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Exploring Growth Patterns in Mathematical Ability of Students with Mathematics Difficulties: Focusing on the Cognitive Correlates of Word Problem Solving

수학 학습장애 위험 아동의 수학 능력 발달 특성에 관한 연구:
수학 문장제 해결 능력과
인지 변인 간의 관계를 중심으로

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Abstract

Exploring Growth Patterns in Mathematical Ability of Students with Mathematics Difficulties: Focusing on the Cognitive Correlates of Word Problem Solving

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Although many students show proficiency on assessments of general mathematics skills, many of them experience difficulties in solving word problems in mathematics. This may be attributed to a lack of the required skills necessary for the solution of these word problems. In order for students to be able to solve these problems, they

first have to be able to read and understand the problem. Next, they must have the ability to translate the texts of the problem described in natural language into arithmetic operations expressed in mathematical language. Finally, they should be able to compute the equations without any error and to check that their calculations and computations are correct. In other words, word problem solving in mathematics requires the integration of computation and application knowledge with basic reading skills in terms of language comprehension.

Students with certain learning disabilities are especially less successful in word problem solving in mathematics compared to other students without learning disabilities. The former group has difficulty understanding word problems because of the fact that they lack the basic reading and computation skills, coupled usually with the condition of diminished working memory. Specifically, they are not capable of representing the relationship in the problem to some comprehensible form which can lead them to solve the problem mathematically. Moreover, students with learning disabilities use even more inefficient strategies for problem solving than typically achieving students.

The notion of learning disabilities has traditionally indicated unexpected underachievement due to a disorder in one or more of the basic psychological process or processes. This notion has been ratified within an intrinsic processing deficits model and has been used as an indicator for identifying learning disabilities. This model attempts to evaluate psychological process or capacity weakness directly, because these two variables form the basis on which learning problems are determined. Although the intrinsic processing deficits model provides the primary criterion for identifying learning disabilities, and although it is a direct approach to characterize and measure learning disabilities as opposed to the indirect methods using exclusionary clauses, it has not been a mandatory requirement for identification of learning disabilities in South Korea. That is not only because theoretical and empirical support for the notion of psychological process deficits and its influence on learning is still vague, but also because there is no agreement as to how to accurately measure cognitive ability This renders results acquired through this test invalid at best.

This research attempts to explore in a defined heterogeneous population whether students' difficulties in learning mathematics is based on their growth patterns and/or cognitive abilities. The research attempts this by, first of all, exploring the possibility that students' growth trajectories might be a determinant of the mathematical learning problem as it pertains to solving word problems. . Then, the cognitive predictors of group membership are identified and the similarities and differences in the cognitive characteristics by subtypes among students with mathematical difficulties are determined. For these purposes, the following research questions were established: 1) Are there any identifiable groups within the given population whose mathematical word problem solving ability show correlation with those students' growth patterns (intercept and slope) 2) Given that there are multiple growth patterns, what are the effects of students' cognitive abilities on their growth patterns in terms of word problem solving in mathematics? 3) Do cognitive abilities differ among students with mathematical difficulties identified by their growth patterns, depending on whether there is an accompanying problem in computation, in reading, in both computation and in reading, or in neither computation or reading? The implications in conjunction with the interventions, as well as some of the limitations of the study, are discussed at the end of this study.

Research Question 1 was used to examine identifiable subgroups based on growth patterns of word problem solving in mathematics. In order to explore the heterogeneity in growth trajectories of students' repeatedly measured data, latent class growth analysis (LCGA) was conducted. As a result of LCGA, four distinct classes emerged based on characteristics of growth patterns (i.e.

performance levels and growth rates). Class 1 (15.2%) was characterized as high intercept and slow progress. Class 2 (25.6%) was characterized as average intercept and fast progress. Class 3 (43.1%) was characterized as low intercept and slow progress. Class 4 (16.1%) was characterized as lowest intercept and little progress. The four groups classified by exploratory methods were labeled as high achieving students (HAS), average and fast growing students (AFG), low but steadily growing students (LSG), and students with mathematics difficulties (SDD) respectively.

Research Question 2 was used to examine the relationship between growth patterns of word problem solving and cognitive abilities in mathematics. To investigate this relationship between students' learning progress in word problem solving and their cognitive abilities, a growth mixture modeling approach was used. In the case of setting up a HAS group as a reference, the lower the students' working memory ability is, the higher the possibility for students to be categorized into an AFG group (odds ratio=0.570) and a LSG group (0.582). Also, when the values of a processing speed and a language decrease (odds ratio=0.344 and 0.477, respectively), the probability of being in a SDD group increases. If the AFG group is set up as a reference, the lower the students' processing speed (odds ratio=0.664), the higher the possibility for students to be classified into a LSG group.

