The Value of Time and the Interaction of the Quantity and the Quality of Children

Chengze Simon Fan*

Based on Becker's (1965) insight that consumption is costly in terms of both time and money, this paper extends the existing literature by assuming that the consumption of quantities is relatively time-intensive, while the consumption of qualities is relatively money-intensive. The analysis implies that the quantity of children is positively correlated with income only when incomes are relatively low. When people's incomes increase to a certain threshold level, fertility rate will decrease and will ultimately approach to a certain stable level as incomes rise. On the other hand, the analysis implies that the quality of children is not strongly correlated with income with incomes are low. However, when people's incomes reach a certain threshold level, further income increases will mainly lead to the increases of quality, rather than quantity, of children. Thus, this paper provides a unified theory that simultaneously explains why fertility is positively correlated with income when people's incomes are relatively low and why only the quality, but not the quantity, of children is positively correlated with income when people's incomes are relatively high. (*JEL* classification: J13)

I. Introduction

Almost for the whole human history two hundred years ago, fertility rates and people's income had been fairly strongly positively correlated (Becker 1991; Wrigley and Schofield 1981). In fact, this observation seems to have served as the empirical foundation of the

*Department of Economics, Lingnan College, Tuen Mun, Hong Kong. (Tel) 852-2616-7206, (Fax) 852-2891-7940, (E-mail) fansimon@ln.edu.hk. I am grateful to Robert Pollak, Junsen Zhang, William Chan, Eddy Fung, XiangDong Wei, Changyong Rhee, and an anonymous referee for helpful comments. Any remaining errors are mine.

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"classical" population theory by Malthus(1933). However, this positive correlation between fertility and income disappeared for the more recent human history. For example, the economic growth in the developed countries for the past 150 years has brought about a secular decline of fertility rates, although the fertility rates became stable as they decreased to a certain level (e.g. Schultz 1985; Becker 1991).[1] Also, we do not observe that rich families generally have more children than poor families in developed countries. The evidence of more recent history contradicts the prediction of the "classical" population theory by Malthus(1933) and motivates the "modern" theory of fertility.

One of Becker's important contributions is that he introduces the idea that an individual obtains utility from the quality as well as the quantity of his offspring (Becker 1960), where the quality of children can be interpreted as the children's certain characteristics (e.g. educational attainment) desired by the parents. The exploration of the interaction of the quantity and the quality of children yields the interesting theoretical *possibility that* individuals *may* spend more on the improvement of the quality, rather than on the increase of the quantity, of their children as their incomes rise (Becker and Lewis 1973; Becker 1991). Furthermore, Becker and Tomes(1976) argue that people's income increase will only (mainly) lead to an increase in the quality rather than the quantity of children *if* the quality income elasticity is relatively high and the quantity income elasticity is relatively low. Thus, these models extend Malthus' population theory and significantly improve our understanding of the observed demographic transition experienced in most developed countries. In fact, these models also shed light on the evidence that people's educational attainment improved significantly during the demographic transition (Becker 1991).

However, a more satisfactory theory of fertility demands an intuitive explanation why the quality income elasticity is relatively high and the quantity income elasticity is relatively low in the first place. More importantly, because the evidence suggests that the non-positive correlation between fertility and income is only a recent phenomenon of human history, and in fact, there seems to exist a threshold level of income, before which fertility increases with income, after which quality instead of quantity of children increases with income, a better

[1]For eample, the fertility rates in the U.S. seemed to have remained stable after 1974 (e. g. Aulette 1994).

understanding of individuals' fertility behavior entails a theory that can *simultaneously* explain why fertility is positively correlated with income when people's incomes are low and why only the quality, but not the quantity, of children is positively correlated with income when people's incomes are high.

This paper will try to build such a *unified* framework, based on a seminal contribution by Becker (1965), "*A Theory of the Allocation of Time*," which argues that consumption is costly in terms of both time and money (i.e. material resources). This paper extends Becker's theory and other existing literature by distinguishing the different factor intensities of time input and money input (i.e. material resource input) in the consumption of the quantity and the quality of both material goods and children. More specifically, this paper assumes that the consumption of the quantity of both material goods and children is time intensive, while the consumption of the quality is money intensive. The assumption is explained as follows.

First, as for the consumption of material goods, the assumption is very straightforward. The consumption of the quantity of material goods requires both time input and money input. For example, seeing a movie, travelling abroad, eating food, etc. are costly to an individual in terms of both time and money. Clearly, the more quantity of movies, or travels, or food an individual consumes, the more time input as well as money input is required for his (her) consumption. However, if the individual just wants to consume higher quality of the same amount of goods (e.g. better movie, better accommodation in travelling, better food, better cars, better seat in a basketball game), it will cost him(her) more money, but will not cost him (her) any extra amount of time.

Second, as for the case of children, in much literature on fertility (e.g. Becker 1991), the assumption is commonly made that the "production" of the quantity of children is time intensive, or simply speaking, time consuming. The assumption is both intuitive and strongly supported by empirical evidence. For example, Espenshade (1977) shows that the (opportunity) cost of mother's time contributes about two-thirds of the cost of producing and rearing children. It is necessary for the parents to spend time on raising a child to his (her) adulthood regardless of the quality of the child. Also, mother's time of bearing a child and taking care of her infant is hardly substitutable by money inputs.

