# Note:
# A Further Note on the Theory of Labor Supply with Wage Rate Uncertainty: Comment

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In this *Journal* (1994), Horowitz uses a mean-variance approach to demonstrate that Kim's (1994) comparative statics results concerning the effect of an increase in risk on labor supply established in the framework of the expected utility hypothesis can be also derived when variance is used as a measure of changes in risk. The merit of Horowitz's contribution is that, for small risks, it makes the consistency between two approaches for two-argument utility function more clearly known. However, Horowitz's results obtained in the framework of the mean-variance approach are consistent with prediction reached by Kim *only* when a restriction is imposed on the type of an increase in risk. The purpose of this note is to clarify that Horowitz's conclusions do not hold in general but when a *specific* condition on the random variable in a choice set is assumed or satisfied.

Previous studies indicated that variance is a perfect indicator of risk when the agent is characterized by quadratic utility or when comparing two normal or two uniform distributions in the Two-moment decision models. Three-moments ranking is consistent with expected utility maximization if the agents' preferences are restricted by a cubic polynomial form of the utility function or if the distributions defined on the positive domain represent the first-order stochastic dominance relationship and have equal means.

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However, Meyer (1987) shows that, in one-argument utility function, the expected utility framework is consistent with the mean-standard deviation one if the location and scale parameter condition is satisfied and confirms that it holds in many economic decision models. The results of Meyer can be viewed as ones which improve moment-based decision models in one direction, in the sense that the condition described below does not require any special restrictions on the form of the utility function or on the distribution of random terms. The following definition of Meyer (1987) refers to a formal specification of the location and scale parameter condition.

***Definition***

Two cumulative distribution functions $F_1$ and $F_2$ are said to differ only by location and scale parameters $\lambda_1$ and $\lambda_2$ if $F_1(X)=F_2(\lambda_1+\lambda_2 X)$ with $\lambda_2>0$ for all $X$, where $X$ is the random variable.

It should be noted that in Kim's paper there are two types of increases in risk. One is a particular type of an increase in risk accomplished by linear deterministic transformation used by Sandmo (1971) such as $w=\overline{w}+\gamma\varepsilon$, where $\varepsilon$ represents a random variable with mean zero. Since $c=m+wL=m+(\overline{w}+\gamma\varepsilon)L=\lambda_1+\lambda_2\varepsilon$, where $\lambda_1=m+\overline{w}L$ is a location parameter and $\lambda_2=\gamma L$ is a scale parameter, the members of the choice set will differ only by location and scale parameters and a mean-preserving linear deterministic transformation satisfies the location and scale condition. Therefore, the effect of such a change in $\gamma$ can be also analyzed under the mean-standard deviation framework. The other is the Rothschild-Stiglitz increase in risk which does not satisfy the location and scale condition. Note that the results presented in equation (9) of Kim's paper are *different* from those of Horowitz and *may not* be derived from the mean-variance approach. Therefore, one of Rothschild and Stiglitz's (1970) conclusions that the variance is not a proper measure of increases in risk still holds, but when the location and scale condition holds, the variance can be a proper measure. Variance as a measure of risk variations should be carefully used.

This note reemphasizes here that to ensure the consistency between the ranking based on the expected utility maximization and the ranking based on the moment rule, the type of an increase in risk which differs only by location and scale parameters, not a

general definition of increased risk, termed a mean preserving spread, introduced by Rothschild and Stiglitz, should be assumed and shows that the consistency between two approaches in a model using two-argument utility function has not yet been fully demonstrated in two-moment decision models.

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## References

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