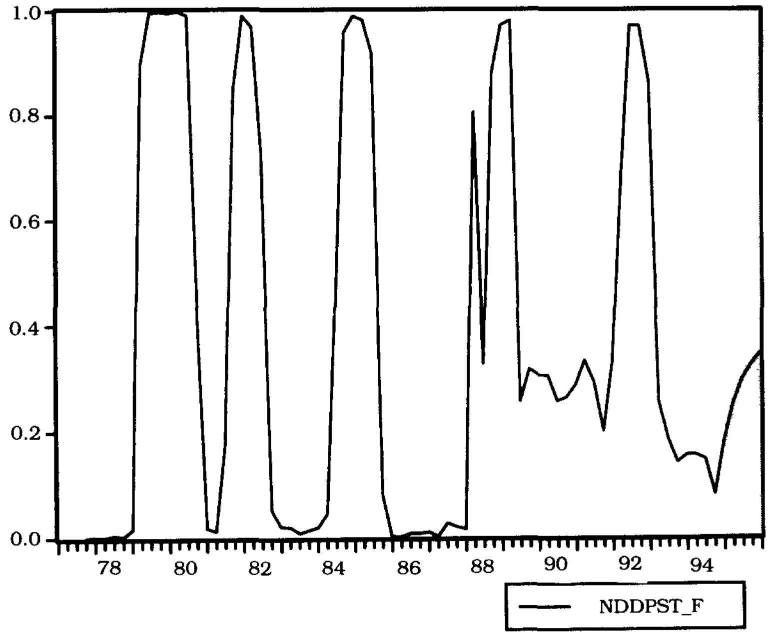



**FIGURE 4B**

BASED ON HAMILTON MODEL WITHOUT DURATION DEPENDENCE USING MSF INDEX AND GIBBS SAMPLER

mation of the latter, this paper adopts the Kim and Yoo's (1995) approximate MLE. These two probabilistic models appear to equally well approximate the recent Korean business cycle chronology.

This paper found evidence of positive duration dependence in expansions and somewhat weaker evidence for contractions: the probability of a transition into the other stage of business cycle increases as the current phase matures. This finding suggests that there possibly be the endogenous force in economic activity that triggers the end of the current phase of business cycle as it ages. This finding has some practical implication to the policy makers and businessmen: it may help economic agents to form their rational expectations about what is ahead, and in particular when to intervene to policy makers. Furthermore, the method in this paper can be extended to address the question like whether exogenous monetary and fiscal policies have helped to end the recessions (or the other way around), in addition to endogenous switches into the next regime, by modifying the transition probability equations. This exercise is left for a future extension.

## Appendix 1

**Conditional Density of $\{S_t, S_t^*\}$ ($t = 1, 2, \ldots, T$)**

Let $S_{-t} = \{S_j: 1 \le j \le T, j \ne t\}$, $Y_t \equiv (y_1, y_2, \ldots, y_t)$, and $\theta$ be all other parameters of the model. Similar to Albert and Chib (1993) and Kim and Kim (1995), the conditional distribution of the latent state variable $S_t$ is given

$$\begin{aligned} Pr(S_t \mid Y_T, S_{-t}, \theta) \propto\ & Pr(S_t \mid S_{t-1}, D_{t-1}) \times Pr(S_{t+1} \mid S_t, D_t) \\ & \times Pr(y_t, \ldots, y_p \mid Y_{t-1}, \tilde{S}_p) \\ & \times \Pi_{k=p+1}^{t+p} f(y_k \mid Y_{k-1}, S_k),\ \ t \le p \end{aligned} \tag{A1}$$

$$\begin{aligned} Pr(S_t \mid Y_T, S_{-t}, \theta) \propto\ & Pr(S_t \mid S_{t-1}, D_{t-1}) \times Pr(S_{t+1} \mid S_t, D_t) \\ & \times \Pi_{k=t}^{t+p} f(y_k \mid Y_{k-1}, S_k)\ \ p + 1 \le t \le T - p + 1, \end{aligned} \tag{A2}$$

$$\begin{aligned} Pr(S_t \mid Y_T, S_{-t}, \theta) \propto\ & Pr(S_t \mid S_{t-1}, D_{t-1}) \times Pr(S_{t+1} \mid S_t, D_t) \\ & \times \Pi_{k=t}^{T} f(y_k \mid Y_{k-1}, S_k),\ \ T - p \le t \le T, \end{aligned} \tag{A3}$$

where $D_{t-1} = D_{j,\,t-1}$ if $S_{t-1} = j$, $j \in \{0, 1\}$. With transition probabilities $Pr(S_t \mid S_{t-1}, D_{t-1})$ and $Pr(S_{t+1} \mid S_t, D_t)$ given by equations (7) and (8) in the text, the sequence of $S_t$, $\{S_t\}$, can be simulated from a Bernoulli distribution.

