Optimal Target Zone and Stability Tradeoff

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Conventional literature on exchange rates largely ignores normative issues of the target zone regime. This paper builds a model for the optimal target zone and considers whether the target zone is desired compared to other exchange rate regimes. The paper finds that when the central bank's objective is to minimize the sum of price and output variances, a target zone regime is superior to a fixed or floating exchange rate. In determining the band width, a tradeoff exists between output and price stability, with a wider band producing smaller output variability and larger price variability. (*JEL* Classification: F31, F33, F41)

I. Introduction

The choice of exchange rate regime continues to be a topic of debate on both the theoretical and practical levels, and generally falls between the two extremes of fully fixed and fully flexible rates. Williamson (1983, 1987), however, criticized both fixed rates and

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their currencies to float freely after the breakup of the Bretton Woods system produced substantial short-term volatility of exchange rates and apparent misalignments of major currencies from their long-run equilibrium levels. In addition, the free-floating regime does not allow countries to coordinate macroeconomic policies. Williamson also criticized fixed exchange rates for their lack of flexibility. His suggestion of a target zone for exchange rates lies between the two extremes, keeping the merits and avoiding some of the problems of both options.

Much research has been done on target zones since Williamson's proposal. Krugman (1988, 1991) first formally modeled a standard target-zone regime where the zone is credibly maintained without future realignments. This modeling was further analyzed by Froot and Obstfeld (1991), among others.

In the standard target zone model, the exchange rate, like other assets, satisfies an asset pricing equation that is assumed to be a log-linear function of the aggregate market fundamental and the expected rate of depreciation. Suppose the central bank imposes a zone for the exchange rate around a pre-specified target. Within the zone, the central bank allows the exchange rate to move freely according to conditions in the foreign exchange market. Whenever the exchange rate reaches the boundaries of the zone, however, the central bank intervenes in the market just enough to keep the exchange rate from moving outside the zone. This marginal intervention is accomplished by restricting the movement of the market fundamentals through regulating monetary policy. In other words, when the exchange rate reaches the upper (lower) boundary, the money supply is decreased (increased) to keep the overall fundamentals and the exchange rate from straying outside the zone. In the presence of the target zone, while the market fundamental follows the regulated Brownian motion, the equilibrium path of the actual exchange rate is different. Assuming perfect credibility and marginal intervention, a rational agent's expectations of the future level of the exchange rate incorporates the exchange rate target zone and becomes negative (positive) when the exchange rate approaches the upper (lower) boundary. This produces an *S*-shaped path when plotting the equilibrium exchange rate against the market fundamentals. This *S* curve has a slope less than one, the slope of the equilibrium path of a free-floating exchange rate, and is tangent to the boundaries of the zone. This implies that the

exchange rate is more stable under a target-zone regime than under a free-floating regime.

Theoretical extensions of the Krugman model have been developed by relaxing one or both of the model's two critical assumptions: Perfect credibility and marginal intervention (Svensson 1992b). Specifically, Bertola and Caballero (1992) introduce discrete intervention and stochastic realignment at the boundaries of the zone with probability p. As p increases, the relationship between the market fundamentals and the exchange rate becomes more convex above the central parity and more concave below the central parity. This produces an inverted S-shaped path of the exchange rate against the market fundamentals. Krugman and Rotemberg (1990), Delgado and Dumas (1990) and Dumas and Svensson (1991) consider speculative attacks and the implications of these attacks on the target zone when the central bank's reserve holdings are limited. Under the European Monetary System (EMS), the central bank's interventions are frequently carried out in the interior of the zone. Delgado and Dumas (1992) formally modeled intra-marginal intervention by introducing a mean-reverting process for the fundamental rate. Flood and Garber (1991) consider a target zone with discrete interventions at pre-specified points inside the band. Also, some target zone models employ an exogenous stochastic jump process for realignments. See, for example, Bertola and Svensson (1991) and Svensson (1991a, 1991b, 1992a) for this line of research.

Two main problems emerge from the conventional literature on target zones. The first problem arises from the rule concerning central bank intervention. Most conventional literature assumes that the central bank's intervention tool is its monetary policy. In the case of intra-marginal intervention, the level of the money supply is adjusted according to the position of the exchange rate within the zone, and in the case of marginal intervention, it is adjusted only at the boundaries of the zone. For both cases, monetary policy is used *without limitation* to keep the exchange rate within the zone. The implicit assumption behind this unlimited intervention is that intervention is costless; frequent changes in the money supply by varying amounts are assumed not to affect any economic indicator other than the exchange rate. This is clearly an unrealistic assumption, since changes in the money supply can lead to changes in price and other variables. Therefore, the exclusive

and unlimited use of monetary policy for targeting the exchange rate would sacrifice other objectives of the central bank such as price or output stability.

