Credit Rationing with a Moral Hazard Problem

Jeeman Jung [1]

This paper examines an alternative model of credit rationing when moral hazard is present in the credit market. Two regimes are considered: one with a continuous trading assumption and the other with a restriction on trading. Continuous trading enables one to construct a riskless hedging portfolio and therefore leads to market failure. Under restrictions on trading, however, the entrepreneur of a firm does not undertake an extremely risky activity and the optimal strategy depends on the amount of debt: the larger the amount of debt, relative to the value of a firm's assets, the greater the entrepreneur's incentive to follow a risky strategy. In this situation, credit rationing is beneficial to lenders.

Keywords: Moral hazard, Credit rationing, Continuous trading

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I. Introduction

Credit rationing is a well-known phenomenon in capitalistic economies with banking systems. It is often the case that there are limited funds that banks can lend. Credit rationing can be observed when usury laws restrict the lending rate of interest. Since bankers are not allowed to increase the interest rate on their loans above a ceiling rate, they will allocate the credit to their customers with discretion. Keynes (1930) discusses the possibility that the market for bank loans is imperfect, so that banks can influence the

*Assistant Professor of Economics, Sangmyung University, Seoul 110-743, Korea, (Tel) +82-2-2287-5189, (Fax) +82-2-396-5702, (E-mail) jmjung@pine.sangmyung.ac.kr. I wish to acknowledge the financial support of the Sangmyung University.

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volume of the economy's investment without any changes in the interest rate. Later, this view is incarnated in the availability doctrine.[1]

Hodgman (1960) was the first who explained credit rationing as an equilibrium phenomenon. Limited liability makes the lender to be concerned about the risk of default on his loan. Hodgman's idea is that, once a critical loan size is reached, the lender will not increase the size of the loan accommodation regardless of how high an interest rate the borrower offers. The argument is based on the assumption that the amount of return from the borrower's investment is independent of the size of the loan. In this situation, the default risk on the loan may not be compensated by an increase in the interest rate. However, as Freimer and Gordon (1965) point out, with an open-end investment the supply of loans can increase indefinitely with the lending rate of interest. For example, if the borrower's investment project shows constant returns to scale, the lender does not have to ration credit as in Hodgman's model.[2]

One distinctive trait of credit markets is that the risk characteristics of the borrowers are very heterogeneous, and it may be too costly for the lender to acquire accurate information concerning the borrowers. Thus, modern theories of credit rationing focus on informational asymmetry in the credit markets, implying adverse selection and moral hazard. Considering the moral hazard problem, Jensen and Meckling (1976) assume that there are only two possible investment strategies and obtain a corner solution. Jaffee and Russell (1976) set up a simple two-period Fisherian consumption model to account for credit rationing in the consumer loan market. They consider two types of borrowers: "honest" and "dishonest" borrowers. Lenders, however, are neither able to distinguish between these two types of consumers, nor can they enforce liquidation in the case of default. Consequently, there exists a rationing equilibrium with a smaller loan size, which reduces the likelihood of default by the dishonest borrowers.

Stiglitz and Weiss (1981, 1983) develop a model of credit rationing based on informational asymmetry in the loan market.

[1]See Roosa (1951).

[2]Jaffee (1971) shows that with decreasing returns to scale, the supply of loans can be backward bending, and credit rationing can be explained. My model in the paper assumes constant returns to scale, but shows that credit rationing can still take place.

Their model shows that the lender may not increase the interest rate charged on the loans even if there exists excess demand for loans in the market. If the lender charges a higher interest rate on the loan, less risky borrowers may abandon their investment projects and the probability of default increases as a consequence. This is the dilemma of adverse selection. Moral hazard is also an important concern to the lender, since a higher interest rate induces the borrowers to make their investment strategies riskier. The increase in riskiness of the loans is not beneficial to the lender. As a result, the lender is reluctant to charge a higher interest rate on its loan even if there exists excess demand and the phenomenon of credit rationing can persist as an equilibrium. Their model, however, considers only two possible outcomes for each investment project. Moreover, since the size of the loan is assumed constant for each project, they cannot account for the effect of the loan size on the decision making of borrowers and lenders. Another question with respect to their model (Stiglitz and Weiss 1983) is that the solution of the model requires an arbitrarily decreasing expected rate of returns as the riskiness of the investment project increases.