With simulations of $\{S_t\}(t = 1, 2, \ldots, T)$, the duration variables $\{D_{0,t-1}, D_{1,t-1}\}(t = 1, 2, \ldots, T)$ can be constructed. Given $\{S_t, D_{0,t-1}, D_{1,t-1}\}$ ($t = 1, 2, \ldots, T$) and $\tilde{g} \equiv (g_0, g_1, g_2, g_3)'$, the latent variable $\{S_t^*\}$ can also be simulated from the following truncated standard normal distributions using a rejection sampling:

$$S_t^* \sim \begin{cases} N(g_0 + g_1 + g_3 D_{1,t-1}, 1)_{I[S_t^* \ge 0]} & \text{when } S_t = 1 \text{ and } S_{t-1} = 1, \\ N(g_0 + g_1 + g_3 D_{1,t-1}, 1)_{I[S_t^* < 0]} & \text{when } S_t = 1 \text{ and } S_{t-1} = 1, \\ N(g_0 + g_2 D_{0,t-1}, 1)_{I[S_t^* < 0]} & \text{when } S_t = 1 \text{ and } S_{t-1} = 0, \\ N(g_0 + g_2 D_{0,t-1}, 1)_{I[S_t^* \ge 0]} & \text{when } S_t = 1 \text{ and } S_{t-1} = 0, \end{cases} \tag{A4}$$

where the symbol $I[S_t^* \ge 0]$, for example, refers to the indicator function that takes on value of one if $S_t^* \ge 0$. $\{S_t^*\}$ is stored to draw $\tilde{g}$ in the later step.

**Conditional Distribution of $\tilde{\phi} \equiv (\phi_1, \phi_2, \ldots, \phi_p)'$**

Given $\tilde{\mu}$ $(\mu_0, \mu_1)'$ and $\{S_t\}$, equation (1) in the text can be written

$$y_t^* = \phi_1 y_{t-1}^* + \ldots + \phi_p y_{t-p}^* + e_t, \quad \text{(A5)}$$

where $y_t^* = y_t - \mu_0 - \mu_1 S_t$. Define $\tilde{Y}^*$ to be the vector of $y_t^*$'s and $\tilde{W}^*$ to be the matrix of the right-hand side variables. The posterior distribution of $\tilde{\phi} \equiv (\phi_1, \ldots, \phi_p)'$ has the following multivariate distribution:

$$\tilde{\phi} \sim N((\Pi + \tilde{W}^{*\prime}\tilde{W}^*)^{-1}(\pi\Pi + \tilde{W}^{*\prime}\tilde{Y}^*), \sigma_e^2 (\Pi + \tilde{W}^{*\prime}\tilde{W}^*)^{-1}), \quad \text{(A6)}$$

where the conjugate prior distribution for $\tilde{\phi}$ has the multivariate normal form:

$$\tilde{\phi} \sim N(\pi, \Pi^{-1}). \quad \text{(A7)}$$

A rejection sampling can be used to simulate values of $\tilde{\phi}$ so that all roots of zero of the determinant of $\phi(L)$ in (1) lie outside the complex unit circle.

**Conditional Distribution of $\sigma_e^2$**

The full conditional distribution of $\sigma_e^2$ is given by:

$$\sigma_e^2 \sim IG\left\{\frac{v+T}{2}, \frac{\delta + \sum_{t=1}^{T}(y_t^* - \phi_1 y_{t-1}^* - \ldots - \phi_p y_{t-p}^*)^2}{2}\right\}, \quad \text{(A8)}$$

where $IG$ denotes the inverse gamma distribution and the prior distribution is given by $IG(v/2, \delta/2)$, and the hyperparameters $v$ and $\delta$ are known. $v$ reflects the strength of the prior of $\sigma_e^2$.

**Conditional Distribution of $\tilde{\mu} \equiv (\mu_0, \mu_1)'$**

Given $\tilde{\phi}$ and $\{S_t\}$, equation (1) can be written

$$(y_t - \phi_1 y_{t-1} - \ldots - \phi_p y_{t-p}) = \mu_0^* + \mu_1(S_t - \phi_1 S_{t-1} - \ldots - \phi_p S_{t-p}) + e_t, \quad \text{(A9)}$$

where $\mu_0^* = \mu_0(1 - \phi_1 - \ldots - \phi_p)$. If we define $\tilde{Y}^{**}$ to be the vector of left-hand-side variables of the above equation and $\tilde{W}^{**}$ to be the matrix of right-hand side variables, the posterior distribution of $\tilde{\mu} \equiv (\mu_0^*, \mu_1)'$ has the following truncated multivariate normal distribution:

$$\tilde{\mu} \sim N((M + \tilde{W}^{**\prime}\tilde{W}^{**})^{-1}(mM + \tilde{W}^{**\prime}\tilde{Y}^{**}), \sigma_e^2 (M + \tilde{W}^{**\prime}\tilde{W}^{**})^{-1})_{I[\mu_1 > 0]}, \quad \text{(A10)}$$

where the conjugate prior distribution for $\tilde{\mu}$ has the multivariate normal form

$$\tilde{\mu} \sim N(m, M^{-1}) \quad \text{(A11)}$$

and the symbol $I[\mu_1 > 0]$ is an indicator function on $\mu_1 > 0$. The value of $\mu_0$ is then computed as $\mu_0^*/(1 - \Sigma_{i=1}^{p}\phi_i)$.

**Conditional Distribution of $\tilde{g} \equiv (g_0, g_1, g_2, g_3)'$**

Define $\tilde{X}$ and $\tilde{S}^*$ to be the right-hand-side variables in (6) and the vector of $S_t^*$, respectively. $\tilde{g} \equiv [g_0, g_1, g_2, g_3]'$ can be simulated using the following posterior distribution:

$$\tilde{g} \sim N[(G + \tilde{X}'\tilde{X})^{-1}(\gamma G + \tilde{X}'\tilde{S}^*), (G + \tilde{X}'\tilde{X})^{-1}), \qquad (A12)$$

where the conjugate prior distribution for $\tilde{g}$ has the multivariate normal form:

$$\tilde{g} \sim N(\gamma, G^{-1}). \qquad (A13)$$

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