Debt Financing with Limited Liability and Quantity Competition

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In this paper we extend some results of Brander and Lewis (1986). In particular, we show that their main theorem, as it is stated, is not true. We provide the correct and extended version of that result and also discuss how this approach can be used to probe into some well known Industrial Organisation results. (*JEL* Classification: L13)

I. Introduction

Till the mid 1980s, the industrial organisation literature assumed that in choosing its competitive strategy the firm's objective is to maximise total profits. The finance literature, on the other hand, focussed on maximisation of equity value while generally ignoring product market strategy. The linkages between financial and output market decisions were largely ignored until Brander and Lewis (1986).

Models of oligopoly dealing with the interlinkages between financial structure and product market decisions of firms (with limited liability) started appearing in the literature over the past decade and a half. This literature mainly evolved around the pioneering paper of Brander and Lewis (1986). Abstracting from the well known determinants of capital structure, these models show that firms with limited liability choose a positive amount of debt in

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equilibrium. Leverage makes a firm more aggressive in quantity competition and this gives debt a strategic advantage.

Brander and Lewis (1986) consider a homogeneous product duopoly in which financial and output market decisions follow in a sequence. They show that any firm with limited liability, competing in a Cournot framework with an exogenous demand shock, would always find it optimal to become leveraged. The *limited liability* nature of debt forces a firm to behave more aggressively in the product market. In recent years a number of papers like Maksimovic (1988), Glazer (1994), Campos (1995), Showalter (1995) and Dastidar and Sengupta (1998) have formalized the ways in which product market decisions may both influence and be influenced by corporate financing decisions.

Though Brander and Lewis (1986) is very influential, it has several weak points. For example, their main proposition, as it is stated, is not true (see also Campos 1995). However their model can be used successfully to analyse several issues in Industrial Organisation. In this paper we attempt to tie the loose ends of such an approach and derive certain results. In particular we generalise the Campos (1995) counterexample and provide the correct and extended version of the main Brander and Lewis (1986) result. Later we also briefly discuss some features of the model which can be applied in many areas of Industrial Organisation.

The plan of the paper is as follows. First we spell out the framework of our analysis in Section II. In Section III we present the model. In Section IV we give our principal results and in the last section we provide the concluding remarks.

II. The Framework

We closely follow the structure of Brander and Lewis (1986). We consider a homogeneous product, symmetric cost duopoly where each firm with limited liability is owned by a group of risk neutral shareholders. It is assumed that firms can raise funds in a competitive capital market. In such a capital market, a group of risk neutral debtholders with identical outside options, is willing to supply firm i with a loan with face value D^i, which is payable once the product market profits materialise.

In stage 1 each firm chooses a level of debt in order to maximise its expected total market value, where total value is equity value plus debt value. Debt is understood, in general, as any kind of monetary obligation which the firm must pay back before dividends can be distributed to shareholders. In the second stage the firms choose output levels taking as given the debt levels chosen in the first stage. Here Brander and Lewis (1986) assume that the manager of the firm is free to choose whatever output level he desires after debt is issued. In the second stage output is chosen to maximise expected returns to the shareholders. The equilibrium concept is sequentially rational Nash equilibrium in debt levels and output levels. In other words, the second stage outcome is a Cournot equilibrium in output which is correctly anticipated by firms when choosing debt levels in the first stage. The output decisions of firms are made before the realisation of a random variable reflecting variation in demand. Once profits are determined, firms are obliged to pay debt claims out of operating profits, if possible. If profits are insufficient to meet the debt obligations, the firm goes bankrupt and its assets are turned over to the debtholders.

In this set up and under certain general assumptions Brander and Lewis derive two basic results. (i) In a subgame perfect equilibrium, both firms will always select a strictly positive level of debt. However, Campos (1995) has shown that this result is not necessarily true. In fact Campos provides a counterexample to illustrate this point. (ii) The second result is that the leveraged firm in the product market is more aggressive relative to an unleveraged firm.

III. The Model

Consider a homogeneous product duopoly with symmetric costs. Let $R^i(q^i, q^j, z^i)$ denote the revenue function of the ith firm, where $z^i \in [\underline{z}, \bar{z}]$ is a random variable with distribution function $F(z^i)$ and density function $f(z^i)$. z^i and z^j are independently and identically distributed.

The following assumptions are made.

(a) $R^i(q^i, q^j, z^i)$ is concave in q^i.

(b) $R^i(q^i, q^j, z^i)$ and $MR^i(q^i, q^j, z^i)$ are decreasing in q^j and increasing in z^i. The uncertainty is resolved only after the output decisions have been made by the firms.

Let D^i be the face value of the debt raised by firm i.