The paper develops a generalized model of credit rationing when moral hazard is present in the credit market. The model in the paper considers a continuous stream of possible outcomes, and a continuous stream of possible strategies with different risks, but with a constant expected return. Also, the model makes allowance for variable investment projects with constant returns to scale and examines the effect of the loan size on the riskiness of a firm's business. The paper shows that it is the amount of debt liability itself, rather than the level of the interest rate on the loan *per se*, which causes the moral hazard problem. With this problem in the credit market, the Modigliani-Miller theorem does not hold.

II. Theoretical Models

This paper examines credit rationing when moral hazard is present in the credit market, and gives a theory of the amount of credit rationing and the amount of response to the moral hazard. Considering the moral hazard problem, Stiglitz and Weiss (1981, 1983) suggest that higher interest rates induce borrowers to undertake riskier business. In their model, each project is assumed to have only two possible outcomes.[3] A weakness in their study, however, is that their model cannot be solved with constant expected returns, that is, with mean-preserving distributions of outcomes. The solution of their model requires that the expected return should decrease arbitrarily as the riskiness of the borrower's business increases.[4] But they do not explicitly refer to this fact.

The model in this paper considers a continuous stream of possible outcomes, and a continuous stream of possible strategies with different risks, but with a constant expected return. I show in this paper that it is the amount of the loan itself,[5] rather than the level of the interest rate on the loan, which determines the amount of response to the moral hazard problem. The larger the amount of debt, the greater the entrepreneur's incentive to undertake a risky activity. With this problem in the credit market, the Modigliani-Miller result does not hold as the amount of debt affects the firm's managerial strategy and hence the outcome of the business.

Asymmetric information is often cited as a reason for inefficient resource allocation or market failure. In the credit market, an entrepreneur who manages a firm's business is likely to have some private information and may affect the prospects of the business. The moral hazard problem is present in the credit market, because the entrepreneur can switch his/her managerial or investment strategy after he/she borrows funds from a bank.

There are always incentives for the entrepreneur to use debt for his own benefit. Limited liability puts a bound on the damage that one may occur from undertaking a risk that results in a bad outcome. That is, the value of a firm's equity cannot be negative even in the event of disaster. On the other hand, the lender to the firm cannot claim more than the principal plus fixed interest payments. Because of limited liability in the contract, the entrepre-

[3]Their model considers a continuous stream of possible investment strategies with different risks.

[4]See Stiglitz and Weiss (1983). If we consider a constant expected return for each project, their model cannot be solved for the optimum choice of an investment project since their first order condition, equation (4), is always positive. Indeed, increasing the riskiness of the firm's investment project is always beneficial to the firm in their model.

[5]Strictly speaking, it is the amount of debt at the maturity date, which includes both the principal and the interest payments.

neur's decision can be distorted in favor of a riskier business. Riskier strategies tend to give higher returns to borrowers and lower returns to lenders.

Once the lender or bank recognizes the incentive effects of its loan, it becomes important to monitor the action of the entrepreneur so as to discourage the undertaking of highly risky businesses. Even though it cannot directly control the business strategy of the entrepreneur, the bank understands how the terms of the contracts influence the decision of the entrepreneur. Sometimes the amount of debt has positive effects on the riskiness of the firm's business. Hence, the bank limits the amount of credit that the entrepreneur can obtain for the firm's business.

This paper considers two regimes to investigate the problem of moral hazard. Regime A assumes continuous trading. Firms are evaluated in an efficient market and are traded continuously. The price of a firm's equity is derived in the same way as the Black-Scholes' call option price. Under this assumption, the bank will not finance the firm's investment project. In Regime A, as with some earlier models, the amount of credit rationing is determined as a corner solution.

In fact, however, firms cannot trade continuously, and this brings us to Regime B, in which we impose a restriction that firms cannot be traded continuously. Under this restriction on trading, the risk-averse entrepreneur does not want to venture into an extremely risky business. The optimal strategy depends on the level of debt the firm currently holds. The larger the amount of debt relative to the value of the firm's assets, the greater the entrepreneur's incentive to venture into a risky business. This is the moral hazard problem with post-contractual informational asymmetry. Consequently, the optimal lending policy of the bank is to ration credit even below a level that is justified by the current degree of riskiness of the project. Credit rationing is beneficial to the bank, but results in a reduction of the firm's investment.

