

Labor Contract, Asymmetric Information and Business Cycles

Ilsoon Shin*

In this paper, a heterogeneous agent model of workers and employers is developed with differing information structures. In particular, the static implicit contract is introduced into otherwise standard dynamic equilibrium business cycle model by embedding the state-contingent wage-hour outcomes from the labor contract with the firm's capital accumulation decision. With this model, it is discussed whether this model's implications are different from those of standard general equilibrium business cycle model and implicit contract model. Compared to the prototype general equilibrium model, the labor contract feature, especially under asymmetric information, improves the predictions for the relative fluctuation and the correlation between hours and real wages. On the other hand, the dynamic general equilibrium consideration also help to overturn the unsatisfactory features in the previous contract theory. (JEL Classifications: J41)

I. Introduction

Business cycle phenomena are typically found in time-series data on total hours and real wages. According to the data, hour fluctuations are considerably greater than wage fluctuations. Furthermore, real wages seem not to have systematic cyclical movement.

*Korea Information Society Development Institute, 1-33 Juam-dong Kwachun, Kyunggi-do, 427-070, Korea. This paper is a modified version of the chapter 1 in my dissertation submitted to University of Rochester. I have benefited from discussions and comments by Mark Bills. Special thanks are due to Tom Cooley for his support as well as numerous suggestions which significantly improved this paper. Any remaining errors are my own. Financial support from National Science Foundation grant SES-9224440 is gratefully acknowledged.

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Cooley and Prescott (1995) report that for the postwar U.S. economy, percentage standard deviations of total work hours are 1.59 and 1.69 according to the household and establishment surveys, while percentage standard deviation in average hourly earnings in establishment survey and average hourly compensation based on National Income Accounts are 0.75 and 0.55 respectively. The correlations of the latter measure of real wage with output is 0.03.

Many recent studies have attempted to explain these phenomena. Most of these studies fall into equilibrium business cycle theory in which business cycles are the result of individual agents optimizing in a competitive environment. Standard real business cycle models are typically based on the intertemporal substitution of leisure to account for large fluctuation in employment with small fluctuations in real wages. This idea, often called the intertemporal substitution theory, stems from the contribution of Lucas and Rapping (1968). Their basic hypothesis is that leisure is easily substitutable across periods so that individuals are willing to supply a large amount of effort in periods of high wages. If this is true, small transitory movements in perceived real wages can have large effects on the path of labor supply. The ability of the intertemporal substitution theory to explain observed fluctuations in employment and real wages thus depends heavily on the presence of strong substitutability in labor supply across periods. However, most of the empirical studies found that the elasticity of intertemporal substitution in labor supply is very low.¹ On the other hand, equilibrium models driven by technology shocks have strong implications that real wages should be procyclical since a shock to productivity drives employment and output, and the real wage is proportional to productivity.

The foregoing arguments suggest the need to develop richer models of individuals' behavior in the labor market. One potential resolution to these problems has been to introduce "indivisible labor" into real business cycle constructs. Assuming that agents' labor supply can take on only one of a finite set of values, Hansen (1985) and Rogerson (1988 consider agents not as selling labor, but as trading lotteries). The indivisibility of labor, along with the trading of lotteries can amplify the variability of aggregate hours relative to productivity as shown by Hansen (1985). While the

¹See Altonji(1986), Ham(1986), and Alogokoufis(1987) among others.

introduction of indivisibility increases the ability of real business cycle models to confront observations, such an introduction does not address the complete set of issues. In particular, since agents do not sell labor but instead trade lotteries in this model, there is no obvious counterpart to a wage rate. Furthermore, the inconsistency of real wage's cyclicity between model and real data remains.

Meanwhile, the above stylized facts related to real wages can be dismissed as irrelevant if, for some reason, real wages do not vary with the marginal productivity of labor. When the real wage is not equal to the marginal productivity of labor, whereas the competitive equilibrium ensures efficient allocation of resources, workers vary their labor supply in response to shifts in the marginal productivity of labor even without wage movements. While one can be agnostic about the behavior of real wages and focus on real allocations (for example, Prescott 1986) based on the above arguments, a more demanding way is to propose some reasons for the divergence, and the investigate real wage movements as well as real allocations in a general equilibrium model.

This paper adopts a different strategy for enhancing the ability of real business cycle models to predict labor market variables. It begins by taking a fairly standard real business cycle model and introduces a labor contract between workers and firms, which is proposed as a reason for the divergence of real wage from marginal productivity of labor. Particularly, risk aversion and asymmetric information are identified as important factors in explaining the employment relationship. The risk aversion is concerned with the following two behavioral characteristics of firms and workers. First, worker's access to the financial market is limited since their main wealth is in the form of human capital, yet the latter cannot collateralize loans. Second, as argued by Azariadis (1975) and Bailly (1974), firms are usually less risk averse than workers. With these two features, firms have an incentive to insure workers against wage fluctuations by a risk-sharing arrangement. More specifically, firms introduce optimal wage-employment contracts that specify compensations and hours for workers. Both the efficient allocation of labor and the efficient allocation of risk will result from this contract, and the latter implies a smooth labor income over business cycles. Smoothing means that the marginal productivity of labor will vary more than real wage, and at any point in time these

two need not equal.

In regard to the asymmetric information, we will assume that firms have an informational advantage in the sense that only they see the ex-post realization of random conditions that affect productivity. A fairly standard incentive problem due to asymmetric information is resolved by having firms introduce an incentive-compatible contract. The efficient allocation of labor and optimal risk sharing are no longer possible to attain simultaneously, but they are traded off with each other. Thus, work hours are typically different from what they would be with symmetric information.

The equilibrium relation between aggregate state variables and wage-hour outcomes from symmetric or asymmetric information contracts are embedded into the consumption-saving problem by the firm owner. In this way, the general equilibrium with labor contracts is modeled, calibrated and investigated numerically using standard techniques.

The optimal contract resolving the risk aversion and asymmetric information have been analyzed in previous "implicit contract" literature, but not well addressed in dynamic general equilibrium models. For examples, while Danthine and Donaldson (1992) and Greenwood and Gomme (1993) consider the resolution of risk by implicit contract features in a real business cycle framework, they do not analyze the optimal contract, nor do they investigate the asymmetric information.² In this paper, hours and wages are determined directly within an optimal contract between firms and workers under either symmetric or asymmetric information. Note that this direct specification of labor contracts has the implication

²Danthine and Donaldson (1992) present a model in which workers are divided into two groups, young and old, with the latter only being covered by a contract. The full employment to the old workers is guaranteed by the assumption of inelastic labor supply, while the young enter and exit the employment relation according to spot market demand but have their income protected through a minimum wage. Therefore it is hard to investigate the role of the labor contract. On the other hand, Greenwood and Gomme (1993) follow the approach in which work hours of both workers and entrepreneurs are determined in an optimizing framework by holding financial securities in a complete market. Then the labor income includes the amount of borrowing-lending that workers carry out together with the usual marginal productivity payment. Thus, the optimal contract is not studied directly. Their methodology implicitly assumes that the introduction of labor contract only change observed factor payment but have no impact on the real allocation, as in Wright (1988).

that the behavior of real aggregate variables as well as factor payments are affected by labor contracts.

Findings are summarized as follows. First, the asymmetric information model outperforms the symmetric information model in predicting fluctuations of labor market variables, while the latter predicts slightly better than the standard business cycle models. Furthermore, the general equilibrium consideration helps to overturn the unsatisfactory features of contract theory, that is, underemployment in the bad states is consistent with normal leisure, and the utility of workers is higher in good states. Second, the acyclical real wage observed in the data is replicated through an aggregation effect when spot market workers are assumed to coexist with contract workers under asymmetric information. Third, the difference between the firm's and the worker's risk aversion is an important parameter affecting the fluctuations of labor market variables, especially hour fluctuations. Furthermore, the sign and magnitude of the effects depends on the information structure.