Meanwhile, in much literature on human capital theory (e.g. Becker and Tomes 1976; Loury 1981), the assumption is also commonly made that the "production" of the quality of children is money intensive, that is, financially costly. In other words, it emphasizes that the quality of a child is greatly affected by his parents' expenditure on his (her) education, etc. Indeed, if we measure an individual's educational attainment by the number of years of his (her) schooling and the quality of the school that he (she) attends,[2] parents' expenditure on education is clearly the key factor that determines their children's quality. For example, rich parents can give their children more and better education by spending more money to send their children to a good private school, a good college, or to hire a good tutor etc.

Then, the basic logic of this paper is as follows: When people are financially poor, they are "time abundant". In this case, they have plenty of time to complement their scarce material resources to consume. Thus, when people are poor, they are only constrained by their financial resources, but they are not effectively constrained by their time endowments. Therefore, under the conventional assumption that the number of children is a normal good, fertility will increase as people's incomes rise. This prediction is consistent with the observed positive correlation between income and fertility two hundreds years ago.

When their incomes increase,[3] people will become more and more "money abundant", and hence more and more "time scarce", for the time endowment is constant for every individual. Because of the complementarity between time and material resources in consumption, the model shows that when one's income reaches a certain threshold level, *ceteris paribus*, his (her) endowment of time will become a scarce resource. In this case, he (she) will be subject to both time constraint and money constraint.

Because the consumption of the quantities of both material goods and children is time-intensive, the increase of people's consumption of the quantities will be limited by their constant time endowments. On the other hand, because more money input can always increase qualities, the increase of incomes will mainly lead people to increase their expenditure on the qualities, and particularly the qualities of their children. Therefore, the model provide an explanation for the observation

[2]For example, Card and Kruger (1992) shows that school quality plays an important role in individuals' educational attainment.

[3]In this paper, economic growth is modelled as an exogenous increase of a family's income. So, the model is a static one.

that in more recent history, people spend disproprtionately on the quality, rather than on the quantity, of children as their income rise. In other words, this analysis explains why only when people's incomes reach a certain threshold level, the positive correlation between the quantity of children and income turns into a positive correlation between the quality of children and income.

II. The Model

As in much literature on fertility, the basic decision making unit in this model is a family (or a couple). Following Becker (1960, 1991), we assume that a couple obtains utility from two sources: (1) their material consumption, and (2) the quantity and the quality of their children. Let n, and q denote the quantity of the children, and the quality of each child respectively, and let Z represent a vector of material commodities, then, a couple's utility function can be expressed as,

$$U = U(n, q, Z). \tag{1}$$

A couple is assumed to be endowed with Y amount of income (or simply speaking, money) and T amount of time. In this model, we assume that a family's labor supply is inelastic. Thus, we will treat both Y and T as constants. The purpose of this simplification is to allow us to focus on the analysis of how households allocate their resources into different types of consumption. As will be discussed later, relaxing this assumption will be a natural and interesting extension of this model and deserves a separate paper in future research.

Following Becker (1965), we assume that consumption is costly in terms of both time and money. This approach, in fact, has already been applied in some existing literature on fertility (e.g. Willis 1973). The current paper extends Becker's insight and other literature by distinguishing different factor intensities of time and money used in the consumption of quantity and quality. Specifically, as discussed in the introduction, we assume that the consumption of the quantities (of both material goods and children) is time-intensive, while the consumption of the qualities is money-intensive. In the following, we will try to formalize this idea.

First, to specify the production function of the quantity of children, we follow the formulation by Becker, Murphy, and Tumura (1990), who assume that it takes τ amount of time and ν amount of money to produce and raise a child to his (her) adulthood, where both τ and ν are both positive constants. Thus, we can express the production function of the quantity of children as,

$$n = \min\left(\frac{t_n}{\tau}, \frac{x_n}{\nu}\right), \tag{2}$$

where t_n and x_n are the time input and the money input devoted to producing the quantity of children respectively. This formulation is interesting because it seems to be the only simple functional form that emphasizes that both material resources and time are *necessary* to have and raise a child to his (her) adulthood. Specifically, the formulation captures the following important observations: (1) To have and raise a child to his (her) adulthood, it is necessary to spend a certain amount of material resources on the child, regardless of his (her) quality. For example, if a household is too poor to buy enough food, cloth, medicine, etc for a child of theirs, the child cannot survive to his (her) adulthood no matter how much time the parents are with the child. (2) Time input, particularly from the mother, is *necessary* to bear and raise a child (particularly when the child is an infant), no matter how rich the household is. In fact, even when the child is no longer an infant, it is necessary for the parents to have close interaction with the child in order to establish intimate family relationship.

Second, we will try to formulate the production function of the quality of each child. As discussed in the introduction, this formalization will follow Becker and Tomes (1976) and Loury (1981), who emphasize that parents' expenditure on children's education as the key factor that determines children's educational attainment. Specifically, the production function of a child's quality is assumed to take the following form,

$$q = Ax_q^{\sigma}, \tag{3}$$

where x_q denotes the input of money to producing the quality of each child; σ and A are both positive coefficient.

