Note: A Note on the So-Called 'Double Counting' Problem in the Transformation Procedure

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The purpose of this paper is to clarify the problem of double counting of the profit contained in the elements of constant capital. This problem is getting more and more important because most recent vindicators of the labor theory of value maintain that the value of constant inputs should be defined as the price at which they were purchased. I will show that the value definition newly adopted does not hold without unreal assumptions which have already been explored in the value debate in 1970s. (*JEL* Classification: B24)

I. Introduction

The purpose of this paper is to clarify the problem of double counting of the profit contained in the produced inputs (in Marx's term, constant capital), which has been treated in the context of the so-called 'transformation' of Marxian values into prices of production. This issue, originated from Lipietz (1982), has been explored by Glick & Ehrbar (1987), Saad-Filho (1994) and Ramos-Martinez & Rodriguez-Herrera (1996) etc. Lipietz (1982) pointed out that the traditional view, in which the aggregate equalities between value and price and surplus value and profit refer to the money value and price of the gross product, is misleading because the profit on the production of means of production is counted twice. So he argued that the aggregate equality must be amended to one with respect to net product. Its implication was

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explained more concretely by Glick & Ehrbar(1987) and Saad-Filho (1994) respectively. But recently Ramos-Martinez & Rodriguez-Herrera (1996) tried to refute this argument.

The problem of double counting is getting more and more important because most recent vindicators of the labor theory of value maintain that the value of constant inputs should be defined as the price at which they were purchased.[1] This paper will show that the value definition newly adopted does not hold in general without an improbable assumption. Presenting a simple example, this assumption will be shown to be nothing but a roundabout confirmation of conditions already established in the value debate in 1970s.

II. Compatibility Condition (1)

The condition in which double counting will not arise can be derived as follows.

According to the view that value transferred from the constant portion of capital must be calculated as its purchasing price, value system can be defined in the following formula.

$$\lambda = pA + l, \tag{1}$$

where λ, p, A and l denote, respectively, the value vector, the price vector, the physical-technological input coefficient matrix (non-singular and indecomposable) and the labor input coefficient vector.

The system of prices of production is, in general, defined as following.

$$p = (1 + r)(pA + wl), \tag{2}$$

where w and r denote the wage per unit of labor and the rate of profit.

Substituting (2) into (1) yields[2]

$$\begin{aligned}\lambda &= pA + l \\ &= (1 + r)(pA + wl)A + l\end{aligned}$$

[1]The authors of the articles in Freeman and Carchedi eds.(1996) are included here. They named themselves 'non-dualists'. Lee (1993) also adopts this position.

[2]According to some reading of Marx's texts, this procedure may be suspected for keeping the so-called Smithian dogma. But, in my opinion, this mathematical method itself is not relevant to the Smithian dogma. More detailed discussion is out of the scope of this paper.

$$= \cdots\cdots$$
$$= pA^n + wlA + wlA^2 + \cdots + \pi A + \pi A^2 + \cdots + l,$$

where π is the profit vector.

Under a certain condition,[3] $\lim_{n \to \infty} A^n = 0$, so the above equation can be summarized as

$$\lambda = w \sum_{k=0}^{\infty} lA^k + \sum_{k=1}^{\infty} \pi A^k + (1 - w)l. \tag{3}$$

On the other hand, the conventional value calculation, $\lambda = \lambda A + l$, can be transformed into

$$\begin{aligned} \lambda &= l(I - A)^{-1} = w(1 + e) \sum_{k=0}^{\infty} lA^k \\ &= w \sum_{k=0}^{\infty} lA^k + ew \sum_{k=0}^{\infty} lA^k, \end{aligned} \tag{3$'$}$$

where e denotes the rate of exploitation. The second equality holds if, following Lipietz (1982), we define the value of labor power as labor equivalent of money wage.

From (3) and (3)$'$, the condition guaranteeing the compatibility between the new and the conventional definition is

$$\begin{aligned} ew \sum_{k=0}^{\infty} lA^k &= \sum_{k=1}^{\infty} \pi A^k + (1 - w)l \\ &= \sum_{k=1}^{\infty} \pi A^k + ewl \end{aligned}$$

$$\therefore\ ew \sum_{k=1}^{\infty} lA^k = \sum_{k=1}^{\infty} \pi A^k. \tag{4}$$

Equation (4) requires the sum of surplus values resulted from labor quantities regressed from period $t - 1$ to infinity to be equal to the sum of profits distributed to constant inputs regressed infinitely.

