Tax-Rate Uncertainty, Tax Exemption, and Tax Evasion

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This paper examines comparative statics predictions for the response to first-order stochastic dominance (FSD) and mean preserving spreads (MPS) shifts about the tax rate on tax evasion when the distributions of the tax base and of the tax rate are stochastically dependent. For MPS changes, in this more general setting, we confirm a previously reported result that, under plausible assumptions on risk preferences, greater tax-rate risk decreases reported income, thereby increasing tax evasion. We also provide comparative statics effects of FSD and MPS changes in the tax exemption on tax evasion.
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I. Introduction

There has been received a considerable attention in recent years as regards the issues of uncertain tax policies and their economic impact. This interest stems from pervasive concerns that the fiscal environment has been destabilized by either the frequent revisions of tax codes or the variability of effective tax rates due to the interaction of inflation with an unindexed tax structure (Feldstein and Summers 1979). Alm (1988), for example, referred that tax-rate uncertainty is attributed to frequent changes in tax rate schedules, failure to index fully the base and rates of income tax in a world of unpredictable inflation, and proposals for tax reform. Watson (1992) indicated that most work overlooked the uncertainty

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taxpayers anticipate a change in tax laws. Hess (1993) argued that tax rates are too volatile so that they fluctuate in excess of variations of economic fundamentals. Weiss (1976), Stiglitz (1982), Alm (1988), Watson (1992), and Kim, Snow, and Warren (1995), among others, have investigated the effect of uncertainty about the tax rate or tax base on factor supplies. In particular, Watson (1992) concluded that the impact of increased tax-rate uncertainty on optimal saving depends on the magnitude of the relative risk aversion coefficient. Unlike previous studies, Kim, Snow, and Warren (1995) analyzed an increase in uncertainty about a tax that applies comprehensively to interest and wage incomes by considering a model of individual labor supply and saving in which these decisions are made simultaneously. In a macroeconomics context, Basu (1995) examined the effect of income tax rate uncertainty on the degree of consumption smoothness. However, none of these papers considers decision models with tax exemption level or multiple sources of randomness including cases where the risks are not independent of one another.

Specifically, Alm analyzed the effects of increased tax-rate or tax-base uncertainty on individual tax evasion behavior, without the probability of detection, and concluded that, under plausible restrictions on risk preferences, greater tax-rate (tax-base) risk reduces (increases) declared income, thereby increasing (decreasing) tax evasion. In discussing caveats to these conclusions, however, Alm (1988, p. 244) noted that "tax base and tax rate risk are examined as separate cases, with base (rate) risk absent when rate (base) uncertainty is analyzed. The joint effects of both types of uncertainty may well be different from their individual effects." In this paper, we provide commonly accepted conditions on a decision maker's willingness to bear risk which are sufficient to affirm results presented by Alm (1988) concerning the effect of increased tax-rate uncertainty or the uncertain exemption level on tax evasion, but in an empirically more realistic setting, in which the tax rate and tax base are stochastically dependent.

In the framework adopted by Alm and other authors, the stochastic distributions of the tax base and tax rate are assumed to be independent of one another. It seems likely, however, that individuals with higher marginal tax rates will have a lower fraction of unreported income that is detected by the revenue authorities. One reason for this relationship between one's tax rate and effective tax base is that individuals in higher tax brackets generally have more complex tax returns. This complexity, in turn, makes it more difficult for unreported income to be detected by a tax audit. Moreover, at the aggregate level, the same political forces that generate randomness in tax rates simultaneously produce uncertainty about the tax base. Indeed, Fullerton (1994) chronicles the evolution of U.S. federal tax policy, giving special attention to the frequent changes in the tax structure — and the accompanying changes in the tax base — that have occurred in the last thirty years.

In this paper, we extend the framework used by Alm and other authors in three new directions. First, whereas Alm and other authors assumed that the random variables characterizing the uncertainty over these parameters were independent, we consider the case where the random variables are dependent. Second, whereas most of the previous studies focused exclusively on the case of mean preserving spreads (MPS), we also consider the case of first-order stochastic dominance (FSD) shifts. Third, we incorporate the exemption level into the tax structure for progressivity. This paper examines the impact of two types of changes in tax-rate risk on tax evasion when the distributions of the tax base and of the tax rate are stochastically dependent, which is a point admitted neglected by the previous studies. We use a result derived by Meyer and Ormiston (1994) to show that, under the restrictions on risk preferences, greater uncertainty about the tax rate decreases reported income, thereby increasing tax evasion. Thus, we are able to confirm Alm's conclusion but in a more general framework that allows for a stochastic dependent relationship between the tax rate and tax base. We also show that the effects of FSD and MPS changes in the tax rate on tax evasion do not influence the degree of progressivity and the comparative statics effects of changes in the tax exemption on tax evasion is unpredictable.