The second problem arises from the direction of research. The standard Krugman-type model and its extensions focus on positive issues, mainly the dynamics of the exchange rate in the presence of the target zone. As Svensson (1992b) points out, however, such models hardly deal with the basic issue of whether the target zone is desired compared to other exchange rate regimes. In a later article, Svensson (1994b) argues that the target zone regime is better than a regime of fully fixed exchange rates, since the former not only stabilizes the exchange rate but also allows some degree of monetary independence, such as control over domestic interest rates even under full capital mobility. By controlling the position of the exchange rate within the zone, the central bank can affect the expected rate of depreciation, and thus from the interest rate parity condition, the domestic interest rate. Such control is only plausible, however, when the zone is fully credible. If not, such a policy might convey the unintended signal of the central bank's unwillingness to defend the zone and thereby instigate speculative attacks as the exchange rate deviates from the central parity. Also, the interest rate and the exchange rate are intermediate targets, rather than the ultimate goals of the central bank (Williamson and Miller 1987). As such, the two indicators are not proper measures for the optimality of exchange rate regimes. Svensson (1994b) is also unable to avoid the problem of the zero intervention cost assumption mentioned above. Further, Svensson (1994b) does not consider the free-floating regime. In contrast to Williamson (1983, 1987), Frenkel (1987) argues that the volatility and misalignment of exchange rates are the result of a lack of synchronization of fiscal policies among major countries, rather than the result of the failure of the current floating regime. Conventional literature, however, makes no comparison between the target-zone and the free-floating regimes.

The purpose of this paper is to specify intervention costs and the ultimate targets of the central bank and to analyze the normative issues of whether the target zone regime is desired and, more specifically, the tradeoffs in determining the band (zone) width.

In Section II, I develop a simple static model and obtain the motions of the exchange rate, the regulated money supply, price

and output under the assumptions of marginal intervention and perfect credibility. To get a tractable solution I initially assume that shock follows a triangular distribution. Given these assumptions, the variances in price and output are obtained in Section III. I show that a tradeoff exists between output stability and price stability in choosing the band width, due to the cost of intervention.

In Section IV, I relax the assumption of triangular distribution. I simulate the variances in price and output under the assumption that the random shocks follow the standard normal distribution, and show that a tradeoff still exists between price and output stability in choosing the band width.

In Section V, an optimal band width is considered assuming that the central bank's objective is to minimize the sum of the variances of price and output. By analyzing the optimal band width I consider the optimality of the target zone compared to the free-floating and the fixed exchange rate regimes. The paper ends with conclusions in Section VI.

II. The Model

Assume an open economy summarized by the following set of equations:

$$y_t = a(m_t - p_t), \tag{1}$$

$$y_t = cp_t + v_t\,, \tag{2}$$

$$x_t = p_t - p_t^*, \tag{3}$$

where x is the log of the spot exchange rate;
p is the log of the domestic price level;
y is the log of the domestic real output;
m is the log of the domestic money supply and is defined as the deviation of the money supply from its normal level;
a and c are constants; and
the asterisk represents foreign prices, which are assumed to be exogenous and normalized to zero.

The first equation serves as an aggregate demand function, where demand is driven by the real money supply. The second equation is a sticky wage aggregate supply function with a composite shock v.

The central bank has to determine the equilibrium exchange rate before it initiates the target zone. The third equation says that the central bank sets the equilibrium exchange rate, the central parity of the target zone, *ex ante*, allowing for a differential between domestic and foreign prices.[1]

From equations (1) and (2) we can obtain the equilibrium price and output as follows:

$$p_t = \frac{a}{a+c} m_t - \frac{1}{a+c} v_t, \tag{4}$$

$$y_t = \frac{ac}{a+c} m_t + \frac{a}{a+c} v_t. \tag{5}$$

Assume that the shock v is transitory and the equilibrium level of the exchange rate is constant and zero with normalization. Then from equations (3) and (4) the deviation of the exchange rate from the equilibrium is governed by the deviation of the money supply from its normal level and the random shock v.