Admittedly, there are some other factors, such as the externality effect of parental human capital or the societal level of human capital, that also affect an individual's "quality". However, it should be noted that these externalities, albeit important, do not compete for the scarce resources with a household's other consumption need. Thus, this consideration is not essential for the purpose of this paper. In fact, because of the positive correlation between income and human capital, we could incorporate these externality effects into the model by assuming the coefficient A is an increasing function of the household's income Y. As will be clear, this consideration will only reinforce the results of the paper.

One may also argue that parents' time input to teach their child can play a role in the child's school performance. However, empirical research in education suggests that parents affect their child's cognitive ability mainly when child is an infant or very little (e.g. Sahota 1978; Van der Eyken 1977). Clearly, during this period, the parents have to take care of the child even only for the necessity of raising the child and establishing intimate relationship with the child. Recalling that we have assumed earlier that it is necessary for the parents to spend τ amount of time on each child for the necessity of raising a child, we might as well interpret that τ includes the period when a child is an infant or very little. When a child grows older, the literature in educational psychology (e.g. Bandura 1989) and in Sociology (e.g. Ballantine 1993) indicates that the effects of the home environment on children's learning is through children's *observation* on parents' behavior (e.g. a comment the parents make on the presidential campaign when watching TV), which can be an interpretation of the externality effect of parental human capital, rather than so much through parents' purposeful teaching. In fact, parents' teaching their children excessively can be even counter-productive because it reduces the opportunities of the children's independent thinking and their making their own decisions.[4]

Now, we will discuss how a family devotes their endowment of money and time to the consumption of material goods. This discussion will be based on Becker (1965), who assumes that a family combines time supplied by family members with goods and services purchased in the market to produce within the household the more "basic commodities" that are the true objects of utility. In fact, this idea is also consistent with Lancaster's (1966) theory that

[4]Alternatively, we can assume that parents' time input after a certain level will not increase their child's quality. However, as will be clear, this alternative assumption would only significantly complicate the exposition, but would not materially change any result of the paper.

individuals obtain utility from the "characteristics" of a good by consuming the good. Clearly, the total "characteristics" of "basic commodities" of a good is determined by both the quantity and the quality of the good. So, we define Z as

$$Z \equiv (Z_1, Z_2),$$

where Z is explained in Equation (1), Z_1 and Z_2 represent the quantity and the quantity of the material goods respectively.

The formulation of the "production functions" of both the quantity and the quality of material goods is similar to the cases of those of children.

First, the consumption of the quantity of material goods requires both time input and money input. As discussed in the introduction, although it may only need money to purchase goods (e.g. by delivery), to consume the goods one must need time. Similar to the discussion of the quantity of children, we also choose the simple formulation that the consumption of the quantity of material goods requires a fixed proportion of time input and money input. For simplicity of notation, we normalize the time reguirement to consume one unit of material goods to be one. Then, we can expressed the "production" function of the quantity of material goods as,

$$Z_1 = \min(t_1, x_1) \tag{4}$$

where t_1, x_1 are the time input and the money input devoted to consume the quantity of the material goods respectively. Intuitively, we may interpret x_1 as the "quantity purchased", and Z_1 as the "quantity consumed".

Second, the input to the consumption of the quality of material goods only includes money. In particular, it is reasonable to interpret the quality of a commodity as an increasing function of the price of the commodity, that is, the higher the price of a commodity is, the better the quality of the commodity is.[5] We might as well assume that there is a single aggregate material good, and let x_2 denote the price of the material good, then, the "production" function of the quality of the material good can be expressed as,

$$Z_2 = Gx_2^k, \tag{5}$$

where G and k are both positive coefficients.

[5]In fact, there is a Chinese saying, "one penny, one quality."

To obtain the closed form solutions of the endogenous variables of the model, we will assume the utility function (1) take the following specific form,

$$U = \ln n + \alpha \ln q + \beta \ln Z_1 + \gamma \ln Z_2, \tag{6}$$

where α, β, γ are all positive coefficients. Clearly, these coefficients represent the weights of the quality of each child, the quantity of the material good, and the quality of the material good relative to the quantity of children in a couple's utility function.

Plugging (2), (3), (4), (5) into (6), and rearranging, we get

$$U = \ln AG + \ln \left[\min\left(\frac{t_n}{\tau}, \frac{x_n}{\nu}\right)\right] + \alpha\sigma \ln x_q + \beta \ln[\min(t_1, x_1)] + \gamma k \ln x_2. \tag{7}$$

Now, we consider a couple's budget constraints. In this model, a couple faces two budget constraints: a time constraint and a money constraint. Recall that we assume a couple is endowed with Y amount of money and T amount of time. Then, the time constraint facing a couple is

$$t_1 + t_n \leq T, \tag{8}$$

the money constraint is

$$x_n + nx_q + x_1 x_2 \leq Y, \tag{9}$$

To explain the money constraint, note that the left hand side of the above inequality represents a couple's total expenditure: the first item, x_n, is the expenditure on the quantity of children; the second item, nx_q, is the total expenditure on the quality of children; the third item, $x_1 x_2$, is the total expenditure on the material good (recall that x_1 is the quantity purchased, x_2 is the price). Thus, the summation of these three expenditures is a couple's total expenditure.

