Money and Interest in a Simple Production Economy

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In this paper, I study the effects of government open market operations on households' production and interest rates. I show that if the degree of relative risk aversion is less (greater) than one, then a known, temporary increase in future money growth increases (decreases) a bond holder's production and decreases (increases) a money holder's production. However, the distributional effects tend to cancel out when aggregated. I also show that mean money growth increases bond market participation; the variance of money growth rates increases (decreases) bond market participation if the degree of relative risk aversion is less (greater) than one. Finally, if the degree of relative risk aversion is less than one and the initial inflation is low, then an increase in money growth may increase real interest, a reversal of the Mundell-Tobin effect. If the initial inflation is high or the degree of relative risk aversion is greater than one, the Mundell-Tobin effects tend to hold. (*JEL* Classification: E40)

I. Introduction

In transferring resources over time, households typically rely on financial assets. It is well known that the distribution of households' asset holdings is far from uniform: for instance, some rely more heavily on bonds, others more on money. Grossman and Weiss (1983) and Rotemberg (1984) have shown that such non-uniform distribution of financial asset holdings result in non-uniform effects of government open market operations on asset returns. This in turn implies that

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open market operations affect asset holders' productive activities also in a non-uniform way, especially when such activities incorporate the expectations of future events. In this paper, I study such distributional output effects of open market operations.

As open market operations affect asset returns non-uniformly, they cause demands for financial assets to change. Thus, open market operations would affect asset returns in two ways. Open market operations affect asset returns directly due to non-uniformity of asset holdings. As asset returns change, the distribution of financial asset holdings changes, which, in turn, affects asset returns.

Recently, Chatterjee and Corbae (1992) have studied the effects of a change in the growth rate of government monetary transfers on real interest in an exchange economy, where a change in money growth affects households' demands for financial assets. They have found that an increase in money growth affects real interest negatively—the Mundell (1963)—Tobin (1965) effect. In this paper, I study how government open market operations affect real interest in a production economy with particular emphasis on the possibility of a reversal of the Mundell-Tobin effect.

For simplicity, there are only two financial assets: default free government securities with positive nominal interest and government money with zero nominal interest. To explain the possible rate of return dominance of government money, I assume that agents incur transaction costs when participating in the government securities market. In this paper, the transaction or participation costs are measured in terms of a fraction of agents' lifetime expected utility forgone. The participation costs play the role of the brokerage fees in the classic inventory theoretic models such as Baumol (1952), Tobin (1956), and Miller and Orr (1966). With positive transactions costs, some agents may decide not to participate in the government securities market and decide to hold non-interest bearing money between periods. That is, at each moment, only a fraction of agents may go to the government securities market.

In this paper, current production is determined solely by the decisions of currently productive agents—there are no exogenous technology shocks; no inelastically supplied factors of production; and no previously determined factors of production. Thus, an expected increase in future inflation would affect an agent's current production decisions. An agent's decision concerning participation in the government securities market would also affect his production decisions, since government securities are a better inflation hedge than money. These features distinguish this paper from other studies such as Lucas and Stokey (1987) and Lucas (1990). In these models, the expectations of future inflation do not affect an agent's current production, and, thus, they do not affect the marginal rate of intertemporal substitution in equilibrium. This explains why the Fisher hypothesis on nominal interest holds at least on the average in these models.

The specific environment considered in this paper is an overlapping generations economy with young producers and old consumers. All the goods are perishable, so that to consume goods in their old age, young agents must hold money or government securities—there are no other means of saving in this economy. The only government securities considered are one period discount bonds. At each moment, government chooses stochastically the dollar amount of government bonds to be traded at the government bond market. Thus, money growth is stochastic since the government can change the money stock only through open market operations in this paper. In the model, all young agents are ex ante identical; whether they are bond holders or money holders just depends on whether they have forgone a fraction of their lifetime expected utility to participate in the government bond market. Young agents decide whether to participate in the bond market before observing the actual amount of government bonds to be traded. I can calculate the expected utility of a bond holder and the expected utility of a money holder. If the former is greater than the latter (taking account of the cost of participating in the government bond market), some agents switch from being money holders to being bond holders. As more agents become bond holders, the expected utility of bond holders falls relative to money holders. This results from the fact that given the supply of government bonds, as more agents want to hold bonds, the price of bonds increases and the return to bonds decreases. Thus, the fraction of agents who participate in the government securities market is endogenously determined in this paper.

I summarize the main results. First, a known temporary change in future money growth affects a bond holder's production differently from a money holder's production. However, such a change tends not to affect aggregate production. That is, the distributional effects of a known, temporary change in money growth tend to cancel out when aggregated.

Second, transactions costs (or participation costs) increase with potential bond holders' production. This is consistent with a result in the Baumol-Tobin model, where total brokerage costs increase with income. With positive transactions costs, the model is consistent with the ideà in the literature that money saves transactions efforts, and thus allows money holders more time for other activities—typically leisure. See for example Dutton and Gramm (1973), Lucas (1980), and McCallum and Goodfriend (1987). This paper shows that the degree of relative risk aversion determines whether a money holder enjoys leisure more or works more. More specifically, since the return to labor of a bond holder is greater than that of a money holder, the cost of leisure of a bond holder is greater than that of a money holder. Thus, if substitution effects dominate wealth effects—that is, if the degree of relative risk aversion is less than one, then a money holder enjoys leisure more than a bond holder. However, if the degree of relative risk aversion is greater than one, then a bond holder enjoys leisure more than a money holder.

Third, mean money growth increases the mean of government bond returns without affecting their coefficient of variation. Thus, mean money growth increases bond market participation. The variance of money growth rates increases both the mean and the coefficient of variation of government bond returns. The variance of money growth rates increases bond market participation if the degree of relative risk aversion is less than one; it decreases bond market participation if the degree of relative risk aversion is greater than one.

Finally, if both bond holders and money holders co-exist in equilibrium, the nominal interest rate is greater than the rate of inflation on the average, even though the Fisher hypothesis does not hold. The Fisher hypothesis does not hold because a known change in future money growth affects asset returns nonuniformly, which, in turn, affects agents' participation decisions. I find that if the degree of relative risk aversion is less than one, an increase in money growth increases nominal rates. An increase in money growth may even increase real interest when inflation is low; it may decrease real rates when inflation is high. If the degree of relative risk aversion is greater than one, an increase in money growth decreases both nominal and real interest.