The estimates of the multinomial logistic regression show that the probability of being in a SDD group increases as the values of a processing speed and a language decreasing (odds ratio=0.340 and 0.540, respectively). When a LSG group is the reference group, having lower processing speed (odds ratio=0.513) and language (odds ratio=0.640) increased the estimated odds of being a SDD group student compared to other groups. When a SDD group is set up as a reference, when the values of a processing speed (odds ratio=1.950) and a language (odds ratio=1.562) increase, the probability that students belong to a LSG group increases. Attention and nonverbal reasoning are not related to group contrasts.

In research Question 3, the different growth patterns and cognitive abilities among subtypes of students with difficulties in word problem solving were explored. Based on the hypothesis that students would have different cognitive characteristics depending on whether they have difficulties in computation, in reading, in both computation and reading, or in neither computation nor reading, multivariate analysis of variance (MANOVA) was used to explore their differences on cognitive abilities. For group formation, 25th percentile and 40th percentile were selected as cutoffs and four subgroups were identified by difficulty status – word problem solving difficulty (PD), computational difficulty with word problem solving difficulty (CPD),

reading difficulty with word problem solving difficulty (RPD), computational and reading difficulty with word problem solving difficulty (CRPD). Observed mean trajectories revealed CPD, RPD, and CRPD showed significantly lower growth levels compared to PD. CRPD showed the lowest growth levels among the four groups. Significant differences between PD and RPD were found in working memory, processing speed, language, and nonverbal reasoning. Working memory, processing speed, and language between PD and CRPD, and working memory and language between CPD and RPD/CRPD were significantly different. No difference in attention was found in any contrast. In neither PD versus CPD, nor RPD versus CRPD, were cognitive profile differences found.

Key words: word problem solving, cognitive ability, mathematical learning disabilities, identification, latent class growth analysis, growth mixture modeling

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I. INTRODUCTION

1. Statements of Purpose

Mathematics is considered to be the first field of study in human history (Mankiewicz, 2000). It has been emphasized as a basis of all learning and a core of development in technology from ancient times to the present (Dossey, 1992). For many years, mathematics has been regarded as one of the disciplines for qualification to be a contributing member of society, and for developing the human ability to think logically and to solve a variety of problems in life using reasoning.

Achieving mathematical skills begins to be emphasized from the elementary school years, and mathematics education accounts for a great part of the school curriculum in South Korea It is because mathematics not only plays an important role in learning and cognitive development in the childhood (Kim, Hur, Kim, & Lee, 2009), but the acquisition of mathematical competence also has an influence on daily functioning in today's society, including success in the labor market (Fuchs & Fuchs, 2002; Geary, Hoard, Nugent, & Bailey, 2012; NCTM, 2000; National Mathematics Advisory Panel, 2008).

Despite the fact that mathematics possesses this great significance, many students struggle to achieve mathematical competence. Approximately 7% of students were reported to have experienced a substantive learning deficit in at least one area of mathematics before graduating from high school (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Shalev, Manor, & Gross-Tsur, 2005), and another 5% to 10% of students, possibly more, were found to suffer mild but persistent difficulties in learning mathematical concepts (Berch & Mazzoco, 2007). In South Korea, according to the basic accountability plan for low achievers in schools, the percentage of students who did not meet the basic standards in mathematics of national diagnostics assessment of basic competency was 4.4%, while the percentage of underachieving students was 2.4% in reading and 2.0% in writing (The Ministry of Education, Science and Technology, 2008). Although the underachievement percentage has been going down year after year, the percentage of low achievers in mathematics is almost double that in other subjects.

Learning problems in mathematics for students is not a recent phenomenon. Because of its persistence teachers, therefore, have become concerned with how they could help students to be more successful in mathematics. With this intent, problem solving began to be adopted as an approach to mathematics instruction to make learning and teaching of mathematics more meaningful using the authentic contexts (NCTM, 1980). Emphasis on problem solving was intensified in the updated guideline, *Principles and Standards for School Mathematics* (NCTM, 1989, 2000), and since then, problem solving has been considered an ultimate goal in mathematics (Miller & Mercer, 1993; 1997; NCTM, 2006). The aim of mathematics on the current 7th education curriculum in South Korea is the improvement of ‘mathematical power,’ which means an enhancement in the ability and attitude to solve problems in real life rationally through developing the mathematical way of thinking based on the basic knowledge and skills in mathematics. It is an extended concept of the major goal of 6th curriculum, 'development of problem solving skills.'