The paper is organized as follows. Section II presents the model and Section III analyzed static contract solutions. In particular, compared to previous studies, more general cases about the worker's and the firm's preferences are studied and their implications under asymmetric information are discussed. The equilibrium of this economy is described in Section IV, and according to this definition, a dynamic general equilibrium model is parameterized, calibrated and numerically solved in Section V. Section VI reports three results about three different versions of this model: implicit contracts under symmetric information, and asymmetric information, and spot market workers are introduced in the second version of the model. Section VII concludes.

II. The Model

There are two types of agents in this economy, workers and firm owners. Workers are infinitely living agents and we assume that there are n workers for each firm. Workers are assumed not to have access to financial markets. They cannot own shares in firms, nor can they borrow and lend. Thus the worker's preference is defined as $v(b_t, h_t)$, where $b_t - w_t h_t$ is consumption and h_t is labor hours. Two observations suggest this form for preference. First, a

large fraction of the population does not own stocks. Mankiw and Zeldes (1991) report that only one quarter of all households own stocks in the U.S. economy. Second, a significant proportion of households are constrained in cyclical borrowing-lending possibilities (Campbell and Mankiw 1989). With these assumptions just made, we can introduce heterogeneity of agents, workers and firms, without having to keep track of the wealth distribution. We further assume that $v(b, h)$ is twice continuously-differentiable, increasing in b , decreasing in h , and $v_{11} < 0$, $v_{22} < 0$, $v_{12} < 0$.

Firms are owned by infinitely living shareholders, and undertake all investment and hiring decisions. All firms produce a unique commodity with the same constant returns to scale (CRS) technology as described by a production function of the form, $z_t f(k_t, nh_t)$, where k_t represents an individual firm's capital stock, z_t is the economy-wide shock to technology common to all firms and nh_t denotes the total amount of labor services from workers. The shocks z_t lie in some subset Z of R and are assumed to evolve over time according to a first order Markov process, represented by the transition function $G(z', z)$.

Shareholders receive the value of output net of the wage payment. This income of a firm owner is defined as $\pi(k_t, K_t, z_t)$ when individual capital is k_t , and the state of the economy is summarized by the aggregate capital stock K_t and the technology shock z_t . With this notation, the representative shareholder's decision is assumed to solve the following problem:

$$\max_{\{c_t\}, \{k_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (P1)$$

subject to

$$C_t + X_t \leq \pi(k_t, K_t, Z_t)$$

$$k_{t+1} = (1 - \delta)k_t + X_t, \quad k_0 \text{ given}$$

where the two constraints in (P1) are the budget constraint and the capital accumulation equation, respectively. The utility function $u(c_t)$ is assumed to be twice continuously differentiable, increasing, strictly concave, $\lim_{c \rightarrow 0} u'(c) = \infty$, and $\lim_{c \rightarrow \infty} u'(c) = 0$.

The hypothesis of workers' restricted access to financial markets prevents an optimal allocation of risks via financial markets. The lack of markets to diversify the risk in the labor incomes ensures that it will, in general, be optimal for workers and firms to agree to a contract, since without a contract workers in this model consume

all their labor income and are likely to experience substantial income risk. Thus the important feature of the labor contract is that it provides workers with insurance during bad states and firms with a source of extra funds during good states. Given the state of the economy at the end of period t , (k_t, K_t, z_t) , the labor contract $\{b(s_t), h(s_t)\}$ is a pair of functions $b(s_t):S \rightarrow R^+$ and $h(s_t):S \rightarrow [0, 1]$, where S is the space spanning all triples of (k, K, z) , and b, h are wage bill and hour, respectively. Note that each worker is endowed with one unit of time.

Given $\{b(s_t), h(s_t)\}$ determined by this contract process, the firm owners receive $\pi(s_t) = z_t f(k_t, nh(s_t), -nb(s_t))$. Then the firm owners solve the consumption-saving problem described in (P1). In this way, the equilibrium relation between the wage-hour outcome and aggregate state variables is embedded into a standard dynamic general equilibrium model. Note that the introduction of labor contracts will not only change observed factor payment but have an impact on the real allocation. Particularly, the real allocations from (P1) will depend on labor contract $\{b(s_t), h(s_t)\}$, since the latter affects shareholder's income $\pi(s_t)$. In this respect, the present paper follows a different approach from Wright (1988) and Gomme and Greenwood (1992).³

III. Characterization of Labor Contracts

The implicit contract literature teaches us that the crucial properties of contracts depend on the nature of available information. If all individuals are fully informed about the state of nature, they may share risks indirectly by trading in contingent claims markets or directly by writing detailed contingent contracts. In either case, both the productive decisions and the allocation of risks will be Pareto efficient. It is well-known, however, that these properties may fail to hold when information is private, since efficient allocative mechanisms will not provide the individuals with sufficient incentives to reveal truthfully to others information that may be in their exclusive possession. On the other hand, some individuals may find it in their interest to manipulate these mechanisms in their favor by lying. As we will see later, a natural

³See footnote 2.

way around incentive problems is to write contracts with truth-telling. Such contracts, however, restrict the choice set available to individuals under asymmetric information and typically result in second-best allocations.

A. Contract with Symmetric Information

The information structure here is that firms and workers observe the productivity shock. At the time of contract, each worker has an alternative source of income that yields a utility of \tilde{V} .⁴ Thus, no worker will accept a contract unless it guarantees at least \tilde{V} units of expected utility. The contract $\{b(s), h(s)\}$ under symmetric information is defined as a solution of the following problem,

$$\max_{b(\cdot), h(\cdot)} \int_z u(c(s)) G(z, dz) \quad (1)$$

subject to

$$\int_z v(b(s), h(s)) G(z, dz) \geq \tilde{V}.$$

This problem is equivalent to the following optimal contract, which lies on the expected utility possibility frontier :

$$\max_{b(\cdot), h(\cdot)} \int_z \{ \lambda u(c(s)) + (1 - \lambda) v(b(s), h(s)) \} G(z, dz), \quad (P2)$$

where λ picks a particular outcome from Pareto optimal sets. In particular, the magnitude of parameter λ will reflect the bargaining strength of each party. The solutions $\{b^0(s), h^0(s)\}$ are characterized by first order conditions for this contract problem, which are

$$-\frac{v_2(b^0(s), h^0(s))}{v_1(b^0(s), h^0(s))} = z f_2(k, n h^0(s)) \quad (2)$$

$$\frac{v_1(b^0(s), h^0(s))}{u'(c^0(s))} = n \frac{\lambda}{1 - \lambda}. \quad (3)$$

Equation (2) says that the marginal rate of substitution of the worker is equal to the marginal product of labor. It reflects the efficiency condition for the allocation of labor. Equation (3) is the Arrow-Borche condition for the efficient allocation of risk which is

⁴In general, \tilde{V} is determined from alternative wages available to workers, and varies from time to time. For example, the spot market wage rate or institutional feature like unemployment benefits will affect \tilde{V} . But, it is rather complicated to discuss the determination process of \tilde{V} , so that we will regard \tilde{V} as exogenously given by assuming that the an outflow of contract workers is excluded.

generally not satisfied in spot market equilibrium. Some facts should be noted before going on to the asymmetric information case. First, if firms are risk neutral so that $u(c)=c$ and v is separable between b and h , equation (3) implies that labor income, b is equalized across all the states. Second, with symmetric information, there is no conflict between efficiency in the allocation of resources and efficiency in the allocation of risks. Both goals can be accomplished and the result is Pareto optimal *ex ante* and *ex post*.

B. Contract with Asymmetric Information: Benchmark Preferences

Next, let us consider the asymmetric information case in which only firms can observe the technology shock z_i .⁵ Since a two state Markov process considerably clarifies the analysis, we adopt this assumption. Assume that $Z=\{z_B, z_G\}$ with $z_B < z_G$ where the subscripts G and B denote good and bad state. One lesson from the implicit contract literature is that the asymmetric information solutions depend on the specification of preferences, in particular, the difference of the worker's and the firm's risk aversion and the income elasticity of leisure. In this subsection, we will first investigate the characteristics and limits of two well-known cases for preferences because these can give clear analytic results and be used as a benchmark for more general preferences, which will be discussed later.