(c) Our formulation assumes that the firm has a limited liability contract vis-a-vis the debtholders, that is, the debtholders receive whatever is left over from the revenue, net of the costs incurred in the production stage upto a maximum of D^i. Here it may be noted that for simplicity we have assumed that the asset value of the firm is zero, as if assets are completely used up in the production of output. Creditors can, therefore, collect only current operating profits if the firm becomes insolvent.

In the second stage firms choose quantities to maximise the shareholders' expected return (the equity value). For firm i this is equal to the following:

$$V^i = E[\max\{R^i(q^i,\ q^j,\ z) - D^i,\ 0\}].$$

As before, in the above expression $E[\cdot]$ stands for the expected value. The above indicates that after production and sales take place and the uncertainty regarding the firm's revenue is settled, the firm is obliged to pay creditors D^i out of its current revenue. If the firm is unable to meet its debt obligations, its creditors are paid whatever revenue is available and the shareholders get zero. Note that

$$V^i = \int_{\hat{z}^i}^{\bar{z}} \{R^i(q^i,\ q^j,\ z) - D^i\} f(z) dz. \tag{1}$$

In the above $\hat{z}^i(q^i,\ q^j,\ D^i)$ is the critical bankruptcy threshold of z^i such that firm i's revenue is just enough to repay its outstanding debt. That is, we have the following:

$$R^i(q^i,\ q^j,\ \hat{z}^i) = D^i. \tag{2}$$

The following may be noted.

$$\frac{d\hat{z}^i}{dD^i} = \frac{1}{R_z^i(\hat{z}^i)} > 0, \tag{3a}$$

$$\frac{d\hat{z}^i}{dD^j} = 0, \tag{3b}$$

$$\frac{d\hat{z}^i}{dq^i} = \frac{-R_i^i(\hat{z}^i)}{R_z^i(\hat{z}^i)}, \tag{3c}$$

$$\frac{d\hat{z}^i}{dq^j} = \frac{-R_j^i(\hat{z}^i)}{R_z^i(\hat{z}^i)}. \tag{3d}$$

In the second stage the debtholders[1] can expect to be paid back the following amount, which is the debt value of the firm.

$$W^i = \int_{\underline{z}}^{\hat{z}^i} R^i(q^{i^*}, q^{j^*}, z^i) f(z^i) dz^i + D^i(1 - F(\hat{z}^i)). \tag{4}$$

In the above q^{i^*} represents the equilibrium choice of output in the second stage. The first term in (4) represents the revenue of the firm in states of the world when this revenue is insufficient to completely cover debt obligations. The second term represents those states of the world in which the creditors of the firm are paid in full.

In the first stage the firms choose D^i to maximise $(W^i + V^i)$, where $i = 1, 2$. In the second stage they choose q^i to maximise V^i, where $i = 1, 2$.

IV. The Results

A. Characterisation of the Second Stage Equilibrium

The first order condition for the second stage equilibrium is as follows.

$$V^i_i = \int_{\hat{z}^i}^{\bar{z}} R^i_i(q^i, q^j, z^i) f(z^i) dz^i = 0. \tag{5}$$

As usual, subscripts denote the partial derivatives. The second order condition is that $V^i_{ii} < 0$. It may be noted that

$$V^i_{ij} = \int_{\hat{z}^i}^{\bar{z}} R^i_{ij}(q^i, q^j, z^i) f(z^i) dz^i + f(\hat{z}^i) \frac{R_j^i(\cdot) R_i^i(\cdot)}{R_z^i}.$$

From the assumptions it follows that the first term of the above is negative and the second term is positive. Hence the sign of V^i_{ij} is uncertain. It may be noted that the slope of the second stage reaction function is $-V^i_{ij}/V^i_{ii}$. From the second order condition the denominator is negative but the numerator's sign is not known. Hence the sign of the slope of the numerator is uncertain. Brander and Lewis (1986) assume that the slope will be negative but it *does not follow from the primitives of their model.* This has been noted independently by Campos (1995) and Dastidar (1993). In fact, Dastidar and Sengupta (1998) provide a specific numerical example

[1] It may be noted that we assume debtholders to be risk neutral.

to illustrate this point. Note that had the firms been completely equity financed (i.e. $D^i=0$) then the reaction functions would have been downward sloping, given our assumption of $R^i_{ij}<0$.

***Remark* 1**

The standard literature indicates that any outcome in oligopolistic interaction depends crucially on whether products are strategic substitutes or complements (see Bulow *et al.* 1985). For example we can say that for output competition, reaction functions are downward sloping if the products are strategic substitutes whereas they are upward sloping for strategic complements. The shapes of the reaction functions in general will depend on the primitives of the models including the nature of competition (that is price or quantity competition), the form of the demand curves, the cost structures and others. Our analysis suggests that such classification may no longer remain valid when one considers the possibility of debt financing by the firms.[2] This can create problems for the analysis since the extent of debt financing typically cannot be taken as a primitive of any model but has to be solved for in the context of a particular model.