Thus, if the degree of relative risk aversion is less than one and the initial rate of inflation is low, the Mundell-Tobin effect can be reversed. This results from the following. Due to limited participation, a one percent increase in future inflation brought about through higher current borrowing on the part of the government results in a more-than-one percent increase in the nominal receipts of the government and, thus, a decrease in the price of bonds. This decrease in the price of bonds encourages more participation in the bond market, which raises the price of bonds. When inflation is low, participation in the government bond market is low. The subsequent increase in the price of bonds tends to be outweighed by the initial increase if the degree of relative risk aversion is less than one and the initial rate of inflation is low. Otherwise, the Mundell-Tobin effect holds.

The rest of the paper is organized as follows. Section II describes the structure of the model. Section III discusses the properties of equilibrium, given the fraction of agents who hold bonds. The fraction is endogenized in section IV. Section V discusses steady state relationships between money growth, inflation, and interest rates. Concluding remarks on how to incorporate liquidity effects and the model's implications for optimal money growth are in section VI.

II. The Model

Consider an infinitely lived economy, populated by many two period lived agents. Each generation is identical in size, containing a continuum of agents with unit mass. There is only one factor of production, labor, used to produce one type of perishable consumption good. Neither young nor old agents have endowments of the consumption good. Each young agent is endowed with labor, N. If a young agent supplies n_t units of labor, he produces the same n_t units of the consumption good. (For all practical purposes, $n_t < N$ is assumed.) A young agent's utility is decreasing in n_t, strictly concave, and twice differentiable. To simplify the exposition with little cost in generality, I assume that only an old agent cares about current consumption. An old agent's utility is increasing in consumption c_t, strictly concave, and twice differentiable. More specifically, each agent has the following utility function:

$$W(n_t, c_{t+1}) = -u(n_t) + \beta v(c_{t+1}),$$

where $\beta \varepsilon (0, 1)$. As indicated, $u'(\cdot) > 0$, $u''(\cdot) > 0$, $v'(\cdot) > 0$, and $v''(\cdot) < 0$. Each young agent maximizes expected utility, $E\{W(n_t, c_{t+1}) \mid \Omega_t\}$, where $E\{\cdot \mid \Omega_t\}$ is the mathematical expectations operator, conditional on the information set Ω_t.

There are two kinds of financial assets, both are government issued: non-interest bearing fiat money and interest bearing government bonds. A government bond entitles its owner one dollar at the beginning of the next period. Government bonds are auctioned off in the security market at a price q_t. The nominal interest rate r_t is defined as $-1 + 1/q_t$; r_t is approximately equal to $-\log q_t$ for $0 < r_t < 1$.

As indicated, an agent may hold interest bearing government bonds if he forgoes a constant fraction of lifetime expected utility. An agent decides whether to purchase government bonds in the beginning of the period. This assumption makes the fraction of young agents who want to hold bonds not susceptible to current shocks. However, it may create a time consistency problem. Suppose an agent has already decided to hold bonds. Yet, if q_t happens to be greater than one and thus the nominal rate of return from holding bonds is negative, then the agent would rather hold on to money. This paper avoids such complication simply by restricting the stochastic processes governing bond prices so that nominal rates are always positive. The fraction of agents who decide to purchase government bonds will be endogenously determined in Section IV. In Sections II and III, assuming that such decisions have been already made, I study how the bond holders and the money holders would respond to known future money growth in terms of their production of the consumption good and their demand for the government bonds.

All the agents behave competitively. When producing output, young agents have the following information set:

$$\Omega_t = \{M_t, B_t, P_t, q_t, \hat{\Omega}_{t-1}; \lambda\}, \tag{2}$$

where M_t is the amount of money stock, B_t is the amount of bonds that the government sells in the security market per bond holder in period t, P_t is the price of a young agent's labor-output, $\hat{\Omega}_{t-1}$ is the set of all the shocks realized through period $t-1$, and λ is a measure of bond holders among the young.

For convenience, I assume the following sequence of events. A young agent with n^j_t units of output goes to the goods market and sells his entire output to old agents at P_t:

$$M^j_t = P_t n^j_t \ \text{ for } j = b, m. \tag{3}$$

M^j_t is the agent's nominal balances. The superscripts b and m indicate whether the particular agent holds bonds or money between periods.

If an agent has decided to hold bonds, he now goes to the security market with M^b_t money balances to purchase bonds at q_t:

$$M_t^b = q_t B_t, \tag{4}$$

where B_t is the amount of the one period, dollar denominated government bonds supplied per bond holder. In the beginning of period $t + 1$, the agent receives B_t dollars from government. Thus, his period $t + 1$ money balances become:

$$M_{t+1}^b = B_t. \tag{5}$$

At the goods market, the agent now old spends all the money to purchase the goods produced by the current young:

$$M_{t+1}^b = P_{t+1} c_{t+1}^b, \tag{6}$$

where c_{t+1}^b denotes a bond holder's old age consumption.

If an agent has decided not to purchase government bonds, he begins period $t + 1$ with the same M_t^m amount of money. Thus,

$$M_{t+1}^m = M_t^m = P_{t+1} c_{t+1}^m, \tag{7}$$

where c_{t+1}^m is a money holder's old age consumption.

Note that this paper is consistent with Lucas's (1980) cash-in-advance model since agents have money before purchasing goods. It is also consistent with Wallace's (1983) legal restrictions theory since agents do not exchange bonds for goods directly.

A. A Bond Holder's Maximization Problem

Given P_t and q_t, a bond holder maximizes (1) subject to the budget constraints (3), (4), (5), and (6), and the information set (2). Substituting eqs. (3) through (6) into (1), then differentiating it with respect to n_t^b, I get the following first order condition:

$$1 = \beta E\left\{ \frac{v'(c_{t+1}^b)}{u'(n_t^b)} \frac{P_t}{P_{t+1}} \frac{1}{q_t} \middle| \Omega_t \right\}. \tag{8}$$

Eq. (8) relates an agent's marginal rate of substitution between current labor and future consumption to the inflation rate $\pi_{t+1} = P_{t+1}/P_t - 1$ and the nominal interest rate $r_t = 1/q_t - 1$.