III. Compatibility Condition (2)

To clarify the meaning of (4), let us assume the three-department model as below. C_i, V_i, P_i and W_i denote, respectively, the constant capital, the variable capital, the profit and the price of production of the i-

[3]If r_M is the eigenvalue of A which is maximum in modulus, $|r_M| < 1$ is this condition. For the proof of this theorem, see Pasinetti (1977), 264-5.

th department. Note that each variable is measured in price terms, not by labor-time unit. As will be shown, the deviation of values from prices does not matter here. For analytical simplicity, I, II, III are assumed to represent, respectively, department producing means of production, wage goods and luxury goods.

$$\begin{aligned} &\text{I. } C_1 + V_1 + P_1 = W_1 \\ &\text{II. } C_2 + V_2 + P_2 = W_2 \\ &\text{III. } C_3 + V_3 + P_3 = W_3. \end{aligned}$$

Firstly, using the above system, the sum of surplus values resulted from labor quantities regressed from period $t-1$ to infinity can be calculated.

We start with C_1. C_1 represents the constant capital used up in department I, in which the profit distributed to the capitalist who produced it in the previous period and sold it in the beginning of current period are contained. For example, if steel is an input of car production, in the purchasing price of steel are contained the constant and variable capital used and the profit of the steel-producing industry.

Let α_1, β_1, γ_1 which are, by definition, $0 < \alpha_1, \beta_1, \gamma_1 < 1$ and $\alpha_1 + \beta_1 + \gamma_1 = 1$ denote, respectively, the proportion of the constant and the variable portion of capital used up and the profit to C_1. Then, the quantity of the variable capital used up in the production process of C_1 will be $\beta_1 C_1$,[4] from which resulted the surplus value $e\beta_1 C_1$.[5]

On the other hand, the constant capital used up in the production process of C_1 will be $\alpha_1 C_1$. And as the variable capital, $\alpha_1\beta_1 C_1$ is contained in the $\alpha_1 C_1$, the resulted surplus value is $e\alpha_1\beta_1 C_1$.[6] In this way, the sum of surplus values created at each stage of production is

$$\begin{aligned} &e\beta_1 C_1 + e\alpha_1\beta_1 C_1 + e\alpha_1^2\beta_1 C_1 + \cdots \\ &= e\beta_1 C_1 (1 + \alpha_1 + \alpha_1^2 + \cdots) \\ &= \frac{e\beta_1 C_1}{(1-\alpha_1)}. \end{aligned}$$

[4]This quantity is measured in price terms. But, as we adopt the assumption that the value of labor power is equal to money wage, it can also be regarded as value magnitude insofar as its dimension is suitably transformed.

[5]Following the convention of the transformation debate, we assume that the sectoral rates of exploitation are equalized. Therefore, subscript i is not necessary here.

[6]For example, if steel is an input of car and iron is an input of steel, this means that the iron contains the variable capital consumed in iron producing sector. And for simplicity, assuming the organic composition of capital be the same between industries which belong to department I, we use coefficient β_1.

Applying this procedure to the constant inputs of department II and III, the total sum of surplus value infinitely regressed is

$$e\beta_1 C_1/(1-\alpha_1) + e\beta_2 C_2/(1-\alpha_2) + e\beta_3 C_3/(1-\alpha_3). \qquad (5)$$

Secondly, the sum of profits distributed to constant inputs regressed to infinity can also be calculated.

The profit distributed in the production process of C_1 is $\gamma_1 C_1$. And as the constant capital used up in the production of C_1 is $\alpha_1 C_1$, the profit distributed in the production process of $\alpha_1 C_1$ is $\gamma_1 \alpha_1 C_1$. In this way, the sum of profits distributed at each stage of production is

$$\begin{aligned} &\gamma_1 C_1 + \gamma_1 \alpha_1 C_1 + \cdots \\ &= \gamma_1 C_1 (1 + \alpha_1 + \alpha_1^2 + \cdots) \\ &= \frac{\gamma_1 C_1}{(1-\alpha_1)}. \end{aligned}$$

Applying this procedure to the other departments, the total sum is

$$\frac{\gamma_1 C_1}{(1-\alpha_1)} + \frac{\gamma_2 C_2}{(1-\alpha_2)} + \frac{\gamma_3 C_3}{(1-\alpha_3)} \qquad (6)$$