The model considers two types of economic agents in the credit market: a risk-averse entrepreneur and a risk-neutral bank. The entrepreneur in the model is either an owner-manager of a firm, or the entrepreneur can be considered to maximize the value of the firm's equity on behalf of equity-holders.[6] That is, the entrepreneur

[6]Although highly plausible, this is not always true in the real world. In

acts in the interest of equity-holders. The bank represents the lender or the financial intermediary that offers a loan to the firm.

Suppose that the entrepreneur of a firm does not have sufficient internal funds to finance an investment project, and must rely on external funds, such as those obtained from issuing equity and/or bonds. It is often observed that the value of the firm falls by a considerable amount when the firm announces an equity issue (Asquith and Mullins 1986; and Myers and Majluf 1984), and that the price of an initial public offering is substantially lower than the fundamental value (Shiller 1988). For simplicity, we ignore the new equity issue in order to deal with the problem of moral hazard in the credit market,[7] and assume that the entrepreneur can borrow the firm's investment fund only from the bank.

The problem of asymmetric information takes place in the credit market, since the firm can switch its management or investment strategy after it borrows funds from the bank. Observing its balance sheet position, the entrepreneur can select a business strategy that has a certain degree of riskiness.

The bank holds a well-diversified portfolio and behaves as if it is risk-neutral. Indeed, the bank is the institution that specializes in acquiring information about the default risk of firms. The bank, however, cannot directly monitor the firm's operating strategy, but tries to affect the firm's strategy by controlling the amount of lending at the given rate of interest. For simplicity, it is postulated that both the entrepreneur and the bank have a common prior belief on the probability distribution of the outcome.

This model assumes that the expected rate of return is the same for each strategy, while the degree of riskiness can be different. We suppose that the period of the investment project coincides with the term to maturity of the borrowing, which is denoted as τ, and that there are no interest payments before maturity so that borrowing is in fact equivalent to issuing zero coupon discount bonds to the bank at a discount rate of ρ. Also there are no dividend payouts prior to the maturity of the project.

At the maturity date, $T=t+\tau$, the entrepreneur sells the assets

order to consider implications of the asymmetric information problem in the credit market, we disregard the principal-agent problem between the manager and the owners. See Myers and Majluf (1984).

[7]A low debt-equity ratio often implies a higher cost of capital, and debt financing is often necessary to maximize the firm's value.

of the firm and settles its account.[8] Let B represent the amount owed to the bank at the maturity date. If the value of the firm at time T, denoted V_T, is greater than the amount of the debt liability, then the difference, $V_T - B$, is given to the share-holders of the firm in return.[9] But if the value of the firm is less than the debt, the firm goes bankrupt and the bank claims all the assets of the firm. It is clear that equity-holders of the firm have a claim on the firm's assets, which is like a call option with an exercise price of B.

The values of a firm's debt, equity, and assets at the maturity date are represented as equations (1), (2), and (3), respectively:

$$D_T = \min(V_T,\ B), \tag{1}$$

$$F_T = V_T - \min(V_T,\ B) = \max(V_T - B,\ 0), \tag{2}$$

$$V_T \equiv D_T + F_T\ . \tag{3}$$

We assume that the factors determining the value of the firm's assets evolve in such a way that the value of the firm follows an Ito process:

$$\frac{dV}{V} = \alpha dt + \sigma dz, \tag{4}$$

where α is the instantaneous expected rate of return on the firm's business, dz is a standard Brownian motion (Wiener process), and σ represents riskiness of the firm's business. Here, we assume that the expected rate of return, α, is constant and not affected by changes in σ. Also we assume that σ can be made arbitrarily large. The entrepreneur chooses σ for his business strategy.[10]

[8]Our model does not assume the cost of observation. Gale and Hellwig (1985) take the observation costs into account and show that the standard debt contract is optimal and incentive compatible.

[9]It is assumed that the firm does not default if the value of the firm at the maturity date is greater than the amount of debt. On the contrary, Jaffee and Russell (1976) consider the moral hazard problem with a dishonest borrower who can default even if he has a sufficient fund to cover the debt.

[10]The actual practice of bank credit, not considered in this model, is that loan contracts are repeated over many periods. In this case, reputation can

Under this stochastic process, the value of the firm, *i.e.* the sum of the values of its equity and its debt, seems independent of its debt-to-asset ratio, and the Modigliani-Miller theorem appears to hold true. However, as will be shown later, the level of the firm's debt influences the firm's choice of investment strategy, and thus the value of the firm.

