공학박사학위논문

Mathematical Modeling of Cake Filtration Using Filter Number and Experimental Verification

FN을 이용한 케이크 여과의 수학적 모델링과 실험적 검증

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Doctor of Philosophy

Mathematical Modeling of Cake Filtration Using Filter Number and Experimental Verification

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Abstract

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Recently, dewatering has received great attention in many fields, including; particle separation, wastewater sludge treatment, pharmaceutical industries, wineries and food waste management in which solid and liquid separation is important. Among the several types of sludge dewatering techniques, the most widely used is cake filtration. In cake filtration a positive pressure is applied to force liquid through the cake and filter medium. The cake filtration can be divided into two phases; filtration phase and expression phase. In filtration phase liquid pass through the cake and filter medium while a free suspension is still left on top of the cake. However

the expression phase focuses in the removal of the water by squeezing the cake. In expression phase the sludge and filter resistances are stabilized. For decades, the performance of cake filtration has been modeled based on the cake resistance and/or filter blockage. However further study and development of less complicated model is required to appropriately assess the relationship between the cake and the filter resistance, especially filter clogging, as well as the conditions of both the filtration and expression phases.

In this study, a new model for evaluating the filtration and expression phases of a one-dimensional cake filtration process is proposed. The effect of the filter~~ing~~ medium resistance was considered applying energy law and using a new concept of filter number (FN). The effects of the governing parameters on the model predications for both the filtration and expression phases were also investigated. Based on sensitivity analysis of the model on the governing parameters, during the filtration phase the effect of the cake hydraulic conductivity was found to be negligible whereas the compacted sludge layer and filter medium plays the primary role. On the other hand, the hydraulic conductivity of the filtration cake is the dominant factor during the expression phase. Moreover the findings of the model sensitivity analysis are also used to make some cake filtration suggestions.

Four filtration experiments with two types of filter at two pressures

(300 kPa and 500 kPa) were performed for wastewater sludge from Gawchon wastewater plant and repeated (totally 8 tests). The Experimental data obtained in this study were used to verify the proposed model. The experimental data and model prediction were compared and regression analysis was made to find the agreement and disagreements part of the model with data. Based on observations form laboratory tests, we found that the new model provides a mean relative error (MRE) of 21%, 31%, 28% and 29% between predicted and observed values of the filter types I and II at 300 and 500 kPa respectively.

The model can be used to predict the time for achieving the certain percent of sludge dewatering which can be very useful in many industrial applications. Pressure and time in the real operational field means energy and money. In some cases, pressure is the limiting factor and sometimes time is the limiting factor, depending on the type of sludge and condition. Based on the demand to achieve the certain degree of dewatering and also by considering the limiting factors of the company, it is possible to plot dewatering operational graph based on model prediction into the most economical way that the company can have.

Keywords: Sludge dewatering, Cake filtration, wastewater, filter number

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Nomenclature

Symbols	Parameter	Units
A	Cylinder area	m^2
A_f	Filter medium area	m^2
A*	Total open area of filter	m^2
AOS	Aperture opening size	-
a	Filter opening ratio	-
α_0	Initial specific resistance of cake	m^{-2}
α_{av}	Average specific resistance of cake	m^{-2}
(a)	Point (a) located on top of sludge	-
(b)	Point (b) located immediately after filter	-
D	Expression phase coefficient	m^2
dh	Sludge height drop during time dt	m
dt	Time variation	s
Δh	Sludge height drop during Δt	m
Δt	Time step of numerical solution	s
FN	Filter Number	-
FN_0	Filter number of a clean filter	-
FN_c	Filter number during expression phase	-
E_a	Total energy head at point (a)	m
E_b	Total energy head at point (b)	m
ΔE_{cake}	Head loss due to cake resistance	m
ΔE_{filter}	Head loss due to filter medium resistance	m
f	Filter medium loss coefficient	-
g	Gravitational acceleration	ms^{-2}
h	Sludge height	m

Symbols	Parameter	Units
h_0	Sludge initial height	m
h_i	Cake height at time i	m
h_{i+1}	Cake height at time i+1	m
μ	Viscosity of the fluid phase	Pa.s
m	Clogging coefficient of filtration phase	-
n	Material characteristics for compressibility	-
L	Cake length	m
k	Hydraulic conductivity	ms^{-1}
k_s	Hydraulic conductivity of filtration phase	ms^{-1}
$\bar{k}_c$	Hydraulic conductivity of expression phase	ms^{-1}
Q	Filtration discharge	ms^{-3}
t	Filtration time	s
t_c	Shifting stage from filtration to expression	s
t_i	Filtration time at i moment	s
t_{i+1}	Filtration time at i+1 moment	s
P	Applied pressure	Pa
P_L	Liquid pressure	Pa
P_s	Solid stress	Pa
P_a	Scaling factor	-
U	Filtration velocity	ms^{-1}
Z	Elevation of a point above a reference	m
γ	Specific weight of the fluid	Nm^{-3}

CHAPTER 1:

INTRODUCTION

1.1. Background

The treatment of wastewater and/or contaminated soil is a well-known process in environmental and geo—environmental engineering. Several techniques have been developed in this regard and executed for a long time. However, large amounts of sludge are produced in this process. Subsequently, management of the produced sludge is a challenging issue for environmental engineers. The direct disposal of the sludge to the environment has been found to impact on the environment and human health, due to the presence of heavy metals, pathogens and pesticides in the sludge (Singh et al., 2004).

Sludge dewatering is a physical (mechanical) unit operation used to reduce the water content of sludge so that it can be handled and/or processed as a semi-solid instead of as a liquid. The unite processes that are most often used for sludge are: (1) Cake filtration, (2) Vacuum filtration, (3) Centrifugation and (4) Drying beds (Spellman, 1997). Recently, dewatering has received great attention, in the civil and environmental engineering fields.

The significance of sludge dewatering is summarized in following (Metcalf & Eddy, 1991):

- The cost of transporting sludge to the ultimate disposal site is greatly reduced when sludge volume is reduced;
- Sludge dewatering allow for easier handling;
- Sludge dewatering (reduction in moisture content) allows for more efficient incineration;
- If composting is the beneficial reuse choice, dewatered sludge decreases the amount and therefore the cost of bulking agents;
- With the USEPA's new 503 rule, sludge dewatering is required to render the sludge less offensive;
- When land filing is the ultimate disposal option, dewatering is required to reduce leachate production.

Dewatering is used in a variety of fields including sewage and wastewater (Robles et al., 2013; Yuan et al., 2011), agriculture (Szögi et al., 2006), pharmaceuticals (Golla and Johnson, 2006), food waste (Mohmoud et al., 2010) or tofu sheet (Xia et al., 2003), wineries (Rayess et al., 2011), mineral processing (Pearse, 2003), construction and dredging (De Maeseneer, 1997; Hajjar et al., 1998), pulp and paper production (Chen et al., 2002; Rojas and Hubbe, 2004), and biotechniques and bioreactors (Grassia et al., 2008).

Of the several types of dewatering methods available, the most widely used and simple method is cake filtration dewatering. When compared with other dewatering techniques, such as vacuum filtration, centrifuging or thermal dewatering (evaporative processes) for water reduction, cake filtration dewatering is often selected due to its simple application, high efficiency and low energy requirement (Vaxelaire et al., 1999).

In cake filtration, a positive pressure is applied whereas the cake and filtering medium resist it (Fig.1.1). As the liquid phase flows through the sludge particles and filtering medium, the height of sludge, h drops with time. The cake filtration can be divided into two phases: i) Filtration phase: the clogging of filter medium starts right after the filtration process starts but for a short time. The retention of the solid particles by the filtering medium results in the small particles of sludge accumulates on the surface of the filter medium and creates “Skin Layer”. The skin layer acts like a filter and raises the sludge resistance dramatically during filtration phases. In this phase, a free suspension is still left on top of the cake (Curves et al 2007), (Fig 1.1.a). ii) Expression phase: After disappearance of the free suspension, the remained liquid left in the cake pores is forced out from the cake. This phase is called the expression phase or cake compression phase (Curves et al 2007). The expression consists in the removal of the water by squeezing of the cake

(Oliver et al, 2007). In this phase, the cake has already been formed and both the sludge and filter resistances have stabilized (Fig 1.1.b).

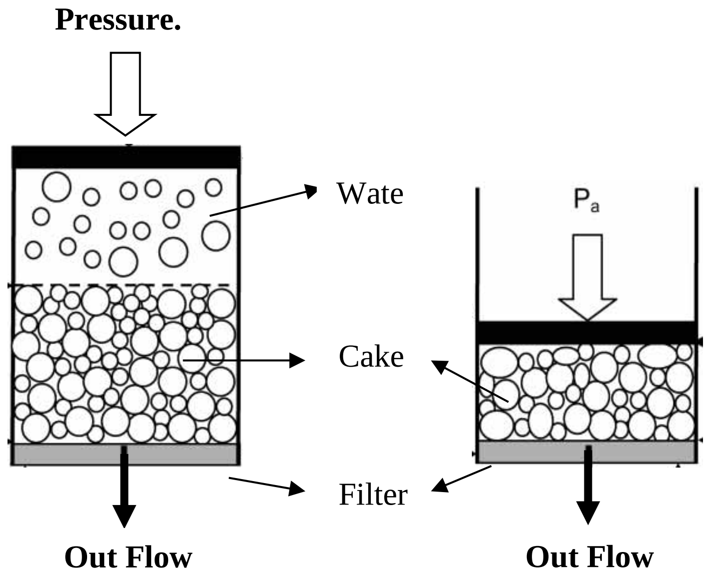


Figure 1.1 Filtration and expression processes

The objective of cake filtration modeling is to correctly predict the change in sludge height with time under the physical-chemical conditions of the sludge and the applied pressure. Despite the relative simplicity of the process, the design and the verification of the installations are still largely empirical (Oliver et al, 2007).

Cake filtration modeling has been a subject of interest for decades, and many models have been proposed by researchers from different scientific backgrounds, especially in chemical engineering, soil mechanics, civil and environmental engineering. Some detailed review papers on the subject have been published (Wakeman 1981, Shirato et al 1985, Lee and Wang 2000). Considering that the performance of cake filtration depends on both the property of cake and filtering medium it is necessary to appropriately assess the constitutive relationship between the cake and the filter resistance, especially filter clogging as well as the conditions of both the filtration and expression phases. However the existing models do not provide adequate information about the resistance and the changes caused by clogging. In the majority of works on cake filtration modeling, the applied medium resistance is based on a filter medium resistance coefficient R_m, using the Darcy equation, which does not adequately represent the effects of the filter on the filtration process.

The purpose of the present study is to develop a new cake filtration model of a one-dimensional, vertical filtration process mathematically and to verify with laboratory experimental cake filtration data. The model considers the energy losses made by both cake and filter resistances. The formulation of the governing equation of the model entailed the Bernoulli principle, Darcy's law and the continuity condition, resulting in a differential equation. In order to take the medium resistance into consideration in the model and obtain a numeric solution, a new dimensionless number termed the Filter number (FN) was introduced, which can represents the filter properties (e.g.

the opening size and shape) during cake filtration process. The equation was numerically solved for the filtration phase, and analytically solved for the expression phase. The effects of the governing parameters on the filtration and expression phases were also analyzed. The application of the new model to predict the filtration time at different physical and chemical condition is investigated on wastewater sludge management and operational diagram for predict wastewater dewatering time is developed.

1.2. Research objectives

This work aims to

1. To developing an analytical cake filtration model for one dimensional, one side filtration to simulate and predict the change of sludge height with time under the physical-chemical conditions of the sludge and the applied pressure.
2. To introduce dimensionless Filter Number (FN) to characterize filter parameters both for clean filter and clogged filter during cake filtration.
3. To analysis the model sensitivity to governing parameters.
4. To evaluate the performance of the developed model with filtration experimental data on wastewater sludge.
5. To investigate the applications of the developed model on predication of

the wastewater sludge dewatering and suggest operational diagram for wastewater treatment proposals.

1.3. Dissertation organization

This dissertation consists of 6 chapters (Figure 1.2): The first chapter deals with introduction about general cake filtration back ground and objectives of this study. Chapter 2 provides a detail literature review on existence cake filtration modeling research interest and statues. Chapter 3 presents the proposed model derivation and model sensitivity to governing parameters. Chapter 4 consists of model verification with laboratory experimental data and model error analysis. Chapter 5 contains the model application to wastewater sludge dewatering technology. In this chapter the some operational graph is developed to predict sludge dewatering time. Finally, chapter 6 gives a summary of the dissertation and concludes overall the study and is mentioned the limitations and further study.

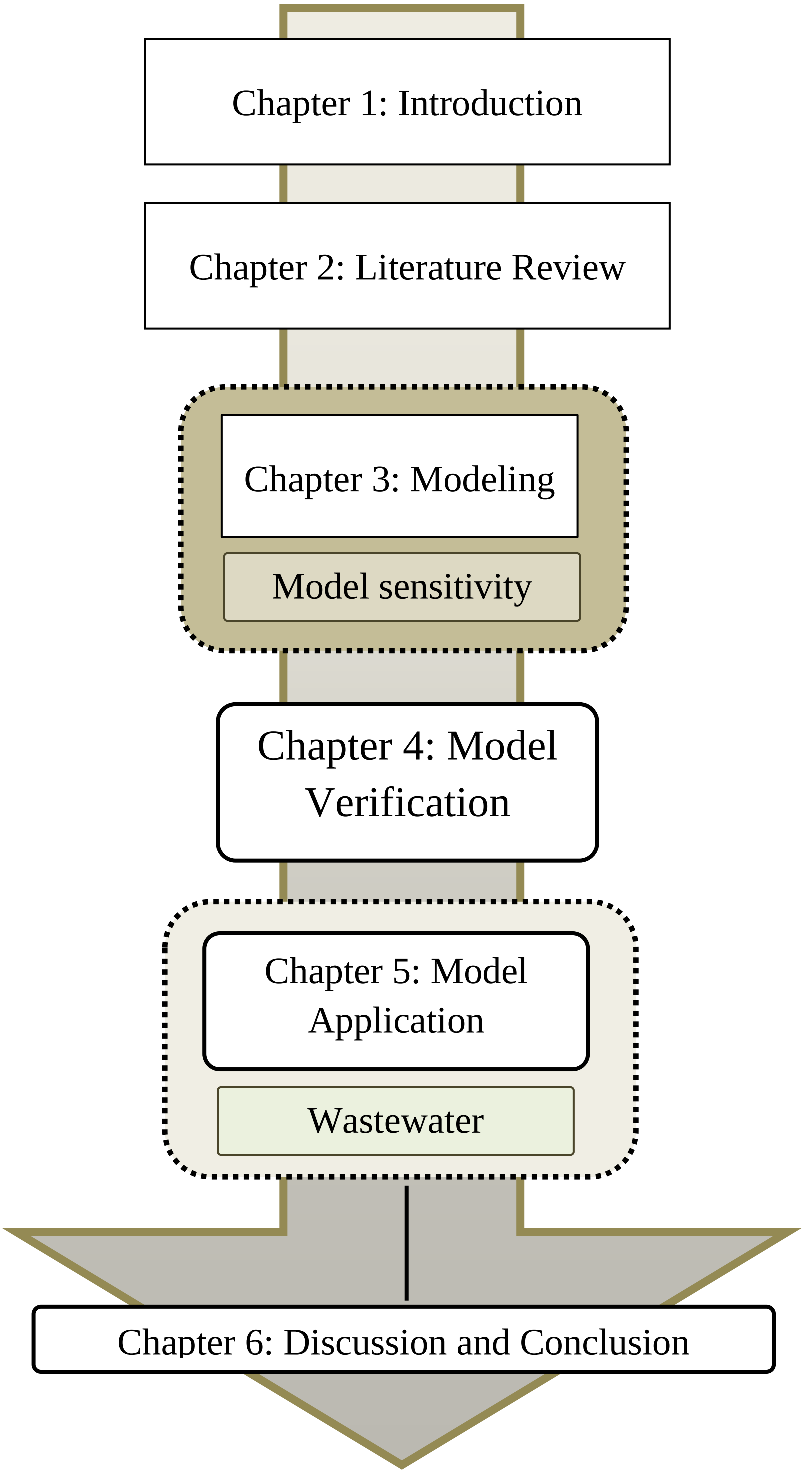


Fig 1.2. Structure of the Dissertation

CHAPTER 2:

Literature Review

2.1. Introduction

Modeling of cake filtration has been a matter of interest for decades, and many filtration models from different scientific backgrounds, especially in chemical engineering, soil mechanics, civil and environmental engineering. Some detailed review papers on the subject have been published (Wakeman, 1981, Shirate et al, 1985, Lee and Wang, 2000, Yu¨kseler et al, 2005 and J.Olivier and Vaxelair 2007). Overall the different filtration models are all derived on the same basis with fundamental equations of continuity equation and momentum balance, and material properties parameters of compressibility and permeability. The discrepancies between the different approaches are essentially due to the way of these equations and parameters derived (J.Olivier and Vaxelair 2007).

Modeling of sludge filtration and expression can be divided into five groups. The five methods are described extensively by Lee and Wang (2000) and Oliver and Vaxelair (2007). The following section provides a summary of the exiting cake filtration model.

2.2. Conventional Modeling (Specific resistance)

The conventional filtration and consolidation theory originates from Ruth's pioneering work (Ruth et al., 1933a,b; Ruth, 1935, 1946), The model is based on analogy with Ohm's law and the total resistance is the sum of the cake and filter medium resistance:

$$\frac{dh}{dt} = \frac{1}{\mu\alpha_{av}v}\frac{\Delta P}{h} + \frac{\Delta P}{\mu R_{fm}} \qquad (1)$$

WhereΔP is the total pressure drop across the whole system (cake + filter medium), V_f is the volume of filtrate, μ is the liquid viscosity, v is the ratio between the volume of cake deposited and the volume of filtrate collected, α_{av} is the cake average specific resistance in volume expressed in m^{-2}, and R_{fm} is the resistance of the filter medium (m^{-1}). Tiller, Shirato and coworkers widely developed the contemporary two-resistance theory on the basis of compressible cake filtration. Wakeman (1981a,b) reviewed pertinent literature prior to 1981. He et al. (1997a,b) later reviewed related investigations on filter cake characteristics.