Note that if production were exogenous, the production process would determine the marginal rate of substitution, and the money supply process would determine the inflation rate. In this case, (8) would generate nominal rates consistent with the Fisher hypothesis. In this paper, however, both inflation rates and bond prices affect the marginal rate of substitution; thus, nominal rates do not in general follow the Fisher hypothesis.[1]

B. A Money Holder's Maximization Problem

Given P_t, a money holder maximizes (1) subject to the budget constraints (3) and (7), and the information set (2). Substituting eqs. (3) and (7) into (1), then differentiating it with respect to n_t^m, I get the following first order condition:

$$1 = \beta E\left\{\frac{v'(c_{t+1}^m)}{u'(n_t^m)}\frac{P_t}{P_{t+1}}\middle|\Omega_t\right\}. \tag{9}$$

Eq. (9) is essentially identical to the first order condition of a bond holder (8) except for the term involving the price of bonds. Since the agent holds money between periods, his intertemporal decision depends on the purchasing power of money (the inverse of the price level) overtime, but not on the nominal interest rate.

C. Aggregation and Equilibrium Conditions

From the first order conditions (8) and (9), each agent's demand for money, demand for bonds, and production of the consumption good can be derived. Since the asset returns differ, a bond holder's production differs from a money holder's. For simplicity, this paper uses geometric averages to aggregate each agent's choice variables. Let $\overline{n}_t$ denote aggregate output in period t, a geometric average of the goods produced by bond holders n_t^b and those by money holders n_t^m:

$$\log \overline{n}_t = \lambda \log n_t^b + (1-\lambda)\log n_t^m, \tag{10}$$

where $\lambda \varepsilon [0, 1]$ and λ is a measure of bond holders among the young.[2]

[1]This does not imply that the Fisher hypothesis never holds. In Section III, I have indicated a special case where it holds.

[2]In this footnote, I discuss a relationship between geometric averaging and arithmetic averaging. Consider $\lambda^* \varepsilon [0, 1]$ such that $\overline{n}_t = \lambda^* n_t^b + (1-\lambda^*)n_t^m$. The relationship between the weight λ^* in arithmetic averaging and the weight λ in geometric averaging (see (10)) is as follows. First, if $\lambda^* = 0$, $\lambda = 0$; if $\lambda^* = 1$, $\lambda = 1$; and if $\lambda^* \varepsilon (0, 1)$, $\lambda \varepsilon (0, 1)$. Second, for $\lambda^* \varepsilon (0, 1)$, $\lambda^* = \{(n_t^b/n_t^m)^\lambda - 1\}/\{(n_t^b/n_t^m) - 1\}$. Thus, in steady state, there is a one-to-one and onto relationship between λ and λ^*. Interestingly, $\lambda \approx \lambda^*$ for $n_t^b < 2n_t^m$, since $\lambda = \log (n_t^b/n_t^m)^\lambda / \log (n_t^b/n_t^m) \approx \{(n_t^b/n_t^m)^\lambda - 1\}/\{(n_t^b/n_t^m) - 1\} = \lambda^*$ for $n_t^b < 2n_t^m$; the accuracy increases with λ and decreases with n_t^b/n_t^m.

In equilibrium, the stock of money M_t equals the nominal value of aggregate output, $P_t \overline{n}_t$:[3]

$$M_t = P_t \overline{n}_t. \tag{11}$$

Before proceeding, let us derive a relationship between money growth and nominal interest rates. First, recall that when period $t + 1$ comes, each bond holder receives B_t dollars from the government and each money holder still holds M^t_m dollars. Using the same weights used in (10), I get the period $t + 1$ money supplied per young agent:

$$\log M_{t+1} = \lambda \log B_t + (1 - \lambda) \log M_t^m. \tag{12}$$

Thus, each agent in period t knows the money stock in period $t + 1$, M_{t+1}. Since $M_t^b = q_t B_t$ (see (4)), (12) can be re-written as:

$$\begin{aligned} \log M_{t+1} &= \lambda \log\left(\frac{M_t^b}{q_t}\right) + (1 - \lambda) \log M_t^m \\ &= \log M_t - \lambda \log q_t. \end{aligned} \tag{13}$$

Let $m_{t+1} \equiv M_{t+1}/M_t$. Since $q_t \equiv 1/(1 + r_t)$, (13) can be re-written as:

$$\log m_{t+1} = - \lambda \log q_t \equiv \lambda \log (1 + r_t), \tag{14}$$

which is approximately equal to λr_t for $r_t \varepsilon$ (0, 1). Thus, for any given λ, government monetary policy completely determines nominal interest rates. The rate of time preference, $- \log \beta$, does not affect nominal interest rates—this arises from the simplifying assumption that young agents do not consume. However, as will be seen below, an increase in β increases the young agent's aggregate saving, which is equal to the aggregate production in this paper. Obviously, without discussing how λ is determined, any discussion of nominal interest rates is incomplete. I thus postpone the discussion until section IV.

D. *Specific Assumptions on Preferences and Money Supply*

To get closed form solutions, I assume the following parametrized versions of $u(\cdot)$ and $v(\cdot)$:

[3]In equilibrium, aggregate consumption is also equal to aggregate production. From (10) and (11), $M_t = P_t(n_t^b)^\lambda (n_t^m)^{1-\lambda}$. Now, from (4), (6), (7) and (12) below, $M_t = (B_{t-1})^\lambda (M_{t-1}^m)^{1-\lambda} = (M_t^b)^\lambda (M_t^m)^{1-\lambda} = (P_t c_t^b)^\lambda (P_t c_t^m)^{1-\lambda} = P_t (c_t^b)^\lambda (c_t^m)^{1-\lambda}$. Thus, $(n_t^b)^\lambda (n_t^m)^{1-\lambda} = (c_t^b)^\lambda (c_t^m)^{1-\lambda}$.

$$u(n_t) = \frac{1}{1+\alpha} n_t^{1+\alpha} \quad and \quad v(c_{t+1}) = \frac{1}{1-\sigma} c_{t+1}^{1-\sigma}, \tag{15}$$

where $\alpha > 0$, and $\sigma > 0$ and $\sigma \neq 1$.[4] α is the percentage increase in a young agent's marginal disutility in response to a one percent increase in labor; σ is the Arrow-Pratt measure of the degree of relative risk aversion. I also assume the following stochastic process that governs money growth rates:

$$\log m_{t+1} = \mu + \log x_t. \tag{16}$$

Log x_t is a temporary money growth shock. It is independent and has a stationary normal distribution with mean zero and constant variance σ_x^2. A change in μ can be considered as a permanent change in net money growth rates, since E (log m_t) is μ. Now, according to (14), actual net money growth rates need to be strictly positive so that a would-be bond holder actually purchases bonds in the government bond market. This together with (16) implies that to avoid any time inconsistency problem that potential bond holders might encounter, μ needs to be sufficiently large so that the probability of log m_t being less than zero is very small. For the most part of this paper, only such μ's are considered.[5]