A couple will choose t_1, t_n, x_n, x_q, x_1, x_2 to maximize (7) subject to the time constraint (8) and the money constraint (9). To solve this optimization problem, we need the following technical assumption,

Assumption 1

$$1 > \alpha\sigma, \text{ and } \beta > \gamma k$$

Remark: Recall that α, β, γ represent the weights of the quality of each child, the quantity of the material good, and the quality of the material good relative to the quantity of childrenin a couple's atility function. So, simply speaking, this assumption means that quantity (of both children and material goods) has a heavier weight than quality in a couple's utility function. As we will see, this assumption is important for the model to yield realistic implications when people are poor.

For the clarity of the presentation of the most important part of the model, the optimization problem is solved in the appendix. The analysis yields several interesting implications. The first result is the following lemma.

***Lemma* 1**:
There exists a unique threshold level of income Y^c, such that a household is not effectively constrained by time, or the time constraint (8) is not binding, if and only if the family's income $Y < Y^c$.

Remark: The intuition of this lemma is that when people's are low, they will have plenty of time to complement their scarce material resources for their consumption. Thus, in this case, households are not effectively constrained by their time endowment. Indeed, "time is money," which highlights the opportunity cost of time, is a motto only in the modern affluent society. But for the most part of the human history, most people have lived on merger material resources and have often been on the verge of starvation. For them, the purpose of the activity of their material consumption is just to survive. Clearly, the time devoted to this kind of simple consumption activities is very little. Also, when people are poor, the number of their children who can survive into adulthood is determined by the amount of their material resources. Thus, in this case, people is only constrained by the lack of material resources. In fact, even in modern society, it is difficult to conceive that the very poor (e.g. the homeless) is constrained by their time in their consumption activity.

When the economy develops and people's incomes increase, however, they will become more and more "money abundant," and hence more and more "time scarce," for the time endowment is constant for every individual. Because of the complementarity between time and material resource in consumption, time will be more and more valuable in individuals' utility maximization. Thus, a household's income reaches a certain threshold level, *ceteris paribus*, the household's endowment of time will become a scarce resource. In this case, the household will be subject to a time constraint as well as a money constraint.

Based on Lemma 1, in the following, we will show that people have very different patterns of consumption (in both children and material goods) before and after their incomes reach the threshold level. First, when people's incomes are below the threshold level, Lemma 1 indicates that their consumption (of both material goods and children) is effectively constrained only by the their material resources. In this case, the pure "income effect" implies the following theorem.

Theorem 1

When $Y \leq Y^c$,

$$\frac{dn}{dY} > 0, \ \frac{dq}{dY} = 0, \ \frac{dZ_1}{dY} > 0, \ \frac{dZ_2}{dY} = 0$$

Remark: This theorem, together with its proof in the appendix, implies that when people's income is below a certain threshold level, only the consumption of the quantity (of either children or material goods) increases with income, while the quality remains at a constant or minimal level as income rises. The essence of this theorem is that when people's income is low, their income increase mainly leads to increase of the consumption of the quantities of both material goods of children. In other words, the theorem implies that when people's incomes are low, the income elasticities of quantity (of both children and material goods) are much larger than that of qualities. Thus, this theorem provides an explanation for evidence in the human history two hundreds years ago.

What is the basic logic of the results here? The answer is that (1) the interaction of the log-linear utility function and the non-linearity of the money constraint (which is "quasi log-linear") yields "corner" solutions; (2) Assumption 1 implies that quantities (of both children and material goods) carries a heavier weight than qualities in a couple's utility function (when they are poor). So, the optimal solutions are that the qualities of both children and material goods are "corner" (or "quasi-corner") solutions, while the quantities increase with income. The rigorous proof of the theorem is provided in the Appendix.

One may argue that the result that the qualities of children remain constant as incomes rise is overly strong. However, we have two further comments here: (1) Recall that we have discussed that there may be other factors that also affect an individual's quality, such as the externality effect of the parental human capital or the societal level of human capital. Because income and human capital is positively correlated, an individual's quality will increase with his parental or societal human capital, and hence income even though his parents' expenditure on his education remains constant. In fact, as discussed earlier, we may extend the model to assume that the coefficient A is an increasing function of the household's income Y. Then, both the quality and the quantity of children will increase with income, although, in this case, by the same logic of Theorem 1, the income elasticity of quantity is still much greater than that of quality. (2) The corner solution is mainly generated by the assumption that a couple has a log-linear utility function, which is described by (6). If we assume that the utility function is more convex than the log-linear utility function, then we will get "interior" solutions, which implies that both quantity and quality will increase with income. However, as long as we make a similar assumption as Assumption 1 that quantity carries a heavier weight than quality in a family's utility function, we will still obtain the conclusion that the income elasticity of quantity is much greater than that of quality when people are poor.

Now, we will turn to discuss the case when a household's income $Y \geq Y^c$, that is, when an economy develops and people's incomes reach a cartain threshold level. In this case, Lemma 1 implies that households are subject to not only money constraints, but also binding time constraints. Interestingly and quite surprisingly, the interaction of these two constraints generates the following theorem, which almost exactly reverses the results of Theorem 1.

Theorem 2:

(1) When $Y \geq \max\left(Y^c, \dfrac{(1+2\beta - \gamma k)\nu}{(1+\beta - \alpha\sigma - \gamma k)\tau}\tau \right)$, the quantity of children will decrease as incomes rise, while the quality of children will increase as incomes rise.