The development of conventional theory consists of two steps: (1) combining the mass balance equation and the momentum balance equation (Darcy's law) for liquid phase in the cake to form the governing equation with both porosity and liquid pressure as dependent variables and (2) assuming that only point contacts exist between particles. With the assistance of some empirical equations related to solid pressure and porosity, the number of dependent variables in the differential equation obtained from step (1) reduces to only one, usually representing the solid pressure. Under appropriate initial/boundary conditions, the governing equations can be analytically or numerically solved. Modeling works in previous literature differ in terms of the different constitutive equations and boundary/initial conditions used (Lee and Wang, 2000).

As it is mentioned conventional modeling is widely based on two key parameters, the specific cake resistance (α), and the solid pressure (PS), to describe the liquid flow through the cake and the consolidation stages, respectively. The notion of specific cake resistance comes from pioneer works on filtration mechanics, such as the ones of Ruth et al. (Ruth et al., 1933a,b; Ruth, 1935) whereas the origins of the solid pressure concept mostly refer to soil mechanics field. The solid pressure (also called “compressive stress” (Tiller and Huang, 1961, Shirato et al., 1969, Stamatakis and Tien 1991, Wu 1994, Lee et al 2000), compressive drag pressure (Tiller and Green 1973), cumulative stress or accumulative drag pressure (Tiller and Crump 1977, Tiller et al. 1981), contact pressure (Sorensen et al. 1996), structure stress (Sorensen et al. 1997) represents a core concept in conventional filtration modeling. It considers that a force (F_S)

due to the flow of the liquid through the filter cake is transmitted by friction with the solid particles. This force transmitted by the liquid to the solid builds up from particle to particle, up to a maximum at the filter medium level. At the same time, the liquid loses some of its energy and its pressure decreases to reach its lowest at the filter medium level (Oliver et al 2007).

The average specific cake resistance (α_{av}) of the filter cake, which is the quantitative measurement of the filterability of suspension. It is assumed that the flow rate is uniform throughout the entire cake in filtration of dilute suspension. With this assumption, a_{av} is defined by the expression (Tiller et al. 1960)

$$\alpha_{av} = \frac{P_S - P_m}{\int_0^{P_S - P_m} 1/\alpha dp_s} = \frac{\Delta P_C}{\int_0^{\Delta P_C} 1/\alpha dp_s} \quad (2)$$

where P_m is the pressure drop across the membrane, α is the local specific cake resistance of the filter cake, P_s is the local solid compressive pressure, and ΔP_c is the pressure drop across the filter cake. By Eq. (2), α_{av} can be seen to be a function of the effective pressure ΔP_c across the filter cake. The average specific cake resistance α_{av} varies with the pressure drop ΔP_c across the filter cake in the case of the compressible filter cake. In calculations of cake filtration process it is necessary to relate α_{av} to ΔP_c by means of empirical formulas. The power law expression has been generally utilized for moderately compressible filter cake as follows (Sperry 1921):

$$\alpha_{av} = \alpha_1 \Delta P_c^n \tag{3}$$

Where α_1 and *n* are the empirical constants and n is especially termed the compressibility coefficient, reflecting the degree of the compressibility of the filter cake. The higher the n-value, the more compressibility the cake is. Tiller and Cooper (Tiller and Cooper, 1962) assumed that the local specific cake resistance α may be constant below some low pressure pi, and proposed the empirical equations representing a as a power function of the solid compressive pressure P_s leading to

$$\begin{cases} \alpha = \alpha_0 P_s^n & P_s \geq P_i \\ \alpha = \alpha_0 P_i^n & P_s \leq P_i \end{cases} \tag{4}$$

Where α_0 and P_i are the empirical constants, on the basis of Eq. (4), the relation of ΔP_c to α_{av} becomes (Tiller and Lue 1980)

$$\alpha_{av} = \frac{\alpha_0 (1-n) \Delta P_c^n}{1 - n(P_i / \Delta P_c)^{1-n}} \tag{5}$$

Later, Tiller and Leu (Tiller and Lue 1980) proposed another empirical equation relating a to P_s as follows

$$\alpha_{av} = \alpha_0 \left(1 + \frac{P_s}{P_a}\right)^n \tag{6}$$

Where *n* and P_a are the empirical constants. Based on Eq. (6), the relation between α_{av} and ΔP_c becomes (Tiller and Lue 1980)

$$\alpha_{av} = \frac{\alpha_0(1-n)(\Delta P_c/P_a)}{(1+\Delta P_c/P_a)^{1-n}-1} \quad (7)$$

By considering the analogy between Eqs. (3) and (4), a new empirical equation similar in form to Eq. (6) is proposed as

$$\alpha_{av} = \alpha_1 \left(1 + \frac{\Delta P_c}{P_a}\right)^n \quad (8)$$

where α_1 represents the empirical constant. In this paper, the applicability of these empirical equations is examined based on the experimental data of the pressure dependence of α_{av}.

2.3. Diffusional modeling (Piezometric head)

The modeling proposed by Smiles et al. (Smiles, 1970, Smiles and Kirby 1987, Smiles 2000) is quite different to the two approaches presented above. It used the "piezometric potential" of liquid (which can be directly measured by using a piezometric tube) instead of liquid pore pressure in the Darcy's law and derives the filtration equation in the form of a diffusion-like equation. Neglecting the gravity effect they produce the following g equation:

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial w}\left(D(e)\frac{\partial e}{\partial w}\right) \quad (9)$$

where e is local void rate, ω is arbitrary position in cake (m) and $D(e)=(k/\mu(1+e))(d\Phi/de)$ and $\Phi=\Psi+gx$ in which Φ is piezometric head (m) and Ψ is piezometric potential. Eq. (9) resembles a nonlinear diffusion equation in which diffusivity D is required for completing the formulation. The experimental procedures for determining D are available in many works (e.g. Meeten and Sherwood, 1994). Furthermore, Smiles approach did not define the interface between cake and slurry, which requires an extraneous equation in his formulation. In addition to modeling the filtration process as a diffusional problem, Atsumi and Akiyama (1975) and Wakeman (1978, 1986) introduced a moving singular boundary to differentiate the slurry as well as the cake regimes. The approach adopted in Wakeman (1978, 1986) was based on space coordinates, while for Atsumi and Akiyama (1975), it was based on material coordinates. Their constitutive equation of D is:

$$D(e) = a_s \exp(1.6(e^* - e)) \tag{10}$$

and that for Wakmean (1978,1986) is:

$$D(\varepsilon_L) = a_6 \exp(b_6(\varepsilon_i - \varepsilon_L / \varepsilon_i - \varepsilon_m)) \tag{11}$$

where e* is initial void rate, a_5, a_6 and b_6 are empirical constant as fitting parameters, ε_i and ε_m are the porosity at cake/slurry interface and at the filter medium surface, respectively. Diffusional modeling is not significantly used through the filtration literature, apparently because its development is conceptually difficult (Wakeman 1986, Tosun 1986).

2.4. Multiphase (3D)

Another way of cake modeling is developed by Willis and co-workers (Willis and Tosum, 1980, Willis et al., 1984, Chase and Willis 1992 and Tosun et al., 1995). Their works is based on the multiphase theory. They derived the fundamental Equations in 3D at the scale of each of the phases (each one assumed to be continuous). Then to overcome the poor knowledge of cake structure they carried out a change of scale by volume averaging (Gray 1975, Whitake 1977). The link between the relations established on each phase is ensured by the usual boundary conditions holding for continuous media (velocity and stress continuities). This averaging technique consists in integrating the equations written at the phase scale, on a characteristic averaging volume for which all physical quantities are defined by their average on the averaging volume. The changeover to this new scale called local (or macroscopic) allows the averaging volume to be considered as a material point of an equivalent homogenous medium.

Later Willis and Tosun (1980) formulated their mass and momentum balances in filtration on the basis of multi-phase theory. The momentum balance equations for incompressible, Newtonian fluid flowing through an isothermal porous medium can be stated in the following

$$\nabla\left(\frac{P_L}{\varepsilon_L}\right) = -\frac{\varepsilon_L \mu}{\underline{K}^{TP}}.(\bar{u}_L - \bar{u}_s) \tag{12}$$

Where $\underline{K}^{TP}$ is the permeability tensor based on two-phase theorem (m^2). The point contact assumption can only be achieved by setting ε_L= 1.0. On the basis of their calculations, most of the drag force occurs at the cake/septum interface rather within the cake body.

Multiphase modeling is used little through the literature; the conventional approach and the model based on the Buscall and White works (1987) are the most used currently (Stickland, 2005).

2.5. Particle dynamic approach

Simulations of particle dynamics originate from studies of physics and chemistry since the development of digital computers. In particulate technology, Houi and Lenormand (1986) used a sticking model to describe deposition of spheres on filter fibers. Tassopoulos and Rosner (1992) employed a rolling scheme for 3D simulations.

Lu and coworkers thoroughly elucidated the cake formation in filtration using particle dynamics modeling (Lu and Hwang, 1993, 1995; Hwang and Lu, 1997; Hwang et al., 1997a, b; Lu et al., 1997a). In those investigations, particles were released in slurry, whose trajectories were determined by Newton's 2nd law of motion. They also considered the force balance, including gravity force, inertial force, lift force, hydrodynamic drag force and others. Notably, they assumed a parameter, i.e. the critical friction angle, to determine the static stability of the particles. The direct information obtained from these modeling works is mainly the porosity distribution within and on the top surface of the filter cake. Evaluating permeability or local specific resistance requires extraneous models, such as the Kozeny equation employed in Lu and Hwang (1995). Owing to that only a local force balance is considered, there is no necessity for a priori assumption of relationship between solid stress and local porosity. In addition, no definite initial and boundary conditions are necessary in modeling. Fig. 3 summarizes the simulation results of the study of Liao (1997).

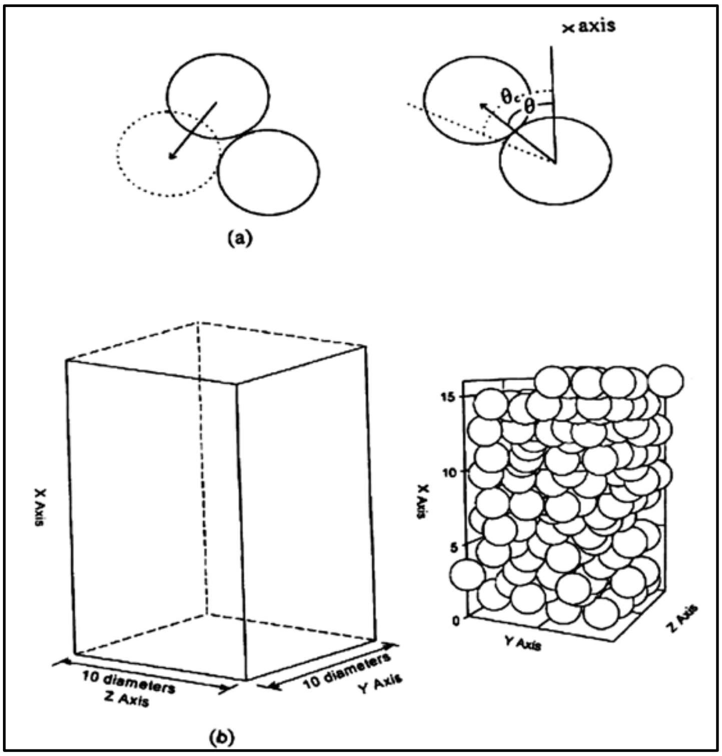


Fig 2.1. (a) Algorithm for dynamic particle simulation (b) some simulation results, data from Liao (1997)

2.6. Consolidation Theory

If we assume that the consolidation theory can be applied to sludge (Chu and Lee, 1999; Chang and Lee, 1998), water drainage in STW can be explained by the pressure exercised by the residual sludge layer. If the sludge layer compared well to an unsaturated soil, the process could be simulated by means of a porous flow model using the Richards equation. However, if we assume that the sludge layer is saturated for some days after loading, then the consolidation theory can be applied (Uggetti et al. 2012).

The process of consolidation is here explained in an ideal system composed of a spring, a container with a valve in its cover and water (Fig. 1). In this system, the spring represents the compressibility of the sludge and water represents pore water in the sludge. Initially, the container is full of water and the valve is closed, representing fully saturated sludge (Fig. 1a). If a certain load is applied to the cover when the valve is still closed, water pressure is developed (Fig. 1b). This corresponds to the pressure exercised by the weight of the sludge layer in STW. As soon as the valve is opened (Fig. 1c), water starts draining through the valve due to excess pore water pressure. This represents water percolation in STW. When excess pore water pressure is fully dissipated, water drainage stops and the spring alone resists the load (Fig. 1d).

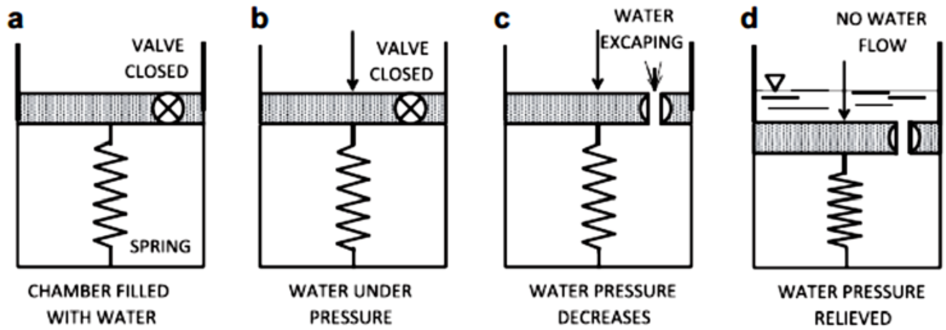


Fig 2.2. Diagram representing the process of consolidation; a) The container is completely filled with water and the hole (valve) is closed (fully saturated soil); b) a load is applied onto the cover, while the hole is still closed; c) the hole is opened and water drains out due to excess pressure; d) when excess pore water pressure is fully dissipated, flow through the soil pores ceases, and the system reaches an equilibrium position, with a compressed soil skeleton resisting the applied force (adapted from Lambe and Withman, 1979).

The one-dimensional consolidation theory of Terzaghi (Terzaghi and Peck, 1967) is based on the following assumptions:

1. Soil is homogenous (uniform in composition throughout).

2. Soil is fully saturated (zero air voids due to the high water content).

3. Solid particles and water are incompressible (water density is constant and any change of soil volume is only due to change in void ratio).

4. Compression and flow are one-dimensional.

5. Strains in the soil are relatively small.

6. Flow of water in the soil voids is one-dimensional (Darcy's

Law is valid for all hydraulic gradients).

7. The coefficient of permeability and the coefficient of volume compressibility remain constant throughout the process.

8. There is a unique relationship, independent of time, between the void ratio and effective stress. Equation (13) is derived from Terzaghi's consolidation theory:

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{13}$$

Where C_v is the consolidation coefficient (m^2/s), u is the interstitial pressure ($N/m\ s^2$), z the distance (in our case, sludge height) (m) and t is time (s). Equation (13) can be expressed as a dimensionless equation:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial U}{\partial T} \tag{14}$$

Where $U=u/\Delta\sigma$ is the consolidation ratio, $Z=z/H$ is the point where consolidation is considered, $T=c_v t/H^2$ is the time factor and $\Delta\sigma$ the pressure variation (H is the initial cake height).

2.7. Conclusion

Table 2.1 summarize the literatures on cake filtration models. Although, using specific resistance factor in conventional modeling, produces some good and simple models, however because the models equations do not describe the cake structure very well, it is highly debatable, whether it works for compressible materials. Other cake filtration models such as pizometric, multi-phase and particle dynamics models attempted to consider the cake structure and changes during filtration and expression phases but their model are conceptually difficult and consist many complicated equations.

Table. 2.1 Summary of literature on cake filtration modeling

Theory	Researcher	Year	Constitutive equations	Sludge particle resistance	Filter clogging effect
Conventional modeling	Ruth et al. Tiller Shirato et al.	1933 1953 1969	Power-law &derivatives	O	X
Diffusional (Piezometric)	Smiles Wakeman Tosun	1970 1987 1986	Experimentally measured	O	X
Multi-phase Theory	Whitaker Gray et al. Willis, Tosun	1967 1975 1980	Experimentally measured	X	X
Particle-Dynamics	Houi et al. Lu & Hwang Liao	1986 1995 1997	Power-law &Kozeny eq.	X	X
Energy Eq. and FN	Present Study		Energy Law.& Filter Number	O	O

O = considered in the model and X= not considered in the model

Considering that the performance of cake filtration depends on both the property of cake and filtering medium it is necessary to appropriately assess the constitutive relationship between the cake and the filter resistance, especially filter clogging as well as the conditions of both the filtration and expression phases. However the existing models do not consider the effect of filter clogging and compacted skin layer appropriately and do not provide adequate information about the change of resistance caused by skin layer and clogging. In the majority of works on cake filtration modeling, the applied medium resistance is based on a filter medium resistance coefficient R_m, using the Darcy equation, which does not adequately represent the effects of the filter on the filtration process.

CHAPTER 3:

Cake Filtration Equation

3.1. Introduction

Fig.3.1. shows the schematic diagram of cake filtration process. The purpose of cake filtration modeling is to correctly predict the filtration rate during filtration and expression phases. On the other hand, the cake filtration modeling attempts to calculate the change in sludge height *h* with time *t*, under the physical-chemical conditions of the sludge and the filter medium and the applied pressure (Fig. 3.1). In cake filtration process, pressure is the main driving force of the dewatering, whereas the sludge and filter resist it.