III. An Equilibrium Relationship between Money, Price, and Output

This section discusses a relationship between money growth, prices, and individual and aggregate production for any given λ. First, according to (8) and (15), the amount of goods produced by a bond holder is:

$$n_t^b = \left[\beta E\left\{\left(\frac{P_t}{P_{t+1}}\frac{1}{q_t}\right)^{1-\sigma} \middle| \Omega_t\right\}\right]^{\frac{1}{\alpha+\sigma}}. \tag{17}$$

[4]For $\sigma = 1$, let $v(\cdot)$ be $\log(\cdot)$. Then, from (8) and (9), a bond holder's production is equal to a money holder's production: $n_t^b = n_t^m = \beta^{1/(\alpha+\sigma)}$. Using the definition of equilibrium in Section IV, one could show that bond holders and money holders can not co-exist in equilibrium. This does not seem interesting; thus, the case of $\sigma = 1$ is omitted in the main text.

[5]Even with $\mu = 0$, it is possible that $m_t > 1$ for every t if the log-normal distribution shifts to the right. That is, suppose $\log(x_t - 1) \sim N(0, \sigma_x^2)$. Then, x_t is defined only on $(1, \infty)$; thus, even with $\mu = 0$, net money growth rates are strictly positive. However, it is not pursued here since it will make the model unnecessarily complicated.

As the discount factor β increases, a bond holder values his future consumption more, and thus, works harder. The terms inside the parenthesis indicate the return to labor. For each unit of labor, a bond holder produces a unit of the consumption good. In exchange for a unit of the consumption good, the agent gets P_t dollars. Using this money, a bond holder purchases bonds at q_t. In period $t + 1$, the agent gets the proceeds from the government and then purchases the consumption good produced by then young agents at P_{t+1}. According to (17), a bond holder's production increases with return to labor if the degree of relative risk aversion is less than one, $\sigma\varepsilon(0, 1)$—in this case, (intertemporal) substitution effects dominate wealth effects. If $\sigma > 1$, however, wealth effects dominate, and the production decreases with the return to labor.

According to (9) and (15), the amount of goods produced by a money holder is:

$$n_t^m = \left[\beta E\left\{\left(\frac{P_t}{P_{t+1}}\right)^{1-\sigma} |\Omega_t\right\}\right]^{\frac{1}{\alpha+\sigma}}. \tag{18}$$

A money holder does not purchase government bonds; thus, his return to labor is not a function of bond prices. Except for this, (18) is essentially identical to (17), and does not warrant discussion.

Substitution of (10), (17), and (18) into the equilibrium condition (11) results in the following: for any $\lambda\varepsilon[0, 1]$,

$$\frac{M_t}{P_t} = \left[E\left\{\beta\left(q_t^{-\lambda}\frac{P_t}{P_{t+1}}\right)^{1-\sigma} |\Omega_t\right\}\right]^{\frac{1}{\alpha+\sigma}}. \tag{19}$$

In appendix, I show that if the degree of relative risk aversion σ is between 0 and 1 or between 1 and $2 + \alpha$ (recall that α is the percentage increase in a young agent's marginal disutility in response to a one percent increase in labor), then there is a unique equilibrium price solution that is forward looking, uniformly bounded, and market fundamental: for any $\lambda\varepsilon[0, 1]$,

$$P_t = \gamma M_t, \tag{20}$$

where $\log \gamma = - 1/(\alpha + \sigma) \log \beta$. From now on, this paper only considers $\sigma\varepsilon(0, 2 + \alpha)$ and $\sigma \neq 1$. According to (11) and (20), the aggregate output becomes:

$$\bar{n}_t = \gamma^{-1} = \beta^{\frac{1}{\alpha+\sigma}}. \tag{21}$$

Thus, an increase in the discount factor β increases aggregate production, which is the aggregate saving of the young. The equilibrium solution above has the quantity theoretic property: the current money stock M_t affects the price level proportionally and does not affect aggregate production. This results from the fact that a temporary change in future money growth moves a bond holder's production and a money holder's production in opposite directions; the resulting distributional effects cancel out when aggregated. (As indicated, this section presupposes a constant λ. Section IV will show that permanent changes in future money growth, changes in μ, affect λ.)

To see why a known temporary change in future money growth does not affect aggregate production, first, consider a case of $\sigma\varepsilon(0, 1)$. In this case, substitution effects dominate wealth effects; thus, an agent's return to labor and his production move in the same direction. According to (14), the nominal return to bonds, the inverse of the price of bond is $(m_{t+1})^{1/\lambda}$. Each agent knows that the future price level P_{t+1} will increase proportionally to the future money growth rate m_{t+1}. Thus, from (17) the percentage change in a bond holder's return to labor with respect to a one percent increase in the future money growth rate becomes $-1 + 1/\lambda$. For any $\lambda\varepsilon(0, 1)$, the return to labor of a bond holder increases; thus, production increases.

Now, according to (18), the percentage change in a money holder's return to labor with respect to a one percent increase in the future money growth rate becomes -1. That is, the return to labor of a money holder is inversely proportional to the future money growth rate. Thus, an increase in future money growth induces a money holder to decrease production. Note that in the case of $\sigma\varepsilon(1, 2 + \alpha)$, wealth effects dominate; an increase in return to labor decreases production. In this case, an increase in future money growth decreases a bond holder's production and increases a money holder's production.

Thus, a change in future money growth moves the production of a bond holder and that of a money holder in opposite directions; hence, its effects on aggregate production tend to be small. Since λ is the measure of bond holders and $1 - \lambda$ is the measure of money holders, the change in the aggregate return to labor with respect to a one percent change in money growth is the weighted average of a change in the return to labor of a bond holder, $-1 + 1/\lambda$, and a change in the corresponding return of a money holder, −1. Multiplying λ and $1 - \lambda$ by the respective returns, then summing them up, I get zero. Thus, the distributional effects resulting from a known temporary change in future money growth cancel out when aggregated. It is not certain whether this exact cancellation result holds in general.[6] It is, however, certain that temporary changes in future money growth do not have significant effects on aggregate production due to offsetting forces.