In order to obtain a tractable solution in the presence of a target zone, I assume that the shock v follows an triangular distribution with range $(-\lambda, \lambda)$. The advantage of employing the triangular distribution is being able to get the explicit solution for the variances of price and output, while preserving the property that large shocks occur less frequently than small shocks. The p.d.f. of v is as follows:

$$f(v) = \begin{cases} \frac{1}{\lambda^2} v + \frac{1}{\lambda} & \text{if} \quad -\lambda < v < 0, \\ \frac{1}{-\lambda^2} v + \frac{1}{\lambda} & \text{if} \quad 0 \le v < \lambda, \end{cases} \tag{6}$$

so that v has the highest density at 0 and the lowest density at $-\lambda$ and λ.

Now, suppose the central bank imposes a symmetric target zone around the equilibrium exchange rate, with the band width $\pm b$ less than the limit of the range of the shocks. Further assume that the accompanying intervention rule of the central bank is the following.

[1]Note that equation (3) does not imply that the exchange rate is determined by the market according to the long-run purchasing power parity condition. It just describes how the central bank determines the central parity before implementing the target zone. Therefore this is not inconsistent with the short-run supply schedule in equation (2).

If the magnitude of the shock v is small and the deviation of the exchange rate from central parity is less than the band width b, the central bank does not intervene in the foreign exchange market. In this case, the money supply, the central bank's tool of intervention, is kept at its normal level, i.e. $m=0$. However, if the magnitude of the shock v is large enough that the deviation of the exchange rate from central parity is greater than the band width b, central bank intervention is called for. The money supply is adjusted so that the magnitude of the deviation of the exchange rate from the central parity is exactly the band width b, i.e. the exchange rate remains at the boundary of the zone. Thus intervention is marginal and the zone is perfectly credible.

III. A Tradeoff between Price and Output Stability

Given the above rule for intervention, the movements of the exchange rate and the regulated money supply are the following:

$$x_t=\begin{cases} -b & \text{if} \quad v_t>(a+c)b, \\ -\dfrac{1}{a+c}v_t & \text{if} \quad -(a+c)b\leq v_t\leq(a+c)b, \\ b & \text{if} \quad v_t<-(a+c)b, \end{cases} \tag{7}$$

$$m_t=\begin{cases} \dfrac{v_t-(a+c)b}{a} & \text{if} \quad v_t>(a+c)b, \\ 0 & \text{if} \quad -(a+c)b\leq v_t\leq(a+c)b, \\ \dfrac{v_t+(a+c)b}{a} & \text{if} \quad v_t<-(a+c)b. \end{cases} \tag{8}$$

The above equations imply the following. Suppose the magnitude of the shock is positive and large such that $v>(a+c)b$. Without intervention, this forces the exchange rate to move below the lower boundary of the zone. Therefore the central bank intervenes and partially offsets the shock by setting $m_t=\{v_t-(a+c)b\}/a$ so that the exchange rate remains at the lower boundary, i.e. $x_t=-b$. By the same token, if the magnitude of the shock is negative and large, such that $v<-(a+c)b$ and there is no intervention, the exchange rate will move outside the upper boundary of the zone. Therefore, to defend the zone, the central bank sets $m_t=\{v_t+(a+c)b\}/a$ and the exchange rate remains at the upper boundary of the zone, i.e.

$x_t = b$. Finally, if the magnitude of the shock is in between $-(a+c)b$ and $(a+c)b$, the exchange rate will remain within the zone without intervention. In this case, the central bank does not intervene ($m_t = 0$) and, as a result, the deviation of the exchange rate is fully determined by the random shock, i.e. $x_t = -\{1/(a+c)\}v_t$.

From the regulated movement of the exchange rate and the density of v given above, the variance of price using simple statistical analysis is shown by the following equation:

$$\begin{aligned} var(p_t) = var(x_t) &= \int_{(a+c)b}^{\lambda} b^2 \left(-\frac{1}{\lambda^2}s + \frac{1}{\lambda}\right) ds + \int_0^{(a+c)b} \left(\frac{s}{a+c}\right)^2 \left(-\frac{1}{\lambda^2}s + \frac{1}{\lambda}\right) ds \\ &\quad + \int_{-(a+c)b}^{0} \left(\frac{s}{a+c}\right)^2 \left(\frac{1}{\lambda^2}s + \frac{1}{\lambda}\right) ds + \int_{-\lambda}^{-(a+c)b} b^2 \left(\frac{1}{\lambda^2}s + \frac{1}{\lambda}\right) ds \qquad (9) \\ &= \frac{(a+c)^2}{2\lambda^2} b^4 - \frac{4(a+c)}{3\lambda} b^3 + b^2. \end{aligned}$$

Since foreign prices are normalized to zero, the variance of price is identical to the variance of the exchange rate.