The first case is when the firm is risk-neutral ($u(c)=c$) and the worker's utility is separable in b and h . This case has been studied by Green and Kahn (1983), Chari (1983), Cooper (1983). The second case is when the firm is risk-averse and the income effect is zero in the worker's preference. Azariadis (1983), and Grossman and Hart (1981) analyzed this case. Then Proposition 1 says that the Pareto-efficient symmetric information contract is *not imple-*

⁵At first, it might look extreme to assume asymmetry in observing an aggregate shock. But the nature of a shock should be distinguished from the informational assumption of agents. Even though an aggregate shock affects an agent directly, he cannot observe this shock (for example, asymmetric information model in monetary policy). Note that the firm tend to have the informational advantage since they organize the production process, while the worker will not effort to have the observation other than to improve the contractual outcome since they consume whatever they earn in this model.

mentable under asymmetric information, and that the timing of the firm's lying depends on preferences.⁶

Proposition 1

(1) If the firm is risk neutral and v is separable, the firm has incentive to lie in the bad state, but not in the good state.

(2) If the firm is risk averse and v assumes no income effect on leisure, i.e., $v_{12}=(v_2/v_1)v_{11}$, the firm has an incentive to lie in the good state, but not in the bad state.

Suppose that the firm were to lie about the true value z . At the time of negotiating the contract, the worker knows that the firm will lie if it has an incentive to do so. The contract designed between the firm and the worker will therefore be such that the firm will not have an incentive to lie. Thus the optimal contract must satisfy incentive compatibility or self-selection constraint,⁷ which are of the form,

$$\int_z \pi(s, b, h) G(z, dz) \geq \int_z \pi(s, b', h') G(z, dz) \quad (4)$$

$$\forall s \in S, \forall b' \in R^+, h' \in [0, 1]$$

Equation (4) indicates that the firm's profits, if it reports the true value of z , are at least as large as if it lies.

Let α denote the probability of good state and set up a two-state version of the labor contract under asymmetric information. The optimal contract with asymmetric information must solve

$$\max_{b_G, h_G} \alpha \{u(c_G) + \lambda v(b_G, h_G)\} + (1 - \alpha) \{u(c_B) + \lambda v(b_B, h_B)\} \quad (P3)$$

subject to

$$z_G f(k, nh_G) - nb_G \geq z_G f(k, hn_B) - nb_B \quad (5)$$

$$z_B f(k, nh_B) - nb_B \geq z_B f(k, nh_G) - nb_G. \quad (6)$$

The two constraints in (P3) are two state versions of the incentive compatibility condition (4).⁸ Next, we will characterize the solutions

⁶All the proofs are in the appendix.

⁷By revelation principle, any allocation resulting from a contract that is not incentive compatible can also be attained from one that is.

⁸We have ruled out intertemporal resolution of the incentive problem. However, the literature suggests that it might be difficult for a firm to lie period after period if reputation effect is taken care of. This result depends on the firms caring sufficiently about the future.

(denoted by asterisk $*$) by means of the Lagrange multipliers μ_G , μ_B of equation (5) and (6).

Lemma 1

- (1) If the firm is risk neutral and v is separable, then $\mu_B^* > \mu_G^* > 0$.
- (2) If the firm is risk averse and v assumes no income effect on leisure, then $\mu_B^* > \mu_G^* > 0$.

By the incentive compatible constraints, the profit of truth-telling and the profit of lying become equalized in the state when the firm tends to lie. This implies that the Lagrange multiplier is strictly positive in that state.

Using the above results, we can show that the "employment problem" also depends on the preference. When overemployment (underemployment) is defined such that the marginal rate of substitution is larger (less) than the marginal product of labor, Proposition 2 shows that the income elasticity is an important factor to the result.

Proposition 2

- (1) If the firm is risk neutral and v is separable, then there is *overemployment* in the *good* state.
- (2) If the firm is risk averse and v assumes no income effect on leisure, then there is *underemployment* in the *bad* state. Furthermore, $v(b_G^*, h_G^*) < v(b_B^*, h_B^*)$.

Proposition 2 also shows limits of both preferences in the following reasons, even though both can give clear analytic solutions. Given that macroeconomists are generally more concerned about inefficiently low employment in bad states, Proposition 2 says that underemployment in bad states occurs only when leisure is a non-normal good.⁹ Furthermore, worker's utility is higher in bad states.

Finally, compared with the symmetric information case, hours variation under asymmetric information is increased only in (case 2) as the following Proposition shows.

Proposition 3

- (1) If the firm is risk neutral and v is separable, then $h_G^0 > h_G^*$ and $h_B^0 > h_B^*$.

⁹Nosal, Rogerson, and Wright (1992) discussed the role of home production in generating underemployment with normal leisure.

(2) If the firm is risk averse and v assumes no income effect on leisure, then $h_G^0 = h_G^*$ and $h_B^0 > h_B^*$.

C. Contract with Asymmetric Information: Further Discussion and General Preferences

Analyzing (case 1) and (case 2) preferences has an advantage to obtain clear results, but it contains some limits. First, non-normal leisure is a necessary condition for underemployment. Second, the worker's utility is higher in the bad state. Finally, the asymmetric information case does not imply a significant increase in hours fluctuations.¹⁰ Thus, it might be interesting to study more general preferences instead of the extreme assumption of zero income effect on leisure or risk neutral firms.¹¹ For this, functional forms for worker's and firm's preferences are borrowed from what have been often used in general equilibrium business cycle models. First, the firm owner's preference is specified as CRRA form, so that firms risk averse. Second, the worker's utility is separable in consumption and hour, so that leisure is a normal good.

$$v(b, h) = \frac{b^{1-\gamma}}{1-\gamma} - \frac{h^{1+\phi}}{1+\phi}$$

$$U(c) = \frac{b^{1-k}}{1-k}$$

where V is the worker's utility, and U is the firm owner's utility. ϕ is fixed to 0.5, which means that (consumption-fixed) labor supply elasticity is 2, and $k \in (0, \gamma)$, where the lower bound implies risk neutrality and the upper bound is set because the firm is assumed to be less risk averse than the worker. The usual subroutine solving nonlinear simultaneous equation is used to derive the symmetric and asymmetric information solutions for $\gamma \in [0.5, 1, 1.5]$ and $k \in [0.1, 0.2, \dots, \gamma]$ for each γ . Several results emerge from this experiment.

Followings are three major results.¹² The first result addresses the effects of the difference between the firm's and the worker's risk aversion on the hour and wage fluctuations under symmetric information.

¹⁰The numerical method and results are explained in Shin(1995).

¹¹To my knowledge there is no study to analyze these intermediate case.

¹²The numerical method and detailed explanations are found in Shin (1995).

Result 1

The difference between the firm's and the worker's risk aversion is *positively* related to the hour fluctuations, and *negatively* related to the wage fluctuation under *symmetric information*.

The second result is about the characteristics of asymmetric information solutions.

Result 2

When the difference between the firm's and the worker's risk aversion is small, the firm has a tendency to lie in *good* states under asymmetric information. Contrary to the cases in Section III-B the asymmetric information contract implies *underemployment in the bad state* even though leisure is a *normal* good.

The next result discusses the effects of the difference between the firm's and the worker's risk aversion on the hour and wage fluctuations under asymmetric information, and it turns out that the effects are drastically different from the symmetric information case.

Result 3

The difference between the firm's and the worker's risk aversion is *negatively* related to the hour fluctuations and *positively* related to the wage fluctuations under asymmetric information. Furthermore, the worker's utility is *higher* in the good state.

Combining these results and summing up, given the preferences based on the calibration exercises, the asymmetric information contract overturns the previous results. In particular, first, normal leisure coexists with underemployment, second, worker's utility is higher in good states. Given the small difference between the firm's and the worker's risk aversion, the asymmetric information contract may yield a significant increase in hour fluctuation.