Finally, I consider two extreme cases to show that the model in this paper captures some well known, standard results. First, if virtually no agents participate in the government bond market, $\lambda = 0$, then a one percent increase in future money growth reduces the return to labor of a money holder by one percent. This is the standard inflation tax result in a representative agent setup. Second, if virtually all the agents participate in the bond market, $\lambda = 1$, then a change in future money growth has no effect on the return to labor of a bond holder. This is a case where all the agents have perfect inflation hedge, a well known monetary neutrality result in a standard representative agent setup. In this case, there is a Fisher effect: the nominal interest rate increases with an expected increase in the future inflation point-for-point.

IV. Equilibrium Determination of λ

What I have discussed so far is the equilibrium relationship between money, prices, nominal interest, and production for a given λ. In this section, I discuss how λ is determined using the following equilibrium concept. (i) The economy with $\lambda \varepsilon (0, 1)$ is in equilibrium if the expected

[6]The neutrality result also holds even if there is capital instead of bonds. Suppose the government receives a unit of capital each period and sells the ownership of the capital, guaranteeing its nominal return. As before, agents hold either money or capital between periods. One period later on the behalf of the owners, the government hires workers to produce goods with capital. After the owners of capital receive the guaranteed nominal rental income, the government sells the goods to the old, and pays the young workers. If for simplicity, capital lasts only for one period and the production technology is of Cobb-Douglas, then the economy is almost identical to the one described in the main text (if the share of capital is zero, then they are identical). It is not difficult to see that neutrality holds in such an environment.

However, suppose arithmetic averages are used to aggregate variables. Then, neutrality holds up to the first order Taylor series approximation as long as a bond holder's production is less than twice of a money holder's production, $n_t^b < 2n_t^m$ (see footnote 2).

utility of a bond holder is equal to that of a money holder at $\lambda\varepsilon(0, 1)$. (ii) The economy with $\lambda = 0$ is in equilibrium if the expected utility of a bond holder is less than that of a money holder at $\lambda = 0$. (iii) The economy with $\lambda = 1$ is in equilibrium if the expected utility of a bond holder is greater than that of a money holder at $\lambda = 1$.

Given λ, a young agent decides whether to hold fiat money with zero nominal interest or default free government bonds with positive nominal interest. Participating in the government bond market is costly, and the decisions must be made prior to the knowledge of the dollar amount of government bonds to be sold in period t. That is, prior to observing x_t and producing output, each young agent decides whether to hold fiat money or government bonds with the knowledge of the average money growth rate μ, the variance of temporary money growth shock σ_x^2, and the actual λ. The economy is in equilibrium if given λ, a money holder has no incentive to become a (potential) bond holder and a (potential) bond holder has no incentive to become a money holder. After each young agent has decided whether to participate in the bond market or not, a young agent observes x_t and produces output. Thus, the equilibrium value(s) of λ is determined by comparisons of unconditional expected utility—average utility over all the possible realizations of a temporary future money growth shock x_t.

Let $\overline{W}_\lambda^b$ and $\overline{W}_\lambda^m$ denote the average utility of a bond holder and that of a money holder, given λ. For any given $\lambda\varepsilon(0, 1]$, the average utility of a bond holder becomes:

$$\overline{W}_\lambda^b \equiv (1-k)E\left[E\left\{-\frac{1}{1+\alpha}(n_t^b)^{1+\sigma} + \beta\frac{1}{1-\alpha}(c_{t+1}^b)^{1-\sigma}|\Omega_t\right\}\right]$$

$$= (1-k)E\left[\frac{\alpha+\sigma}{(1+\alpha)(1-\alpha)}\left\{\beta E\left(\frac{P_t}{P_{t+1}}\frac{1}{q_t}\right)^{1-\sigma}|\Omega_t\right\}^{\frac{1+\alpha}{\alpha+\sigma}}\right], \tag{22}$$

according to eqs. (3), (4), (5), (6), and (17). Note that k is the fraction, of which expected utility an agent must forgo to participate in the government bond market. According to (22), for any $\sigma\varepsilon(0, 1)$, the average utility is positive. Thus, it is natural to assume $0 < k < 1$. However, if $\sigma > 1$, each agent's utility is defined as a negative real number (see also (1) and (15)). In this case, a positive k is incompatible with the notion of forgone utility. That is, the term $(1 - k)$ in (22) must be strictly greater than one to be consistent with the postulate that an agent incurs transactions costs when purchasing bonds. Thus, for $\sigma > 1$, I assume

$-1 < k < 0$. Now, the participation cost incurred by a bond holder is positively related to his production on the average (see (17)). This is consistent with a result in the Baumol-Tobin model, where total brokerage costs increase with income.[7]

For any given $\lambda \varepsilon [0, 1)$, the average utility of a money holder becomes

$$\overline{W}_{\lambda}^{m} \equiv E\left[E\left\{-\frac{1}{1+\alpha}(n_t^m)^{1+\sigma} + \beta\frac{1}{1-\alpha}(c_{t+1}^m)^{1-\sigma}\middle|\Omega_t\right\}\right]$$
$$= E\left[\frac{\alpha+\sigma}{(1+\alpha)(1-\alpha)}\left\{\beta E\left(\frac{P_t}{P_{t+1}}\right)^{1-\sigma}\middle|\Omega_t\right\}^{\frac{1+\alpha}{\alpha+\sigma}}\right], \tag{23}$$

according to eqs. (3), (7), and (18).

Now, from (22) and (23), for any given $\lambda \varepsilon [0, 1)$,

$$\frac{\overline{W}_{\lambda}^{b}}{\overline{W}_{\lambda}^{m}} = (1-k)E\left\{(q_t^{-1})^{\frac{(1+\alpha)(1-\alpha)}{\alpha+\sigma}}\right\}. \tag{24}$$

The expected utility ratio is a function of k and q_t^{-1}, the nominal return to holding bonds. Eqs. (22), (23), and (24) have the following interpretation. Due to transactions costs, the return to labor of a bond holder is greater than that of a money holder. That is, the cost of leisure of a bond holder is greater than that of a money holder. Thus, if (intertemporal) substitution effects dominate wealth effects—that is, if the degree of relative risk aversion is less than one, then a money holder enjoys leisure more than a bond holder. However, if wealth effects dominate substitution effects—that is, if the degree of relative risk aversion is greater than one, then a bond holder enjoys leisure more than a money holder. Thus, the degree of relative risk aversion determines whether a money holder enjoys leisure more than a bond holder or a money holder works harder than a bond holder.