The remainder of the paper is organized as follows. In Section II, we set out the tax evasion model with tax exemption. Our main results on comparative statics predictions for FSD and MPS changes after introducing the *ceteris paribus* assumption we impose on changes in uncertainty are derived in Section III. Conclusions are presented in the final section.

II. The Model

We generalize the tax evasion model of Alm to encompass the case in which the fraction of income detected by the authorities is stochastically dependent upon tax rate and to incorporate tax exemption allowing for a progressive tax rate system. An individual allocates fixed income I_0 between declared income D and evaded income I_0-D. Declared income above an exemption level m is subject to an uncertain tax rate t, while an uncertain fraction α of the evaded income is detected and subject to a penalty. We assume that the penalty π is imposed on each amount of unreported income that is detected. Thus, the uncertain income of the individual is given by $I = m + (D - m)(1-t) + (I_0-D)(1-\alpha\pi)$. The consumer chooses declared income D to maximize

$$EU(I) = \int_0^1\int_0^1 U(I) \cdot d^2H(t,\alpha;\phi), \tag{1}$$

where E denotes the expectation operator, $H(t,\alpha;\phi)$ is the joint cumulative distribution function (CDF) for t and α defined on the interval [0, 1], and d^2H denotes $[\partial^2 H/\partial t \cdot \partial\alpha] \cdot dt \cdot d\alpha$. Increases in the index ϕ change the CDF $H(t,\alpha;\phi)$ in a prescribed manner, as discussed below. We assume that the utility function is non-decreasing, strictly concave, and three times continuously differentiable in I.

To simplify the analysis, the initial joint distribution $H(t,\alpha;\phi_0)$ of t and α is assumed to be such that the consumer attains a unique, regular, and interior maximum at D_0 satisfying $0<D_0<I_0$. The first-order condition for a maximum is

$$V(D_0) = \int_0^1\int_0^1 \rho(t;\alpha,D_0) \cdot d^2H(t,\alpha;\phi_0) = 0, \tag{2}$$

where $\rho(t;\alpha,D_0) = \partial U(I)/\partial D = U_I(I) \cdot (\alpha\pi - t)$ denotes the derivative of utility with respect to D and the subscripts on functions here and later denote differentiation. It is also assumed that the second-order condition is always satisfied at $D=D_0$; that is,

$$V'(D_0) = \int_0^1\int_0^1 \frac{\partial\rho(t;\alpha,D_0)}{\partial D} \cdot d^2H(t,\alpha;\phi_0) < 0. \tag{3}$$

We consider changes from an initial CDF $H(t,\alpha;\phi_0)$ to a final CDF $H(t,\alpha;\phi_1)$, where $H(t,\alpha;\phi_1) \equiv H(t,\alpha;\phi_0)$ if and only if $\phi_1 = \phi_0$. The following assumption requires that these changes satisfy the Meyer and Ormiston (1994) *ceteris paribus* condition for comparative statics involving changes in uncertainty about t.

Assumption (Ceteris Paribus)
If $\phi_1 \neq \phi_0$, then the change from $H(t,\alpha;\phi_0)$ to $H(t,\alpha;\phi_1)$ changes the marginal distribution for t but not the marginal distribution for α.

For an environment where two random variables are stochastically dependent, Meyer and Ormiston (1994) identified the *ceteris paribus* assumption appropriate for the study of comparative statics analyses when one random variable undergoes a change. The *ceteris paribus* assumption requires that the marginal cumulative distribution for the second random variable remain unchanged when the first random variable is altered. Extending this assumption to encompass an additional random variable, we examine increases in uncertainty for one random variable by requiring that the joint marginal distribution for the other two random variables remains unchanged. For example, a shift in the marginal distribution for t must also shift either the marginal distribution for α or its conditional distributions, or both. Thus, we can write the conditional distribution for t as $F(t|\alpha;\phi)$, which is dependent of ϕ and the marginal distribution for α as $G(\alpha)$, which is independent of ϕ, for changes satisfying the *ceteris paribus* assumption. By focusing on shifts that meet this condition, and defining first-order stochastic dominance (FSD) shifts and mean preserving spreads (MPS) in terms of the conditional distributions for the original random variable, we derive restrictions on preferences that are sufficient to determine an individual's response to FSD and MPS relations.

We are interested in two family sets of changes in the initial CDF, FSD shifts, which increase the mean value of the random variable, and MPS shifts, which increase risk in the general sense of Rothschild and Stiglitz (1970). With reference to t as the random variable, we have

FSD with respect to t: For all α, if $\phi_1 > \phi_0$, then

$$F(t\,|\,\alpha;\phi_1) - F(t\,|\,\alpha;\phi_0) \leq 0;$$

and MPS with respect to t: For all α, if $\phi_1 > \phi_0$, then

$$\int_0^s \{F(t\,|\,\alpha;\phi_1) - F(t\,|\,\alpha;\phi_0)\} \cdot dt \geq 0 \text{ for all } s \text{ in } [0,\ 1],$$
$$\text{with } \int_0^1 \{F(t\,|\,\alpha;\phi_1) - F(t\,|\,\alpha;\phi_0)\} \cdot dt = 0,$$

where $F(t|\alpha;\phi)$ is the relevant conditional distributions for t, and the intial and final distributions are indexed by ϕ_0 and ϕ_1, respectively.