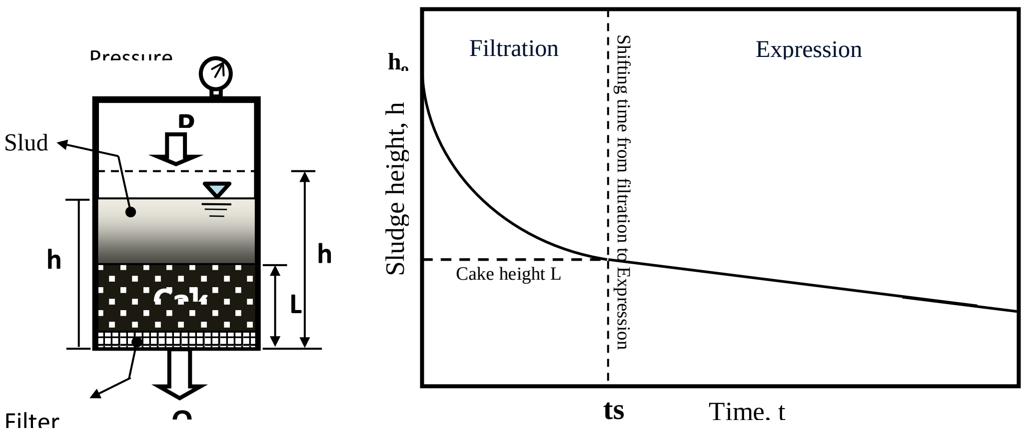


Figure 3.1 Schematic diagram of cake filtration. And h-t curve during filtration and expression phases

Developing a comprehensive model which includes both the property of cake and filtering medium especially considering the clogging effect is the need to progress in the cake filtration and expression modeling. t_s is the shifting time which separate filtration phase and expression phase. This time differs with the type of sludge and pressure.

This chapter describes the modeling theory and development of the new cake filtration model of a one-dimensional, vertical filtration process both for filtration and expression phases. The proposed model considers the energy losses made by both cake and filter resistances. The formulation of the governing equation of the model entailed the Bernoulli principle, Darcy's law and the continuity condition, resulting in a differential equation. In order to take the filter medium resistance into consideration in the model and obtain a numeric solution, a new dimensionless number termed the Filter number (FN) was introduced, which can represents the filter properties (e.g. the opening size and shape) during cake filtration ~~cake filtration~~ process. The equation was numerically solved for the filtration phase, and analytically solved for the expression phase.

3.2. Cake filtration Model

3.2.1. Model theories

The conceptual diagram of the cake filtration process is shown in Fig. 3.2. By applying Bernoulli equation between the point (a) at the sludge surface and the point (b) at the bottom of filter medium, and including the head loss through cake (ΔE_{cake}) and filter medium resistance (ΔE_{filter}), we have:

$$E_a = E_b + \Delta E_{cake} + \Delta E_{filter} \tag{1}$$

$$\left[\frac{P}{\gamma} + \frac{U^2}{2g} + z\right]_a = \left[\frac{P}{\gamma} + \frac{U^2}{2g} + z\right]_b + \Delta E_{cake} + \Delta E_{filter} \tag{2}$$

where E_a and E_b are the total energy heads at points (a) and (b), respectively (m); P is the pressure head (~~Pa~~ N/m^2); γ is the specific weight of the fluid at all points in the fluid (Nm^{-3}); U is the flow velocity (ms^{-1}); g is the acceleration due to gravity (ms^{-2}); and Z is the elevation of the point (m) above a reference plane (e.g. filter medium).

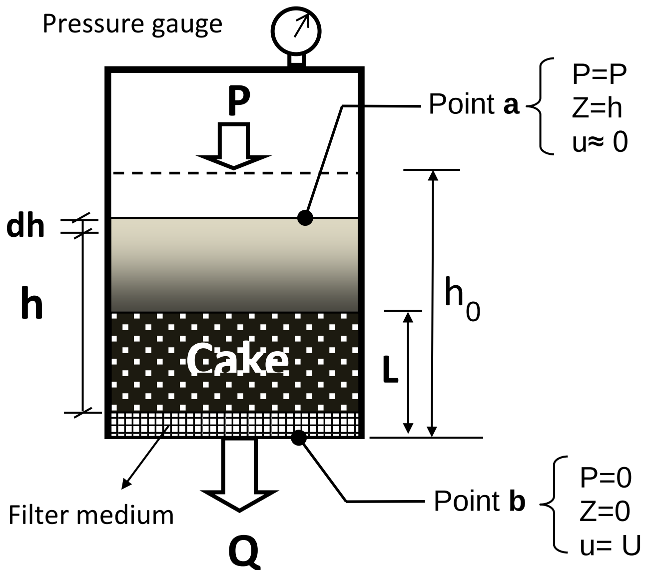


Figure 3.2 Conceptual diagram of cake filtration process (h_0 is the initial height of the sludge) the point a is on the sludge surface, whereas point b is at the bottom of the filtering medium)

The fluid velocity head at point (a) is negligible ($[U^2/2g]_a \approx 0$). If point (b) is considered as the reference plan ($[Z]_b = 0$), then the distance to point (a) would be equal to the length of the sludge ($[Z]_a = h$). The pressure at point (a) is equal to the applied filtration pressure ($[P]_a = P$). Because point (b) is considered to be the lower part of the filtering medium, the pressure is zero ($[P]_b = 0$) which reduces Eq. (2) to

$$\frac{P}{\gamma} + h = \frac{u^2}{2g} + \Delta \mathrm{E}_{cake} + \Delta \mathrm{E}_{filter} \quad (3)$$

Where $u$is the flow velocity through the filter medium (ms^{-1}).

3.2.2. Head loss through cake

Darcy's law (1856) is generally used to predict flows through porous media with negligible solid velocity. On this theoretical basis, various equations for simulating the flow through a filter cake for filter designs or interpretation of experimental data have been developed (Li et al. 2011). One of the assumptions of darcy equation which limits the application of it for sludge is that the solid particles are assumed as incompressible or barely compressible. It is highly debatable, whether it works for those materials for which the hypothesis of a rigid cake is not acceptable e.g. waste biological sludge of harvested algae. In the present study, Darcy's law was used to predict the energy head loss due to the cake resistance.

$$Q = kA\frac{\Delta \mathrm{E}_{cake}}{L} \tag{4}$$

where Q (m^3s^{-1}) is the total discharge, which is equal to the product of the cake hydraulic conductivity k (ms^{-1}), and the cross-sectional area A (m^2), and the head drop $\Delta \mathrm{E}_{cake}$ (m), divided by the length over which the head drop occurs, which is the cake height L (m). The head loss of the cake can thus be obtained as

$$\Delta \mathrm{E}_{cake} = \frac{LQ}{kA} \tag{5}$$

In the case of compressible sludge such as biological waste or harvested algae the average specific cake resistance α_{av} which is described in chapter 2 can be used instead of hydraulic conductivity k. $\alpha_{av}=\gamma/\mu k$, where μ is the viscosity of the fluid phase (Pa.s). (Oliver et al., 2007, Cheng Lin et al., 2013, Iritani et al., 2012).

The total pressure applied P on the sludge comprises the liquid pressure P_L and the solid stress P_S (Tiller et al., 1972; Walkman and Tarleton, 1999):

$$P = P_L + P_S \quad (6)$$

During the filtration phase, the solids touch each other without exerting solid stress ($P_S = 0$). The solid stress begins at t_s and reaches P after a long period.

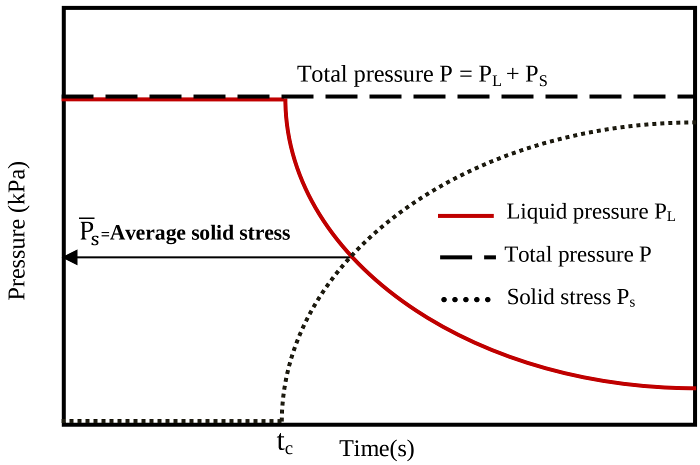


Figure 3.3 Distribution of total pressure between liquid pressure P_L and solid stress P_S

Therefore, during filtration phase there is no pressure of solid parts of the cake and the hydraulic does not change much. The theoretical analysis of Lee et al. (Lee et al 2000) the porosity change during infiltration is minimal.

In the expression phase, the compression of the cake layer may cause the hydraulic conductivity to be time dependent (Bugge et al., 2012; Jørgensen et al., 2012). However the hydraulic conductivity of the sludge may change suddenly during the shift time from filtration to expression (Lee et al 2011). The hydraulic conductivities of various kinds of sludge have also been previously investigated (Chung et al., 2006; Glendinning et al., 2010; Inzumi, 2003). In our model, we considered the average hydraulic conductivity of the cake k_{av} , during expression phase which can be calculated using the power-law relationships as formulated by Tiller and Leu (1980) The method is most frequently use for modeling the dewatering of extremely compressible materials:

$$\frac{k_0}{k_{av}} = \left(1 + \frac{\bar{P}_s}{P_a}\right)^n \Longrightarrow k_{av} = k_0 \left(1 + \frac{\bar{P}_s}{P_a}\right)^{-n} \qquad (7)$$

where, $\bar{P}_s$ is the average solid pressure, and P_a and n are empirical constants. Therefore $\bar{P}_s$ is assumed to be half of the total pressure P during the filtration phase (Fig 3.3).

There are two broad categories of determining hydraulic conductivity:

(1) Empirical approach by which the hydraulic conductivity is correlated to sludge properties like pore size and particle size (grain size) distributions, and soil texture. Several equations are developed in this regards.

(2) Experimental approach such as constant-head method and falling head method by which the hydraulic conductivity is determined from hydraulic experiments using Darcy's law.

3.2.2.1. Determination of hydraulic conductivity

(1) Constant-head method (ASTM D5084-01)

The constant-head method is typically used on granular soil (Fig. 3.4.a) this procedure allows water to move through the soil under a steady state head condition while the quantity (volume) of water flowing through the soil specimen is measured over a period of time. By knowing the quantity *Q* of water measured, length *L* of specimen, cross-sectional area *A* of the specimen, time *t* required for the quantity of water *Q* to be discharged, and head *h*, the hydraulic conductivity can be calculated:

$$k = \frac{QL}{Aht} \tag{8}$$

(2) Falling-head method (ASTM D5084-03)

The falling-head method is totally different from the constant head methods in its initial setup (Fig. 3.4.b). However, the advantage to the falling-head method is that it can be used for both fine-grained and coarse-grained soils. The soil sample is first saturated under a specific head condition. The water is then allowed to flow through the soil without maintaining a constant pressure head.

$$k = \frac{2.3aL}{At}\log\left(\frac{h_1}{h_2}\right) \tag{9}$$

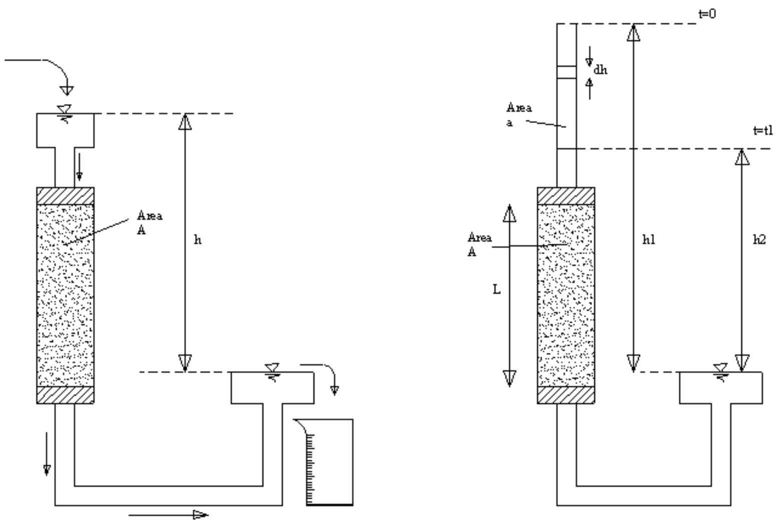


Figure 3.4 Determination of hydraulic conductivity by experimental approach a. constant head method, b. falling head method

(3) Tracing method

The common hydraulic conductivity determining methods, like constant head methods or falling head methods are not appropriate for sludge especially high compactable sludge like wastewater sludge. The particles of sludge are significantly small and breakable also due to the specific characteristics of sludge, like bad smell, the common use methods of hydraulic conductivity measurement are hard to be operated. Therefore, a specific method for calculating the hydraulic conductivity has been designed and executed in this research which called tracking method. In this method some tractable liquid like milk is used instead of water with sludge.

3.2.2.2. Sample calculation of hydraulic conductivity *k*

Falling head method:

The first experiment is to find $k_{0,}$ which is the hydraulic conductivity of sediment sludge. For this purpose, a falling head experiment has been applied on the sludge. Figure 3.5 is presenting the schematic and practical view of the falling head apparatus. The cylinder is filled with 5 cm of cake sample and back pressure method was applied to saturate the specimen up to 300 kPa. And the barrier used had a larger hydraulic conductivity than the sludge.

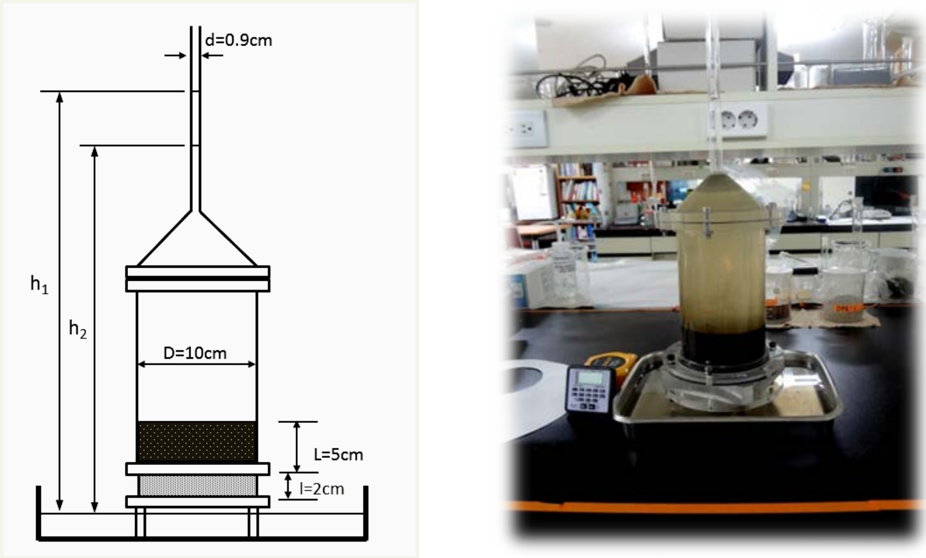


Figure 3.5 (a) Schematic and (b) practical view of Cake Hydraulic Conductivity Falling Head Experiment Set-Up

The falling of water level was recorded between time t. Based on Eq. (9), the hydraulic conductivity of sludge is being calculated as:

$$k = \frac{2.3aL}{At} = \log\left(\frac{h_1}{h_2}\right)$$

$$k_0 = \frac{2.3(0.9cm)^2 \times 5\times10^{-2}m}{(10cm)^2 \times 86S} \log\left(\frac{44}{40}\right) \Rightarrow k = 5.54 \times 10^{-7} m/s \tag{10}$$

And for the average hydraulic conductivity the results is:

$$k_{av} = \frac{2.3(0.9cm)^2 \times 5\times10^{-2}m}{(10cm)^2 \times 755S} \log\left(\frac{44}{40}\right) \Rightarrow k = 6.1 \times 10^{-10} m/s \tag{11}$$

Tracking method

Also, the tracking method has been applied for the same sludge. 50 ml glass tube (Fig. 3.6.a) is filled with fresh wastewater sludge for 10 cm depth of sediment cake and the free water on top of the cake is removed. 20 ml diluted milk is added to the top of the sludge. Because of the black color of sludge, the milk was used so that it is easier to track its movement. In this case, to make the viscosity of the milk closer to water, it was diluted with seven times volume of water. Pressures (300 kPa) have been applied to the system and the milk move through the sludge, the time when the first drop of mild comes out is recorded as Δt.

Figure 3.6 (a) Schematic and (b) practical view of Cake Hydraulic Conductivity Tracking Method Experiment Set-Up

The hydraulic conductivity of the sediment cake can be calculated through the Eq. (12)

$$\begin{cases} \mathrm{u} = \mathrm{ki} \Longrightarrow u = k\frac{\Delta P}{L} \\ u = \frac{L}{\Delta t} \end{cases} \Longrightarrow \frac{L}{\Delta t} = k\frac{\Delta P}{L} \Longrightarrow \mathrm{k_0} = \frac{\mathrm{L}^2}{\Delta \mathrm{tP}} \quad (12)$$

In this case: $\mathrm{k_0} = (0.1\mathrm{m})^2/(380\mathrm{s})(31\mathrm{m}) = 8.5\mathrm{E} - 7\mathrm{m}/s$ The second experiment is to find the average hydraulic conductivity of the filtered cake (k_{av}). The same glass tube is filled with 50% dewatered sludge and milk is added on top of the cake and pressure is applied. In this case because it takes significantly long time that the milk travel all through the cake and comes out, the movement of the milk has been tracked and the time of the movement of milk for short distance is measured as *dt* (Fig.5.3.b).

The k_{av} can be calculated as follow

$$\mathrm{k_{av}} = \frac{\mathrm{dL}^2}{\mathrm{dtP}} \quad (13)$$

In this case the mild moved in to cake very slowly and within one and half hour it just move 1 cm:

$$\mathrm{k_{av}} = (0.01\mathrm{m})^2/(5400\mathrm{s})(31\mathrm{m}) = 5.9\mathrm{E} - 10\,\mathrm{m}/s$$

Same experiments can be performed for other sludge and other different pressures.