Problem solving is defined as a problem solvers’ cognitive ability to operate given situations for a given purpose even when they do not have any clear solution (Eysenck, 1994). In other words, problem solving ability refers to one of the higher-order thinking skills for identifying the problem, collecting necessary information, finding the way to solve it, and getting the answer when there is no solution prompt to the problem; in short, it means solving the problem using proper methods or strategies (Sim, 2007).

Word problems are considered to be "the most common form of problem-solving" assignment in school mathematics curricula (Jonassen, 2003, p. 267). In other words, in school mathematics, problematic situations are mainly presented in the form of word problems, which have been used for improving problem solving skills. Word problems are comprised of some sentences that include problems for solution based on various cases from real life experiences (Kim, Lee, & Shin, 2009). Students are able to learn how to mathematically express many problems laid in authentic situations, and to develop their ability in problem-solving by giving meaning to the mathematical activities and realizing the usefulness of those, through the relevance to their actual life (Chang, 2002; Kim, 2004). Moreover, a significant amount of research indicated that students could strengthen their previous knowledge in mathematics such as understanding underlying arithmetic operations, distinguishing between types of word problems on a basis of mathematical operations, and using effective strategies (Swanson, Jerman, & Zheng, 2008; Van de Walle, 2004).

Although the importance of achieving word problem solving skills in mathematics has been emphasized and a body of research about mathematical word problems has been accumulated over several years, many students, including even those who show proficiency on assessments of general mathematics skills still have trouble solving the word problems because of the complexity of the solution process (Jonassen, 2003; Lucangeli, Tressoldi, & Cendron, 1998; Schurter, 2002). Word problems require students to integrate several cognitive processes. Above all, students need to be able to understand the language and factual information presented within the word problem. Next, students should have the ability to translate the texts of the problem described in natural language to arithmetic operations expressed in mathematical language (Ilany & Margolin, 2010). This means that they have to translate the problem to create an appropriate mental representation, which allows them to devise and monitor a solution plan. Finally, students should be able to execute adequate procedural calculations without error and then to check the calculations (Desoete, Roeyers, & De Clercq, 2003; Kim, Lee, & Shin, 2009; Kim, 2004; Mayer, 1999). In other words, word problem solving in mathematics requires the integration of computation and application knowledge and basic reading skills (Leh, Jitendra, Caskie, & Griffin, 2007). This inability to integrate these two aspects of word problem solving in mathematics may explain why students feel uncomfortable with solving word problems. Some students attempt to avoid doing these problems, and deprive themselves of the benefits which may have accrued to their education. Word problem solving has some features which might have aroused students’ interest, and it is also helpful for improving their mathematical ability.

Students with learning disabilities are particularly less successful in word problem solving compared to other students without learning disabilities (Parmar, Cawley, & Frazita, 1996). They have trouble in understanding word problems because of the fact that they lack the basic skills in reading and computation, as well as possessing diminished working memory (Andersson, 2007; Geary, 1994; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Swanson, 1993; Swanson & Sachse-Lee, 2001). Above all, they are generally incapable of representing the relationship in the problem to comprehensible form which can lead them to solve the problem mathematically (Hutchinson, 1993; Montague & Aapplegate, 1993). Moreover, students with learning disabilities more often use inefficient strategies for problem solving than typically achieving students do (Montague, 1997; Parmar, Cawley, & Frazita, 1996).

Therefore, it is a prerequisite to providing appropriate educational services to students with learning disabilities that their disabilities in learning be identified in a valid and reliable way. Before any identification of learning disability can occur, one must be clear as to what a learning disability is. This has been a matter of debate for a long time and still remains one in the quest for a workable definition.

There are three major models which can be used to identify learning disabilities: Intelligence Quotient- (IQ)-achievement discrepancy model, response-to-intervention (RTI) model, and intrinsic processing deficits model (Kim, Lee, & Shin, 2009). These models reveal the different ways to conceptualize the unexpected underachievement of learning disabilities. Since the Samuel Kirk's 1963 definition of learning disabilities, the IQ-achievement discrepancy model has been the longest universal model for identification (Hong & Kim, 2006). This approach, however, faced criticism and the 2004 federal reauthorization of the Individual with Disabilities Education Act (IDEA) introduced the responsive to intervention model as an alternative way to identifying learning disabilities. This was because The IQ-achievement discrepancy model employs reactive rather than proactive strategies for remediation, since it depends on the incidence of failure before intervention was provided, and also because of its technical difficulties in terms of the IQ-achievement discrepancy (Fletcher, Lyon, Fuchs, & Barnes, 2007). With this alternative approach, the unexpected underachievement is defined as non-responsiveness to validated instruction, so it has the advantage of offering early intervention as prevention before resulting in inaccurate identification of learning disabilities (Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012). In spite of these advantages, the response-to-intervention model has encountered some opposition in that this model does not involve the essential concept of learning disabilities (Lee, 2007). Namely, even though the basic psychological processes of the learning disabilities has been initially attributed to central nervous system dysfunction, any information in terms of the primary cause of the learning disabilities is not provided by the responsive to intervention model.