To analyze how the variance of price is affected by changes in the band width, consider the minimization of the variance of price with respect to the band width. The F.O.C. yields two real roots, b_1 and b_2, where:

$$b_1 = 0, \quad b_2 = \frac{\lambda}{a+c} \text{ (inflection point).} \qquad (10)$$

From equation (10) and the curvature of equation (9), we can construct the path of the variance of price. Figure 1 shows the path of the variance of price against the band width ($0 < b < \lambda$). We see that the variance of price increases monotonically as the band width increases from zero to λ.

The movement of output is obtained from equations (5) and (8) as follows:

$$y_t = \begin{cases} -cb + v_t & \text{if } v_t > (a+c)b, \\ \dfrac{a}{a+c} v_t & \text{if } -(a+c)b \le v_t \le (a+c)b, \\ cb + v_t & \text{if } v_t < -(a+c)b. \end{cases} \qquad (11)$$

From the movement of output in equation (11) and the density of v in equation (6), we can calculate the variance of output as follows:

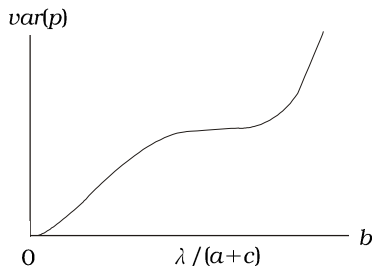

FIGURE 1
THE VARIANCE OF PRICE

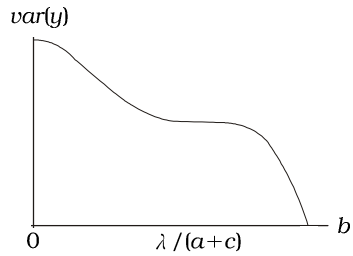

FIGURE 2
THE VARIANCE OF OUTPUT, $A<0$

$$var(y_t)=\int_{(a+c)b}^{\lambda}(s-cb)^2\left(-\frac{1}{\lambda^2}s+\frac{1}{\lambda}\right)ds+\int_{0}^{(a+c)b}\left(\frac{as}{a+c}\right)^2\left(-\frac{1}{\lambda^2}s+\frac{1}{\lambda}\right)ds$$

$$+\int_{(a+c)b}^{0}\left(\frac{as}{a+c}\right)^2\left(\frac{1}{\lambda^2}s+\frac{1}{\lambda}\right)ds+\int_{-\lambda}^{-(a+c)b}(s+cb)^2\left(\frac{1}{\lambda^2}s+\frac{1}{\lambda}\right)ds \qquad (12)$$

$$=\frac{1}{6\lambda^2}\{(-2a^3c-3a^2c^2+c^4)b^4+4\lambda(a^2c-c^3)b^3+6\lambda^2c^2b^2-4\lambda^3cb+\lambda^4\}.$$

To get the relationship between the band width and the variance of output, consider minimizing the variance of output with respect to the band width. The F.O.C. produces the following two real roots:

$$b_1=\frac{\lambda}{a+c}\ \text{(inflection point)},\qquad b_2=\frac{\lambda}{-2a+c}. \qquad (13)$$

Denote the coefficient of b^4 as A,

$$A=\frac{c(-2a+c)(a+c)^2}{6\lambda^2}.$$

When $A<0$, i.e. $-2a+c<0$, then $b_1>0$ and $b_2<0$. The curvature of equation (12) is determined by the signs of A, b_1 and b_2. In this case the variance of output decreases monotonically as the band width increases from zero to λ, as shown in Figure 2.

When $A>0$, i.e. $-2a+c>0$, both b_1 and b_2 are positive. The path of the variance of output in this case is shown in Figure 3. In order for the variance of output to decrease with the band width in its relevant range ($0<b<\lambda$), b_2 should be greater than or equal to λ. This requires that $c\leq 2a+1$.

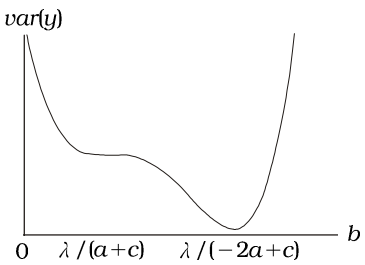

FIGURE 3

THE VARIANCE OF OUTPUT, $A>0$

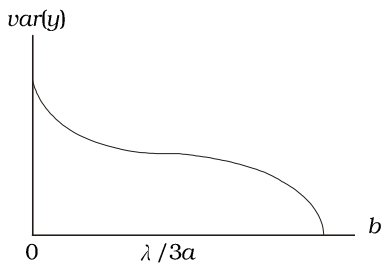

FIGURE 4

THE VARIANCE OF OUTPUT, $A=0$

When $A=0$, i.e. $-2a+c=0$, the variance of output becomes

$$var(y_t)=\frac{1}{6\lambda^2}(-24\lambda a^3b^3+24\lambda^2a^2b^2-8\lambda^3ab+\lambda^4), \tag{14}$$

and the F.O.C. yields the unique root, $b=\lambda/3a$. The variance of output decreases monotonically with the band width, as can be seen from Figure 4.