IV. Equilibrium

The static contract problem is combined with the shareholder's problem to make this economy dynamic. The way these features are combined can be described as follows. At the end of the

previous period (prior to the realization of productivity shock z) firms and workers enter into a contract represented by $\{b(s), h(s)\}$ as a solution of the symmetric information contract problem (P2) or the asymmetric information contract problem (P3). After the exogenous state z is revealed at the beginning of period t , actual employment and wages are determined (these are determined upon the report of firm about z in the asymmetric information case). Following this, goods are produced and sold, and then the income to shareholders $\pi(k, K, z) = zf(k, nh(k, K, z)) - nb(k, K, z)$ is determined given the previous contract. Then the shareholder solves the consumption and savings decisions (P1), which determines next period's capital stock.

The general equilibrium for this economy will now be formulated. To this end, let the state variables be denoted by s where $s = (k, K, z)$. The formal definition of equilibrium is described as follows.

Definition

An equilibrium in this economy is summarized as $\{x(s), h(s), b(s)\}$ such that:

(E.1) The hour-wage pair, $\{b(s), h(s)\}$, solves the static contract problem ((P2) for the symmetric information case and (P3) for the asymmetric information case). Let C denote the space of bounded and continuous functions, and for $s \in S$, define the operator $\Phi(\cdot): S \rightarrow C \times C$ by

$$\Phi(s) = \{h(s), b(s)\} \quad (7)$$

where $\{b(s), h(s)\}$ solves (P2), or (P3).

(E.2) Given k_0 and $\{b(s), h(s)\}$, the investment function $x(s)$ solves the problem (P1). For $\{b(s), h(s)\} \in C \times C$, let us define another operator $\Gamma(\cdot): C \times C \times S \rightarrow C$ by

$$\Gamma\{s, h(s), b(s)\} = x(s), \quad (8)$$

where $x(s)$ solves equation (P1) given the contract solutions, $\{b(s), h(s)\}$. In particular, $x(s)$ is solved from the following:

$$\max_{\{c\} \{x\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c)$$

subject to

$$c + x \leq \pi(s) = zf(k, nh(s)) - nb(s)$$

$$k' = (1 - \delta)k + x, \quad k_0 \text{ given}$$

(E.3) Individual and aggregate quantities coincide, i.e., $k=K$.

Given the definition of equilibrium, the numerical methods are explained as follows. First, with symmetric information, $\Phi(\cdot)$ in the equilibrium definition in (E.1) is approximated log-linearly by solving the first order condition of the contract problem. This gives a log-linear relation of hour and compensation with state variables.¹³

With asymmetric information, the solution is approximated differently since there is no unique relation between hour-wage and state variables. The capital stock and technology shock are discretized according to the way in Section III-C. That is, continuous technology shock is approximated by a two-state Markov chain. The values of good and bad states are 1.025 and 0.975 and $\Pr(z_{G,t+1}|z_{G,t}) = \Pr(z_{B,t+1}|z_{B,t}) = 0.975$. This is consistent with an AR(1) process of $z_{t+1} = 0.95 z_t + \varepsilon_t$ with a standard deviation of innovation of 0.007.¹⁴ Next, the capital stock is discretized around the steady state of this economy. The number of grids is 200, and distance between grids is 0.05. Then using the first order conditions, h_B^* , h_C^* and b_B^* , b_C^* is solved in each (z, k) by a usual subroutine solving nonlinear simultaneous equations. The log of these solutions are then regressed on the logs of state variables to render log-linear approximations, resulting $\Phi(\cdot)$ with asymmetric information.

The next step is to approximate $\Gamma(\cdot)$ given $\Phi(\cdot)$ calculated in the first step. The shareholder's problem is approximated by linear-quadratic methods in Hansen and Prescott(1995). Since this model does not assume representative agent, it is necessary to differentiate the aggregate and individual states.¹⁵ Furthermore due to log linearization in the first step, the log transformation has to be done before approximation.

V. Model Calibration and Numerical Procedure

Since the paper is concerned with the prediction of labor market variables of labor contracts compared to the standard real business

¹³Casual comparison with the solution using the usual subroutine in nonlinear simultaneous equations shows that log linearization is a good approximation.

¹⁴Markov process can be approximated by AR(1) processes. See Tauchen (1985).

¹⁵See Hansen and Prescott(1995) for details.

cycle, we will keep the commonly used parameter values where possible.

First, let tastes and technology be specified in the following way:

$$U(c) = \frac{c^{1-k}}{1-k}$$

$$V(b, h) = \frac{b^{1-\gamma}}{1-\gamma} - \frac{\mu h^{1+\phi}}{1+\phi}$$

$$f(k, h) = k^\theta (nh)^{1-\theta}$$

The stochastic process governing the technology shock G is parameterized as an AR(1) stochastic process with ρ as the autocorrelation coefficient and σ as the standard deviation of the innovation. To compute and simulate the model, values must be assigned to the following parameters:

preferences : ϕ, μ, γ, k
 technology : $\delta, \theta, \beta, \alpha, n$
 shock process : ρ, σ
 contractual relation : λ

Among these parameters, we set $\delta=0.025$, $\theta=0.4$, $\beta=0.99$, $\rho=0.95$, $\sigma=0.007$ as in Cooley and Prescott (1994). The choice of remaining parameters are explained as follows. The relative risk aversion parameter in worker's preference, γ , is set to 1 so that the implied preference is log utility in consumption. This log utility in consumption together with separability between b and h guarantee that income and substitution effects on labor supply cancel out in the long-run so that is no trend in labor supply as observed in the data. γ is also restricted to be larger than firm's relative risk aversion parameter, k . ϕ is an important parameter since the labor supply elasticity is $1/\phi$. Based on the study of McCurdy (1981) that labor supply elasticity for prime-age men is 0.14-0.35, it is adjusted upward to 0.5. The parameter k , which determines the degree of risk aversion of the firm owner, is set less than γ and in $(0, 1)$. In particular, without convincing evidence about k , all values of this range with interval of 0.1 are used for simulations. The number of workers per firm, n is set to be 1. Note that workers in our economy are not only non-stockholders but also constrained in their cyclical borrowing-lending possibilities.

Related to the former, Mankiw and Zeldes (1991) report about 25 percent of all families own stocks, and the literature on consumption smoothing and market incompleteness (e.g. Campbell and Mankiw 1989) reports 20-30 percent of people are constrained in borrowing-lending possibilities. Reflecting these two studies, we set n as 1.

The remaining parameter, μ , will be determined using the condition that steady state hours are one third of total time endowment, which is normalized to 1. The parameter λ represents the relative bargaining power between firms and workers, but this parameter is hard to measure. In principle, λ will vary between lower and upper bound. The lower bound is associated with guaranteeing that the solution of contract problem provides the firms with the same level of expected utility they receive under spot market equilibrium while the upper bound guarantees to the workers their expected utility under spot market equilibrium. Choosing either of these bounds is an extreme assumption, therefore we use a different strategy to calibrate λ . That is, the steady state of the model is assumed to be consistent with the labor contract. Then, both λ and μ are determined in the first order conditions in the contract problem when the variables have steady state value. One important steady-state value is total labor incomes, nb^* . Extending above results on the Mankiw and Zeldes (1991) and Campbell and Mankiw (1989), I assume that number of the worker is half of the population.¹⁶ With this assumption and using income distribution table in Wolff and Marley (1989), the lower 50 percent of people receive about 25 percent of total income. Thus we set $nb^*/y^*=0.25$. The first order conditions are

$$\begin{aligned} [z^*(k^*)^\theta (nh^*)^{1-\theta} - b^* - x^*]^{-\kappa} &= \frac{1}{\lambda} (b^*)^{-\gamma} \\ \frac{n(h^*)^\phi}{(b^*)^{-\gamma}} &= (1-\theta)z^*(k^*)^\theta (h^*)^{-\theta} \end{aligned}$$

From these two equations, we can choose two parameters, λ and μ .

¹⁶In many case, worker can own stocks. Here I made this assumption for the convenience of analysis.