A. Regime A

Continuous trading is an assumption frequently used in the economics literature. In Regime A, continuous trading is allowed both for the equity of the firm and for the value of the firm itself. Thus a riskless arbitrage portfolio can be constructed. The return dynamics on the equity are also of the Ito type:

$$\frac{dF}{F} = \alpha'(V,\ t,\ T)dt + \sigma'(V,\ t,\ T)dz', \tag{5}$$

where F represents the current value of the firm's equity. It is known that using the Black-Scholes call option pricing formula, we can derive the value of the equity and the value of the bond with τ periods to go before maturity.[11] The current value of equity in the market is:[12]

$$F = F(V,\ \tau;\ B,\ r,\ \sigma) = VN(h_1) - Be^{-r\tau}N(h_2), \tag{6}$$

where h_1 and h_2 are defined in equations (7a) and (7b), respectively:

$$h_1 = \frac{\log\left(\dfrac{V}{Be^{-r\tau}}\right) + \dfrac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}, \tag{7a}$$

$$h_2 = h_1 - \sigma\sqrt{\tau}. \tag{7b}$$

help to alleviate the moral hazard problem. In addition, the collateral requirement can lead the entrepreneur to choose a less risky strategy. Refer to Bester (1985) and Hellwig (1987).

[11]We ignore bankruptcy costs.

[12]Refer to Black and Scholes (1973) and Merton (1974).

Note that $N(\ )$ is the cumulative standard normal density function, and that r is the rate of return on riskless assets. The value of debt at time t with τ period to go before maturity, denoted as D, is consequently:

$$D = V - F = V\{1 - N(h_1)\} + Be^{-r\tau}N(h_2). \tag{8}$$

B. Regime B

In Regime B, we impose a restriction that firms cannot be traded before the end of the investment project. Since the market is not perfect, it may not be able to evaluate the value of the firm effectively. We assume the same stochastic process for the value of the firm as in equation (4). From the process of equation (4), we know that the $\log(V_T/V)$ is distributed as normal with the mean, $(\alpha - (1/2)\sigma^2)\tau$, and the variance, $\sigma^2\tau$:

$$\log\frac{V_T}{V} \sim N((\alpha - \frac{1}{2}\sigma^2)\tau,\ \sigma^2\tau), \tag{9}$$

and hence V_T is log-normally distributed:

$$\frac{V_T}{V} \sim LN(e^{\alpha\tau},\ e^{2\alpha\tau}(e^{\sigma^2\tau} - 1)). \tag{10}$$

A variable which is lognormally distributed has a very convenient property when we want to calculate moments of truncated distributions.[13] Hence, the present expectation of the value of equity at time T, defined as W, is:

$$\begin{aligned} W &= E_t[\max(V_T - B,\ 0)] \\ &= \int_B^{\infty} V_T f(V_T)dV_T - B\int_B^{\infty} f(V_T)dV_T \\ &= Ve^{\alpha\tau}N(d_1) - BN(d_2) \\ &= e^{\alpha\tau}\{VN(d_1) - Be^{-\alpha\tau}N(d_2)\}, \end{aligned} \tag{11}$$

[13]Refer to Aitchison and Brown (1957).

where d_1 and d_2 are defined in equations (12a) and (12b), respectively:

$$d_1 = \frac{\log(\frac{V}{Be^{-\alpha\tau}}) + \frac{\sigma^2 \tau}{2}}{\sigma\sqrt{\tau}}, \tag{12a}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}\ . \tag{12b}$$

Note that W in equation (11) is not the current price of the firm's equity. Under restrictions on trading, a riskless hedging portfolio cannot be constructed and the entrepreneur of the firm should bear the risk of bad outcomes.

III. Credit Rationing and Determination of Equilibrium

A. Regime A

In Regime A, continuous trading enables one to construct a riskless hedging portfolio which characterizes the current price. Pricing of a derivative asset is preference-free and risk aversion cannot play a role in determining the price of the derivative asset, that is, equity. Another problem in pricing is the fact that the firm is controlled by the derivative asset holders, that is, the equity holders or their agents. To maximize the value of the firm's equity, F, the entrepreneur has an incentive to increase the variance of the investment strategy after obtaining an amount of bank loan.