An important insight, formalized by this model, is that there exists a tradeoff between output stability and price stability in choosing the band width, as long as $c\leq 2a+1$. Recall that the zone is perfectly credible, so the chosen band width is defended fully. Therefore, if a wider zone is chosen, intervention becomes less intensive, i.e. the money supply becomes less variable, and output becomes more stable. This, however, leads to greater variability of the exchange rate and price. If a narrower zone is chosen, the exchange rate and price become more stable, but intervention then becomes more intensive, leading to higher variability of the money supply and output.[2] In this model intervention is costly.

[2]In traditional economic theory price moves together with money supply. It seems to be unrealistic at a glance that price is more stable when the money supply is more volatile. Note that price is composed of the two components, the money supply and the random shock. An increase in the volatility of money supply, the control variable of the model, arises from central bank intervention to offset the movement of the random shock. That is, the sum of the money supply and random shock that jointly determine price becomes stable due to central bank intervention even

IV. The Case of the Standard Normal Distribution

Thus far I have assumed that the shock v follows a triangular distribution. While useful in obtaining an explicit solution for the variances of price and output, the triangular distribution has the shortcoming that the range of the shocks is limited ($-\lambda < v < \lambda$ in this case). In this section, I assume the standard normal distribution for the density of v. Then I calculate the variances of price and output given values of a and c, the slopes of aggregate demand and supply schedules, and see whether a tradeoff still exists between price and output stability.

Given the assumption of the standard normal distribution for the density of v, along with movements of price and output in equations (7) and (9), the variances of price and output are calculated with different values of a and c as a function of the band width.[3] Figures 5 through 7 show the relationship between the band width and the variance of price given different combinations of a and c.[4] The vertical axis represents the variance of price and the horizontal axis represents the band width in units of standard deviation of v. The variance of price increases with the band width. Also, for a given band width, the variance of price decreases as either a or c increases. For any a, the larger c is, i.e. the flatter the aggregate supply schedule, the smaller the variability of price. Also, for any c, the larger a is, i.e. the flatter the aggregate demand schedule, the smaller the variability of price. In

though the money supply alone becomes more volatile.

[3]In the case of the standard normal distribution, the variances of price and output are obtained from the following equations with different values of a and c. Mathematica is used in calculation.

$$var(p) = \int_{(a+c)b}^{\infty} b^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds + \int_0^{(a+c)b}\{s/(a+c)\}^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds$$
$$+\int_{-(a+c)b}^{0}\{s/(a+c)\}^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds + \int_{-\infty}^{-(a+c)b} b^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds,$$

$$var(y) = \int_{(a+c)b}^{\infty}(s-cb)^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds + \int_0^{(a+c)b}\{as/(a+c)\}^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds$$
$$+\int_{-(a+c)b}^{0}\{as/(a+c)\}^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds + \int_{-\infty}^{-(a+c)b}(s+cb)^2\{(1/\sqrt{2\pi})\exp(-S^2/2)\}ds.$$

[4]Since the use of normal distributions with a different mean and variance produces the same results qualitatively, I do not present them here. Note that the values a and c in the simulation are 0.01, 0.1, 1, 10 and 100. Since different combinations of a and c do not change the qualitative result in that the variance of output (price) decreases (increases) with the band width, I present the pictures with selected combinations of a and c.