TABLE 1
KEY SECOND MOMENTS OF U.S. ECONOMY AND THE PREDICTION OF
STANDARD RBC MODEL

	Quarterly U.S. Time Series		Hansen's Divisible Labor Economy	
	Standard Deviation	Correlation with Output	Standard Deviation	Correlation with Output
Output	1.72	1.00	1.35	1.00
Consumption	1.27	0.83	0.42	0.89
Investment	8.24	0.91	4.24	0.99
Capital	0.63	0.04	0.36	0.06
Hours	1.59-1.69	0.86-0.92	0.70	0.98
Real Wage	0.55-0.76	0.03-0.68		
Productivity	0.73-0.90	0.34-0.41	0.68	0.98
Labor Income Share	0.80	-0.37	0	0

Notes: 1. First two columns are from statistics reported in Cooley and Prescott (1993) except that the statistics of labor income share are from Gomme and Greenwood (1992). Two measures in hour and productivity are from household survey and establishment survey. Two measures in real wage are from average hourly earnings in establishment survey and average hourly compensation based on National Income Accounts.

2. Last two columns are from Hansen (1985).

VI. Model Evaluations

The performance of three versions of the model are discussed in this section. First, labor contract with symmetric information, second, labor contract with asymmetric information, and finally, labor contract with asymmetric information where spot market workers coexist with contract workers.

Each model's statistics were constructed in the following manner. First, decision rules were computed using LQ methods and the static contract problem described above. Next, 50 artificial samples of 154 observations, the numbers of quarters from 1954:1 to 1991:2, were generated by simulating these decision rules. The data from each sample was then detrended using the Hodrick-Prescott filter. Finally, these sample moments were averaged over 50 simulations undertaken.

Before proceeding with the discussion, key second moments of the U.S. economy reported in Cooley and Prescott (1995) are reproduced in Table 1 to show the following stylized facts about labor market variables. First, the standard deviation of hour and output are almost same. The ratio of hour to output Variation is 0.92-0.98 according to hour measure from household survey and establishment survey. Second, hours fluctuate with small variation in real wage and productivity. The ratio of the standard deviation of hours to that of real wage is 2.09-3.07 and the relative standard deviation of hours to productivity is 1.77-2.31. Hour and productivity measures are from household surveys and establishment surveys, and real wage measures are from average hourly compensation based on National Income Accounts and average hourly earnings in establishment survey. Third, the real wage does not appear to have systematic cyclicity. The correlation between real wage output is 0.03-0.68. The moments of Hansen's (1985) divisible labor are also presented in Table 1 to evaluate the predictions of the standard business cycle model about these stylized facts. Briefly, the standard model predicts a low relative variation of hours to productivity and an extremely procyclical productivity.

A. Symmetric Information Case

The sample moments under symmetric information are shown in Table 2 when k is set to 0.1.¹⁷ The first thing to notice is that the model displays less volatility of aggregate output than what is observed in the data. This is a feature shared with most business cycle models that calibrate the aggregate shock process to the observed sample moments of the Solow residual. Broadly speaking, this model is successful in capturing several stylized facts of the aggregate economy. For example, aggregate consumption varies less than aggregate output while aggregate investment varies more, although the model tends to reduce the extent. Another characteristic of the data is that aggregate variables tend to move together, as measured by large, positive correlation with output. Again, the model captures these broad facts. However, consumption of shareholder is negatively correlated with output, reflecting that shareholders act as insurance agents in this model.

¹⁷Experiments are performed in the range of 0.1-0.9 for values of k , and the case which gives the largest output standard deviation is reported.

TABLE 2
SYMMETRIC INFORMATION CASE

	Standard Deviation	Correlation with Output
Output	1.37(0.08)	1.00(0.00)
Consumption (total)	0.39(0.06)	0.89(0.03)
Consumption (shareholder)	0.68(0.14)	-0.22(0.05)
Consumption (worker)	0.18(0.01)	1.00(0.00)
Investment	2.58(0.19)	0.99(0.00)
Capital	0.35(0.06)	0.08(0.04)
Hour	0.80(0.05)	1.00(0.00)
Real Wage	0.62(0.03)	-1.00(0.00)
Productivity	0.57(0.04)	1.00(0.00)
Labor Income Share	1.19(0.07)	-1.00(0.00)

Notes: 1. functional form of worker's preference and firm's preference as assumed as

$$V(B, h) = \ln(B) - \frac{h^{1+\phi}}{1+\phi}$$

$$U(c) = \frac{c^{1-k}}{1-k}$$

and $\phi=0.5$, $k=0.1$ are assume.

2. The numbers in parentheses are standard errors over 50 simulations.

Next, we will focus on the predictions about moments of labor market variables. Generally, the predictions under symmetric information are not satisfactory. In particular, even though fluctuations of hours increase relative to the real wage and productivity compared to Hansen's divisible labor model, it is still too small compared to the data.¹⁸ Also hours fluctuate too little compared to

¹⁸Direct comparison with Hansen model is not based on the same characteristics of models. Note that the preference for worker specified in this paper will lead to predict a zero variation in hours when spot labor market is assumed. Standard real business cycle models, including Hansen model depend on the intertemporal substitution to explain the labor market variations. On the other hand, the contract model in this paper exclude

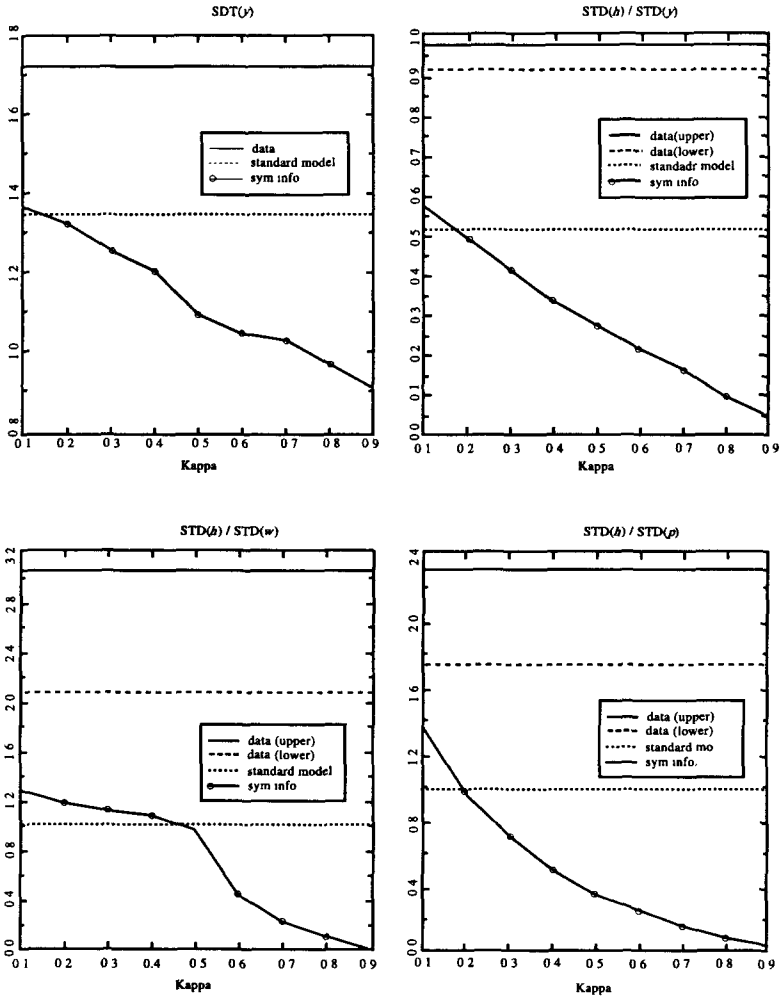


FIGURE 1
SEVERAL MOMENTS IN SYMMETRIC INFORMATION

output. The relative standard deviation of hours to output is 0.58, and the relative standard deviation of hours to productivity and real wage are 1.29 and 1.40, respectively. Note that the latter two mea-

both the spot market and intertemporal substitution assumptions.

tures are different because wages under contract have no direct relationship to marginal (and thus average) productivity. Furthermore, the real wage is perfectly negatively correlated with output.