Let $r^i(q^j)$ be the second stage reaction function of the ith firm. Let $q^{i*}(D^i, D^j)$, where $i=1, 2$, denote the Cournot equilibrium output levels in the second stage. We now give our first result.

***Proposition* 1**

(i) $r^i(q^j)$ is increasing in D^i.
(ii) dq^{i*}/dD^j has the same sign as V^i_{ij}.
(iii) $dq^{i*}/dD^i>0$.

***Proof*:**

(i) $r^i(q^j)$ is the solution in q^i of the following, which is the first order condition.

$$\int_{\hat{z}^i}^{\bar{z}} R^i_i(q^i, q^j, z^i)f(z^i)dz^i=0$$

$$\Rightarrow \int_{\hat{z}^i}^{\bar{z}} R^i_i(r^i(q^j), q^j, z^i)f(z^i)dz^i=0. \tag{6}$$

[2]Dastidar and Sengupta's (1998) example shows that products which are strategic substitutes under equity financing may become strategic complements under debt financing.

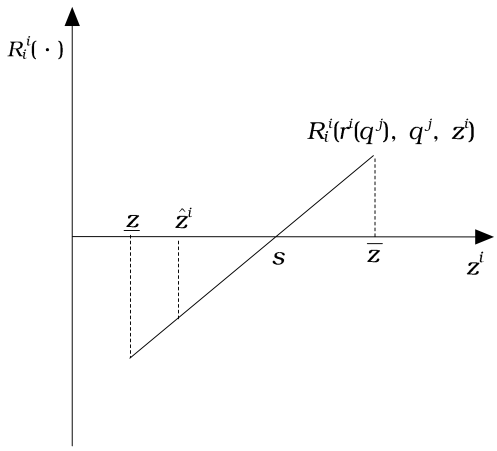

FIGURE 1

Since $R_i^i(\cdot)$ is increasing in z^i from (6) we have $R_i^i(r^i(q^j), q^j, \hat{z}^i) < 0$ and $R_i^i(r^i(q^j), q^j, \bar{z}) > 0$. Now R_i^i is continuous in z^i. Therefore given q^j and hence $r^i(q^j)$,

$$\exists\, s \in (\hat{z}^i, \bar{z}) \text{ such that } R_i^i(r^i(q^j), q^j, s) = 0.$$

Therefore we have

$$\int_{\hat{z}^i}^{s} R_i^i(r^i(q^j), q^j, z^i) f(z^i) dz^i < 0,$$

$$\text{and} \quad \int_{s}^{\bar{z}} R_i^i(r^i(q^j), q^j, z^i) f(z^i) dz^i > 0.$$

From (6) we also have

$$\int_{\hat{z}^i}^{s} R_i^i(r^i(q^j), q^j, z^i) f(z^i) dz^i = -\int_{s}^{\bar{z}} R_i^i(r^i(q^j), q^j, z^i) f(z^i) dz^i.$$

Figure 1 portrays $R_i^i(r^i(q^j), q^j, z^i)$ as a function of z^i (given q^j).

Now, $\hat{z}^i$ is increasing in D^i (see 3a). In Figure 1, note that this means $\hat{z}^i$ shifts to the right and comes closer to s. Now for equation (6) to be satisfied, $R_i^i(r^i(q^j), q^j, z^i)$ must shift down. This is only possible if $r^i(q^j)$ increases as $R_i^i(\cdot)$ is decreasing in q^i. Hence our claim follows.

(ii) Since $\hat{z}^i$ is unaffected by D^j, $r^i(q^j)$ does not change with D^j. Now $r^i(q^j)$ will be positively (resp. negatively) sloping depending on whether $V_{ij}^i > 0$ (or < 0). Now $r^j(q^i)$ shifts to the right with an increase in D^j. Therefore q^{i^*} increases (resp. decreases) with D^j if $r^i(q^j)$ is upward sloping (resp. downward sloping).

(iii) Since $r^i(q^j)$ shifts to the right with an increase in D^i and $r^j(q^i)$ is unaffected by any change in D^i this follows. Hence the proposition is proved.

Q.E.D.

***Remark* 2**

The above result indicates that any unilateral increase in D^i unambiguously increases the equilibrium output of firm i. However the effect on the equilibrium output of firm j is uncertain. Proposition 1 also vindicates the Brander and Lewis result that debt financed firms with limited liability are more aggressive in quantity competition than firms which are completely equity financed.