3.2.3. Head loss through filtering medium

The head loss should be proportional to (1/2g) U^2, which has the same dimensions, because it follows the expression of the kinetic energy per unit volume (Streeter and Wylie 1909). The head loss due to the filtering medium can be quantified by a dimensionless coefficient known as the filter loss factor (*f*):

$$\Delta E_{filter} = f\frac{U^2}{2g} \tag{14}$$

The filter loss factor *f* is constant and depends on the filter parameters such as the shape, opening size and materials and changes with time.

The flow velocity through the filter medium, U is obtained by dividing of the flow discharge Q by the total area A* of the filter opening through which the flow occurs. As shown in Fig 3.7, A* is a percentage of the total filter medium area A_f :

$$A^* = \alpha A_f \tag{15}$$

$$U = Q/A^* = Q/\mathrm{a}A_f \tag{16}$$

where a is the filter opening ratio and changes with time during cake filtration process due to clogging.

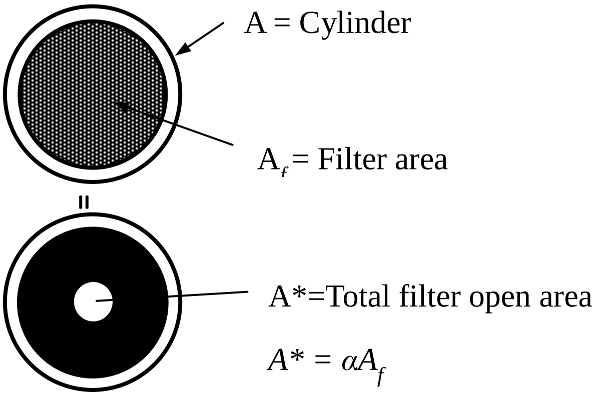


Figure 3.7 Schematic diagram of filter medium opening area

3.2.4. Basic equations

By considering the two head losses expressed by Eqs. (5) and (8), Eq. (3) can be rewritten as

$$\frac{P}{\gamma} + h = \frac{U^2}{2g} + \frac{LQ}{kA} + f\,\frac{U^2}{2g} \tag{17}$$

Substituting U from Eq. (16) into Eq. (17) yields

$$\left[\frac{1+f}{2g\alpha^2 A_f^2}\right] Q^2 + \frac{L}{Ak} Q - \left(\frac{P}{\gamma} + h\right) = 0 \tag{18}$$

However, the flow discharge Q is equal to the product of the variations of the sludge height with respect to the dewatering time and the cake cross-sectional area ($Q = -A\frac{dh}{dt}$). Hence,

$$\left[\frac{1+f}{2g\alpha^2}\left(\frac{A}{A_f}\right)^2\right]\left(\frac{dh}{dt}\right)^2 - \frac{L}{k}\left(\frac{dh}{dt}\right) - \left(\frac{P}{\gamma} + h\right) = 0 \tag{18}$$

Eq. (18) consists of three parts: (i) the filter resistance, (ii) the sludge resistance, and (iii) the pressure driving force of filtration at a drop rate of the dh/dt. Eq. (18) is valid for all the cake filtration dewatering systems. However, this equation consist filter loss coefficient *f* and the opening ratio a which are hard to measure directly and changes with time.

3.3. Filter Number

3.3.1. Definition of filter number

A new concept of the filter number (FN) was used to incorporate the filter resistance into the cake filtration model. The FN is a dimensionless quantity defined as the ratio of the pressure head to the velocity head through the filter medium:

$$FN = \frac{Pressure\ Head}{Inertial\ Head} = \frac{P/\gamma_w}{Q^2/2gA^2} \tag{19}$$

By writing the Bernoulli equation for before and after the filtering medium using the filter loss factor *f* and the opening ratio α, we have

$$\left[\frac{P}{\gamma} + \frac{U^2}{2g} + z\right]_{\text{before filter}} = \left[\frac{P}{\gamma} + \frac{U^2}{2g} + z\right]_{\text{After filter}} + \Delta E_{filter} \tag{20}$$

The fluid velocity head before filter medium and filter medium height is negligible. The pressure head after filter medium is zero. Substituting $U = \frac{Q}{\alpha A_f}\left(\frac{A_f}{A}\right)$ and ΔE_{filter} from Eqs. (13) into (20) yields

$$FN = \frac{1+f}{\alpha^2}\left(\frac{A}{A_f}\right)^2 \tag{21}$$

For a clean filter it is a specific number FN_0 and it can be measure by simple experiments from Eq. (19). This FN_0 is based on water as liquid however in some filtration presses the liquid is not water e.g winery or dairy etc. In that case FN_0 can be modified by using a correction factor as follow:

$$\left.\begin{array}{l} FN_0 = \frac{P/\gamma_w}{Q^2/2gA^2} \\ FN_{0L} = \frac{P/\gamma_L}{Q^2/2gA^2} \end{array}\right\} \rightarrow \frac{FN_0}{FN_{0L}} = \frac{\gamma_L}{\gamma_w} \rightarrow FN_{0L} = \frac{\rho_L}{\rho_w} FN_0 \quad (22)$$

For example the FN_{0L} for filter type I and correction factor for different liquid are shown in table 3.1

Table 3.1 FN_{0L} for filter type I and correction factor for different liquids

Liquid	**Density (kg/m³)**	$\boldsymbol{\rho_w/\rho_L}$	**$FN_{0L}\times10^8$**
Water	997	1	4.6
Wine	1135	0.87	4
Milk	930	1.1	5.1
Honey	1510	0.65	2.76

During filtration presses FN changes with time. During the filtration phase, the solid particles accumulate on the surface and between the pores of the filtering medium, and the permeability of the filtering medium is dramatically reduced by reduction of a. The filter number (FN) is therefore a time-variable parameter in the filtration phase. However, after the cake formation and clogging (t_s) the filtering medium becomes stable and is characterized by a new filter number, FN_f which can be considered as being constant during the expression stage. Fig 3.8 shows the variation of FN between the filtration and expression phases.

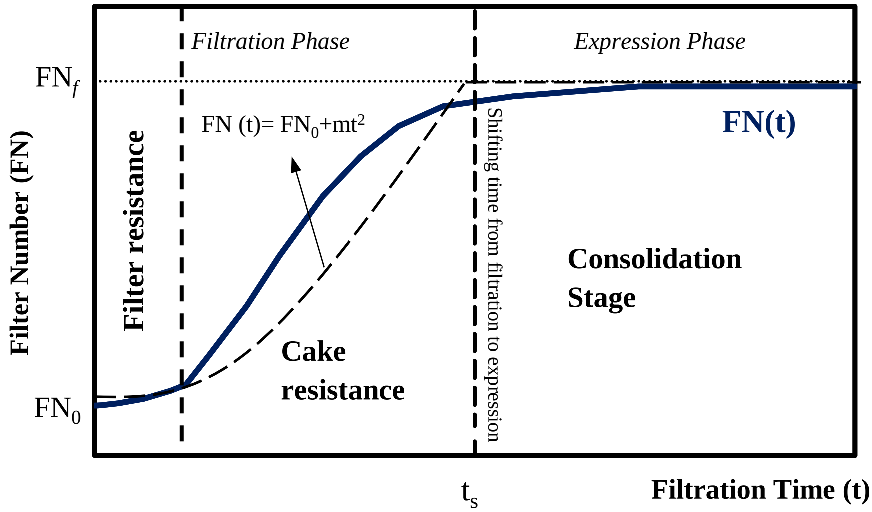


Figure 3.8 Curve of filter number (FN) versus time

The FN function FN (t) is fitted to a logistic cure:

$$FN(t) = \frac{FN_f}{1+\left(\frac{FN_f}{FN_0}-1\right)e^{-mt}} \tag{22}$$

For simplification FN (t) function can be assumed to be a second order exponential function during filtration phases:

$$FN(t) = \begin{cases} FN_0 + mt^2, \; t < t_s \\ \quad hjhg \\ FN_f \quad\quad , \; t \geq t_s \end{cases} \tag{22}$$

where m is total filter resistance coefficient which can be determined experimentally. The FN (t) function can be calculated form Eq. (19) which is the rate of the pressure head to the inertial head and the empirical parameter FN_0, FN_f and m can be determined by simple experiments described in next section.

3.3.2. Sample calculation of FN number

In order to get FN (t) and experimental parameters (Initial filter number FN_0, final filter number FN_f and clogging factor m) for a specific cake filtration system, two simple experiments are needed as in Figure 3.9.

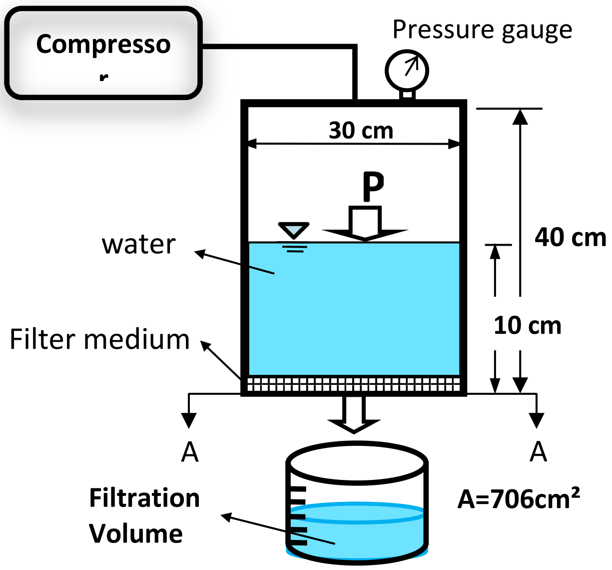


Figure 3.9 Schematic diagram of experimental set up for FN experiments

Clean Filter (FN_0): The first experiment is to find FN_0, which shows the characteristics of a clean filter. The cylinder (D= 30cm, H=40 cm) is filled with clean water for 10 cm depth and pressure (3 bar) is applied. The filtrate volume is measured for a certain time (60 sec) at each time interval. It shows a linear relationship, because there is no change of opening of the filter due to clogging (Fig. 3.10).

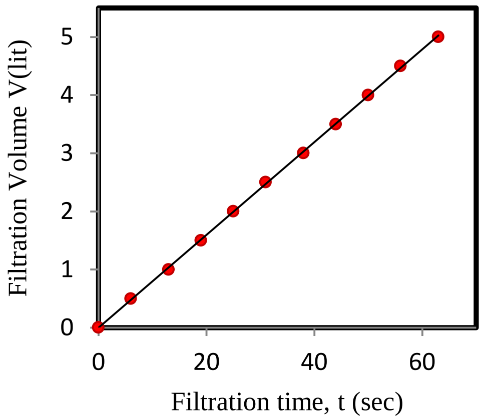


Figure 3.10 Filtration experiment results with water to measure FN_0

FN_0 can be calculated form Eq. (19) which is related to the head loss through the filter.

$$Q = \frac{\Delta V}{\Delta t} \Longrightarrow FN_0 = 2gA^2(P/\gamma)(\Delta t/\Delta V)^2 \qquad (23)$$

In this experiment, $FN_0 = 2\left(\frac{981cm}{s^2}\right)(706\ m^2)^2(3100\ cm)(59\ s/5000\ cm^3)^2 = 4.2E8$ for other pressure such as 1 bar or 5 bar, the result is same. For a filter with small opening, FN_0 become low because of head loss through the filter, and a filter with large opening, FN_0 becomes higher.

Cake Filtration, FN (t): The second experiment is for the system of filter and the cake. The same cylinder is filled with sludge for 10 cm depth and pressure is applied. And the volume of filtrate is measured with time. From this experiment, dV/dt is calculated at each time. (Fig. 3.11)

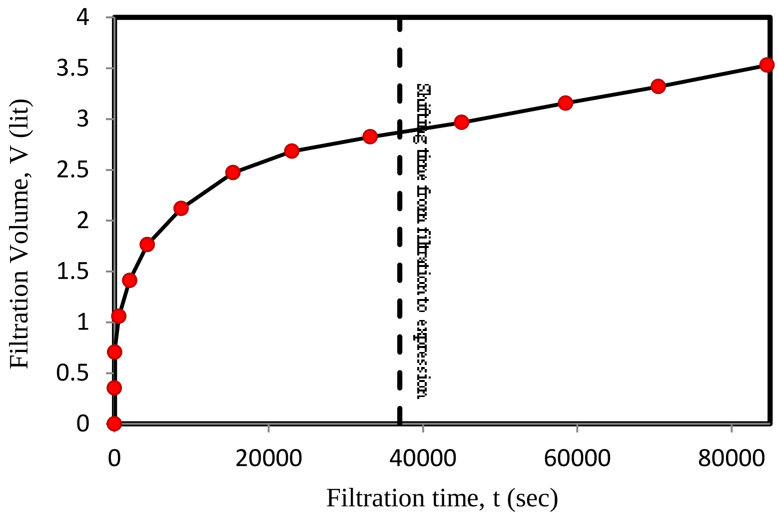


Figure 3.11 Filtration experiment results with cake to measure FN(t)

The shifting time t_s which separate the filtration and expression, is defined as the time which dV/dt becomes constant. In this case t_s=38000 sec. At certain t, FN (t) can be calculated through the equation:

$$FN(t) = 2gA^2Pa(dt^2)/\gamma(dV)^2 \quad (24)$$

For example, in the case of time t=2000, dV/dt is obtained as 2E-7 m^3/s:

$$FN(t = 2000) = 2(9.81\, m/s^2)(0.07\, m^2)^2(31\, m)/(2E - 7\, m^3/s)^2 = 1.45E14$$

FN(t) can be calculated similarly at other time and Figure 3.12 can be drawn.

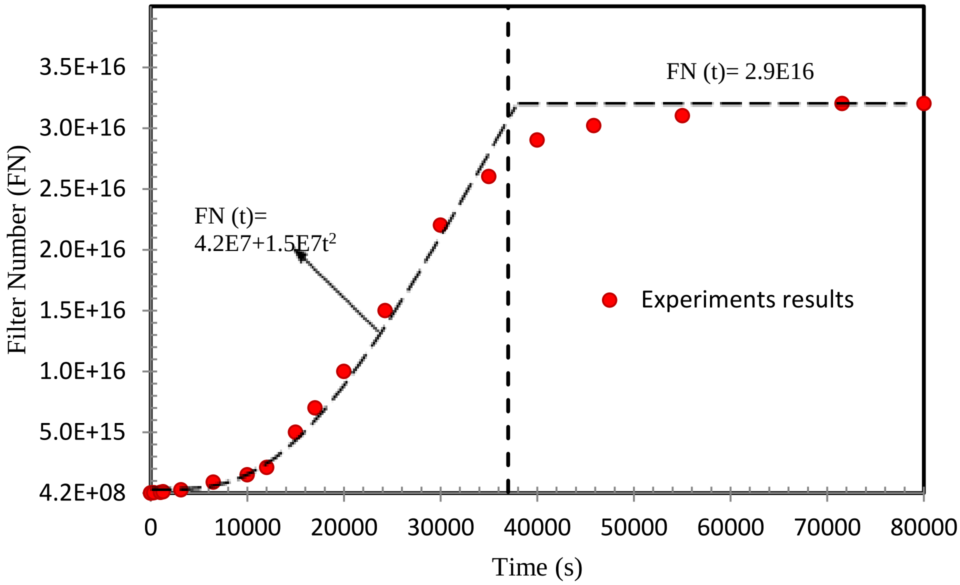


Figure 3.12 Results of calculated FN (t) from filtration experiment

FN(t) curve can be fitted to a second order exponential function can be modeled by three parameter such as FN_0, FN_f and *m*. In this case, FN_0=4.2E8, FN_f=3.2E16 and m=1.5E7 with R^2 number= 0.99:

$$FN(t) = 4.2E8 + 1.5E7t^2 \qquad (25)$$

When we change the experimental conditions such as set up scale, initial sludge height, the result should be the same so the standard experimental condition is set up as in this experiment.

3.4. Governing Equations

3.4.1. Filtration phase

In the filtration phase, the relationship between the filter number and time can be assumed as logistic function Eq.(22). Substituting Eqs. (21) and (22) into Eq. (18) yields

$$\left[\frac{FN_0+mt^2}{2g}\right]\left(\frac{\Delta h}{\Delta t}\right)^2 - \frac{L}{k_0}\left(\frac{\Delta h}{\Delta t}\right) - \left(\frac{P}{\gamma} + h\right) = 0 \tag{26}$$

The simple finite-difference (forward) method can be used to numerically solve Eq. (21) as follows:

$$h_{i+1} = h_i + \frac{g(t_{i+1}-t_i)}{FN(t_i)}\left(\frac{L}{k_0} - \sqrt{\left(\frac{L}{k_0}\right)^2 + 2\left[\frac{FN(t_i)}{g}\right]\left(\frac{P}{\gamma} + h_i\right)}\right) \tag{27}$$

where, h_{i+1} and h_i are the cake lengths at times i and i+1, respectively. The model starts from (t = 0, h = h_0) and runs until h_{i+1}= L. Immediately after h_{i+1} reaches L, then expression phase begins. Provided that Δt is appropriately selected, Eq. (27) would predict the cake filtration rate within an acceptable error.