(2) When income is sufficiently large, the quantity of children and the quantity of material goods will each converge to a constant level as incomes rise, while the quality of both children and material goods will increase as incomes rise.

Remark: This theorem implies that when people's incomes are above a certain level, they will reverse their previous patterns of consumption in both children and material goods, in particular, they will choose to spend more on the quality of children rather than have more children as their incomes rise. The number of children may continue to rise after a household's income reaches Y^c, but it will certainly begin to decrease when incomes reach a certain new level, and ultimately, the fertility rate will be stable as incomes continue to rise. Therefore, the model provide an explanation for the observation that in the developed countries in more recent history, people spend disproportionately on the quality, rather than on the quantity, of their children as their income rise. Also, this analysis shows that only when people's incomes reach a certain threshold level, the quality income elasticity will be high and the quantity income elasticity will be very low, zero, or even negative.

The intuition of the theorem as follows: Because the consumption (production) of the quantities of both material goods and children need both money input and time input, and because the time endowment of every household is constant, the increase of the household's consumption in quantity will be ultimately limited by its time endowment no matter how much income the household has. Meanwhile, the high (perfect) complementarity between time and money in producing the quantities (of both children and material goods) implies that the material resources devoted to the quantity of children will also be determined by the time constraint, and hence will become stable. Thus, most of the material resources will devote to the improvement of the quality of children when people's incomes continue to increase. In other words, income increases will ultimately lead people to spend mostly in quality rather in quantity. From this interpretation, we can see that the basic results and the intuition of Theorem 2 remain regardless of the choice of the specific utility function.

Meanwhile, although the focus of the paper is on the study of households' resource allocation on children, the paper also provide a model that complements the existing literature (e.g. Stockey 1988; Aghion and Howitt 1992) to explain the observation that economic growth ultimately leads to quality improvement much more than quantity increase of people's material consumption.

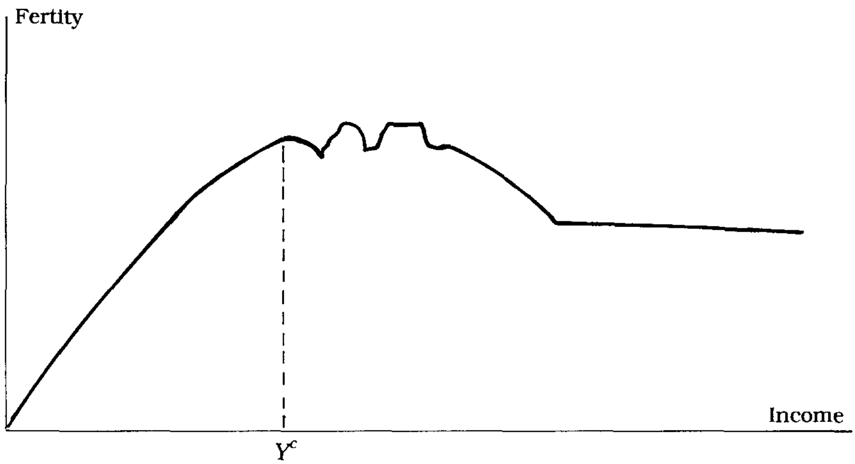

FIGURE 1:
THE RELATIONSHIP BETWEEN FERTILITY AND INCOME

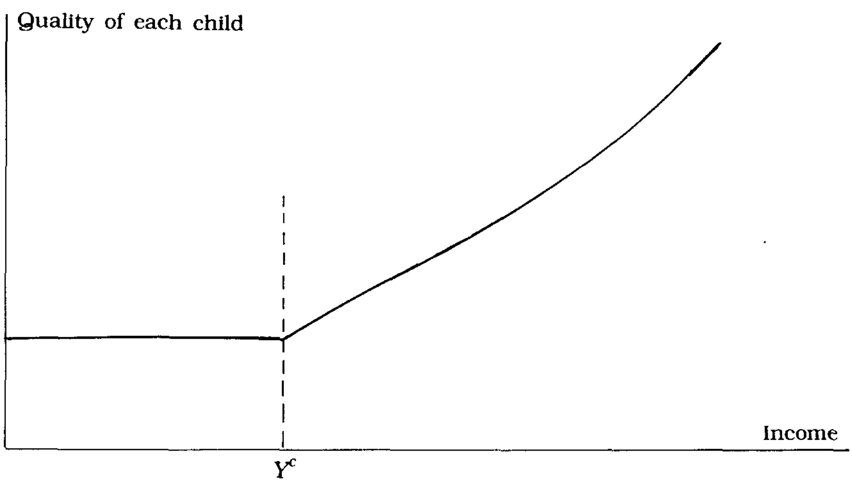

FIGURE 2:
THE RELATIONSHIP BETWEEN THE QUALITY OF CHILDREN AND INCOME

Finally, perhaps the most important contribution of the paper is the combination of Theorem 1 and Theorem 2 (see Figure 1 and Figure 2), which provides a unified theory that simultaneously explains why fertility is positively correlated with income when people's incomes are relatively low and why only quality, but not quantity, of children is positively correlated with income when people's incomes are relativily high. In other words, it explains why after people's incomes reach a certain threshold level, the positive relationship between quantity and income turns into one between quality and income. Thus, this paper complements the existing literature to explain the empirical evidence on the relationship between fertility, and the quality of children, and income.