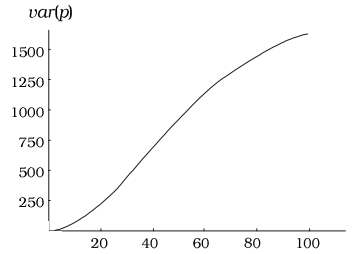

FIGURE 5

THE VARIANCE OF PRICE, $a=0.01$ AND $c=0.01$

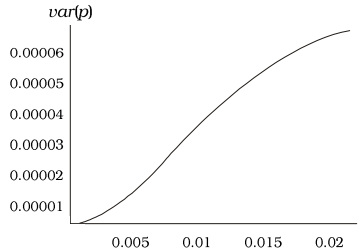

FIGURE 6

THE VARIANCE OF PRICE, $a=0.01$ AND $c=100$

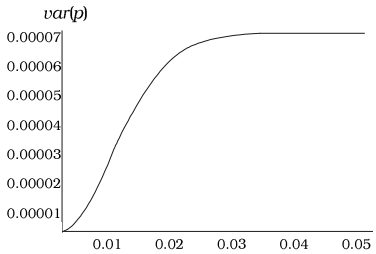

FIGURE 7

THE VARIANCE OF PRICE, $a=100$ AND $c=0.01$

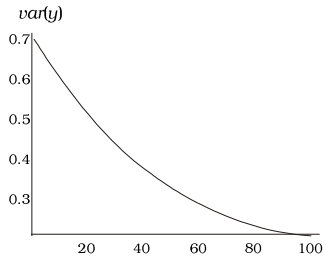

FIGURE 8

THE VARIANCE OF OUTPUT, $a=0.01$ AND $c=0.01$

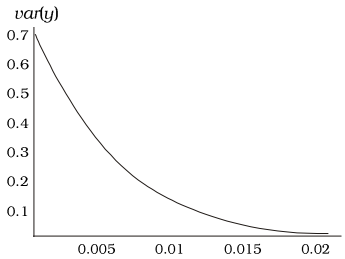

FIGURE 9

THE VARIANCE OF OUTPUT, $a=0.01$ AND $c=100$

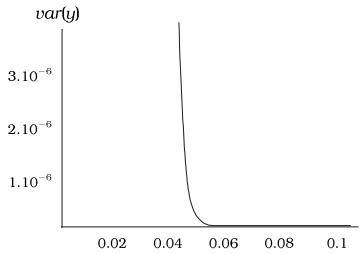

FIGURE 9′

THE VARIANCE OF OUTPUT, $a=0.01$ AND $c=100$

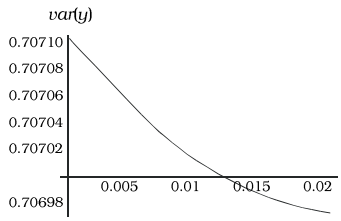

FIGURE 10

THE VARIANCE OF OUTPUT, $a=100$ AND $c=0.01$

the extreme case, if either the supply or demand schedule is horizontal, the price will be fixed and there will be no variability of price.

Figures 8 through 10 show the relationship between the band width and the variance of output given different combinations of a and c. The variance of output decreases with the band width. Note that the variance of output increases with a and decreases with c. The reason for the opposite effects of a and c on the variance of output is as follows.

Consider first the effect of a change in c on the variance of output. Assume that $\bar{a}$ and $\bar{c}$ are the initial slopes of the aggregate demand and supply schedules. These schedules are denoted as AD_0 and AS_0 respectively in Figure 11. At equilibrium, E_0, price is p_0 and output is y_0. Recall that imposing the band on the exchange rate has the same effect on price since foreign prices are normalized to zero in equation (3). The price band, with a band width of $\pm b$, is represented by dotted lines around p_0 in Figure 11. Suppose a large shock takes place. If there is no intervention, the supply schedule then moves to AS_0' and price (the exchange rate) deviates from the band at the new equilibrium, E_1. Under perfect credibility, this leads to intervention and money supply is adjusted to defend the band. Therefore the demand schedule moves to AD_0' and partially offsets the shift in the supply schedule so that price (the exchange rate) stays at the boundary of the zone in the new equilibrium, E_2 (marginal intervention). However, central bank

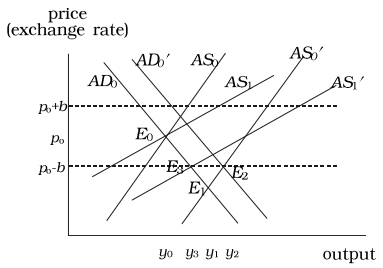

FIGURE 11

THE EFFECT OF AN INCREASE IN c ON THE VARIANCE OF OUTPUT

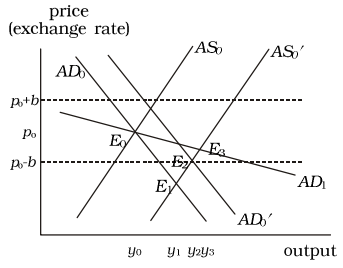

FIGURE 12

THE EFFECT OF AN INCREASE IN a ON THE VARIANCE OF OUTPUT

intervention would not be necessary if c were large. If, for example, the supply schedule is AS_1 with a slope $c' > \bar{c}$, it moves to AS_1' when the same shock occurs. At the new equilibrium, E_3, price (the exchange rate) still remains at the boundary without intervention.