If the expected utility of a bond holder (taking account of the cost of participating in the bond market) is greater than the expected utility of a money holder, some agents switch from being money holders to being bond holders. In equilibrium, money holders have no incentive to become bond holders. In equilibrium where both money holders and

[7]If constant costs were posited, then total costs of transactions would not increase with income, which results from the fact that agents are not allowed to choose the frequency of transactions in overlapping generations models. The setup in this paper is one way to get around this difficulty.

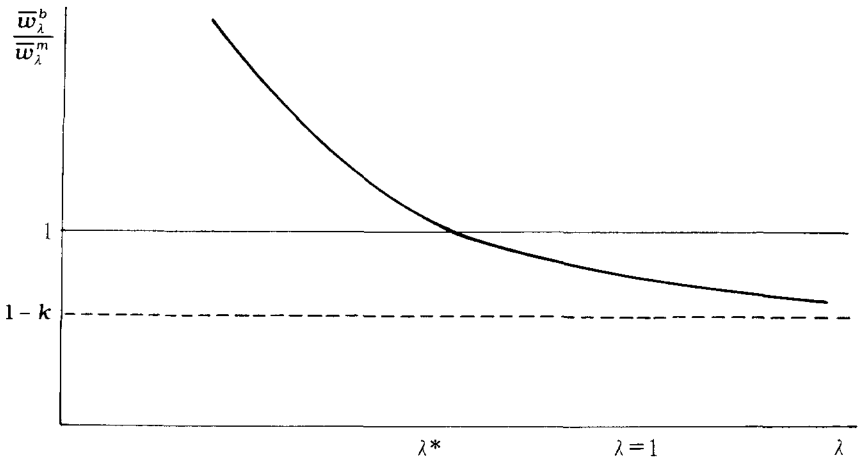

FIGURE 1
A CASE OF $\lambda^{*}\varepsilon(0,1)$ AND $\sigma\varepsilon(0, 1)$

bond holders co-exist, the expected utility of a bond holder is equal to that of a money holder: $W_{\lambda}^{b} = W_{\lambda}^{m}$. Thus, an equilibrium $\lambda\varepsilon(0, 1)$ satisfies the following:

$$\begin{aligned} 1 &= (1-k)E\left\{(q_t^{-1})^{\frac{(1+\alpha)(1-\alpha)}{\alpha+\sigma}}\right\} \\ &= (1-k)\exp\left[\frac{(1+\alpha)(1-\alpha)}{\alpha+\sigma}\frac{\mu}{\lambda} + \frac{1}{2}\left\{\frac{(1+\alpha)(1-\alpha)}{(\alpha+\sigma)\lambda}\right\}^2\sigma_x^2\right], \end{aligned} \tag{25}$$

according to (14), (15), (16), and (24).[8] Suppose $\sigma\varepsilon(0, 1)$. Recall that for $\sigma\varepsilon(0, 1)$, $k\varepsilon(0, 1)$. Differentiation of the right side of (25) with respect to λ indicates that W_{λ}^{b} / W_{λ}^{m} is decreasing monotonically to $1 - k$ (see Figure 1). As more agents become bond holders, the expected utility of bond holders falls relative to money holders. This implies that the larger the value of λ, the smaller the incentive to become a bond holder. The monotonicity implies the uniqueness of an equilibrium.

Some comparative statics results are as follows. First, if k decreases, then the transactions cost of becoming a bond holder decreases, which induces more agents to hold bonds—that is, the equilibrium λ increases.

Note that any change in μ or σ_x^2 affects the statistical properties of

[8]The following facts on the log-normally distribution are repeatedly used: if log $y \sim N(\mu_y, \sigma_y^2)$, then $E(y) = \exp(\mu_y + 1/2\sigma_y^2)$ and $\mathrm{Var}(y) = (E(y))^2(\exp(\sigma_y^2) - 1)$.

returns to bond q_t^{-1}, such as mean and a measure of variation. Typically, variance is used for a measure of variation. However, this paper uses coefficient of variation, since with log-normal distribution, a change in parameter such as μ affects both mean and variance of bond returns. From (14), (16), and footnote 8,

$$E(q_t^{-1}) = \exp\left(\frac{\mu}{\lambda} + \frac{1}{2}\lambda^{-2}\sigma_x^2\right); CV(q_t^{-1}) = \sqrt{\exp(\lambda^{-2}\sigma_x^2) - 1}, \tag{26}$$

where $CV(q_t^{-1})$ denotes the coefficient of variation of bonds returns, $\sqrt{\mathrm{Var}(q_t^{-1})/\{E(q_t^{-1})\}^2}$.

An increase in μ indicates a permanent change in future money growth. According to (26), it increases the mean return to bonds without affecting the coefficient of variation—it affects the mean and the standard deviation by the same amount. This is a case of "a variation-preserving increase in mean." This increases an agent's incentive to hold bonds, and λ increases.

An increase in σ_x^2, the variance of temporary shocks to future money growth rates, increases both the mean and the coefficient of variation of bond returns. Even though the standard deviation of returns to bond increases more than the mean, the right side of (25) increases. Thus, λ increases. This occurs because each agent is relatively tolerant to risk (an agent's degree of relative risk aversion is less than one), so that such changes in the statistical characteristics make bond holding more attractive.

Now, suppose $1 < \sigma < 2 + \alpha$ for any $\alpha > 0$. Recall that for $\sigma > 1$, $k\varepsilon(-1, 0)$. Differentiation of the right side of (25) with respect to λ indicates that $W_\lambda^b \,/\, W_\lambda^m$ decreases initially, then increases monotonically to $1 + |k|$. Thus, there may be two different λ's that satisfy (25) (see Figure 2). In such a case, an equilibrium with smaller λ is unstable, in the sense that a small perturbation of λ does not generate an infinite sequence of moves which converges to the original λ. In this equilibrium, $W_\lambda^b \,/\, W_\lambda^m$ is decreasing. Recall that for $\sigma > 1$, W_λ^b and W_λ^m are negative. Thus, a small increase in λ increases a bond holder's utility even though it reduces the return to bonds (a money holder's utility is not affected by changes in λ). The equilibrium with larger λ is stable since $W_\lambda^b \,/\, W_\lambda^m$ is increasing in λ. Note that differentiation of the right side of (25) indicates that $W_\lambda^b \,/\, W_\lambda^m$ is increasing in λ for $\sigma > 1$ if the following condition holds: for any $\lambda\varepsilon(0, 1]$ and $\mu > 0$,

$$1 + \frac{(1+\alpha)(1-\alpha)}{(\alpha+\sigma)\mu\lambda}\sigma_x^2 > 0. \tag{27}$$