***Proposition* 1**
Under the assumption of continuous trading, the optimal strategy of the entrepreneur is to increase the riskiness of the business indefinitely after obtaining funds from a bank. Therefore, the bank will not finance the firm's investment project and the credit market collapses.

***Proof*:** Differentiating equation (6) with respect to σ, we obtain:

$$\frac{\partial F}{\partial \sigma} = \sqrt{\tau}\, Vn(h_1) > 0, \tag{13}$$

where $n(\ )$ is the standard normal density function. Note that as σ goes to infinity, h_1 increases to infinity and h_2 decreases to minus infinity. Thus, we can see that as σ increases, the values of equity and debt, F and D, approach V and 0, respectively. In addition, it can be shown that the probability of bankruptcy, P_b, converges monotonically to one as σ increases indefinitely:

$$\begin{aligned} P_b &= B\int_{-\infty}^{B} f(V_T)dV_T \\ &= 1-N(h_2) \qquad (14) \\ &\to 1 \quad as \quad \sigma\to\infty, \end{aligned}$$

$$\frac{\partial P_b}{\partial \sigma} = -n(h_2)\frac{-\log(\frac{V}{Be^{-\alpha\tau}})}{\sigma^2\sqrt{\tau}} - \frac{\sqrt{\tau}}{2} > 0. \qquad (15)$$

Q.E.D.

The idea of proposition 1 is that as the riskiness of the business increases, the probability of bankruptcy increases and the current price of its debt decreases. After obtaining some investment funds from a bank, the entrepreneur of the firm has an incentive to increase the variance of business indefinitely so that the current price of the firm's equity increases.

Moreover, this is true for any level of the lending rate of interest. At a higher rate of interest, some firms may turn away from bank credit and abandon their investment projects. However, once the entrepreneur decides to finance the investment project through borrowing from a bank, the higher variance of the firm's business is still in his interest.

In this situation, the bank will not provide the entrepreneur with investment funds and "no credit" ($B=0$) is the equilibrium. Consequently, the firm cannot raise funds for its investment project in the credit market.

This result is partly due to the perfect hedge under the assumption of continuous trading. Thus, traditional corporate finance with the continuous trading assumption relies on another assumption of a constant strategy of business, and ignores the moral hazard problem. On the other hand, the result of this paper takes place in a situation where the underlying asset is controlled by the derivative asset holder, that is, the entrepreneur of a firm. Usual stock options do not allow the option-holder to control the firm. However, even regular option pricing is not always invulnerable to the result of the paper. Call options on stocks are sometimes given to the manager of the firm as rewards. In this case, the manager has an incentive to increase the variance of the firm's stock prices unless it undermines the expected return. Greater fluctuations of stock value can be created by a riskier managerial strategy, which does good to the entrepreneur and owner of the firm, but does harm to the bank and bond-holders.

B. Regime B

In Regime B, the firm cannot be as easily sold to the public, and hence riskless hedging portfolio cannot be formed. This is partly because the market may not be able to evaluate the firm effectively. Continuous trading is not possible, and the value of the firm is settled only at the end of the investment period. The entrepreneur of the firm bears the risk which provides a disincentive to undertaking a risky activity.

A simple form of the entrepreneur's objective function may be as follows:[14]

$$\Pi = W - \frac{k}{2}\sigma^2, \tag{16}$$

where k is a constant. The first term of the objective function is the expected value of the equity (see equation (11)). The entrepreneur of the firm prefers a higher expected return to a lower one. The second term reflects risk-aversion.[15] As he is risk-averse, a less uncertain outcome is preferred. In this case, the managerial strategy of the entrepreneur depends on the amount of borrowing, the value of the firm, and the term to maturity of its debt.[16] The following

[14]This is in a convenient form for our analysis, though the argument of the paper holds generally for various objective functions.

[15]The second term can also reflect extra bankruptcy costs, such as reorganization costs, legal expense, losing reputation, and so on.

[16]If the entrepreneur is extremely risk-averse, the optimal choice can be an investment strategy without any risk. We ignore such trivial case.

proposition can be derived for this regime.

Proposition 2

Under the restriction on trading, the risk-averse entrepreneur does not want to employ an extremely risky investment strategy. However, for a given fixed borrowing rate of interest, the entrepreneur wants to borrow as large amount of funds from the bank as possible unless he or she is unwilling to finance through debt at all.