With a given shock magnitude and band width, price is less likely to deviate from the band as c increases. Therefore, if c is large enough, a narrow band could retain the price within the band for most shocks. Subsequently, less intensive intervention leads to less variability of output. This implies that the variance of output decreases more rapidly as c increases. Compare, for example, Figure 8 with Figure 9. In Figure 8, the variance of output decreases smoothly until the band width reaches 100. In Figure 9, however, it decreases rapidly and almost reaches the lowest point when the band width is 0.02. Further, when c is large, an increase in band width does not induce a large decrease in the variance of output at the margin, since the density of extreme shocks necessitating intervention is very low. As Figure 9′ shows, a further increase in band width produces only a negligible decrease in the variance of output. Also, note that output deviates less from the original level y_0 when c is larger, i.e. the distance between y_0 and y_3 is less than the distance between y_0 and y_2. Therefore, as c increases, the variance of output decreases more rapidly with the band width due to a decrease in the probability of price deviating from the band, and the variance of output becomes smaller for a given band width due to a decrease in the distance.

If a is larger, with a given magnitude of shock and band width, the demand schedule becomes flatter and the probability of price deviating from the band decreases. In Figure 12, when the demand schedule is AD_0, price deviates from the band as the supply schedule moves from AS_0 to AS_0' as a result of a shock (see price at E_1). Then central bank intervention moves the aggregate demand schedule to AD_0' to keep price at the boundary of the band (see price at E_2).

This scenario is not realized when a is large. If the demand schedule is AD_1, price remains within the band when the same shock occurs (see price at E_3). As in the case of an increase in c, price is less likely to deviate from the band as a increases. In this case, however, output stability decreases. In Figure 12, the distance between y_0 and y_3 is larger than the distance between y_0 and y_2. When a is larger, the variance of output decreases more rapidly with the band width, due to a decrease in the probability of deviation from the band. Conversely, for a given band width, output variance becomes larger due to an increase in the distance.

Under the standard normal distribution there still exists an unambiguous tradeoff between price and output stability in choosing the band width. Under perfect credibility, costly intervention leads to greater variability of output (price) when the central bank tries to achieve less variability of price (output) by setting a narrower (wider) band width.

V. Optimal Band Width

Assume that the objective of the central bank is to minimize the sum of the variances of price and output. Formally stated, this objective becomes:

$$\underset{b}{\text{Min}} \quad var(y_t)+var(p_t). \tag{15}$$

The results from Section IV are used in calculating the sum of the variances.[5] Figures 13 through 22 show the path of the sum of the variances against the band width. The optimal band width

[5]Since imposing different weights on the two variances does not qualitatively change the result, I consider the case where the variances of output and price have the same weight in the central bank objective function.