To develop the comparative-statics analysis, the relevant measures of risk aversion with respect to the uncertain income I are

$$A(I) = -\frac{U_{II}(I)}{U_I(I)},$$

the Arrow-Pratt index of absolute risk aversion, and

$$R(I) = -\frac{I \cdot U_{II}(I)}{U_I(I)},$$

the index of relative risk aversion.

III. Analysis

In this section we derive the comparative statics results concerning the effects of an increase in uncertainty about the tax rate or the tax exemption on declared income by examining general sets of changes in the underlying joint cumulative distribution function. As usual, deciding how the optimal value for D changes as the random variable t is altered involves determining the sign of $V(D_0)$ after t is changed.

Following the *ceteris paribus* assumption in Meyer and Ormiston (1994), the conditional distribution for t depends on ϕ, while the marginal distribution for α does not, so that we can write $F(t|\alpha;\phi)$ and $G(\alpha)$. Thus, the first-order condition (2) can be restated as

$$V(D_0) = \int_0^1 \int_0^1 \rho(t;\alpha,D_0) \cdot dF(t|\alpha;\phi_0) \cdot dG(\alpha) = 0. \qquad (4)$$

Subtracting equation (4) from a similar expression with $F(t|\alpha;\phi_1)$ replacing $F(t|\alpha;\phi_0)$ results in:

$$V^*(D_0) = \int_0^1 \int_0^1 \rho(t;\alpha,D_0) \cdot \{dF(t|\alpha;\phi_1) - dF(t|\alpha;\phi_0)\} \cdot dG(\alpha). \qquad (5)$$

It follows that $V^*(D_0) > (<) 0$ if

$$\int_0^1 \rho(t;\alpha,D_0) \cdot \{dF(t|\alpha;\phi_1) - dF(t|\alpha;\phi_0)\} > (<) 0 \quad \text{for all } \alpha. \qquad (6)$$

We shall evaluate (6) in the following propositions, using the conditional and marginal CDFs for t and α, for FSD and MPS shifts in turn, in the case of stochastic dependence between the tax rate and tax base.

***Proposition* 1**
Assume that CDF transformations satisfy the *ceteris paribus* condition when t and α are stochastically dependent. Then declared income rises (falls), $D_1 > (<) D_0$, with all FSD shifts of t if relative risk aversion is greater (less) than one.

Proof: With respect to first-order stochastic transformations, condition (6) can be written

$$-\int_0^1 \frac{\partial \rho(t;\alpha,D_0)}{\partial t} \cdot \{F(t\,|\,\alpha;\phi_1) - F(t\,|\,\alpha;\phi_0)\}dt > (<) 0, \tag{6$'$}$$

for all α, after applying integration by parts. Since $\{F(t\,|\,\alpha;\phi_1) - F(t\,|\,\alpha;\phi_0)\} \leq 0$, the comparative statics condition for the effect of an increase in the tax rate is

$$\frac{\partial \rho}{\partial t} = -U_I(I) - (\alpha\pi - t)(D-m)U_{II}(I) > (<) 0. \tag{7}$$

Using the Arrow-Pratt indexes of absolute and relative risk aversion, we can rewrite (7) as

$$\frac{\partial \rho}{\partial t} = -U_I(I) \cdot [1 - R(I) + \{I_0(1-\alpha\pi) + m\alpha\pi\} \cdot A(I)] > (<) 0. \tag{7$'$}$$

A sufficient condition for this expression to be positive (negative) is the assumption that relative risk aversion be greater (less) than one, implying that declared income rises (falls) with greater uncertainty about the tax rate.

Q.E.D.

***Proposition* 2**
Assume that CDF transformations satisfy the *ceteris paribus* condition when t and α are stochastically dependent. Then declared income falls, $D_1 < D_0$, with all MPS shifts of t if absolute risk aversion is decreasing and relative risk aversion is increasing and less than one.