Supposing that (5) and (6) are equal,

$$\begin{aligned} &\frac{e\beta_1 C_1}{(1-\alpha_1)} + \frac{e\beta_2 C_2}{(1-\alpha_2)} + \frac{e\beta_3 C_3}{(1-\alpha_3)} \\ &= \frac{\gamma_1 C_1}{(1-\alpha_1)} + \frac{\gamma_2 C_2}{(1-\alpha_2)} + \frac{\gamma_3 C_3}{(1-\alpha_3)}. \end{aligned}$$

As $\alpha_i = C_i/(C_i + V_i + P_i)$, $\beta_i = V_i/(C_i + V_i + P_i)$, $\gamma_i = P_i/(C_i + V_i + P_i)$ and $\alpha_i + \beta_i + \gamma_i = 1$, above equality can be summarized into

$$\left(\frac{P_1 - eV_1}{V_1 + P_1}\right)\frac{C_1}{\sum C_i} + \left(\frac{P_2 - eV_2}{V_2 + P_2}\right)\frac{C_2}{\sum C_i} + \left(\frac{P_3 - eV_3}{V_3 + P_3}\right)\frac{C_3}{\sum C_i} = 0 \qquad (7)$$

where $\Sigma C_i = C_1 + C_2 + C_3$.

In general, (7) does not hold. Note that it needs very special relation among the allocation of the constant capital ($C_i/\Sigma C_i$) between departments, the wage-profit ratio in each department and the rate of exploitation.

We have interesting results by considering two special cases in which the equation (7) holds.

A rather trivial case is when $P_i - eV_i$'s are equal to 0 for all i's. That is, $e = P_i/V_i$ for $i = 1, 2, 3$. This is when the wage-profit ratio of each department is not only uniform but equal to the rate of exploitation equalized between departments. This is the famous "uniform organic composition of capital" case. In volumes I and II of *Capital*, Marx carried out his analysis under this assumption. In this case, no transformation is needed because value and price are definitionally equal.

Next, changing the equation (7) more slightly, we can find another case in which it holds.

$$\frac{1}{\sum C_i}\left\{\frac{C_1}{V_1+P_1}(P_1-eV_1)+\frac{C_2}{V_2+P_2}(P_2-eV_2)+\frac{C_3}{V_3+P_3}(P_3-eV_3)\right\}=0$$

If $C_i/(V_i + P_i)$'s are equalized between departments, above equation can be changed into

$$\frac{k}{\sum C_i}\{P_1+P_2+P_3-e(V_1+V_2+V_3)\}=0$$

where k is the magnitude of $C_i/(V_i + P_i)$.

The terms in the bracket of the above equation vanishes because the sum of surplus values, $e(V_1 + V_2 + V_3)$ is equal to the sum of profits, $P_1 + P_2 + P_3$. Note that this is another aggregate equality required in the transformation procedure. Therefore, (7) holds. This is the case in which the proportions of means of production (C_i) to net product or value added ($V_i + P_i$) are equalized between each department. This characterizes Sraffa's standard system(Sraffa, 1960). That is, in standard system, (7) holds.

IV. Critique of Critique

Recently, Ramos-Martinez & Rodriguez-Herrera (1996) tried to refute the argument of double counting. It deserves some attention here because there is few who deny the double counting problem with serious treatment.

Assuming a simple reproduction in a three-department model, they presented a simple numerical example as follows (68).

$$\begin{aligned}&\text{I. } 87.7C_1 + 63.2V_1 + 41.7P_1 = 192.6W_1\\&\text{II. } 57.2C_2 + 28.7V_2 + 40.5P_2 = 126.4W_2\\&\text{III. } 47.7C_3 + 34.5V_3 + 20.9P_3 = 103.1W_3.\end{aligned}$$

In the above system, the constant capital of department I (87.7) can be decomposed into the cost price (68.7) and the profit (19.0) in the