3.4.2. Expression phase

In the expression phase, the filter number is assumed to be constant and equal to FN_f (Fig.3.7), and the pressure force is assumed to be $\frac{P}{\gamma} + L$. Consequently, the following can be derived from Eq. (18):

$$\left[\frac{FN_f}{2g}\right]\left(\frac{dh}{dt}\right)^2 - \frac{h}{k_{av}}\left(\frac{dh}{dt}\right) - \left(\frac{P}{\gamma} + L\right) = \tag{28}$$

Where k_{av} is the average cake permeability in the expression stage. Because all the parameters are time independent, Eq. (28) is a simple quadratic equation, and therefore

$$\frac{dh}{dt} = \left(\frac{h}{k_{av}} \pm \sqrt{\left(\frac{h}{k_{av}}\right)^2 + 4\frac{FN_f}{2g}\left(\frac{P}{\gamma} + L\right)}\right)\Big/\frac{FN}{g} \tag{29}$$

Because the sludge height drops during the filtration process ($dh/dt < 0$), the + is not possible physically. The integral form of Eq. (29) becomes

$$\int_L^h \frac{dh}{h - \sqrt{h^2 + 2\frac{FNK^2}{g}\left(\frac{P}{\gamma} + L\right)}} = \int_{t_s}^t \left[\frac{g}{FNK}\right] dt \tag{30}$$

Assuming $D = \frac{2FNK^2}{g}\left(\frac{P}{\gamma} + L\right)$ and simplifying, Eq. (30) becomes:

$$\int_L^h \frac{dh}{h - \sqrt{h^2 + D}} = \frac{g}{FNK}\int_{t_s}^t dt \tag{31}$$

The integral can be solved analytically and yielding:

$$\left[-\frac{1}{2}\ln\left(h+\sqrt{h^2+D}\right)-\frac{h}{2D}\sqrt{h^2+D}-\frac{h^2}{2D}\right]_L^h=\frac{g}{FNK}\left[t\right]_{t_s}^{t} \qquad (32)$$

Substituting the boundary values into the integral yields:

$$\frac{1}{2D}\left[(L^2-h^2)+\left(L\sqrt{L^2+D}-h\sqrt{h^2+D}\right)+D\ln\left(\frac{L+\sqrt{L^2+D}}{h+\sqrt{h^2+D}}\right)\right]=\frac{2K(\frac{P}{\gamma}+L)}{D}\ (t-t_s) \qquad (33)$$

Rearranging the parameters in Eq. (34) yields

$$t=f(h)$$

$$t=t_s+\frac{1}{4K\left(\frac{P}{\gamma}+L\right)}\left[(L^2-h^2)+\left(L\sqrt{L^2+D}-h\sqrt{h^2+D}\right)+D\ln\left(\frac{L+\sqrt{L^2+D}}{h+\sqrt{h^2+D}}\right)\right]$$

(34)

Because the function f is an injective function (for: $h, t \geq 0$) the sludge height can be calculated as follows:

$$h=f^{-1}(t) \qquad (35)$$

3.5. Model sensitivity

3.5.1. Filtration Test

3.5.1.1. Wastewater sludge

The activated sludge which was sampled by Curves et al (2007) at the Ossemeersen Wastewater Treatment Plant in Ghent, Belgium is used as and standard case in our research. Samples of thickened sludge were taken from the sludge buffer. The sludge was stored at 4 ◦C, and was used within 48 h after sampling. The dry matter content, calculated after 24 h drying at 105 ◦C, amounted to 3.47%. Before performing the dewatering tests, the sludge was conditioned by means of a polyelectrolyte, Zetag 7878 FS40, supplied by Ciba Specialty Chemicals, Belgium. Aqueous polyelectrolyte solutions were prepared at a 2 g/L active polyelectrolyte concentration, at least 24 h prior to application. For the conditioning, a Stuart Scientific Jar Test device was used. It was operated at 270 rpm during the first minute for intense mixing of the polyelectrolyte into the sludge. Then, 10 min of slow stirring at 35 rpm was to promote floc growth. Half a kg of sludge was mixed with 65mL of polyelectrolyte solution, resulting in a total of 7.5 g of polyelectrolyte per kg of sludge dry matter. The total sample was then used for the dewatering tests.

3.5.1.2. Experimental set-up and procedure

Dewatering tests were performed by Curvers et al. in a laboratory scale, piston driven, cylindrical filter press with 0.07m diameter (Fig. 3.13). A thorough description of the press can be found in Saveyn et al. (2005b). The press was connected to a personal computer to control the experiment and to record the filtration data (time, filtration pressure, filtrate mass, piston height) as well as other useful data such as the temperature in the cake. During the total dewatering experiment, the filtration pressure was kept constant at 400 kPa. For every dewatering run, a new piece of Nordifa Lainyl MC4/S5/6 polypropylene filter cloth was used. Each dewatering test was started by applying 400 kPa over the filtration cell.

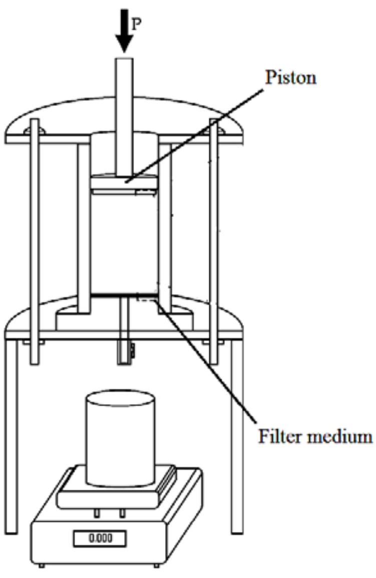


Figure 3.13 Schematic diagram of the filtration equipment used by Curvers et al (2007) together with a more detail presentation of filtration process

3.5.1.3. Initial and experimental parameters

The distance between the piston and the filter medium *h* as a function of the time *t* during filtration test was recorded by curves and et. (2007) and is shown in figure 3.12. The initial distance in the experiments h_0 was 0.145 m. The empirical parameters α_0, *n* and *Pa* were fitted using the experimental data of the first 1800 s of the filtration test. Fitting was done by searching the smallest sum of squared errors using the Nelder–Mead optimization method (Walters et al., 1991). Table 3.2 shows the initial experimental parameter values obtained from Curvers et al. 2007.

Table 3.2 Initial experimental parameter values obtained from Curvers et al.

Parameter	**Symbol**	**Value**	**Unit**
Initial cake height	h_0	0.145	m
Filtration pressure	P	400	kPa
Cylindrical area (diameter)	A (D)	0.0038 (0.07)	m^2 (m)
Cake initial resistance	α_0	1.0623×10^{11}	m^{-2}
Cake height (sludge height at time t_c)	L	0.031	m

Curvers et al. used cake specific resistance α instead of hydraulic conductivity k. However, the fitted initial hydraulic conductivity of the sludge k_0 can be easily calculated using

$$k_0 = \gamma/\mu\alpha_0 = 9806(Pa.m^{-1})/(10^{-3}(Pa.s)1.0621 \times 10^{11}(m^{-2})$$

$k_0 = 9.23 \times 10^{-5}$ (ms^{-1}).

The average hydraulic conductivity of the cake during the expression phase k_{av} can also be calculated using Eq. (7). The empirical constants P_a and n were obtained by the fitting data as 6.4494×10^3 (Pa) and 5.2242, respectively. Using the average solid stress $\overline{P_s}$ of 200 kPa, the average hydraulic conductivity k_{av} of the cake was obtained as 2×10^{-8}. The empirical parameters FN_0, FN_f, and m were calculated from the experiments results as 7×10^8, 1.28×10^{14}, and 0.04 , respectively. Table 3.3 gives the parameter values obtained from the experimental results and used for the model (During the first minute of the experiments the filter was considered to be clean with filter number FN_0).

Table 3.3 Empirical parameter values determined from Curvers et al. 2007

Parameter	**symbol**	**Value**	**Unit**
Initial hydraulic conductivity	k_0	9.23×10^{-5}	ms^{-1}
Average hydraulic conductivity of cake	k_{av}	2.02×10^{-8}	ms^{-1}
Filter number of clean filter	FN_0	7.09×10^8	dimensionless
Filter number during expression phase	FN_f	1.28×10^{14}	dimensionless
Shift time from filtration to expression	t_s	1800	s
Clogging coefficient in filtration phase	m	0.04	dimensionless

The sludge height h changes by time t experimentally determined by Curvers et al. for both the filtration and expression phases as well as the model prediction (Eqs. 16 and 24) are shown in Fig. 3. 14. As can be observed, the sludge heights predicted by the model for the filtration and expression phases are in good agreement with the experimental data. The average hydraulic conductivity of the cake during the expression phase $\bar{k}_f$ was used for the calculations, and this produced errors after a long time. The present results indicate that previous complex cake filtration theories based on the principles mentioned in Section 1, could be simplified using the proposed FN.

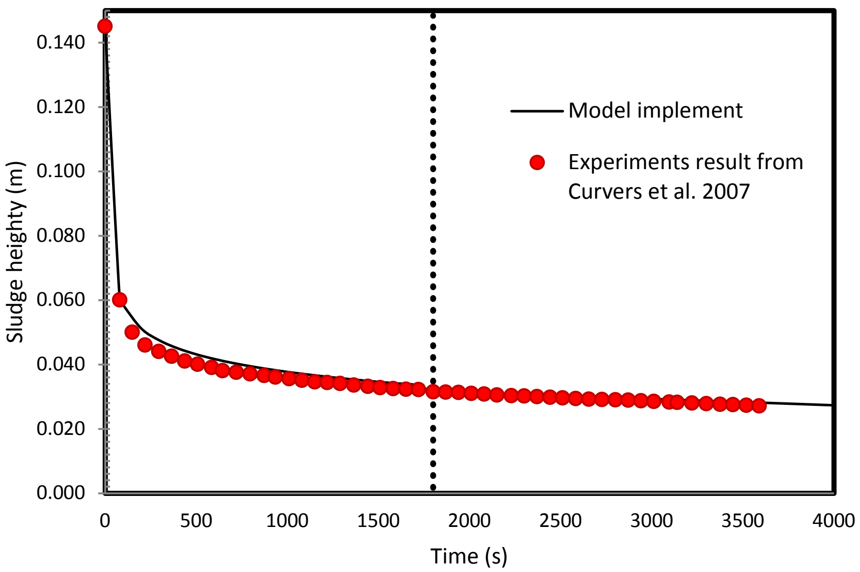


Figure 3.14 Comparison of the sludge height experimentally determined by Curves et al 2007 (dots) with that predicted by the proposed model (full line)

3.5.2. Effect of governing parameters

The applied pressure *P*, sludge hydraulic conductivity *k*, and filter number *FN* are the three governing parameters of cake filtration modeling. In this section, the model sensitivity to the three parameters is investigated. The experimental data obtained by Curvers et al. was considered as the standard case. Table 3.4 gives the standard case values and governing parameters range used for the model sensitivity analysis.

Table 3.4 the standard case values and governing parameters ranges

Fig.	Model sensitivity	Pressure (kPa)	Hydraulic conductivity		Filter Number	
			Filtration	Expression	Filtration*	Expression
4.2	Standard Case	400	9.23E-5	2.02E-8	0.04	1.28E14
4.3	pressure head	100-700	9.23E-5	2.02E-8	0.04	1.28E14
4.4	hydraulic conductivity	400	1E-2-1E-7	1E-8-5E-8	0.04	1.28E14
4.5	filtering medium	400	9.23E-5	2.02E-8	0.01-0.05	5E13-5E14

* Clogging coefficient *m* is the governing parameter during filtration phase instead of FN_0

3.5.2.1. Effect of pressure head

The effects of the pressure head during the filtration and expression phases are clearly shown in Figs 3.15.a and b. Generally, a pressure rise directly results in an increase in the dewatering rate. However the effect of the pressure during the filtration phase is greater than during the expression phase. As mentioned earlier, the total pressure P is distributed between the liquid pressure P_L and the solid stress P_S (Fig 3.3). During the filtration phase, the solid stress is zero and the total pressure acts on the liquid phase. Hence any change in the pressure directly affects the liquid filtration. However, in the expression phase, the total pressure is distributed between the solid and liquid phases, which means that even a significant change in the pressure may be absorbed by the solid phase (the solid velocity is considered to be zero in the model).

The behavior of highly compactable sludge is a bit different. The permeability k varies with the pressure head, resulting significant variation when the pressure head is increased. This behavior is not discussed here because the value of the empirical constants P_a and n in different pressures were not available and power-law relationships (Eq. (7)) could not be used to predict the effect of the pressure on cake hydraulic conductivity during the expression phase.

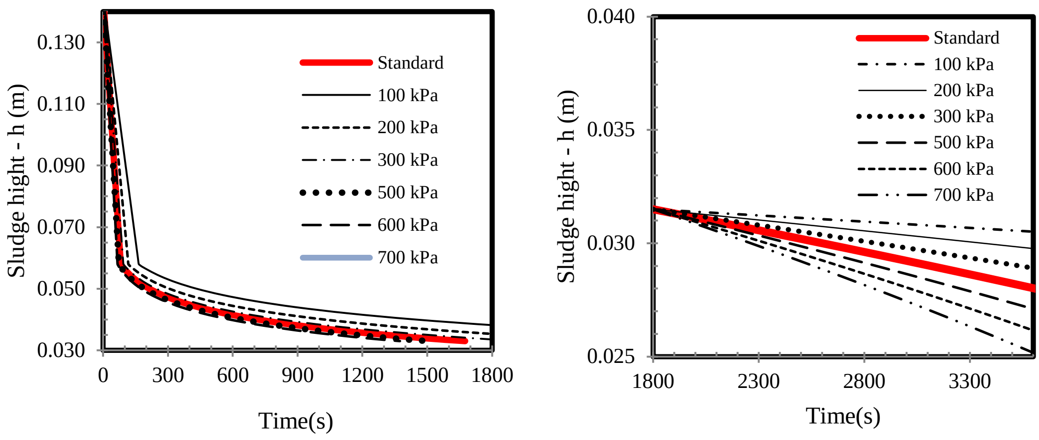


Figure 3.15 Effects of the pressure head on the sludge height a. during the filtration phase and b. during the expression phase

3.5.2.2. Effect of hydraulic conductivity of sludge

It is clear from Fig 3.16 that the hydraulic conductivity of the filter cake significantly affects the expression phase but slightly affects the filtration phase. However when the hydraulic conductivity is extremely small (e.g. clay) it becomes the dominant factor even during filtration phase. The filtration time required to produce a cake height of 0.05 m in the filtration phase and 0.03 m in the expression phase increased from 300 s to 320 s and from 1900 s to 3500 s when the hydraulic conductivity of the sludge was increased from 1E-4 m/s to 1E-6 m/s in filtration phase and 1E-8 to 5E-8 in expression phase from while values of the other parameters were maintained.

During the filtration phase, water simply passed through the sludge particles, but in the expression phase, the flow rate decreased with decreasing hydraulic conductivity, which indicates a negative effect of the cumulative cake particles on the overall permeability of the sludge.

The empirical parameters of the power-law relationships for compressible cakes that account for the effective stress P_s (Eq. (7)) also depend on the material properties of the sludge, which indicates that filtration rate is more sensitive to the cake hydraulic conductivity during expression phase k_c.

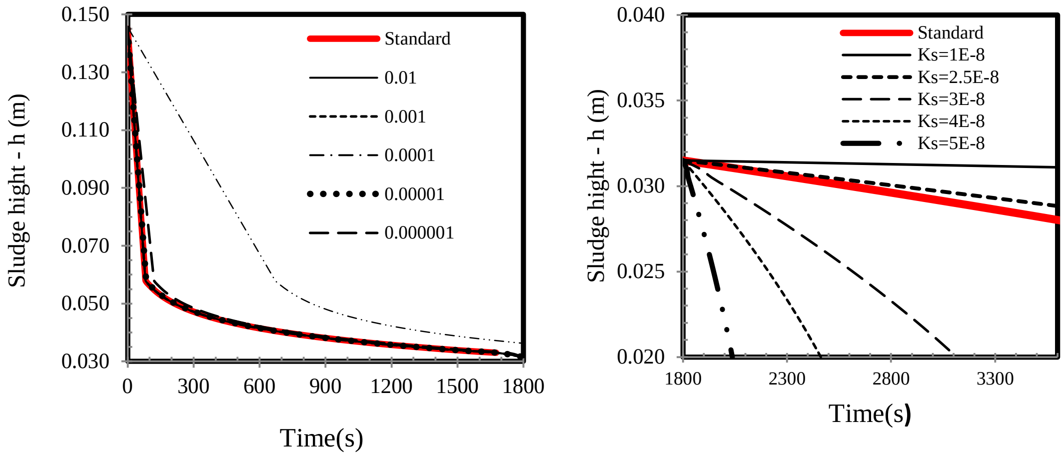


Figure 3.16 Effects of hydraulic conductivity of the filter cake on the sludge height (a) during the filtration phase and (b) during the expression phase

3.5.2.3. Effect of filter number

Fig. 3.17 shows the variation of the cake height and filtration time with the filter number during the filtration and expression phases. Initial filter number FN_0has minor effect on filtration however clogging coefficient *m* is the governing parameter during filtration phase. It is obvious that the filtering medium significantly affects the dewatering during both the filtration and expression phases. Over all, an increase in the filter number (which means smaller filter) reduces the dewatering. However, during the filtration phase, particles accumulate in the pores of the filtering medium, resulting in clogging. This indicates that the filtering medium plays primary role during the filtration phase. During the expression phase, the filtering medium stabilizes and the filter type is more important. During the expression phase, the filtering medium attains a new and stable condition.

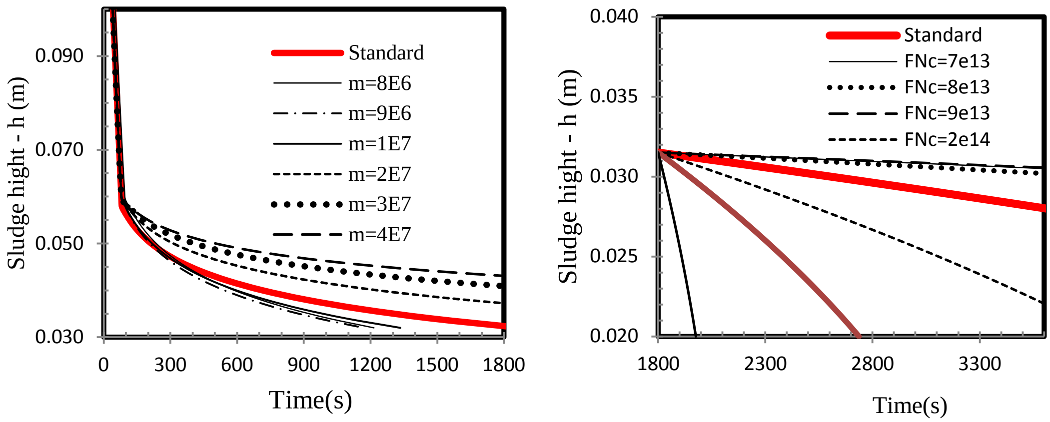


Figure 3.17 Effects of filter number FN on the sludge height (a) during the filtration phase and (b) during the expression phase

3.6. Discussion

The proposed model of a one dimensional one sided filtration process was implement with experimental data on activated sludge filtration reported by Curvers et al (2007) for both the filtration and expression phases. The model assumptions and behavior in the two phases are discussed here.