Proof: With respect to mean-preserving spread transformations, condition (6) can be written

$$\int_0^1 \frac{\partial^2 \rho(t;\alpha,D_0)}{\partial t^2} \cdot \int^t \{F(s\,|\,\alpha;\phi_1) - F(s\,|\,\alpha;\phi_0)\}ds \cdot dt > (<) 0, \tag{6$''$}$$

after applying integration by parts twice. Since $\int^t \{F(s\,|\,\alpha;\phi_1) - F(s\,|\,\alpha;\phi_0)\}ds \geq 0$, the comparative statics conditions for the effect of an increase in the tax rate is

$$\frac{\partial^2 \rho}{\partial t^2} = 2U_{II}(I) + (\alpha\pi - t)(D-m)U_{III}(I) > (<) 0. \tag{8}$$

Introducing the indexes of absolute risk aversion and relative risk aversion, we can rewrite (8) as

$$\begin{aligned} \frac{\partial^2 \rho}{\partial t^2} = &(D-m) \cdot U_{II}(I) \cdot [1 - R(I) + \{I_0(1-\alpha\pi) + m\alpha\pi\} \cdot A(I)] \\ &-(D-m) \cdot U_I(I) \cdot [R_I(I) - \{I_0(1-\alpha\pi) + m\alpha\pi\} \cdot A_I(I)] > (<) 0. \end{aligned} \tag{8}'$$

Since the penalty rate π in income is less than one, a set of sufficient conditions for this expression in (8)′ to be negative is the assumptions imposed by Alm, that relative risk aversion be less than one and increasing ($R(I)<1$ and $R_I(I)>0$), and that absolute risk aversion be decreasing ($A_I(I)<0$) for all $\alpha>0$ and $0<D<I_0$, in which case declared income falls with greater uncertainty about the tax rate. This result implies that $\phi(t;\alpha,D_0)$ is concave in t for all $0<D<I_0$. The result of Rothschild and Stiglitz concerning concave functions indicates that $V^*(D_0)<0$. Therefore, observe that $D_1<D_0$, so that declared income falls and tax evasion increases in response to greater uncertainty about the tax rate if $V^*(D_0)<0$, given first-order and second-order conditions.

Q.E.D.

The restrictions on risk preferences above have been derived for the portfolio problems by Hadar and Seo (1990), Meyer and Ormiston (1994), and Mitchell and Douglas (1997) and for the tax contexts by Alm (1988) and Watson (1992). For example, the following utility function given by a specific form $U(I) = a + b \cdot \log(I+c)$, where $b>0$ and $c \geq 0$ satisfies conditions on risk preferences used in Proposition 2. Notice that, if relative risk aversion is less than one and increasing, then the index of relative prudence introduced by Kimball (1990) is less than two. However, the converse is not true.

This result is consistent with Alm's statement indicating that, "greater tax-rate risk will cause the individual to decrease declared income because declared income becomes riskier than evaded income when the tax rate becomes more random" (p. 242). However, we have shown that this result holds, under plausible assumptions about risk preferences, in a more general setting in which the tax base and tax rate are stochastically dependent.

Finally, the exemption level m has played no role in the analysis thus far. We conclude that predictions for the effects of greater uncertainty about the tax rate and the fraction at which tax evasion is detected do not turn on the degree of progressivity incorporated in the tax rate structure. We turn next to consider the comparative statics effects of changes in an uncertain tax exemption instead of the tax rate.

From the income equation and associated first-order condition (2), we consider first-order stochastic dominance improvements in the distribution for exemption, for which the comparative statics effect turns on the first-order monotonicity property of marginal function $U_I(I) \cdot (\alpha\pi - t)$ with respect to m, as indicated by the sign of $U_{II}(I) \cdot (\alpha\pi - t)t$. Since $(\alpha\pi - t)$ changes sign from negative to positive as α increases, this expression can not have a uniform sign for all α, and therefore no prediction is possible for the comparative statics effect of a first-order stochastic dominance improvements in the distribution for the exemption.

Next the effects of greater uncertainty about the exemption are determined by the second-order convexity property of the marginal function $U_I(I) \cdot (\alpha\pi - t)$ with respect to m, as indicated by the sign of $U_{III}(I) \cdot (\alpha\pi - t)t^2$. Once again, since $(\alpha\pi - t)$ does not have a uniform sign, no prediction is possible for the effect of greater uncertainty about exemptions.

IV. Conclusions

This paper has examined the effects of both FSD and MPS changes in the tax rate on tax evasion when the fraction of income detected by the authorities is stochastically dependent upon the tax rate. By taking the *ceteris paribus* assumption of Meyer and Ormiston (1994), for MPS change, in this more general setting we confirm a previously reported result that, under plausible assumptions on risk preferences, greater tax-rate risk decreases declared income, thereby increasing tax evasion. Results derived in this paper apply to the tax evasion model, regardless of the presence or absence of stochastic dependence between the tax rate and the proportion of evaded income detected. Finally, we show that no prediction is possible regarding the comparative statics effects of FSD shifts and

MPS in the distribution for the exemption level on the amount of tax evasion. Our framework employed in this paper is also generalized in a general model of decision making under uncertainty that includes the portfolio and saving problems as special cases.

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