III. Conclusion

This paper builds on Becker (1965) that the consumption of both material goods and children is costly in terms of both time and money. This paper extends the existing literature by assuming that the consumption of the quantities is relatively time intensive, while the consumption of the qualities are relatively money intensive. The analysis implies that the quantity of children is positively correlated with income only when incomes are low. When people's incomes increase to a certain level, fertility will decrease and ultimately approach a certain stable level as incomes rise. On the other hand, the analysis implies that the quality of children is not strongly correlated with income when people's incomes are low. However, when people's incomes reach a certain threshold level, income increases will mainly lead to increases of quality, rather than quantity, of children. Thus, the paper provides a unified theory that simultaneously explains why fertility is positively correlated with income when people's incomes are relatively low and why only the quality, but not the quantity, of children is positively correlated with income when people's incomes are relatively high. In other words, it explains why after people's incomes reach a certain threshold level, the positive relationship between quantity and income turns into one between quality and income.

In the model, for simplicity, we have assumed that a household's labor supply is inelastic. In future research, we can extend the model to examine households' endogenous labor supply, and particularly the effect of economic development on women's decision on labor market participation, and the feedback on households' choices of the quantity and the quality of children. when womens' wage rates increase and the "gender gap", that is, the difference of wage rates between men and women narrows with economic development (see the evidence by Goldin (1990)), women (i.e. wives) will choose to work in the labor market when their wage rate reaches a certain level. Consequently, a household' total income increases due to the increase of the wife's earning, while the household will have even less time to devote to consumption (of both material goods and children). Since the quantity of children is time-intensive, while the quality of children is money-intensive, people will further increase their expenditure on each child, and reduce the number of children.

Appendix : Mathematical Proofs

Proof of Lemma 1:

From the formulation (2) and (4), clearly, one necessary condition for the optimality of the solutions is

$$\frac{t_n}{\nu} = \frac{x_n}{\nu},\ t_1 = x_1. \tag{A1}$$

Then, the aggregate utility function (7) becomes

$$\begin{aligned} U &= \ln AG + \ln \frac{xn}{\nu} + \alpha\sigma \ln x_q + \beta \ln x_1 + \gamma k \ln x_2 \\ &= \ln \frac{AG}{\nu} + \ln x_n + \alpha\sigma \ln x_q + \beta \ln x_1 + \gamma k \ln x_2 . \end{aligned} \tag{A2}$$

Noticing (2), the money constraint (9) becomes

$$x_n + \frac{1}{\nu} x_n x_q + x_1 x_2 \leq Y \tag{A3}$$

The time constraint (8) becomes

$$\frac{x_n}{\nu} + x_1 \leq Y \tag{A4}$$

To find out the condition that the time constraint is not binding, we will simply maximize (A2) subject to the money constraint (A3). Then, we will check under what condition the solutions will satisfy (A4).

In this case, the Langrangian is

$$L=\ln \frac{AG}{\nu}+\ln x_n+\alpha\sigma\ln x_q+\beta\ln x_1+k\ln x_2+ \lambda\left(Y-x_n-\frac{1}{\nu}x_nx_q-x_1x_2\right). \tag{A5}$$

We denote the price of the material good with the lowest quality by a. Clearly, it is reasonable to assume that $a>0$. So, $x_2\geq a>0$. Then, the Kuhn-Tucker condition is

$$\frac{\partial L}{\partial x_n}=\frac{1}{x_n}-\lambda\left(1+\frac{1}{\nu}x_q\right)\leq 0,\ \text{and}\ x_n\geq 0 \tag{A6}$$

$$\frac{\partial L}{\partial x_q}=\frac{\alpha\sigma}{x_q}-\lambda\frac{1}{\nu}x_n\leq 0,\ \text{and}\ x_q\geq 0 \tag{A7}$$

$$\frac{\partial L}{\partial x_1}=\frac{\beta}{x_1}-\lambda x_2\leq 0,\ \text{and}\ x_1\geq 0 \tag{A8}$$

$$\frac{\partial L}{\partial x_2}=\frac{\gamma}{x_2}-\lambda x_1\leq 0,\ \text{and}\ x_2\geq a. \tag{A9}$$

Any of the above inequalities will hold with strict equality if the solution is interior. From (A8), we have

$$\beta\leq\lambda x_1x_2. \tag{A10}$$

From (A9), we have

$$\gamma k\leq\lambda x_1x_2. \tag{A11}$$

By Assumption 1, $\beta>\gamma k$. So,

$$\gamma k<\beta\leq\lambda x_1x_2. \tag{A12}$$

Namely, the Kuhn-Tucker condition (A9) must hold with strict inequality, which implies the solution is corner, that is, $x_2=a$.