Filtration Phase: the model assumptions and indications regarding the filtration phase are as follows:

i. The thickness and permeability of the sedimentation cake are assumed to be constant. The model sensitivity analysis revealed that the cake hydraulic conductivity *k* and thickness *L* are not a dominant as same as the pressure during the filtration phase.
ii. The relationship between the filter number and the dewatering time can be simplified a second order exponential function. The empirical parameter *m* in Eq. 22 was fitted using experimental data on the filtration phase.
iii. The energy losses due to the share stress force on the cylinder wall can be neglected because of the low flow velocity of the filtration cake.

The migration and capture of the solid particles in the porous media significantly affects the filter number and the correspondingly filtration rate during the filtration phase. The sustainable method for enhancing the cake filtration during the filtration phase is to decrease *m;* e.g. by using electro kinetic filtration instead of adding chemicals or increasing pressure, which are relative expensive and energy intensive.

Expression Phase: The model assumptions and indications regarding the expression are as follows:

i. The filter number *FN* is assumed to be constant during the expression phase. The particle accumulation and cake formation on top of the filtering medium stabilize in the expression phase.

ii. The average hydraulic conductivity of the cake $\bar{k}_c$ is considered as constant irrespective of the time effect. However, the permeability of the filtration cake is dependent on the porosity of the cake during the expression phase. Moreover, the assumption of the permeability of the cake being constant within an average range does not weaken the applicability of the model as confirmed by the verification.

Based on the analysis of the sensitivity of the model to the governing parameters, the hydraulic conductivity of the filtration cake is the dominant factor during the expression phase. The model predictions suggest the use of chemical or electro-osmosis enhancement during the expression phase, as did the experimental of Curvers et al. (2007).

3.7. Conclusion

A new cake filtration model was proposed for predicting the cake filtration. The model considers the energy losses made by both cake and filter resistances. The formulation of the governing equation of the model entailed the Bernoulli principle, Darcy's law and the continuity condition, resulting in a differential equation. To evaluate particle migration and capture in the porous media and the effect of the filter resistance, a new dimensionless filter number FN was introduced which can represents the filter properties (e.g. the opening size and shape) during cake filtration cake filtration process. The equation was numerically solved for the filtration phase, and analytically solved for the expression phase. Compared to previously developed models the models, the proposed model is simple and more accurate.

The effects of the governing parameters on the model predictions were also analyzed. The following is a summary of the findings of the evaluation of the model and its sensitivity analysis:

1. Compared to previously developed models the models, the proposed model is simple and more accurate
2. The effect of the cake hydraulic conductivity was found to be negligible during in the filtration phase. Moreover, the filtration cake and the particles that accumulate front of the filtering medium significantly

reduce the filtration flux.

3. Because the cake permeability is the dominant factor during expression, it is highly recommended that sludge treatment should only be applied during the expression phase to save money and energy.

Further research is necessary to use empirical coefficient to consider the effect of chemical enhancement and electro-dewatering on the model.

CHAPTER 4:

Model Verification

4.1. Introduction

A complete mathematical model should be able to apply in any real cases. Verification and validation is an iterative process that takes place throughout the development of a model. It is necessary to evaluate the validity of a mathematical model with actual experimental results to verify the model prediction. The objective of model verification is to ensure that the implementation of the model is correct. Wastewater sludge from Gwachon wastewater plant was used for model verification. By comparing the data from filtration experiment with the data which model predicts, it is possible to verify the model and analyze the error of it.

In this chapter, the wastewater sludge filtration test has been done under two pressures (300 and 500 kPa) and two types of filters (Filter Type I and II). Experimental parameters such as filter number for clean filter (FN_0), initial hydraulic conductivity (k_0) and average hydraulic conductivity (k_{av}) of the cake were determined by operating pre-experiments and FN_f, m and t_s obtained with cake filtration test as described in chapter 3. These parameters have been used in the new model, to predict the sludge height (h) as a function of time.

The results from filtration experiments has been compared with the model predicted data in order to verify the prediction of the model and based on this, the error of the model was calculated by using regression analysis and MRE method.

4.2. Laboratory Experiment

4.2.1. Materials and Preparation

4.2.1.1. Wastewater Sludge

Samples of wastewater sludge were collected from Gwacheon urban sewage treatment plant in Seoul, Republic of Korea (Figure 4.1). In this wastewater plant, sludge is produced from the treatment of wastewater in on-site (septic tank) and off-site (activated sludge) systems. This is inherently so because a primary aim of wastewater treatment is removing solids from the wastewater. In addition, soluble organic substances are converted to bacterial cells, and the latter is removed from the wastewater.

Figure 4.1 Location of Gwacheon Water Treatment Plant

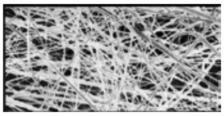
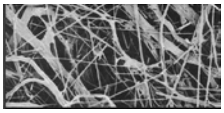
The initial water content of the wastewater sludge is 98.5% with pH=6.8and density=1200 kg/m^3 with zeta potential equal to -10.90mV.The collected sludge was immediately transferred to lab and in order to inactive the active organic and make the sludge condition stable, 0.1g/L Sodium Azide (NaN_3) with concentration of 0.035%w/v is added to sludge for 24hr. The sample was stored in refrigerator at 4°C and allowed to warm to room temperature when required to minimize sludge biological activity.

4.2.1.2. Filters

Fiberglass filter media is widely used in sludge pressure dewatering as it offers high efficiency and high dirt holding capacity. These filters are made from fiberglass mesh. Fiberglass mesh is a neatly woven, crisscross pattern of fiberglass thread that is used to create new products such as tape and filters. When it is used as a filter, it is not uncommon for the manufacturer to spray a PVC coating to make it stronger and last longer. The most common place to find fiberglass mesh is in tape products.

For the laboratory filtration experiments, two common use fiberglass filters were used to evaluate the model assessment for different filter with different filter number. Their characteristics were given in table 4.1.

Table 4.1 Characteristics of filters used in this study

Filter medium	**Materials**	**Opening size (AOS) (mm)**	**Microscopic Scope**
Filter type I	Fiberglass I	0.05	
Filter type II	Fiber glass II	0.07	

4.2.1.3. Experimental set-up

Figure 4.2 illustrates a schematic drawing of batch experimental set up which is similar to that developed by and Tarlton (Tarlton1998a,b). An unstirred cylindrical mould (30 cm diameter and 40 cm height) was utilized. The filter is supported by a perforated panel which makes an effective filter area as 433 cm^2. Sludge in the filtration chamber is dewatered through filter placed in the bottom of the filtration chamber. Connection between the cylinder and filter is with the help of two O-ring seal. A plexiglass window was improvised on the filtration chamber to measure the height of sludge (h) with time. The pressure has been applied by compressed air compressor. The two-in-one P425/HP375-T4i dual pressure portable air compressor has been used which delivers airflow ranging from 0.18 to 0.20 cfm and pressures ranging from 100 to 1100 kPa.

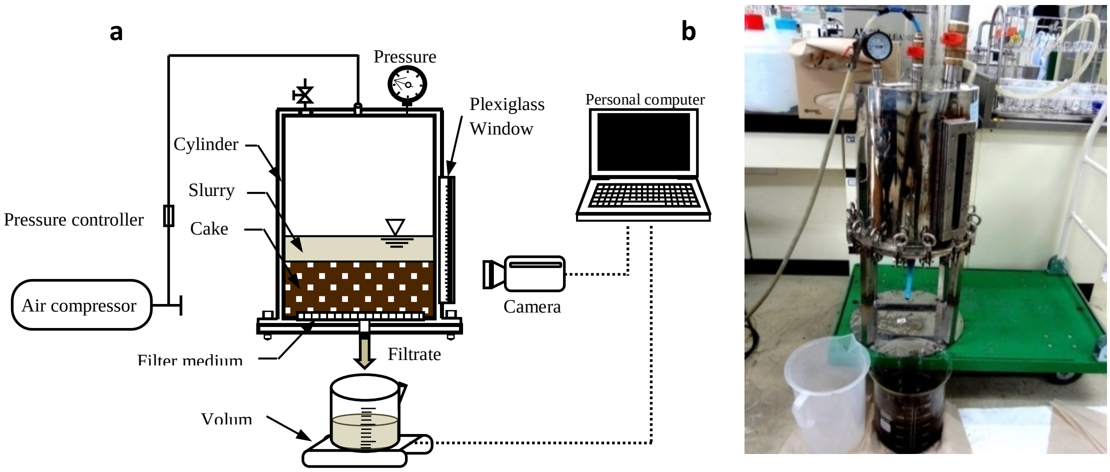


Figure 4.2 (a) Schematic diagram and (b) Photo of experimental set up

4.2.2. Filtration Experiment Condition and Procedure

Filtration experiments were carried out by adjusting the applied filtration pressure automatically by the compressor as it is shown in figure 4.2, and kept constant during test. The height of sludge and cake (h) with time (t) was measured through Plexiglas window by video camera up to h=4cm. Data derived from filtration experiments can be used for model assessment. All the experiments were done in room temperature.

Four cake filtration tests were done with the two filter type, as described in table 4.1, at two pressures 300 and 500 KPa. Table 4.2 is presenting a summary of these experiments.

Table 4.2 Overview of Cake Filtration Experiments

Case Number	**Filter Type**	**Pressure (KPa)**
1	Filter Type I	300
2	Filter Type I	500
3	Filter Type II	300
4	Filter Type II	500

4.2.3. Experiment Results

The results of cake filtration experiments for four cases as described in table 4.2 is presented in figure 4.3 which is showing the gradients of sludge height versus time. Each dot in the graph is showing the sludge height at time t.

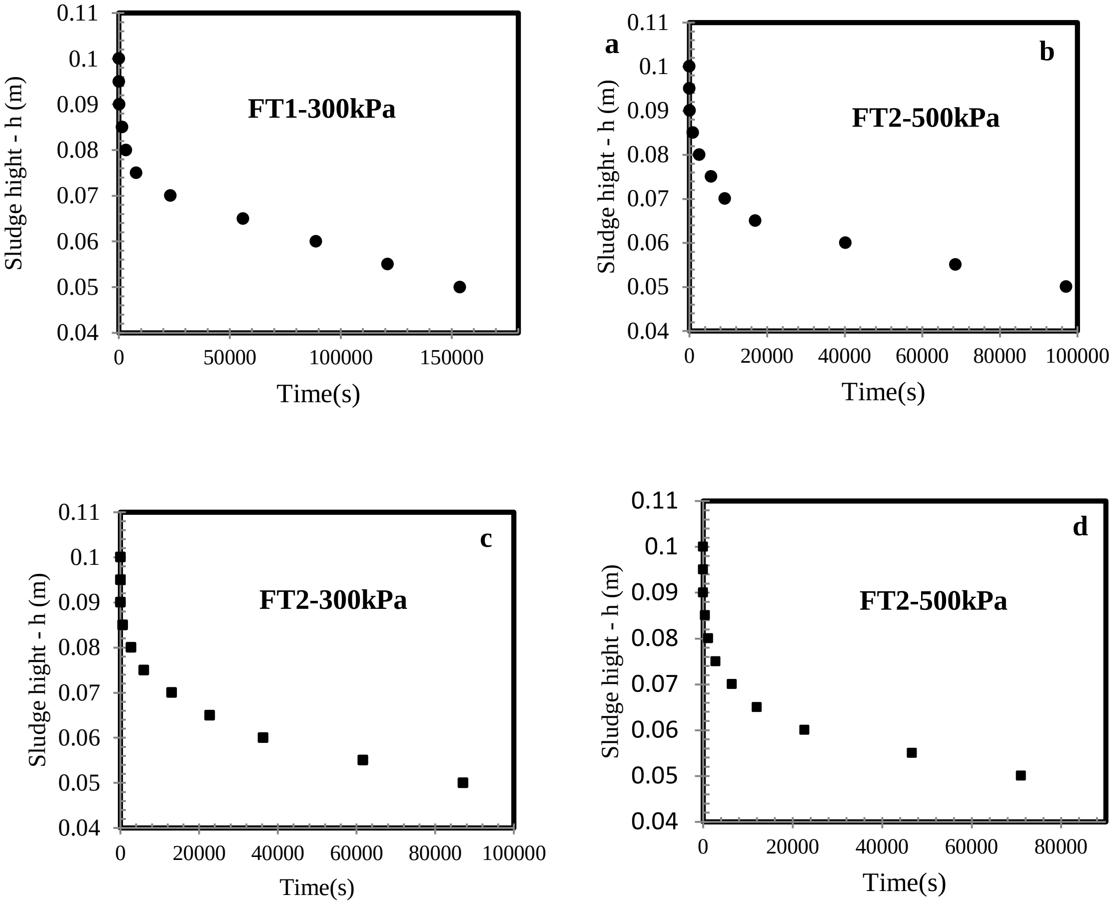


Figure 4.3. Graph of cake filtration experiment results in four described cases

4.3. Model Prediction

4.3.1. Pre-Experiments

4.3.1.1. Filter number for clean filter (FN_0)

The first experiment is to find FN_0, which shows the characteristics of clean filter. The cylinder is filled with clean water for 20 cm depth and 300 kPa pressure applied. In this experiment, to calculate FN_0, a set of experiment has been applied with two different kinds of filters as mentioned in table 4.1.The filtrate volume is measured for a certain time (60 sec) at each time interval. It shows a linear relationship, because there is no change of opening of the filter due to clogging. Results of experiments with water and filter show that this number is always constant for a clean filter.

As an example, filter number of clean filter type I, due to the condition (P = 300 KPa and Time Interval = 60 seconds), is calculated as:

$$FN_0 = 2gA^2P\left(\frac{\Delta t}{\Delta V}\right)^2 = 2\left(981\ {}^{cm}/_{s^2}\right)(706m^2)^2(3100cm)\left(\frac{59s}{5000cm^3}\right) = 4.2E+8$$

Same experiment and method was done to find the FN_0 for filter type II.

Results of filter number expression for clean filters have been presented in table 4.3 and figure 4.4 is showing the graph of radian of volume of water which is discharging from the cylinder versus time for filter type I and II and also the dimensionless graph of P/γ versus $Q^2/2gA^2$ which the slope is defined as FN_0, which is related to the head loss through the filter. Results show a linear relationship, because there is no change of opening of the filter due to clogging.

Table 4.3 Results of Filter Number Calculation for Clean Filters

Filter	**ΔV (lit)**	**Δt (s)**	**Q (lit/s)**	**FN_0**
Filter Type I	5	26	0.19	4.6E+8
Filter Type II	5	63	0.079	7.9E+7

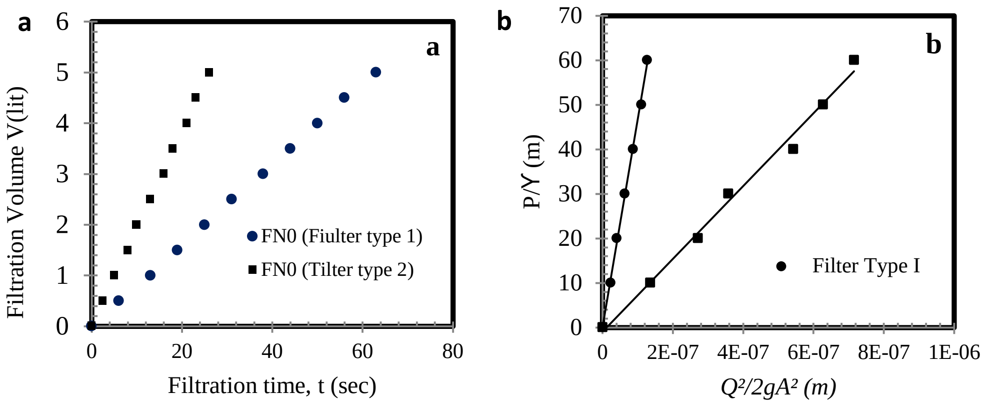


Figure 4.4. Graph of filtration volume versus time under 300 KPa pressure and clean filter for (a) Filter Type I and II and (b) dimensionless graph of P/γ versus $Q^2/2gA^2$ for Filter Type I and II

The scope of line in graph 4.4 a, is the amount of discharge and the scope of graph 4.4 b, is the filter number of the clean filter. The liner form of these graphs show that it is always a constant number. Form this concept, it is shown the Filter Number (FN_0) of clean filter is a constant number which means that FN is a very good indicator for showing the properties of filter in pressure dewatering. Also For the filter with small opening, FN_0 become high because of head loss through the filter, and a filter with large opening, FN_0 becomes lower.

4.3.1.2. Hydraulic Conductivity of Cake

One of the basic parameters of cake is hydraulic conductivity which also can be known as the cake resistance against the water fellow. The particles of sludge are significantly small and also due to the specific characteristics of sludge, like awful smell, the common use methods of hydraulic conductivity measurement, such as falling-head method, are hard to be operated. Therefore, a specific method for calculating the hydraulic conductivity has been designed and executed.

In this case, in a 50 ml glass tube with pressure application property, 20 ml of sludge has been put into the tube and 20 ml diluted milk has been added to the top of the sludge. Because of the black color of sludge, the milk was used so that it is easier to track its movement. In this case, to make the viscosity of the milk closer to water, it was diluted with 7 times volume of water. Different pressures have been applied to the system and the movement of the milk has been tracked and the time of the movement of milk when it comes out from out of the tube has been recorded (Figure 3.5).