Figure 1 draws the standard deviation of output, the relative standard deviation of hours to output, real wage and productivity with various levels of risk aversion for firms. It shows that the symmetric information model requires a huge difference in risk aversion to match the data well. The interesting point is that when the difference in risk aversion is large, the model predictions become better in the sense that relative standard deviations of hours to output, real wage and productivity approach to the data moments. This can be explained by Result 1 in Section III-C. (When firms are less risk averse they would provide more insurance to the worker, i.e. firms are willing to accept more fluctuations of the worker's consumption. This implies that the marginal rate of substitution of workers becomes lower (higher) in the good (bad) state. On the other hand, the efficient condition says that the marginal product of labor is equal to the marginal rates of substitution since there is no conflict between efficient allocation of labor and risk under the symmetric information contract. To equate this efficiency condition, hours become higher (lower) in the good (bad) state, therefore hours fluctuate by a greater extent. Note that contrary to Wright (1989) and Greenwood and Gomme (1992), implicit contract features not only lower the variation of real wages directly, but also affect the real allocations including hour fluctuations.

B. Asymmetric Information Case

The sample moments under asymmetric information case are shown in Table 3 when k is set to be 0.9.¹⁹ One noticeable change from the symmetric information case is that the consumption of shareholders is positively correlated with output. This is caused by the fact that the degree of risk aversion of firms is assumed to be large in the asymmetric information case. When firms become more risk averse, the degree of the insurance role they provide is reduced. Since countercyclicality of the shareholder's consumption is caused by this insurance role in the previous section, the larger

¹⁹ k is chosen as the same criterion as before.

TABLE 3
ASYMMETRIC INFORMATION CASE

	Standard Deviation	Correlation with Output
Output	1.52(0.09)	1.00(0.00)
Consumption (total)	0.85(0.07)	0.99(0.00)
Consumption (shareholder)	1.07(0.17)	0.94(0.02)
Consumption (worker)	0.79(0.05)	0.99(0.00)
Investment	3.81(0.18)	0.99(0.00)
Capital	0.27(0.05)	-0.00(0.00)
Hour	0.97(0.05)	0.99(0.00)
Real Wage	0.34(0.02)	-0.82(0.05)
Productivity	0.54(0.03)	0.98(0.00)
Labor Income Share	0.65(0.04)	-0.99(0.00)

Note: 1. functional form of worker's preference and firm's preference as assumed as

$$V(B, h) = \ln(B) - \frac{h^{1+\phi}}{1+\phi}$$

$$U(c) = \frac{c^{1-k}}{1-k}$$

and $\phi=0.5$, $k=0.9$ are assume.

2. The numbers in parentheses are standard errors over 50 simulations.

risk aversion of firm chosen here plays a role to make the shareholder's consumption procyclical.

The asymmetric information case generally outperforms the symmetric information case in explaining the moments of labor markets. The relative standard deviation of hours to output is 0.64, and the relative standard deviation of hours to real wages and productivity are 2.87 and 1.78 respectively. But, the real wage is still too countercyclical. When we compare Figure 1 and 2, asymmetric information case generally better predicts the moments of labor market variables with various of k . Contrary to the symmetric information case, when $k > 0.6$, the model prediction becomes better when firms are more risk averse, but when $k < 0.6$, the inverse occurs. This can be seen in Figure 2. As mentioned in Section III-C, when k is larger than 0.6, the asymmetric information solution is

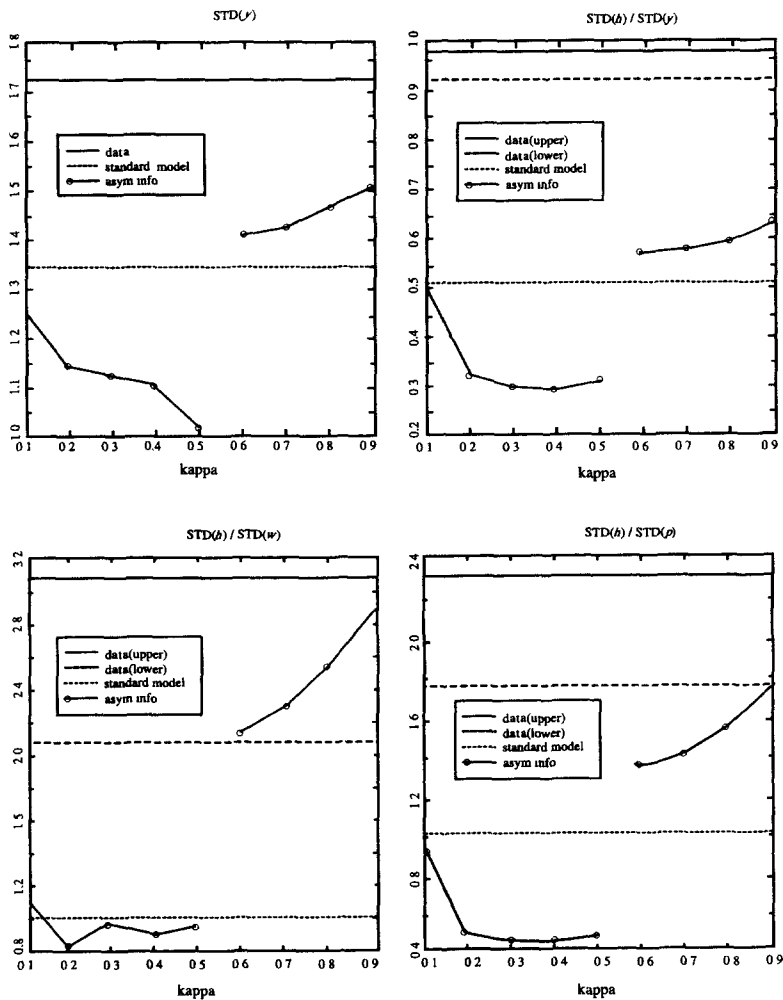


FIGURE 2

SEVERAL MOMENTS IN ASYMMETRIC INFORMATION

qualitatively different from the risk neutral case (Result 2), and in this region, fluctuations of hours tend to be larger when firms are more risk averse to render incentive compatible contract (Result 3).

Without hard empirical studies about the risk aversions of firms and workers, an informal way to evaluate the model is to judge by

the extent to which unreasonable differences of risk aversions are required to generate proper results. In this regard, the asymmetric information case outperforms the symmetric information case. Figure 1 shows that a small k is needed to deliver desirable results under symmetric information. Relative to the symmetric information case, we need small difference in risk aversions to account for labor market fluctuations under asymmetric information.

C. When Spot Market Workers Are Introduced

The real wage is too countercyclical in both models, which is extremely contrary to the prediction of standard RBC models, and is not consistent with the data. In this subsection, we will evaluate the model in which contract workers coexist with spot market workers under asymmetric information. Introducing an additional agent, who is controlled by spot, competitive labor market, may improve the model's ability to replicate the observed acyclical real wage movements since spot market worker wages are proportional to productivity.²⁰ Spot market workers' real wages display procyclical movement since real wages are equal to the marginal productivity of labor, and the latter is positively related to the productivity shock. With the introduction of spot market workers, several modifications are necessary. First, both workers are assumed to have the same preferences. Second, the production function is changed to

$$zf(k, h) = zf(k, \alpha h_1 + (1 - \alpha)h_2),$$

where h_1 denotes the work force organized by implicit contracts, and h_2 denotes the spot market worker. Note that both workers are perfectly substitutable. For simplicity, we assume that the number of both workers are same. With this production function $\alpha/(1 - \alpha)$ is the relative productivity of contract worker to spot market worker and we set this as 4 according to Rios-Rull (1993). Third, the shareholder's budget constraint is changed to

$$c + x \leq \pi(k, K, z) = zf(k, \alpha h_1 + (1 - \alpha)h_2) - b_1 - b_2,$$

²⁰Walter Oi(1984) says that "the heterogeneity in labor market(part being organized under implicit contract and the rest controlled by spot, competitive labor market) must be acknowledged by macroeconomists to understand the cyclical behavior of wages and employment."

where b_i is the wage bill for contract workers ($i=1$), and for spot market worker ($i=2$).