TABLE 1

	Constant Capital Used Up			Variable Capital			Profit			Production Price		
	Sum	Cost Price	Profit	Sum	Cost Price	Profit	Sum	Cost Price	Profit	Sum	Cost Price	Profit
I	87.7	68.7	19.0	63.2	43.0	20.2	41.7	33.2	8.5	192.6	144.9	47.7
II	57.2	44.8	12.4	28.7	19.5	9.2	40.5	32.2	8.2	126.4	96.6	29.8
III	47.7	37.3	10.3	34.5	23.4	11.0	20.9	16.7	4.3	103.1	77.5	25.6
Sum	192.6	150.9	41.7	126.4	85.9	40.5	103.1	82.1	20.9	422.1	318.9	103.1

previous stage of production. Using the notations of this paper, $\alpha_1 + \beta_1 = (87.7 + 63.2)/192.6 \simeq 0.7835$ and $\gamma_1 = 41.7/192.6 \simeq 0.2165$. Therefore, $87.7(\alpha_1 + \beta_1) \simeq 68.7$ and $87.7\ \gamma_1 \simeq 19.0$. In other words, already contained in C_1 is the profit (19.0) of the sector which produced it.

In this paper we decomposed the constant capital contained in C_1 again and regressed infinitely. But Ramos-Martinez & Rodriguez-Herrera (1996) stops here. Instead of regressing to infinity, they apply this method to the variable capital (V_1) and the profit (P_1).[7] In other words, $V_1(63.2)$ is decomposed into the cost price, 43.0 ($\simeq 63.2\alpha_2 + 63.2\beta_2$) and the profit, 20.2 ($\simeq 63.2\gamma_2$). The profit, P_1 can also be decomposed into the cost price, 33.2 ($\simeq 41.7\alpha_3 + 41.7\beta_3$) and the profit 8.5 ($\simeq 41.7\gamma_3$). If this calculation is worked out for all departments, we get Table 1 as a result.

They argue that the vertical sum of the last column of the table clearly indicates that the sum of profits embodied in prices of production corresponds to the sum of profits. The shaded area in the table shows this. For example, they argue that double counting doesn't arise because the profit of department I (41.7) is equal to the sum of profits contained in the constant capital used up in the three departments (19.0 + 12.4 + 10.3).

Using the logic of this paper, the argument of Ramos-Martinez & Rodriguez-Herrera (1996) can be construed in the following manner. Here γ denotes the ratio of total profit to the sum of production prices.

$$(C_1 + C_2 + C_3)\ \gamma_1 + (V_1 + V_2 + V_3)\ \gamma_2 + (P_1 + P_2 + P_3)\ \gamma_3$$

[7]Without doubt, here holds the traditional assumption that wage and profit are only matched to wage goods and luxury goods respectively.

$$= P_1 + P_2 + P_3 = \gamma(\Sigma C_t + \Sigma V_t + \Sigma P_t).$$

Dividing both sides by $\Sigma C_t + \Sigma V_t + \Sigma P_t$, that is the sum of production prices ($\Sigma P_0 P$) and summarizing,

$$\frac{\sum C_t}{\sum P_0 P}\gamma_1 + \frac{\sum V_t}{\sum P_0 P}\gamma_2 + \frac{\sum P_t}{\sum P_0 P}\gamma_3 = \gamma. \tag{8}$$

What they showed is nothing other than the equation (8). What (8) indicates is that the weighted average[8] of the ratio of the profit to commodity price in each department (γ_1, γ_2, γ_3) is equal to the ratio of the sum of profits in the economy to the sum of prices of production (γ). This is self-relevant. But it has nothing to do with the problem of double counting analyzed in this paper.

V. Concluding Remarks

So far, we have shown that the view in which the value transferred from the constant portion of capital should be defined as its purchasing price cannot evade the problem of double counting unless very unreal relation (7) holds. Furthermore, we presented two special cases in which (7) holds, uniform organic composition of capital case and standard system case. But these cases are what Morishima (1973) already pointed out as special cases in which Marx's transformation procedure in volume III of *Capital* does not make an error.

Although nobody argued yet for the compatibility between the two definitions of value, the compatibility condition is worth seeking, at least, on the side of "dualists". Because it can be shown what the theoretical effect of the value definition of "non-dualists" really is. In conclusion, the value definition newly adopted can be refuted by restating one of the key findings of the value debate in 1970s.

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[8]The weight here is nothing other than the relative magnitude of each department in the economy measured by money sum of produced commodities. For example, the weight of r_1 is equal to $\Sigma C_t/\Sigma P_0 P$, because ΣC_t is equal to $C_1 + V_1 + P_1$ under the assumption of simple reproduction.

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