The notion of learning disabilities has traditionally indicated unexpected underachievement due to a disorder in one or more of the basic psychological process or processes (Kim, Lee, & Shin, 2009). It has been referred to as an intrinsic processing deficits model to identify learning disabilities by evaluating psychological process and/or capacity weakness directly which works on the basis of learning problems. The approach of the intrinsic processing deficits model employs a method which identifies the intra- and inter-individual differences in performance on cognitive abilities and how the differences could explain the achievement differences within a specified academic domain (Fletcher, Morris, & Lyon, 2003; Torgesen, 2002).

Although the intrinsic processing deficits model is the core criterion for the identification of learning disabilities and allows for a more direct approach to characterize and measure learning disabilities as opposed to indirect methods via exclusionary clauses, it has not been a mandatory requirement for identification of learning disabilities. That is because of the fact that not only theoretical and empirical supports for the notion of psychological process deficits remain vague still, but also because there is currently no agreement as to how to measure cognitive ability in any valid way (Augustyniak, Murphy, & Phillips, 2005; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Hammill, 1990; Torgesen, 2002). Several considerations which could be applied to the intrinsic processing deficits model in order to identify learning disabilities and to utilize it for developing effective intervention programs include: which cognitive factors or basic learning skills are critical to learn the specific subjects (e.g., reading, mathematics, or calculation/word problem solving in mathematics), what is the relationship between cognitive abilities and basic learning skills, and how the cognitive abilities and basic learning skills can be measured validly and reliably (Lee, 2007).

Since Geary (1993) pointed out that little is known about which cognitive ability has influence on difficulties in mathematics, and to what extent the cognitive abilities are involved in mathematics, the research efforts devoted to identifying the cognitive mechanisms that contribute to the mathematical learning difficulties have substantially increased over the past two decades and have yielded many insights (e.g., Bull, Espy, & Wiebe, 2008; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Fuchs et al., 2006; 2008; Geary, Hoard, Nugent, & Bailey, 2012; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Passolunghi, Mammarella, & Altoé, 2008; Swanson, 2006; Swanson, Jerman, & Zheng, 2008; Taub, Floyd, Keith, & McGrew, 2008). Some of these focused on arithmetic problems (e.g., Bull, Espy, & Wiebe, 2008), others examined univariate determinants (e.g., Swanson, 2006; Swanson, Jerman, & Zheng, 2008), or general cognitive factors for mathematical deficits (e.g., Taub, Floyd, Keith, & McGrew, 2008). Some of them investigated longitudinal changes in cognitive abilities in terms of mathematical competence (e.g., Geary, Hoard, Nugent, & Bailey, 2012).

These studies suggested that students with mathematics learning disabilities have distinctive patterns of cognitive abilities compared to other subtypes with and without learning disabilities. Despite some progress in this area, many aspects of cognitive correlates still remain unresolved. Research dealing with cognitive abilities of students with mathematics learning disabilities is very limited, and most studies have focused on arithmetic in mathematics and mathematical concepts of early ages (Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012). To increase the understanding of mathematics learning disabilities, further research with a focus on other aspects of the curriculum besides arithmetic is required, especially, at the intermediate grade levels.

Meanwhile, the broad-spectrum term, ‘learning disabilities’, embraces heterogeneous learning problems. In many guidelines for identification and diagnosis of learning disabilities, learning disabilities have been primarily classified according to the academic area in which a student has difficulties; these are mentioned as reading disabilities, writing disabilities, mathematics disabilities or their co morbidities (e.g. reading disabilities with mathematics disabilities) (Chung, Kim, & Jeong, 2001). In fact, the classification system of learning disabilities could be more segmented because, for example, there are several sub-domains in mathematics, such as computation, algebra, geometry, or problem solving. Students may have trouble in different domains under the same label, 'mathematics learning disabilities.' Or even if some students have similar difficulties on computation, it could come from different reasons, such as attention deficit or memory problems. Absence of understanding underlying cognitive ability, or learning problems, or learning disabilities, however, makes classification of subtypes impossible.