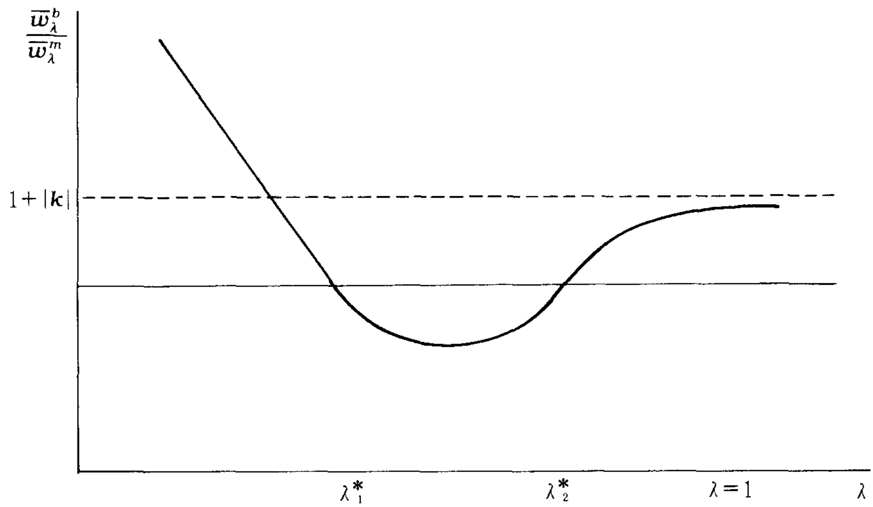

FIGURE 2
A CASE OF λ_1^*, $\lambda_2^* \varepsilon (0, 1)$ AND $\sigma > 1$

The comparative statics results of changing $|k|$ and μ on stable equilibria are consistent with those in case of $\sigma \varepsilon (0, 1)$. A decrease in $|k|$ induces more agents to hold bonds; an increase in μ induces more agents to hold bonds. However, as for σ_x^2, the result is the opposite. An increase in σ_x^2 reduces λ. Again, an increase in σ_x^2 increases the standard deviation of the returns to bonds more than the mean. Since agents are very risk averse $\sigma > 1$, such changes make bond holding less attractive.

For completeness, the following two cases are considered. First, in case of $\lambda = 1$, all the agents want to hold bonds, and no agents have any incentive to become a money holder. Suppose $\sigma \varepsilon (0, 1)$. If the right side of (24) is greater than one at $\lambda = 1$, $W_{\lambda=1}^b > W_{\lambda=1}^m$, then $\lambda = 1$ is equilibrium. Suppose $\sigma > 1$. If the right side of (24) is less than one at $\lambda = 1$, $W_{\lambda=1}^m < W_{\lambda=1}^b < 0$, then $\lambda = 1$ is equilibrium.

Second, in the case of $\lambda = 0$, no agent has any incentive to become a bond holder. Since nobody holds bonds, let the net nominal return to bond is zero, i.e., the price of bond is one. Then, from (24), $W_{\lambda=0}^b < W_{\lambda=0}^m$ and $\lambda = 0$ is equilibrium.

V. Money Growth and Interest Rates in Steady State

In steady state, the average inflation rate is equal to the average money growth rate. That is, $\mu = \bar{\pi}$, where $\bar{\pi} \equiv E(\log P_{t+1} - \log P_t)$. If both money and bonds are held in equilibrium, the average money growth rate and the average nominal interest rate satisfy the following: from (14) and (16),

$$\bar{r} = \frac{1}{\lambda}\mu, \tag{28}$$

where $\bar{r} \equiv E(\log(1 + r_t)) \approx E(r_t)$ for $r_t \varepsilon (0, 1)$, and $\lambda \varepsilon (0, 1)$ satisfies (25). Thus, as long as $0 < \lambda < 1$, the average nominal interest rate is greater than the average inflation rate.

An increase in μ increases the average nominal rate $\bar{r}$ if $\sigma \varepsilon (0, 1)$; it decreases $\bar{r}$ if $\sigma > 1$ and the stability condition (27) holds. To see that from (25) and (28),

$$\frac{d\bar{r}}{d\mu} = \frac{1}{\lambda}\left(1 - \frac{\mu}{\lambda}\frac{d\mu}{d\lambda}\right); \tag{29}$$

$$\frac{d\lambda}{d\mu} = \frac{1}{\left\{\frac{\mu}{\lambda} + \frac{(1+\alpha)(1-\alpha)}{(\alpha+\sigma)}\left(\frac{1}{\lambda}\right)^2 \sigma_x^2\right\}}. \tag{30}$$

Note that (30) is positive (for $\sigma > 1$, the stability condition (27) is needed). From (29) and (30),

$$\frac{d\bar{r}}{d\mu} = \frac{1}{\left\{\lambda + \frac{\alpha+\sigma}{(1+\alpha)(1-\alpha)}\mu\lambda^2\sigma_x^{-2}\right\}}, \tag{31}$$

which is positive if $\sigma \varepsilon (0, 1)$; negative if $\sigma > 1$, according to the stability condition.

Now, define the average real interest rate $\bar{\rho}$ such as the difference between the average nominal interest rate and the average inflation rate, $\bar{r} - \bar{\pi}$. Since $\bar{\pi} = \mu$,

$$\frac{d\bar{\rho}}{d\mu} = \frac{d\bar{r}}{d\mu} - 1. \tag{32}$$

According to (29), (30), and (31),

$$\mathrm{Sgn}\left(\frac{d\bar{\rho}}{d\mu}\right) = \mathrm{Sgn}\left\{-1 + \left(\frac{1}{\lambda} - 1\right)\frac{(1+\alpha)(1-\alpha)}{\alpha+\sigma}\frac{1}{\mu\lambda}\sigma_x^{-2}\right\}, \tag{33}$$

provided the stability condition (27) holds. Thus, if $\sigma \varepsilon (0, 1)$, the initial values of λ and μ determine whether an increase in μ increases real interest rates on the average. Suppose μ is relatively small initially so that not many agents hold bonds between periods (λ is closer to zero than to one). Then, an increase in μ may increase the average real rate, a reversal of the Mundell-Tobin effect. This results from the fact that when money growth is low, inflation is low in the steady state. In this case, participation in the government bond market is low, since the rate of return to bonds is not much higher than that of money. Due to the limited participation in the bond market, a one percent increase in future inflation results in more than a one percent increase in the nominal receipts of the government and therefore a decrease in the price of bonds (see eqs. (14) and (28)). This decrease in the price of bonds encourages more participation in the bond market (see eq. (25) with $\sigma\varepsilon(0,1)$) and therefore tends to raise the price of bonds. When the degree of relative risk aversion is less than one and the initial rate of inflation is low, the subsequent increase in the price of bonds tends to be smaller than the initial increase. Otherwise, the Mundell-Tobin effect holds.