Proof: From the first order condition (FOC), the entrepreneur may find an optimal strategy, σ^*:

$$0 = \frac{\partial \Pi}{\partial \sigma} = \sqrt{\tau} B n(d_2) - k\sigma. \tag{17}$$

We can confirm that the optimal strategy is finite.

$$\frac{\partial \Pi}{\partial \sigma} < 0, \quad as \quad \sigma \to 0. \tag{18}$$

And the second order condition at σ^* is:

$$\frac{\partial^2 \Pi}{\partial \sigma^2} = \sqrt{\tau} B d_2 n(d_2) \left(\frac{\log(\frac{V}{Be^{-\alpha\tau}})}{\sigma^2 \sqrt{\tau}} + \frac{\sqrt{\tau}}{2} \right) - k < 0. \tag{19}$$

The optimal amount of borrowing is then:

$$\begin{aligned} \frac{d\Pi}{dB} &= \frac{\partial \Pi}{\partial B} + \frac{\partial \Pi}{\partial V} \frac{dV}{dB} \\ &= e^{-\alpha\tau} N(d_2) + e^{-\rho\tau} N(d_1). \end{aligned} \tag{20}$$

Note that $N(d_2) < N(d_1)$, and $N(d_1) - N(d_2)$ is increasing as B is increasing. Unless the borrowing rate, ρ, is too high, Π is increasing finally in B. That is:

i) If $\alpha > \rho$, $d\Pi/dB > 0$.
ii) If $\alpha < \rho$, Π is convex in B.

So whenever a firm wants to be indebted, more debt is preferred to less.[17]

Q.E.D.

This second proposition can be contrasted to proposition 1 of Regime A. In Regime A, the continuous trading assumption brings about credit market collapse. In Regime B, however, the restriction on trading gives an internal solution of the model and the credit market does not collapse. The different results are due to the assumption of continuous trading. Sometimes continuous trading is a strong assumption.

Moral hazard is still a problem in the credit market. In Regime B, the entrepreneur's choice of an investment strategy depends upon the amount of credit he can obtain from the bank. That is:

***Proposition* 3**

The optimal degree of riskiness of the investment strategy is an increasing function of the level of debt. The larger the amount of debt, the greater the entrepreneur's incentive to undertake a risky activity.

***Proof*:** Total differentiation of the first order condition, equation (17), with respect to σ and B is:

$$0 = d\sigma\,(\sqrt{\tau}Bn'(d_2)\,\frac{\partial d_2}{\partial \sigma} - k) + dB(\sqrt{\tau}\,n(d_2) + \sqrt{\tau}Bn'(d_2)\,\frac{\partial d_2}{\partial B}).$$

Then,

$$\left\{B(-d_2)n(d_2)\left(-\frac{\log(\frac{V}{Be^{-\alpha\tau}})}{\sigma^2\sqrt{\tau}} - \frac{\sqrt{\tau}}{2}\right) - k\right\}d\sigma$$

$$= -n(d_2)\left(1 - Bd_2\,\frac{Be^{-\rho\tau} - V}{\sigma\sqrt{\tau}BV}\right)dB,$$

[17] If the assumption of the fixed lending rate of interest is replaced with an assumption of a rate which increases with the amount of the loan, then the entrepreneur may not want to borrow an extremely large amount of funds from the bank.

$$\left\{Bd_2n(d_2)\left(\frac{\log(\frac{V}{Be^{-\alpha\tau}})}{\sigma^2\sqrt{\tau}}+\frac{\sqrt{\tau}}{2}\right)-k\right\}d\sigma$$

$$=\frac{-n(d_2)}{2\,\sigma^2\sqrt{\tau}V}\{2(V-Be^{-\rho\tau})\log(\frac{V}{Be^{-\alpha\tau}})+\sigma^2\sqrt{\tau}(V+Be^{-\rho\tau})\}dB. \tag{21}$$

Using the second order condition of equation (19), we obtain:

$$\frac{d\sigma^*}{dB}>0. \tag{22}$$

Q.E.D.

As the amount of debt increases, the entrepreneur is willing to assume a riskier business. Such an increase in the riskiness of the business augments the probability of bankruptcy. Hence, the optimal behavior of the bank under these circumstances is to consider changes in riskiness, and to restrict the amount of lending. That is, credit rationing is beneficial to the bank.