From the utility function (A2), obviously, to maximize the utility, none of the other choice variables will take corner solution (zero). Thus, we can rewrite (A6), (A7), and (A8) as

$$1 = \lambda\,(x_n + \frac{1}{\nu} x_n x_q) \tag{A13}$$

$$\alpha\,\sigma = \lambda\,\frac{1}{\nu} x_n x_q \tag{A14}$$

$$\beta = \lambda\,x_1 x_2. \tag{A15}$$

Noticing (A3), (A13) and (A15),

$$\begin{aligned} \lambda\,Y &= \lambda\,(x_n + \frac{1}{\nu} x_n x_q + x_1 x_2) \\ &= \lambda\,(x_n + \frac{1}{\nu} x_n x_q) + \lambda\,x_1 x_2 \\ &= 1 + \beta\,. \end{aligned} \tag{A16}$$

Thus,

$$\lambda = \frac{1+\beta}{Y} \tag{A17}$$

Plugging this into (A15), and recalling $x_2 = a$, we get

$$x_1 = \frac{\beta\,Y}{\alpha\,(1+\beta)}. \tag{A18}$$

Plugging (A14) into (A13), and rearranging, we get

$$\lambda\,x_n = 1 - \alpha\,\sigma. \tag{A19}$$

Plugging (A17) into (A19), we get

$$x_n = \frac{(1-\alpha\,\sigma)Y}{1+\beta}. \tag{A20}$$

Finally, plugging (A17) and (A20) into (A14), we get

$$x_q\ \frac{\nu\,\alpha\,\sigma}{1-\alpha\sigma}. \tag{A21}$$

Now, we will check the condition when the time constraint is not binding. Noticing (A4), the time constraint is not binding if and only if

$$\frac{x_n}{\nu} + x_1 = \frac{(1-\alpha\sigma)Y}{(1+\beta)} + \frac{\beta\,Y}{\alpha\,(1+\beta)} < T. \tag{A22}$$

Rearranging (A22), we get

$$Y < \frac{a\nu(1+\beta)}{a(1-\alpha\sigma)+\beta(1+\beta)}T. \tag{A23}$$

We denote

$$Y^c \equiv \frac{a\nu(1+\beta)}{a(1-\alpha\sigma)+\beta(1+\beta)}T.$$

Then, we have proved the lemma.

Proof of Theorem 1:

From the proof of Lemma 1, we can see that when $Y \leq Y^c$,

$$x_1 = \frac{\beta Y}{a(1+\beta)},\ x_2 = a,\ x_n = \frac{(1-\alpha\sigma)Y}{1+\beta},\ x_q = \frac{\nu\alpha\sigma}{1-\alpha\sigma}$$

Thus,

$$n = \frac{(1-\alpha\sigma)Y}{(1+\beta)\nu},\quad q = \frac{Y(\nu\alpha\sigma)^\sigma}{(1-\alpha\sigma)^\sigma}, \tag{A24}$$

and

$$Z_1 = \frac{\beta Y}{a(1+\beta)},\quad Z_2 = Ga^\kappa. \tag{A25}$$

Therefore,

$$\frac{dn}{dY} = \frac{1-\alpha\sigma}{(1+\beta)\nu} > 0,\ \frac{dq}{dY} = 0, \tag{A26}$$

and

$$\frac{dZ_1}{dY} = \frac{\beta}{a(1+\beta)} > 0,\ \frac{dZ_2}{dY} = 0. \tag{A27}$$

Proof of Theorem 2:

Noticing (A1) and (8), we can rewrite the utility function (7) as

$$\begin{aligned} U &= \ln AG + \ln\frac{t_n}{\tau} + \alpha\sigma\ln x_q + \beta\ln t_1 + \gamma k\ln x_2 \\ &= \ln\frac{AG}{\tau} + \ln t_n + \alpha\sigma x_q + \beta\ln(T-t_n) + \gamma k\ln x_2. \end{aligned} \tag{A28}$$

Notice that in the above, we already plug the time constraint into the utility function. Also, plugging the time constraint into the money constraint (9), and noting (A1), we get

$$\frac{\nu}{\tau} t_n + \frac{1}{\tau} t_n x_q + (T - t_n) x_2 = Y. \tag{A29}$$

In this case, the Langrangian is

$$L = \ln \frac{AG}{\tau} + \ln t_n + \alpha\sigma \ln x_q + \beta \ln(T - t_n) + \gamma k \ln x_2 + .$$

$$\lambda\,(Y - \frac{\nu}{\tau} t_n - \frac{1}{\tau} t_n x_q - (T - t_n) x_2). \tag{A30}$$

The Kuhn-Tucker condition yields

$$\frac{\partial L}{\partial t_n} = \frac{1}{t_n} - \frac{\beta}{T - t_n} - \lambda \frac{\nu}{\tau} + \frac{1}{\tau} x_q + \lambda x_2 = 0 \tag{A31}$$

$$\frac{\partial L}{\partial x_q} = \frac{\alpha\sigma}{x_q} - \lambda \frac{1}{\nu} t_n = 0 \tag{A32}$$

$$\frac{\partial L}{\partial x_2} = \frac{\gamma k}{x_2} - \lambda\ (\tau - t_n) = 0. \tag{A33}$$

First, noticing (A29), (A32), and (A33), we have

$$\lambda Y = \lambda [\ \frac{\nu}{\tau} t_n + \frac{1}{\tau} t_n x_q + (T - t_n) x_2]$$

$$= \lambda \frac{\nu}{\tau} t_n + \lambda \frac{1}{\tau} t_n x_q + \lambda\,(T - t_n) x_2 \tag{A34}$$

$$= \lambda \frac{\nu}{\tau} t_n + \alpha\sigma + \gamma k.$$

Rearranging (A34), we get

$$\lambda = \frac{\alpha\sigma + \gamma k}{Y - \frac{\nu}{\tau} t_n.} \tag{A35}$$