Hydraulic conductivity of cake filtration has been calculated for two different kinds of sludge: One for the sediment sludge before filtration (k_0) and also for the compacted cake sludge after filtration process (k_{av}). As in it took a long time for the milk to get out of the sludge, the movement of milk has been tracked for a shorter distance (10 cm).

As an example, in the system condition with sediment sludge under 300KPa pressure and the sludge column of 10 cm, experiment shows that it takes 380 seconds that the diluted milk crosses trough the sludge column.

$$k_0 = \frac{L^2}{Pt} = \frac{(0.1m)^2}{(31m)(380s)} = 8.5 \times 10^{-7}\, {}^{m}/_{s}$$

The same experiment was done with the compact cake sludge. And hydraulic conductivity has been measured for sediment sludge and compacted cake. The hydraulic conductivity for sediment sludge and compacted cake will be shown as k_0 and k_{av} respectfully and the arithmetic mean of these two amounts is mentioned as the hydraulic conductivity of the sludge (k_{av}).

4.3.2. Filter Number Function, FN(t)

To determine filter number function FN(t), two filtration experiments with wastewater sludge have been performed with different pressures. Experiments were with using filter type I and II with pressure 300 and 500 kPa.

Experiment is done for the system of filter and the cake. The same cylinder is filled with sludge for 10 cm depth and 300 kPa pressure applied. And the volume of filtrate is measured with time for 80,000 sec. From this experiment, dV/dt is calculated at each time and the curve of filtration volume versus time can be plotted (Figure 4.6a). In case, filter number as a function of time can be calculated from equation 19 in chapter 3. Based on this, the (m) and (FN_f) can be obtained by curve fitting based on the experiment results.

As the results of this experiments, FN(t) can be calculated as:

$$FN(t) = 2gA^2Pa(dt^2)/\gamma(dV)^2$$

For example, in the case of time t=20,000s, dV/dt is obtained as 0.02 cm^3/s:

$$FN(t = 20{,}000) = 2(981\ cm/s^2)(706\ cm^2)^2(5950\ cm)/(0.02\ cm^3/s)^2 = 1.45E + 16$$

In this case, by curve fitting, FN_f and m can be obtained as 3.2E+16 and 0.0013 respectfully with curve fitting method. Figure 4.5b is presenting the FN(t) curve for this experiment. Same experiment can be done for other type of filter and also other amount of pressure.

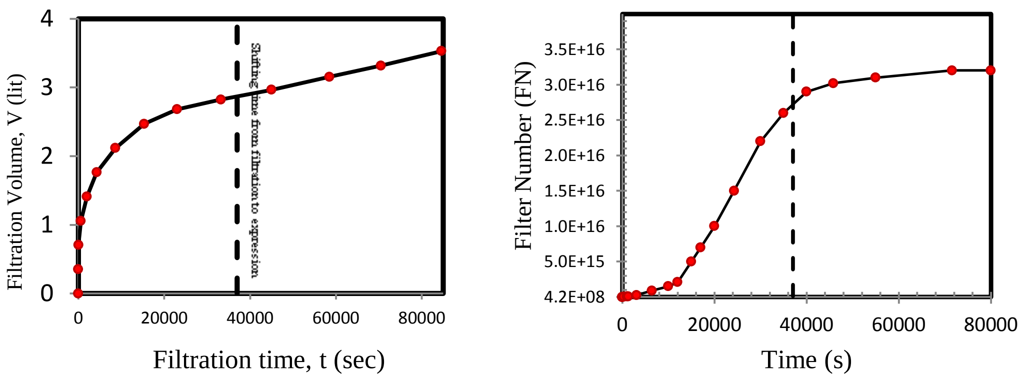


Figure 4.5 (a) v-t curve (b) FN(t) curve for cake filtration process

4.3.3. Results of Model Prediction

By running the model based on data which derived from pre-experiments, it is possible to predict the sludge height versus time graph for a filtration process. Table 4.4 is presenting the summary of data which is derived from pre-experiments for all 4 cases. Same method is also done for other three cases for the purpose of verification.

Table 4.4 Summary of Pre-Experiment Results

Case	FN_0	K_0(m/s)	K_{av}(m/s)	m	t_s(s)	FN_f
1	4.2E8	2.3E-6	5.1E-10	4.2E+7	24000	2.4E+16
2	4.2E8	1.2E-6	3.7E-10	6.5E+7	21000	3.1E+16
3	7.9E7	2.3E-6	5.1E-10	2.8E+7	22500	1.4E+16
4	7.9E7	1.2E-6	3.7E-10	6.E+8	18500	2.2E+16

To find the best Δt and Δh for the numerical solution of the Eq. (27) in filtration phases, error sensitivity analyses is done and the results for case 1 is shown in Figure 4.6.

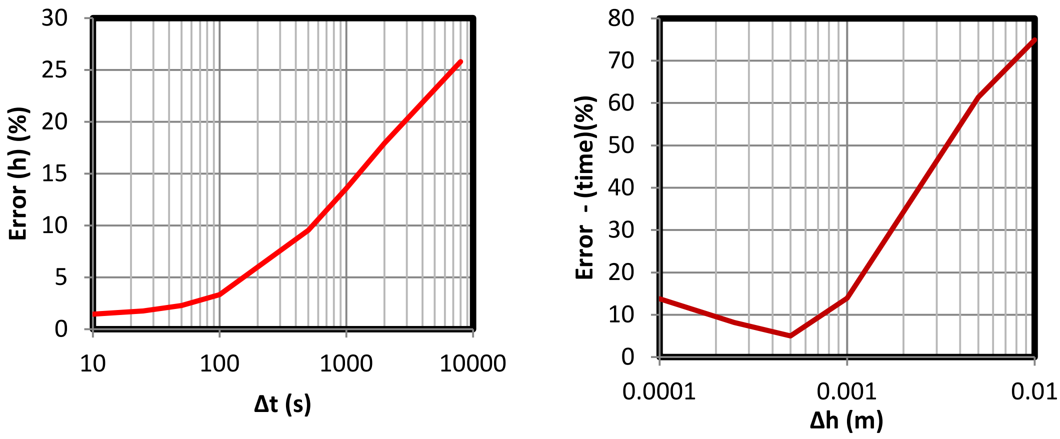


Figure 4.6 Results of sensitivity analysis of model error to (a) Δt (b) Δh

Based on the sensitivity analyses the best Δt and Δh are selected as 50s and 0.006 m respectively.

4.4. Comparing Experimental and Modeling Results

4.4.1. Model Verification

Once the condition of filtration process such as pressure, initial height (h_0) and viscosity of sludge is known and also the empirical parameters such as filter number for clean filter (FN_0), function of filter number (FN(t)) and hydraulic conductivity (K) can be obtained easily from simple pre-experiments, it is possible to predict the filtration process as changes of height versus time (h(t)), without doing any further experiments.

The model calculation and experimental results for four experiments has been presented in figures 4.7 and 4.8. As an example for filter type I, in 300 KPa pressure condition, predicts that it takes 64000 seconds for achieving 40% of the filtration process. However in under 600KPa, this time is reducing up to 46000 seconds. This means that reducing the pressure for 6 times can reduce the filtration time for 20000s.In this case, not only the mode can predict the time for achieving a certain percentage of filtration process, but also it can show the effect of pressure on the time reduction.

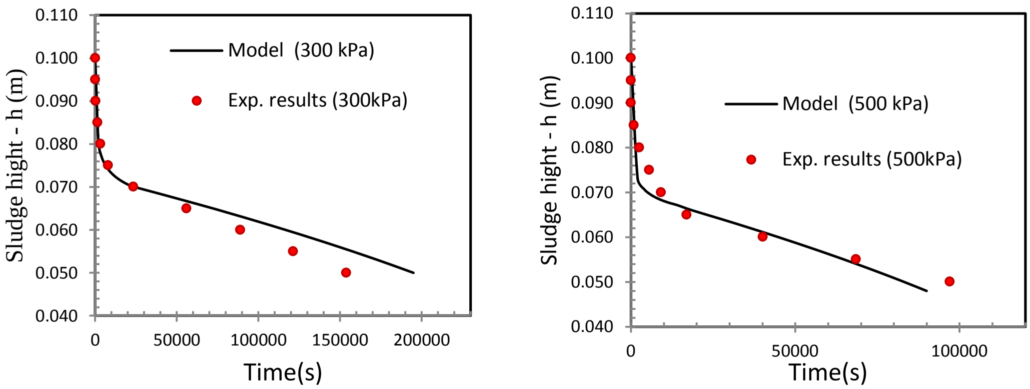


Figure 4.7 Experimental results and model calculation, with filter type I at 300 and 500 KPa pressure

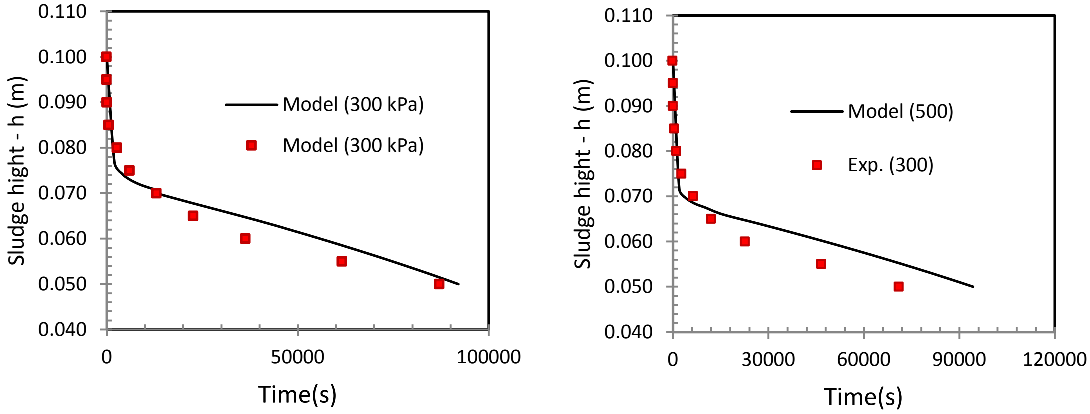


Figure 4.8 Experimental results and model calculation, with filter type II at 300 and 500 KPa pressure

4.4.2. Error Analysis

4.4.2.1. Regression Analysis

Regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. More specifically, regression analysis helps one understand how the typical value of the dependent variable changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables – that is, the average value of the dependent variable when the independent variables are fixed. Less commonly, the focus is on a quintile, or other location parameter of the conditional distribution of the dependent variable given the independent variables. In all cases, the estimation target is a function of the independent variables called the regression function. In regression analysis, it is also of interest to characterize the variation of the dependent variable around the regression function which can be described by a probability distribution.

For purpose of model verification, the regression analysis was done for data derived from experiment and also data which was predicted by the model. Figure 4.9 is showing the result of this analysis.

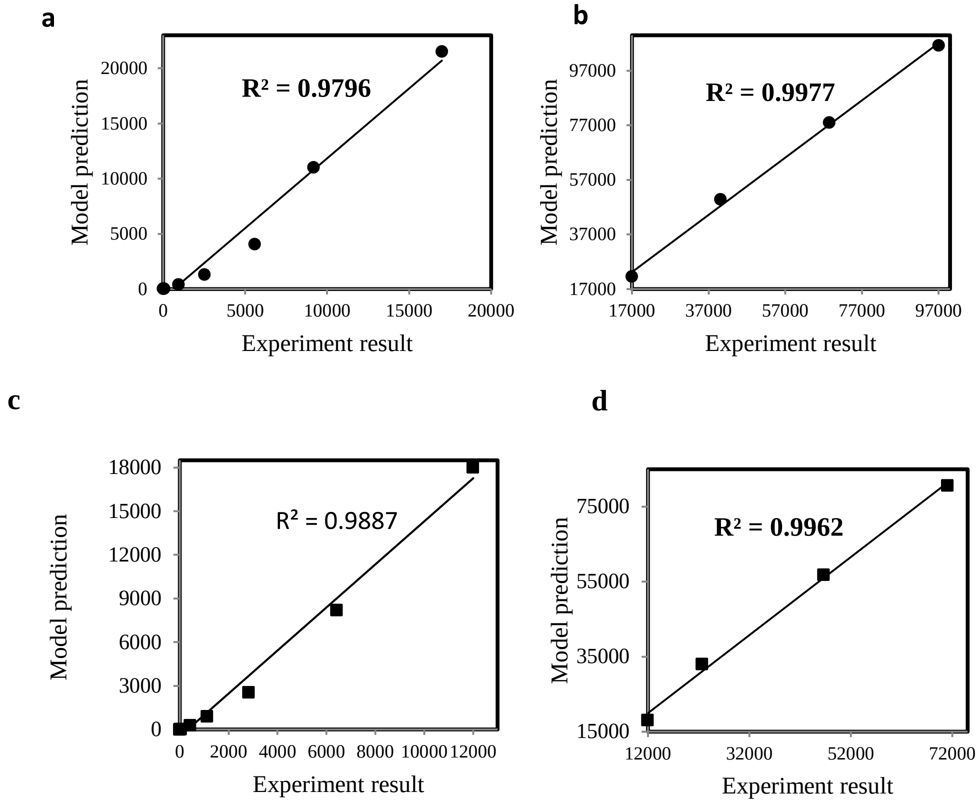


Figure 4.9 Graphs comparing the predicted values and experimental results of filtration time: (a) and (b) Filtration and Expression phases of filter type I at 500 kPa, (c) and (d) Filtration and Expression phases of filter type II at 500 kPa

4.4.2.2. Mean Relative Error

The mean relative error (MRE) is a quantity used to measure how close forecasts or predictions are to the eventual outcomes. MRE is a common measure of forecast error in time series analysis. MRE value can be calculated for each filtration test as equation 1.

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\frac{|Obseved\ time_{\ (i)} - Predicted\ time_{(i)}|}{Obseved\ time_{\ (i)}} \quad (1)$$

The amount of MRE for all described four cases is presented in Table 4.5. This analysis clearly shows that the model can predict filtration process reasonably well.

Table 4.5 Values of MRE for data of four filtration process cases

Model	mean relative error (MRE) %	
	P=300kPa	P=500kPa
Filter type 1	21%	31%
Filter type 2	28%	29%

4.5. Conclusion

In this chapter, the model was verified by comparing the model predicted data with data derived directly from cake filtration process experiment. Initial parameters such as filter number of clean filter and hydraulic conductivity of sludge is needed to be determined. In concept of filter number of a clean filter, it was shown that it is a dimensionless certain number which is Always constant for a filter. Also hydraulic conductivity of the sludge has been measured by a creative method.

The filtration experiments have been performed for two main purposes; first to measure the empirical parapets and the second for model verification .In case of waste water sludge, model has a good fitting with the experimental data which is proved by error analysis. Based on regression analysis, it became clear that model shows better agreement with experimental data in filtration phase under low pressure situation. In high pressure situation, model can predict very well in expression phase.

This model can also be used to predict the time for achieving the certain percent of filtration process which can be very useful in many industrial applications.

CHAPTER 5:

Model Application

5.1. Introduction

The produced sludge from wastewater treatment process which causes environmental pollutions represents the next challenge for water treatment industry. In order to deal with the wastewater sludge, filtration dewatering is required as it can reduce the water content of the sludge, the subsequent treatment and the disposal operations. Moreover, filtered sludge is generally much easier to handle. Dewatering process can be and expensive and inadvisable in concept of energy if do not be known well (Pei et al., 2010).

Cake filtration modeling, which makes it possible to predict the filtration rate and performance, is needed to manage the expenses and energy usage. Therefore, modeling of cake filtration and expression has been a matter of interest for decades. As it is mentioned in chapter 2, several models have been developed to predict the time of filtration to achieve a certain percentage of dewatering. However, due to the presence of several parameters which are related and may govern the filtration process, making a totally accurate model is very hard.

The mathematical cake filtration model presented in this dissertation can be applied for wastewater sludge as well to estimate and predicate the filtration time for reaching a certain percentage of dewatering process at different pressures.

In the real operational practice, time to reach the specific amount of water in wastewater sludge is a very important issue. In many cases, a company is trying to achieve a specific amount of dewatering process in an economical way which can manage energy and money. In this concept, having an operational diagram which can predict the dewatering process by considering the combination role of pressure and time is essential for preventing wastes.

In this chapter, making contour form operational diagram is discussed for the first time as a novel idea for the model application. It is described how to make an operation graph for a specific filter and sludge system in order to predict the dewatering process by considering combination of time and pressure parameters all by using the data that the model predicted for dewatering process and was described in chapter 4.

5.2. Degree of dewatering, %DW

The degree of dewatering or percentage of dewatering, which is mentioned as %DW, is the percentage amount of water which gets out of the system during dewatering process. Degree of dewatering can be calculated from height versus time graph which mentioned in chapter 4 by using equation 1:

$$\%DW = \frac{h_0 - h_t}{h_0} \times 100 \tag{1}$$

As an example, assume a dewatering process under 300 KPa pressure and with sludge and filter type I system as it is described in chapter 4. By doing simple pre-experiment and finding the important empirical parameters of sludge (cake) and filter it is possible to run the model and predict the height of sludge versus time curve. For wastewater sludge, K_0 and K_{av} are 8.5 E-7 m/s and 5.9 E-10 m/s respectfully. Also about the filter m, FN_f and T_s are 4.6 E+7, 2.3 E+16 and 24000s respectfully.

The height of sludge versus time curve can be plotted by running the model using mentioned empirical parameters (figure 5.1 a). Also by using equation 1 on the data derived from height of sludge versus time curve, it is possible to make the dewatering degree versus time curve, which is shown in figure 5.1 b.