Now, consider the bank's behavior. As the bank is risk neutral, its objective is to maximize its expected profit, π:

$$\text{Max} \;\; \pi=(e^{-\alpha\tau}V-W)-Be^{-(\rho-r')\tau}, \tag{23}$$

where r' is the interest rate on the bank's liabilities, or the opportunity cost of its loan. The first term of the equation (23), $e^{-\alpha\tau}V-W$, is the amount the bank expects to receive from the firm at the maturity date. The second term, $Be^{-(\rho-r')\tau}$, is the amount the bank has to pay back to its depositors. Following the FOC gives an optimal amount of loan, B^*, when the bank does not take into account changes in the behavior of the entrepreneur.[18]

$$0=e^{(\alpha-\rho)\tau}+N(d_2)-e^{(\alpha-\rho)\tau}N(d_1)-e^{(r'-\rho)\tau}. \tag{24}$$

[18]If the bank were able to control the behavior of the entrepreneur, B^* would be the first-best solution.

At level of B^*, however, the bank can have the firm decrease the probability of bankruptcy by reducing the amount of lending. Reduction of lending provides the entrepreneur with a disincentive to choose a risky business. Thus, the bank has to consider the effect of credit rationing on the riskiness of the firm's business, $\partial\sigma^*/\partial B>0$. The FOC is then:

$$0=e^{\alpha\tau}\frac{dV}{dB}-\left(\frac{\partial W}{\partial B}+\frac{\partial W}{\partial V}\frac{dV}{dB}+\frac{\partial W}{\partial\sigma}\frac{d\sigma^*}{dB}\right)-e^{(r'-\rho)\tau}\ . \tag{25}$$

Here, the optimal amount of lending, B^{**}, is less than B^*, and $\Pi(B^{**})\geq\Pi(B=0)$. The following proposition is therefore proved.

***Proposition* 4**

The optimal behavior of the bank is to restrict the amount of credit to the level of B^{**}, which is less than the amount of credit when the moral hazard problem does not exist in the credit market.

This model bears an interesting economic implication. In recessionary periods, the debt-to-asset ratio normally increases as the value of assets decreases. In this case, the entrepreneur of the firm has an incentive to undertake a riskier business. This incentive problem in the credit market leads to increase in the ratio of bankruptcy. In this case the bank will ration credit to their customers. Therefore, moral hazard in the credit market results in a reduction of investment, and it can be even more serious in a recession.[19]

IV. Conclusion

Credit rationing has its own long history in the real economy. While many economists have attempted to account for the phenomenon, modern economic theories explain credit rationing as an equilibrium phenomenon, describing informational asymmetry among agents in the credit market. Not only does the entrepreneur of a firm has superior information on his business, but he can also

[19]Bernanke and Gertler (1989), and Greenwald and Stiglitz (1988) argue that high agency cost in recession periods can cause further reduction in aggregate economic activity.

change the managerial or investment strategy after concluding a debt contract. Moreover, the entrepreneur is not fully responsible for the outcome of his business. In this situation, even a risk-averse entrepreneur has an incentive to choose a riskier business if he can obtain a larger amount of loan.

This paper examines theoretical models of credit rationing when moral hazard is present in the credit market. The study considers two regimes: one with a continuous trading assumption and the other with a restriction on trading. In the regime with the continuous trading assumption, a firm may not be able to raise investment funds from a bank because there is incentive for the firm to follow an investment strategy indefinitely variable after receiving some investment funds. Consequently the only possible equilibrium in the regime is a sort of corner solution with a zero level of debt financing, and the credit market therefore collapses.

In Regime B, we impose a restriction that the firm is not tradable until it gets through with its investment project. In this regime, a risk-averse entrepreneur is not willing to venture into an extremely risky business. The optimal strategy of the firm depends on the level of debt the firm holds currently. The higher the amount of debt, the riskier will be the investment strategy. Increasing credit further results in changes in optimal behavior of the firm and increases the possibility of bankruptcy. Because of this moral hazard problem, the optimal lending policy of the bank is to ration credit even below the level that is justified by the current degree of riskiness of the business.

This paper set up a theoretical model of credit rationing and analyzed the moral hazard problem in the credit market. A challenging topic for future research is to investigate empirically the phenomenon of credit rationing with the moral hazard problem.

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