The model moments are reported in Table 4 when $k=0.9$. The results are summarized as follows. First, in regard to labor market predictions, the real wage becomes slightly procyclical. The correlation between wage rate and output is 0.21. The negative correlation in asymmetric information case is improved by the strong procyclicality of spot market worker's real wage, which can be found by impulse response function in Figure 3. On the other hand, hour variation for contract workers become larger compared to the symmetric information case. Notice that for the preferences employed in this section, spot market workers' hours do not fluctuate due to the fact that income and substitution effects cancel each other, and their real wage is strongly procyclical as indicated above. Thus, in good states, firms will hire more contract workers compared to spot market workers since the unit cost of labor (real wage) for contract workers is countercyclical. In bad states, however, firm will hire fewer contract workers for the same reason. These phenomena are translated into larger contract workers' working hours when spot market workers are augmented in the model. The other moments of labor market variables are as good as the asymmetric information case. The relative standard deviation of hours to output is 0.64, while the relative standard deviation of hours to real wages and productivity are 6.2 and 1.72 respectively. Meanwhile, according to the data, the standard deviation of the labor income share is about 0.8, which is similar to the standard deviation of productivity and is in the range between the standard deviations of hours and real wages. Also, observed labor income share is countercyclical as noticed by Gomme and Greenwood (1993).²¹ While the former feature is well duplicated, the latter feature is only replicated qualitatively, even though quantitatively it is far too countercyclical. The correlation between labor income share and output is -0.99 as opposed to the values of -0.37 is the data.

Next, two aspects of data can be investigated from the specific heterogeneity in this model. First, Mankiw and Zeldes (1991) report that the standard deviation of consumption growth of shareholders is about 50 percent more volatile than the standard deviation of

²¹Note that this share is predicted to be a constant number in standard RBC model.

TABLE 4
ASYMMETRIC INFORMATION CASE WHEN BOTH CONTRACT WORKER
AND SPOT MARKET WORKERS ARE PRESENT

	Standard Deviation	Correlation with Output
Output	1.23 (0.09)	1.00 (0.00)
Consumption (total)	0.71 (0.07)	0.99 (0.00)
Consumption (shareholder)	0.78 (0.09)	0.92 (0.03)
Consumption (worker)	0.69 (0.27)	0.99 (0.00)
Investment	2.94 (0.34)	0.99 (0.00)
Capital	0.18 (0.07)	-0.04 (0.09)
Hour (total)	0.77 (0.09)	0.99 (0.00)
Hour (contract worker)	0.79 (0.09)	0.99 (0.00)
Hour (spot market worker)	0.73 (0.09)	0.99 (0.00)
Real Wage (total)	0.06 (0.03)	0.21 (0.04)
Real Wage (contract worker)	0.11 (0.02)	-0.76 (0.09)
Real Wage (spot market worker)	0.46 (0.06)	0.99 (0.00)
Productivity	0.46 (0.06)	0.99 (0.00)
Labor Income Share	0.45 (0.05)	-0.99 (0.00)

Note : 1. functional form of worker's preference and firm's preference are assumed as

$$V(B, h) = \ln(B) - \frac{h^{1+\phi}}{1+\phi}$$

$$U(c) = \frac{c^{1-k}}{1-k}$$

It is assumed that spot market worker and contract worker share the same preference.

2. functional form of production function are assumed as

$$f(k, h) = k^0 (\alpha h_1 + (1 - \alpha) h_2)^{1-\theta}$$

and $\phi=0.5$, $k=0.9$, $\alpha=0.8$ are assume.

3. The numbers in parentheses are standard errors over 50 simulation.

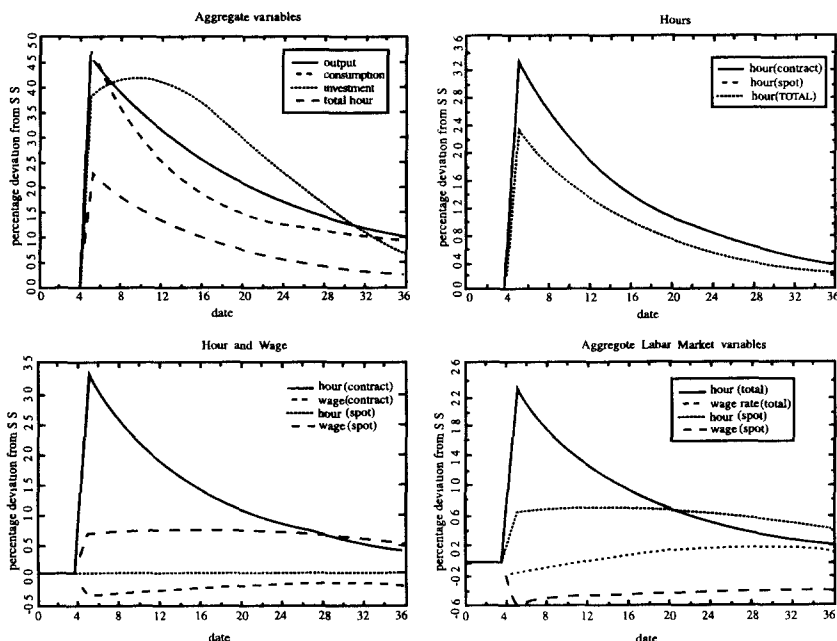


FIGURE 3

IMPULSE RESPONSE FUNCTIONS

consumption growth of non-shareholders. Second, Kydland (1984) and Rios-Rull (1993) report that the fluctuations of hours are inversely related to the level of earnings. In particular, the lowest 20 percent earnings group shows 1.7 times larger standard deviation than the highest 20 percent group (Rios-Rull 1993).

Related to the first, we can compare model moments with Mankiw and Zeldes's result when simulated data is log differenced instead of H-P filtered. The relative standard deviation of shareholder consumption to worker consumption is about 1.36. This is consistent with the number in Mankiw and Zeldes. On the other hand, related to the second point, the ratio of standard deviation between the contract worker's hour and the spot market worker's hour is the opposite of what the data show.

The impulse response of this version of model is in Figure 3. The first panel shows that aggregate variables have desirable responses in the sense that they move together and satisfy the order in

magnitude of response. Each agent's hours are depicted in the second panel. The improvement in the dimension of real wage cyclicity can be seen in the third panel. The spot market worker's real wage goes up about 0.7 percent contemporaneously, and converges to the steady state, while the contract worker's real wage falls about 0.5 percent and again converges to the steady state. As a result of an aggregation of both types of workers, the last panel shows no typical cyclical movement of the aggregate wage. The strong countercyclical labor share can be seen in the last panel with the reduced response of real wage and productivity compared to hours.

VII. Concluding Remarks

In this paper, labor contracts are introduced in a standard equilibrium business cycle model by differentiating firm owners and workers. Risk aversion and asymmetric information are identified to explain the employment relationship. Taking into account that workers are constrained in their cyclical borrowing and lending, the labor contract determining the wage-hour outcome is Pareto-optimal under symmetric information. However, if we assume private information in the sense that workers cannot observe the productivity shock, the optimal contract problem has to involve compatibility constraints. In equilibrium, the firm owner solves the consumption-saving problem given the labor contract, thus the contract outcome affects not only the wage movements but also real allocations including hours.

Under symmetric information, the labor contract implies a smooth labor income over business cycle since the primary role of labor contracts is to allocate risk optimally. Reflecting this, the model's prediction is improved in the dimension of relative fluctuation of hours to real wages compared to standard models such as Hansen's (1985) divisible labor economy. However, other than that, the results about labor market variables are not satisfactory compared to the data. If we place confidence in symmetric information, it suggests that there is some important feature of the economy that is missing.