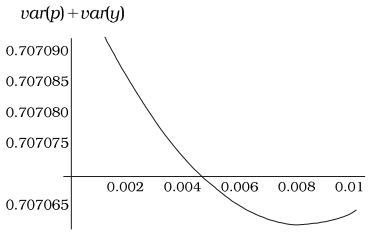

FIGURE 13

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=0.01$ AND $c=0.01$

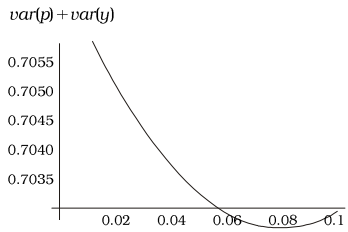

FIGURE 14

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=0.01$ AND $c=0.1$

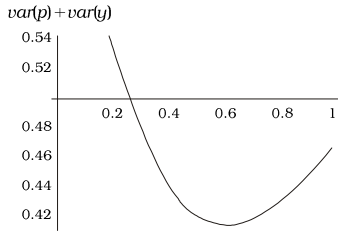

FIGURE 15

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=0.01$ AND $c=1$

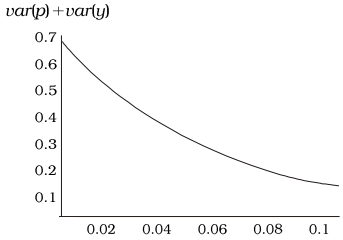

FIGURE 16

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=0.01$ AND $c=10$

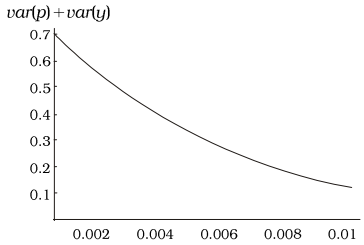

FIGURE 17

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=0.01$ AND $c=100$

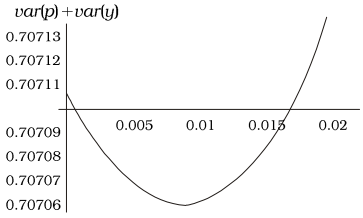

FIGURE 18

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=1$ AND $c=0.01$

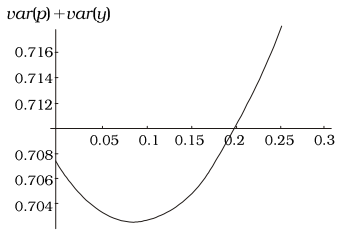

FIGURE 19

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=1$ AND $c=0.1$

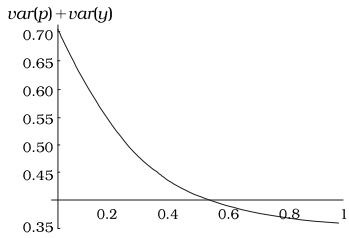

FIGURE 20

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=1$ AND $c=1$

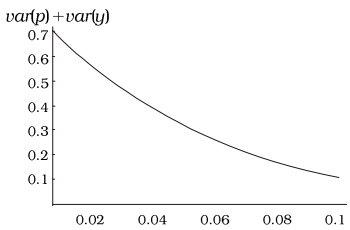

FIGURE 21

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=1$ AND $c=10$

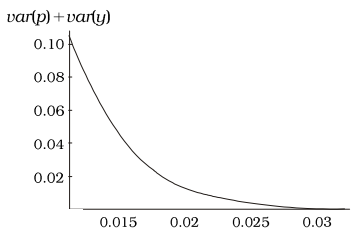

FIGURE 22

THE SUM OF THE VARIANCES OF PRICE AND OUTPUT, $a=1$ AND $c=100$

minimizing the sum of the variances is denoted as b^*. For a given a, b^* initially increases as c increases (supply becomes flatter). When c increases the variance of price decreases more rapidly than the variance of output for a given band width. Therefore b^* increases with c, if all other variables are constant. However, if c increases beyond a critical point, the variance of price becomes negligible and the variance of output dominates. In these cases b^* approaches infinity (see for example Figures 16 and 17), i.e. it is optimal to have floating exchange rates.

When a increases, price variance decreases while the variance of output increases for a given band width. Therefore the critical point of c mentioned in the preceding paragraph decreases as a

increases. The figures clearly show this property. For example, when $a=0.01$, b^* is finite for the cases of $c=0.01$, 0.1 and 1. The variance of output dominates and b^* goes to infinity for the cases of $c=10$ and 100. When $a=1$, however, b^* is finite only for the cases of $c=0.01$ and 0.1.

Under fully fixed exchange rates the central bank intervenes whenever necessary to maintain the parity, allowing no flexibility of the exchange rate. Thus the fully fixed exchange rate can be considered a target zone with zero band width. Under a free floating system, the central bank does not intervene at all. Free floating exchange rates can thus be considered a target zone with an infinite band width. As long as the variance of output does not dominate the variance of price, a target zone with positive band width, like the EMS, appears to be optimal compared to either fixed or free floating exchange rates. However, when the variance of output dominates the variance of price, b^* approaches infinity and free floating exchange rates appear to be optimal in comparison to either fixed rates or a target zone with positive band width.

VI. Conclusion

A simple static model has been developed to deal with two common problems of conventional target zone literature: Costless intervention and a narrow focus on positive issues.

Under the assumptions of marginal intervention and perfect credibility, the model shows that a tradeoff exists between price and output stability in choosing the band width. If a narrower zone is chosen, the exchange rate and price become more stable, but money supply and output become more variable, since intervention then becomes more intensive. This implies that intervention is costly.

Assuming that the objective of the central bank is to minimize the sum of the variances of price and output, the optimal exchange rate regime is considered. As long as the variance of output does not dominate the variance of price, the optimal band width is finite and a target zone with a positive band width is optimal compared to either a fixed or free floating regime. However, the existence of a tradeoff between price stability and output stability does not ensure the optimality of the target zone. With a as the slope of demand

and c as the slope of supply, for a given a (c), a critical value of c (a) exists such that the variance of output always dominates the variance of price. In this case, the optimal band width approaches infinity and the free floating regime is optimal, since it minimizes the variance of output, even though the tradeoff between output stability and price stability still exists.

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