B. Characterisation of Subgame Perfect Equilibrium

In the first stage the firm i chooses D^i to maximise (W^i+V^i). Now we have the following (see Brander and Lewis 1986).

$$\frac{d(W^i+V^i)}{dD^i} = \int_{\underline{z}}^{\hat{z}^i} R_i^i(\cdot)f(z^i)\frac{dq^{i^*}}{dD^i} + \int_{\hat{z}^i}^{\bar{z}} R_i^i(\cdot)f(z^i)\frac{dq^{i^*}}{dD^i} + \left[\int_{\underline{z}}^{\bar{z}} R_j^i(\cdot)f(z^i)\right]\frac{dq^{j^*}}{dD^i}. \tag{7}$$

The main result of Brander and Lewis states that firms will always choose zero debt in equilibrium. However, as noted before, Campos (1995) provides a counterexample to refute this. Propositions 2 and 3, given below, give the correct and extended version of the main Brander and Lewis (1986) result.

***Proposition* 2**

If $V^j_{ji}\geq 0$ then for firm i the optimal debt choice is always zero.

***Proof*:** The second term of (7) is zero (from the first order condition, see (5)). Since $dq^{i^*}/dD^i>0$ (Proposition 1) and since $R_i^i(z^i)<0$ for all $z^i\leq\hat{z}^i$, the first term of (7) is negative. Now $R_j^i(\cdot)<0$. Therefore the last term will be nonpositive if $dq^{j^*}/dD^i\geq 0$. Now $dq^{j^*}/dD^i\geq 0$ iff $V^j_{ji}\geq 0$ (from Proposition 1). Therefore we have if $V^j_{ji}\geq 0$, then $d(W^i+V^i)<0$. This implies that optimal debt choice by the ith firm will be zero.

Q.E.D.

***Proposition* 3**
The optimal debt choice will be positive only if $dq^{j^*}/dD^i<0$.

Proof: Straightforward.

***Remark* 3**
Proposition 2 generalises the Campos (1995) counterexample and also points out the weakness of the Brander and Lewis result. Now $V^j_{ji}\geq 0$ implies that the *j*th firm's product, with debt financing, is a strategic complement to the *i*th firm's product (since reaction functions are upward sloping). So, if debt makes the rival's actions strategic complement then a firm is better off by being completely equity financed.

***Remark* 4**
Proposition 3 shows that the *i*th firm will choose a strictly positive level of debt in a subgame perfect equilibrium only if $V^j_{ji}<0$ (That is only if the *j*th firm's product remains a strategic substitute with debt financing).

We now provide an example to show the *restrictive nature* of one of the assumptions of the Brander and Lewis model.

Example
Consider a homogeneous product duopoly with zero costs. The demand function is given by $P=(A-q^i-q^j)z$ where $z\in[1,\ 2]$ with uniform distribution. Now consider a two stage game in our framework. Here

$$V^i = \int_{\hat{z}^i}^{2} \{q^i(A-q^i-q^j)z-D^i\}dz.$$

In the second stage the first order condition for equity value maximisation is

$$\int_{\hat{z}^i}^{2} (A-2q^i-q^j)zdz = 0 \;\Rightarrow\; r^i(q^j)=\frac{1}{2}(A-q^j).$$

Here note that $r^i(q^j)$ does not depend on D^i and hence is unaffected by any change in it. In this example debt does not make any difference. The reason is that the assumption that R^i_i is increasing in z is violated. In our example, R^i_i is increasing in z if it is positive and decreasing in z if it is negative. The point is to show the restrictive nature of the assumption as it rules out any demand function with a multiplicative random variable.

V. Conclusion

In this paper we derive certain results that attempt to tie the loose ends of Brander and Lewis (1986) model. In particular we provide the correct and extended version of their main result. Their framework, despite its weaknesses, provides an useful approach to analyse many oligopolistic interactions.[3]

An example of this is Dastidar and Sengupta (1998). They recast the entry deterrence model of Dixit (1980) in a debt financing framework. They show, with a specific example, that a completely equity financed incumbent firm will not keep excess capacity to deter entry. In fact, in their example, the incumbent firm (when completely equity financed) is unable to deter entry. However, if the same incumbent is allowed debt financing, it chooses debt and excess capacity strategically and deters entry successfully.

The framework can also be utilised to analyse many issues in trade. For example, Dastidar and Sengupta (1998) show, with the help of two examples, that if the foreign firm is completely equity financed then a tariff on its output reduces imports (as normally expected). However if the same foreign firm is debt financed with a certain face value of debt then imports *may* rise as a result of tariff imposition. In fact, effects of debt financing with limited liability offer exciting scope for research in the future.

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[3]However, the Brander and Lewis (1986) framework is not robust. In a companion paper Dastidar (1998) has shown that a slight change in the game can result in drastically different results.

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