Now, suppose that μ is large initially so that λ is close to one. Then, the terms inside the parentheses on the right side of (33) may become negative. That is, an increase in μ may decrease the average real rate. In this case, with initially high participation in the bond market, the initial decrease in the price of bonds is smaller in magnitude that the subsequent increase. Note that according to (33), if σ is greater than one, an increase in μ always decreases the average real rate.

Therefore, the Mundell-Tobin effect may be reversed if the degree of relative risk aversion is less than one and money growth rates are initially low. The Mundell-Tobin effect is possible if the degree of relative risk aversion is less than one and money growth rates are initially high or the degree of relative risk aversion is greater than one.[9]

VI. Concluding Remarks

I conclude this paper with some remarks concerning how to incorporate a liquidity effect and the model's implications for the optimal money growth rate. If the model is changed so that in addition to deciding whether to become bond holders, agents must also decide how much bonds to purchase before observing shocks as in Lucas (1990),

[9]For other examples of the reverse Mundell-Tobin effect, see Stockman (1981), Gale (1983, Chapter 2.8), and Romer (1986).

then it would exhibit a liquidity effect since agents could not change their money balances for purchasing bonds when the government conducts open market operations.

As for the model's implications for optimal money growth, one can show that using (22) and (23), the average utility is maximized if no agents incur transactions costs associated with purchasing bonds. Thus, the optimal money growth rate is any money growth rate that is sufficiently low so that no agents want to hold bonds at that rate. Since government can change the money stock only through the open market operations in this paper, the money stock must be constant if the government can not sell any bonds. Thus, the optimal net money growth rate is zero. This result is consistent with the one in the general equilibrium versions of Baumol-Tobin models, such as Jovanovic (1982) and Romer (1986). These models also have a feature that higher money growth induces agents to incur higher "brokerage fees."

This result on optimal money growth does not seem robust, however. Suppose current and future states of the economy are costly to observe, but monetary aggregates are freely available. Suppose further that the government is more efficient in gathering information than at least some of the agents; the government provides some garbled information to those agents through changing monetary aggregates. If agents are not very risk averse and the variance of monetary shocks is not large, then the benefit from having additional information on the current and future states could outweigh the transactions costs incurred by some of the agents. If this were the case, the optimal money growth rate could be strictly positive.

Appendix

This appendix shows that if $0 < \sigma < 1$ or $1 < \sigma < 2 + \alpha$, then there is a unique, forward looking, uniformly bounded, market fundamental price equation that satisfies the equilibrium condition (19). First, according to (13), (14), and (16), the information set in (2) can be expressed as:

$$\Omega_t = \{M_t, P_t, x_t, \hat{\Omega}_{t-1}; \lambda\}. \tag{A1}$$

Now, according to (14) , (16), and (A1), the condition (19) can be rewritten as:

$$1 = E\left\{\left(\frac{P_t}{M_t}\right)^{\alpha+\sigma} \beta\left(\frac{e^{\beta} x_t P_t}{P_{t+1}}\right)^{1-\sigma} \mid \Omega_t\right\}. \tag{A2}$$

To exclude non-market fundamental solutions—for example sun spots, I first derive perfect foresight price solutions that satisfy (A2), then apply the expectations operator conditional on Ω_t. From (A2), the perfect foresight price solution satisfies:

$$\begin{aligned}&\log M_t - \frac{1}{\alpha+\sigma}\log\beta - \frac{1-\sigma}{\alpha+\sigma}(\mu + \log x_t)\\ &= \left(-\frac{1-\sigma}{\alpha+\sigma}\right)\left(1 - \frac{1+\sigma}{1-\sigma}L\right)\log P_{t+1},\end{aligned} \tag{A3}$$

where L is the lag operator. Since $0 < \sigma < 2 + \alpha$ and $\sigma \neq 1$, any forward looking solutions with transient terms are not uniformly bounded. A forward looking, perfect foresight, equilibrium price solution without a transient term is:

$$\begin{aligned}\log P_t &= -\frac{1}{\alpha+\sigma}\log\beta - \frac{1-\sigma}{\alpha+\sigma}\left\{\mu + \sum_{j=0}^{\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log x_{t+j}\right\}\\ &+ \frac{\alpha+\sigma}{1+\sigma}\sum_{j=0}^{\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log M_{t+j}.\end{aligned} \tag{A4}$$

Applying the expectations operator conditional on the information set (A1), we get:

$$\begin{aligned}\log P_t &= -\frac{1}{\alpha+\sigma}\log\beta - \frac{1-\sigma}{\alpha+\sigma}(\mu + \log x_t)\\ &+ \frac{\alpha+\sigma}{1+\sigma}E\left\{\sum_{j=0}^{\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log M_{t+j} \mid \Omega_t\right\}.\end{aligned} \tag{A5}$$

Note that according to (14) and (16),

$$\begin{aligned}&\lim_{j\to\infty} E\left\{\left(\frac{1-\sigma}{1+\alpha}\right)^j \log M_{t+j} \mid \Omega_t\right\}\\ &= \lim_{j\to\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log M_t + \lim_{j\to\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j j\mu = 0.\end{aligned}$$

Thus,

$$\begin{aligned}&E\left\{\sum_{j=0}^{\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log M_{t+j} \mid \Omega_t\right\}\\ &= \frac{1+\sigma}{\alpha+\sigma}E\left\{\log M_t + \sum_{j=0}^{\infty}\left(\frac{1-\sigma}{1+\alpha}\right)^j \log m_{t+j} \mid \Omega_t\right\}\end{aligned} \tag{A6}$$

$$= \frac{1+\sigma}{\alpha+\sigma}\left\{\log M_t + \frac{1-\sigma}{1+\alpha}(\mu + \log x_t)\right\}.$$

Substitution of (A6) into (A5) results in (20). The uniqueness results from fact that the price solutions are restricted to be uniformly bounded and market fundamental.

Q.E.D

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