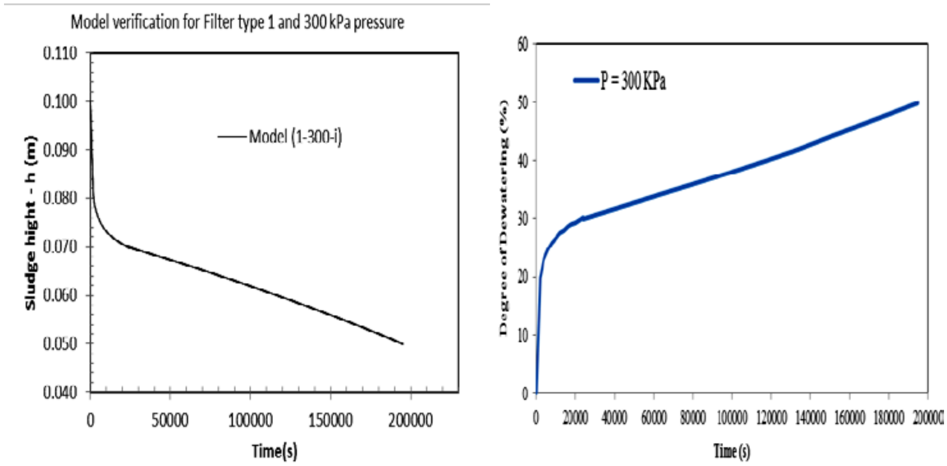


Figure 5.1 (a) height of sludge versus time curve and (b) %DW vs Time Curve for Sludge and Filter Type I under 300 KPa Pressure

This graph shows that, for example, to achieve 30% dewatering, it takes 20500 seconds. Or after 1 day (86400seconds) we can have 38% degree of dewatering.

By repeating the same method for dewatering process under 100 – 600 KPa, it is possible to plot the %DW versus time curve for them. Table 5.2 is presenting the amount of empirical characters for each dewatering process and figure 5.2 is showing the %DW versus time for them.

Table 5.1 Empirical Characters of Sludge and Filter Type I for Dewatering Process under 100 - 600 kPa Pressure

Pressure (kPa)	K_0 (m/s)	K_{av}(m/s)	m	FN_f	ts (s)
100	9.8E-7	7.0E-10	0.0051	1E+16	3000
200	9.1E-7	6.3E-10	0.0043	1.8E+16	26000
300	8.5E-7	5.6E-10	0.0035	2.5E+16	24000
400	8E-7	5.1E-10	0.031	2.9E+16	18000
500	7.6E-7	4.5E-10	0.0024	3.2E+16	14000
600	7.1E-7	3.9E-10	0.015	3.2E+16	12500

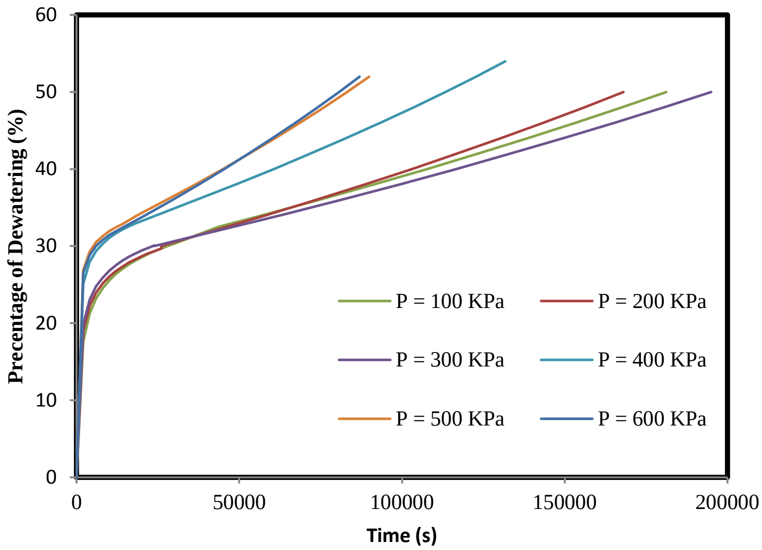


Figure 5.2 %DW vs Time Curve for Sludge and Filter Type I With 300 KPa Pressure

5.3. Operational Diagram

In real operation, the most important parameters are pressure and time. It will be good to estimate the degree of dewatering under the combination of time and pressure. This idea may lead to make operational diagram of a dewatering process.

An operation diagram is a diagram with contour lines at the plane of time and pressure. This kind of diagram can be made from predicted data that is provided by modeling the dewatering process. The main advantages of operational diagram is predicting the degree of dewatering process by considering both parameters of pressure and time without demand of any further experiment. Figure 5.3 is showing the operational diagram for sludge and filter type I system under different pressures.

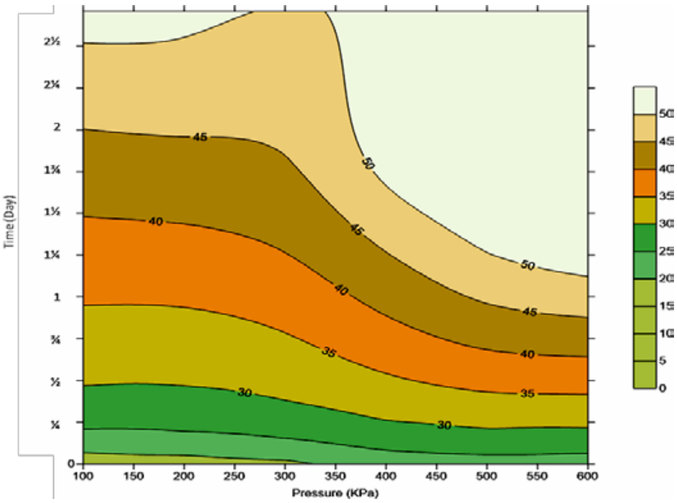


Figure 5.3. Contour Formed Operational Diagram for Sludge and Filter Type I

As some examples for showing the data which can be derived from operational diagrams, in this specific case, it is possible to say:

1. At 300 KPa, to reach 30% dewatering, we need about 3/8 day.
2. At 1 day, to reach 40% dewatering, we need a pressure about 370 kPa.
3. At 500 KPa, after 3/4 day, we can achieve dewatering of 42%.

However, it is possible to make this type of operational diagram for any kind of sludge and filter type regardless to the dimension of dewatering process.

5.4. Discussion and Conclusion

In case of waste water sludge, model has a good fitting with the experimental data which is proved by error analysis. This model can be used to predict the time for achieving the certain percent of filtration process which can be very useful in many industrial applications. In the other hand, pressure and time in the real operational field means energy and money. In some cases, pressure is the limiting factor and sometimes time is the limiting factor, depending on the type of sludge and condition.

Based on the demand to achieve the certain degree of dewatering and also by considering the limiting factors of the company, it is possible to guide the dewatering operation into the most economical way that the company can have.

CHAPTER 6

Conclusion

A new model based on the energy law was proposed for predicting cake filtration. To evaluate particle migration and capture in the porous media and the effect of the filter resistance, a new dimensionless filter number FN was introduced. In concept of hydraulic conductivity, an innovative simple method was presented to calculate the hydraulic conductivity with a good accuracy. Also, the effects of the governing parameters on the model predications for both the filtration and expression phases were found out by sensitivity analysis of the model on the governing parameters.

The model was verified by comparing its predictions with experimental filtration data on wastewater sludge. The effects of the governing parameters on the model predictions were also analyzed. The application of the model to wastewater treatment and harvested algae dewatering field was investigated by performing several laboratory filtration experiments. The following is a summary of the findings of the evaluation of the model and its sensitivity analysis:

1. Compared to previously complicate models the models, the proposed model is simple and more accurate.
2. Based on regression analysis and MRE calculations, the cake height predictions of the model are in good agreement with experimental.
3. The effect of the cake hydraulic conductivity was found to be negligible during in the filtration phase. Moreover, the filtration cake and the particles that accumulate front of the filtering medium significantly reduce the filtration flux.
4. Because the cake permeability is the dominant factor during expression, it is highly recommended that sludge treatment should only be applied during the expression phase to save money and energy.
5. Based on results that this model can predict, it is possible to plot operation diagram for a dewatering process. As pressure and time can be limiting factors in industry, using operational diagrams to predict the dewatering process by assuming the cooperation of both pressure and time is essential in industry.

Further research is necessary to use empirical coefficient to consider the effect of chemical enhancement

APPENDIX

$$t = \int \frac{dx}{x + \sqrt{x^2 - 2ax + b}}$$

$$= \int \frac{dx}{x + \sqrt{x^2 - 2ax + b}} \times \frac{x - \sqrt{x^2 - 2ax + b}}{x - \sqrt{x^2 - 2ax + b}}$$

$$= \int \frac{x - \sqrt{x^2 - 2ax + b}}{2ax - b}\, dx$$

$$= \underbrace{\int \frac{x}{2ax - b} dx}_{I} - \underbrace{\int \frac{\sqrt{x^2 - 2ax + b}}{2ax - b} dx}_{II} \Rightarrow \boldsymbol{t = I - II}$$

$$\boldsymbol{I} = \int \frac{x}{2ax - b} dx = \frac{1}{2a} \int \frac{x - \frac{b}{2a} + \frac{b}{2a}}{x - \frac{b}{2a}} dx = \frac{1}{2a} \int dx + \frac{b}{4a^2} \int \frac{dx}{x - \frac{b}{2a}}$$

$$\Rightarrow \boldsymbol{I} = \frac{x}{2a} + \frac{b}{4a^2} \ln\left(x - \frac{b}{2a}\right)$$

$$\boldsymbol{II} = \int \frac{\sqrt{x^2 - 2ax + b}}{2ax - b} dx$$

$$= \frac{1}{2a} \int \frac{\sqrt{\left((2ax - b) + (b - 2a^2)\right)^2 - (b - 2a^2)^2 + b^2}}{2ax - b} dx$$

$$= \frac{1}{2a} \int \frac{\sqrt{(b - 2a^2)^2 + b^2}\sqrt{\frac{\left[(2ax-b)+(b-2a^2)\right]^2}{(b-2a^2)^2+b^2} - 1}}{2ax - b} dx$$

$$= \int \frac{\sqrt{a^2 - b}}{2ax - b} \sqrt{\frac{(x - a)^2}{a^2 - b} - 1}\, dx$$

By considering $\frac{x-a}{\sqrt{a^2-b}} = Sec\theta \ \Rightarrow x = \sqrt{a^2-b}Sec\theta + a \ \Rightarrow dx =$
$\sqrt{a^2-b}.tan\theta.Sec\theta$

$$\boldsymbol{II} = \int \frac{\sqrt{a^2-b}\sqrt{\sec^2\theta - 1}\sqrt{a^2-b}.tan\theta.Sec\theta}{2a\left(\sqrt{a^2-b}Sec\theta + a\right) - b} d\theta$$
$$= \frac{a^2-b}{2a}\int \frac{\sqrt{\sec^2\theta - 1}\, tan\theta.Sec\theta}{\sqrt{a^2-b}Sec\theta + 1 - b/2a} d\theta$$

$$= \frac{\sqrt{a^2-b}}{2a}\int \frac{tan^2\theta.Sec\theta}{Sec\theta + \frac{2a-b}{2a\sqrt{a^2-b}}} d\theta \ = \frac{\sqrt{a^2-b}}{2a}\int \frac{tan^2\theta}{1 + (\frac{2a-b}{2a\sqrt{a^2-b}})cos\theta} d\theta$$

$$= \frac{\sqrt{a^2-b}}{2a}\int \frac{sin^2\theta}{cos^2\theta\left[1 + (\frac{2a-b}{2a\sqrt{a^2-b}})cos\theta\right]} d\theta$$
$$= \frac{\sqrt{a^2-b}}{2a}\int \frac{1 - cos^2\theta}{cos^2\theta\left[1 + (\frac{2a-b}{2a\sqrt{a^2-b}})cos\theta\right]} d\theta$$

By considering $\cos\theta = \frac{1-u^2}{1+u^2} \ \Rightarrow u = tan(\theta/2) \Rightarrow du = \frac{1}{2}\left(1 + tan^2\left(\frac{\theta}{2}\right)\right) d\theta \ \Rightarrow$
$d\theta = \ \frac{2}{1+u^2}$ so:

$$\boldsymbol{II} = \frac{\sqrt{a^2-b}}{2a}\int \frac{\left[1 - \left(\frac{1-u^2}{1+u^2}\right)^2\right]\left(\frac{2}{1+u^2}\right)}{\left(\frac{1-u^2}{1+u^2}\right)^2\left(1 + \left(\frac{2a-b}{2a\sqrt{a^2-b}}\right)\left(\frac{1-u^2}{1+u^2}\right)\right)} du$$

$$= \ \frac{4\sqrt{a^2-b}}{a}\int \frac{u^2}{(1-u^2)(1+u^2 + \frac{2a-b}{2a\sqrt{a^2-b}}(1-u^2))}$$

By considering $\mathrm{P} = 1 - \frac{2a-b}{2a\sqrt{a^2-b}}$ and $\mathrm{q} = 1 + \frac{2a-b}{2a\sqrt{a^2-b}}$ we will have:

$$\boldsymbol{II} = \frac{4\sqrt{a^2-b}}{a}\int \frac{u^2}{(1-u^2)(Pu^2-q)} du$$
$$= \frac{4\sqrt{a^2-b}}{a}\int \left[\frac{1}{(1+P)(1-u^2)} - \frac{q}{(1+P)(Pu^2-q)}\right] du$$

$$= \frac{4\sqrt{a^2-b}}{a(1+P)} \int \left[\frac{1}{2(1-u)} + \frac{1}{2(1+u)} - \frac{q}{(Pu^2-q)}\right] du$$

$$\boldsymbol{II} = \frac{2\sqrt{a^2-b}}{a(1+P)} \ln(1-u^2) + \frac{4\sqrt{a^2-b}}{a(1+P)} \sqrt{\frac{q}{p}} Arc\tan\left(\sqrt{\frac{p}{q}}u\right)$$

So:

$$\boldsymbol{t} = \boldsymbol{I} + \boldsymbol{II} = \frac{x}{2a} + \frac{b}{4a^2} \ln\left(x - \frac{b}{2a}\right) + \frac{2\sqrt{a^2-b}}{a(1+P)} \ln(1-u^2) + \frac{4\sqrt{a^2-b}}{a(1+P)} \sqrt{\frac{q}{p}} Arc\tan\left(\sqrt{\frac{p}{q}}u\right)$$

In which:

$$x = -\frac{h_s}{K}$$

$\mathrm{P} = 1 - \frac{2a-b}{2a\sqrt{a^2-b}}$ And $\mathrm{q} = 1 + \frac{2a-b}{2a\sqrt{a^2-b}}$

$\mathrm{b} = 2\left[\frac{FN.A^2}{gA*^2}\right]\left(\frac{P}{\gamma}\right)$ And $a = K\left[\frac{FN.A^2}{gA*^2}\right]$

$u = tan(\theta/2)$ And $Sec\theta = \frac{x-a}{\sqrt{a^2-b}} \Rightarrow \mathrm{u}^2 = \frac{x-a-\sqrt{a^2-b}}{x-a+\sqrt{a^2-b}}$

So:

$$\boldsymbol{t} = \frac{\boldsymbol{x}}{\boldsymbol{2a}} + \frac{\boldsymbol{b}}{\boldsymbol{4a^2}} \ln\left(\boldsymbol{x} - \frac{\boldsymbol{b}}{\boldsymbol{2a}}\right) + \frac{2\sqrt{\boldsymbol{a^2-b}}}{\boldsymbol{a(1+P)}}\left[\ln\left(\frac{2\sqrt{\boldsymbol{a^2-b}}}{\sqrt{\boldsymbol{a^2-b}}+\boldsymbol{x}-\boldsymbol{a}}\right) + 2\sqrt{\frac{\boldsymbol{q}}{\boldsymbol{p}}}\tan^{-1}\left(\sqrt{\frac{\boldsymbol{p}}{\boldsymbol{q}}}\,\boldsymbol{tan}\left(\frac{1}{2}\boldsymbol{cos}^{-1}\left(\frac{\sqrt{\boldsymbol{a^2-b}}}{\boldsymbol{x-a}}\right)\right)\right)\right]$$

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FN을 이용한 케이크 여과의 수학적 모델링과 실험적 검증

국문초록

최근에 탈수공법은 고체-액체 분리가 중요한 역할을 하는 입자분리, 하수 슬러지 처리, 제약 산업, 양조장과 음식 폐기물 관리 등 여러 분야에서 많은 관심을 이끌었다. 슬러지 탈수 공법 중에도 케이크 여과는 광범하게 사용되고 있다.

케이크 여과에는 양 압력이 적용되어 슬러지 내의 수분이 제거된다. 수분 제거는 슬러지와 필터에 의해 저항으로 한다. 현재까지 케이크 여과의 거동모형은 케이크 저항이나 필터 차단으로 개발되었다. 본 논문은 1-차원 케이크 여과 과정의 여과 거동과 단계를 평가하는 해석적 모형을 제시하였다. 제시된 모형은 에너지법칙을 이용하여 여과 매체의 저항 영향을 고려하고 새로운 매개 변수인 필터계수 (FN)가 적용된다. 하수 슬러지에 대해 여과 실험으로 제시된 모형을 검토했다. 실험 과

모형을 비교한 결과, 압력 300 kPa 에는 평균상대오차가 21%이고 500 kPa 에는 평균상대오차가 31%이다. 결론으로 본 논문에서 제시된 모형은 여과시간과 케이크 여과 과정의 단계를 예측할 수 있다.

압력 및 시간은 실제 운영에는 에너지 및 비용과 밀접한 관계가 있다. 슬러지의 종류와 상태에 따라 어떤 경우에는 압력이 제한 조건으로 되고 어떤 경우에는 시간이 제한 조건으로 된다. 요구되는 탈수 정도와 제한 조건을 고려해서 탈수 운영 그래프를 플롯팅할 수 있다. 그래프 플롯팅은 예측 모형에 의해 이뤄졌고 이 그래프를 통하여 가장 경제적 방법을 선택할 수 있다.

KEYWORDS

모형화, 케이크 여과, 하수 슬러지, 필터 계수, 수리경, 투수계수