Second, Rearranging (A31), and noticing (A32), we get

$$1 - \frac{\beta t_n}{T - t_n} = \lambda\,(\frac{\nu}{\tau} t_n + \frac{1}{\gamma} t_n x_q) - \lambda\, t_n x_2$$

$$= \lambda \frac{\nu}{\tau} t_n + \alpha\sigma - \lambda\, t_n x_2\,. \tag{A36}$$

Rearranging (A36), and noticing (A33), we get

$$T - t_n - \beta t_n = \lambda \frac{\nu}{\tau} t_n (T - t_n) + \alpha\sigma(T - t_n) - t_n \lambda x_2(T - t_n)$$
$$= \lambda \frac{\nu}{\tau} t_n (T - t_n) + \alpha\sigma(T - t_n) - \gamma k t_n , \quad \text{(A37)}$$

or

$$T - (1 + \beta) t_n = \lambda \frac{\nu}{\tau} t_n (T - t_n) + \alpha\sigma T - (\alpha\sigma T + \gamma k) t_n . \quad \text{(A38)}$$

Plugging (A35) into (A38), and rearranging, we get

$$\frac{(1+\beta)\nu}{\tau} t_n^2 - \left[\frac{(1+\gamma k)\nu}{\tau} T + (1 + \beta - \alpha\sigma - \gamma k) Y \right], t_n - (1 - \alpha\sigma) T Y = 0. \quad \text{(A39)}$$

Totally differentiating (A39), with respect to t_n and Y, and rearranging, we get

$$\frac{dt_n}{dY} = \frac{(1 + \beta - \alpha\sigma - \gamma k) t_n + (1 - \alpha\sigma) T}{2 \frac{(1+\beta)\nu}{\tau} t_n - \frac{(1+\gamma k)\nu}{\tau} T - (1 + \beta - \alpha\sigma - \gamma k) Y.} \quad \text{(A40)}$$

Recalling Assumption 1, we can see that the numerator of the right hand side (RHS) of (A40) is positive. Thus, dt_n/dY is negative if and only if the denominator is negative, namely

$$2 \frac{(1+\beta)\nu}{\tau} t_n - \frac{(1+\gamma k)\nu}{\tau} T - (1 + \beta - \alpha\sigma - \gamma k) Y < 0. \quad \text{(A41)}$$

Because $t_n \leq T$, a sufficient condition that (A41) is satisfied is the following condition is satisfied

$$2 \frac{(1+\beta)\nu}{\tau} T - \frac{(1+\gamma k)\nu}{\tau} T - (1 + \beta - \alpha\sigma - \gamma k) Y < 0. \quad \text{(A42)}$$

Rearranging (A42), we get

$$Y > \frac{(1 + 2\beta - \gamma k)\nu}{(1 + \beta - \alpha\sigma - \gamma k)\tau} T.$$

Finally, recalling Lemma 1, we get that if

$$Y > \max\left(Y^c, \frac{(1 + 2\beta - \gamma k)\nu}{(1 + \beta - \alpha\sigma - \gamma k)\tau} T \right) \quad \text{(A43)}$$

then

$$\frac{dt_n}{dY} < 0\,.$$

Because $n = t_n/\tau$, the quantity of children will increase with income when the above condition (A43) is satisfied.

Finally, from (A40), noticing that $0 \leq t_n \leq T$, it is easy to see that as Y approaches to infinity, dt_n/dY will approach to zero, which implies t_n, and hence n, will approach to a constant level.

Now, let's see the relationship between x_q and Y. From (A32) and (A33), we can get

$$x_2 = \frac{\gamma k t_n}{\alpha\sigma(T - t_n)} x_q\,.$$

Plugging this into the (A29), and rearranging, we get

$$x_q = \frac{Y - \frac{\nu}{\tau} t_n}{\frac{\alpha\sigma + \gamma k \tau}{\alpha\sigma\tau} t_n}. \tag{A44}$$

Thus,

$$\frac{dx_q}{dY} = \frac{\frac{\alpha\sigma + \gamma k \tau}{\alpha\sigma\tau} t_n \left(1 - \frac{dt_n}{dY}\right) - \left(Y - \frac{\nu}{\tau} t_n\right) \frac{\alpha\sigma + \gamma k \tau}{\alpha\sigma\tau} \frac{dt_n}{dY}}{\left(\frac{\alpha\sigma + \gamma k \tau}{\alpha\sigma\tau} t_n\right)^2} \tag{A45}$$

When the condition (A43) is satisfied, $dt_n/dY < 0$. Also, noticing that

$$Y - \frac{\nu}{\tau} t_n = \frac{1}{\tau} t_n x_q + (T - t_n) x_2 > 0\,.$$

Thus, it is easy to see

$$\frac{dx_q}{dY} > 0\,. \tag{A46}$$

Thus,

$$\frac{dq}{dY} = \sigma A x^{\sigma-1}{}_q \frac{dx_q}{dY} > 0\,. \tag{A47}$$

Finally, when Y approaches to infinity, t_n will approach to a constant level. In this case, from the budget constraint (A29) and the utility function (A28), it is easy to see that x_2 and x_q will be a fixed proportion of Y. So,

$$\frac{dZ_2}{dY} > 0 .$$

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