Under asymmetric information, first, we find qualitatively different results from previous implicit contract literature. By numerical

techniques, underemployment in the bad state is shown to exist with fairly general preferences, which include the commonly-used preferences in real business cycle literature. In particular, leisure does not need to be a non-normal good and utility of workers is higher in the good state under these preferences. Second, the results in the general equilibrium show that both the relative fluctuations of hours to real wages and the relative fluctuations of hours to output are increased compared to the standard business cycle model. Furthermore, the model under asymmetric information predicts labor market variables better than they symmetric information case, and the relative fluctuations of hours to real wage replicates the data successfully. The acyclicity of real wages is accomplished through an aggregation effect when this asymmetric model is augmented with spot market workers.

Without hard empirical studies on the risk aversions of firms and workers, and informal way to evaluate the model is to judge the extent to which unreasonable differences in risk aversion are required to generate proper results. It is shown that under symmetric information, the greater the difference between the firm's and the worker's attitudes toward risk, the better the model predicts labor market fluctuations, while under asymmetric information, a smaller difference leads to better predictions. By these results, this paper suggests that labor contract features, especially under asymmetric information are important factors in understanding labor market fluctuations in general equilibrium business cycle model.

Finally, possible extensions of this model are discussed. First, this model may be extended to construct monetary models.²² This monetary model has an advantage over the Fisher (1977) type long-term contract model since contracts are derived in a welfare maximizing framework. Thus, it would be interesting to augment the asymmetric information contract with the cash-in-advance monetary model such as Cooley and Hansen (1989) and compare the results with Cho and Cooley (1992), which combines the Fisher type long-term contract with the cash-in-advance monetary model.

By permitting mobility between contract workers and spot market

²²By converting static, partial implicit contract model into monetary model, Calvo and Phelps (1977) and Canzoneri and Sibert (1990) derive the positive correlation between monetary innovation and real economic activity. But these models do not introduce capital accumulation.

workers, another extension can be thought of. This is reminiscent of the so-called "insider-outsider" model. Assuming the same productivity for both workers, one characteristic of the insider (the contract worker) is to guarantee his status. In this setting, one may imagine richer dynamics since the contract worker's reservation wage will not be constant but depends upon the spot market wage, which is affected by the states of the economy.

Appendix Proofs

Proof of Proposition 1

Suppose z is a continuous random variable. If the firm lies and announce a value that is slightly higher than z , then

$$\frac{d\pi}{dz} = z f_2 \frac{\partial h}{\partial z} - \frac{\partial b}{\partial z}.$$

where z is the realized value of z . We would like to derive that for the (case 1), $d\pi/dz$ is positive, and for the (case 2), $d\pi/dz$ is negative.

Using first order conditions in (3),

$$\frac{d\pi}{dz} = \frac{v_{11} z f_2 + v_{12} \frac{\partial h}{\partial z}}{v_{11} + u''/\lambda}.$$

For the part of (1), $\frac{v_{11} z f_2}{v_{11} + u''/\lambda}$ is positive when v is separable. For the part (2), let z represent the announced value. $z > z$. By the first order condition,

$$v_{11} z f_2 + v_{12} = v_{11}(-v_2/v_1) + v_{12} = 0.$$

Again,

$$0 = v_{11} z f_2 + v_{12} < v_{11} z f_2 + v_{12}.$$

Thus, $\frac{v_{11} z f_2 + v_{12}}{v_{11} + u''/\lambda}$ is negative.

Next we need to see the sign of $\partial h/\partial z$. Again, using the FOC, it is straightforward to show that the sign of $\partial h/\partial z$ is positive.

Q.E.D.

Proof of Lemma 1

Denoting the solutions of (P3) as $\{b_i^*, h_i^*, \mu_i^*\}$ for $i=B, G$ the first order conditions of are

$$\alpha (nu' (c_G^*) z_{Gf_2}(k, nh_G^*) + \lambda v_2(b_G^*, h_G^*)) = \eta f(k, nh_G^*) (\mu_B^* z_B - \mu_G^* z_G) \quad (A1)$$

$$(1 - \alpha) (nu' (c_B^*) z_{Bf_2}(k, nh_B^*) + \lambda v_2(b_B^*, h_B^*)) = \eta f(k, nh_B^*) (\mu_G^* z_G - \mu_B^* z_B) \quad (A2)$$

$$\alpha (nu' (c_G^*) - \lambda v_1(b_G^*, h_G^*)) = n(\mu_B^* - \mu_G^*) \quad (A3)$$

$$(1 - \alpha) (nu' (c_B^*) - \lambda v_1(b_B^*, h_B^*)) = n(\mu_G^* - \mu_B^*) \quad (A4)$$

where μ_i is the Lagrange multiplier of equations (5) and (6).

First, suppose. $\mu_G = \mu_B = 0$. Then solutions are the same as those under symmetric information. But as shown in Proposition 1, it is not incentive compatible, thus not implementable. Second, suppose $\mu_G > 0$, $\mu_B > 0$.

$$z_G[f(k, nh_G) - f(k, nh_B)] = z_B[f(k, nh_G) - f(k, nh_B)] = n(b_B - b_G).$$

From this, $h_G = h_B$ and $b_G = b_B$, which violate the first order conditions. Thus, either $\mu_G > 0$, $\mu_B = 0$ or $\mu_G = 0$, $\mu_B > 0$.

For the part (case 1), since firm has an incentive to lie in bad state according to Proposition 1, equation (6) should be binding, which means $\mu_G = 0$, $\mu_B > 0$. For the part (case 2), since firm has an incentive to lie in good state according to Proposition 1, equation (5) should be binding, which means $\mu_G > 0$, $\mu_B = 0$.

Q.E.D.

Proof of Proposition 2

Using lemma 1 and first order conditions of asymmetric in (A1)–(A4), it is straightforward to show that $-v_2(b_G^*, h_G^*)/v_1(b_G^*, h_G^*) > z_{Gf_2}(k, nh_G^*)$ in (case 1), that is, marginal rate of substitution is larger than marginal product. With the same analogy, there occurs underemployment in the bad state in (case 2). Note that $v(b_G, h_G) - v(b_B, h_B) = v(b_G, h_G) - v(b_B, h_G) + v(b_B, h_G) - v(b_B, h_B)$. Since $b_G - b_B < z_B[f(k, nh_G) - f(k, nh_B)]$ and $h_G - h_B < [f(k, nh_G) - f(k, nh_B)]/\eta f_2$,

$$v(b_G, h_G) - v(b_B, h_B) < [f(k, nh_G) - f(k, nh_B)](z_B v_1 + v_2/\eta f_2).$$

But we know that $(z_B v_1 + v_2/\eta f_2) > 0$ and $[f(k, nh_G) - f(k, nh_B)] > 0$. Thus, $v(b_G, h_G) - v(b_B, h_B) < 0$.

Q.E.D.

Proof of Proposition 3

In (case 1), from the first order conditions, $n z_{Gf_2}(k, nh_G^*) + \lambda v_2(b_G^*, h_G^*) > 0$ and $n z_{Bf_2}(k, nh_B^*) + \lambda v_2(b_B^*, h_B^*) < 0$. Note that since v is separable, both second terms are only function of h . Since LHS is

decreasing function of h given (z, k) and symmetric information case should satisfy above equations with equality. $h_G^0 > h_G^*$, and $h_B^0 > h_B^*$.

Next let us prove (2). From the first order condition, it is straightforward to derive $-\frac{v_2(b_G^*, h_G^*)}{v_1(b_G^*, h_G^*)} = z_G f_2(k, nh_G^*)$ and $-\frac{v_2(b_B^*, h_B^*)}{v_1(b_B^*, h_B^*)} > z_B f_2(k, nh_B^*)$. But since $\frac{\partial \text{LHS}}{\partial b} = \frac{-v_{21}v_1 + v_2v_{11}}{v_1^2} = 0$ and $\frac{\partial \text{LHS}}{\partial h} = \frac{-v_{22}v_1 + v_2v_{12}}{v_1^2} < 0$, LHS is a function of only h and decreasing. Note that symmetric information case should satisfy both efficiency condition with equality. Thus $h_G^0 > h_G^*$, $h_B^0 > h_B^*$.

Q.E.D.

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