Since the response-to-intervention model was introduced as an identification model for learning disability in South Korea, it has come up for lively academic discussion, and many theoretical studies have been conducted (e.g., Cho, Byun, & Choi, 2011; Hur, 2005; Jung, 2007; Kang, Hong, Lee, Kim, & Kim, 2008; Kim & Jung, 2008; Lee, 2008). Most researchers have suggested that the response-to-intervention model should be considered as a promising alternative way to identify students with learning disabilities. Nevertheless, they recommended that it should be given careful consideration before its implementation in South Korea, because the education context is not the same as that of the United States; in addition, there is also a lack of relevant empirical studies for South Korea. Recently, some studies, which applied the response-to-intervention model in practice, reported that some level of effectiveness had been observed (e.g., Hong & Yeo, 2011; Kim, 2012; Lee, 2009; Lee & Choi, 2005; Lee & Hong, 2007; Yeo & Hong, 2011), but these studies number less than 10 and cannot be judged to be conclusive. Among them, Hong and Yeo (2011), Kim (2012), and Yeo and Hong (2011) tried to verify empirically how students' growth patterns were stratified based on the framework of the response-to-intervention model, but their studies only targeted students' computational skills. The empirical research base is still insufficient and their interest is limited to basic mathematical skills. In addition, there is no research that has dealt with cognitive correlates with respect to mathematical problem solving ability in South Korea.

Under the research context mentioned above, this study attempts to explore the heterogeneity of students who have difficulty in learning mathematics based on their growth patterns and cognitive abilities. First of all, the study investigates whether and how students' growth patterns of mathematical problem solving ability are stratified. From this, the study explores the possibility of identifying the mathematical learning problem using students' growth trajectories based on the response-to-intervention model. The study then identifies the cognitive predictors of group membership, and the similarities and differences in the cognitive characteristics by subtypes among students with mathematical difficulties are determined. The results can contribute not only in evaluating the adequacy of interventions for the implementation of response-to-intervention model, but also to establish the theoretical foundation for the intrinsic cognitive deficits model.

In this study, the third grade students were selected as subjects because, according to the standard mathematics curriculum in South Korea, students complete their learning about the four fundamental arithmetic operations at the end of the first semester of third grade. Moreover, from prior research, third graders begin to develop their problem-solving skills based on their previous skills of arithmetic, algorithmic computation, and arithmetic word problems acquired in the first and second grades (Fuchs et al., 2006). Also, with regard to semantic structure of word problems, it was reported that third-grade students were typically able to solve most types of word problems (Geary, 1994).

2. Research Questions

The main purpose of this present study is to identify at-risk students with learning disabilities in mathematics based on the growth pattern of word problem solving ability and to investigate the different cognitive profiles with regard to different development patterns of achievement in word problem solving in mathematics.

The specific research questions are as follows:

Question 1. Are there any identifiable groups in students' growth patterns (intercept and slope) of mathematical word problem solving ability?

Question 2. Given that there are multiple growth patterns, what are the effects of students' cognitive abilities on their growth patterns of mathematical word problem solving ability?

Question 3. How do the cognitive abilities differ among students with mathematical difficulties identified by their growth patterns, depending on whether there exists an accompanying problem in computation, in reading, in both computation and reading, or in neither computation nor reading?

3. Definition of Terminology

1) Students with mathematics difficulties

The term "mathematics learning disabilities" indicates that students show substantially lower achievement in mathematics than what is expected for their age, intelligence, and education (Kim, Lee, & Shin, 2009). The reauthorization of the individuals with disabilities education act (IDEA) in 2004 introduced the notion of response-to-intervention (RTI) as an alternative approach for identifying learning disabilities. With this approach, students with mathematics learning disabilities are defined as students whose achievement in mathematics calculation or mathematics problem solving, are not commensurate with their age when they are provided with learning experiences appropriate for that age (IDEA, 2004).

In this study, under the framework of response-to-intervention, the definition of students with mathematics difficulties refers to students who report consistently lower performance (lower initial score than students' average level and insufficient slope to achieve their expected year-end goal) on repeatedly measured achievement tests than other students in the general student population.

2) Word problem

A mathematical problem is a situation in which students are asked to perform a task for which there is no immediately available algorithm that completely defines a solution method (Ilany & Margolin, 2010). A word problem in mathematics is a unit of text that is comprised of sentences containing verbal and mathematical factors to denote the problematic situations. Word problems are referred to as story problems in some research.

3) Cognitive abilities

Cognitive ability refers to an individual’s psychological functions in the perspective of an information processing theory. It includes mental processes such as attention, memory, processing speed, and producing and understanding language. Cognitive abilities are not affected by learning, but could have an influence on learning (Lee, 2007).

Ⅱ. LITERATURE REVIEW

1. Cognitive abilities related to mathematics learning and mathematics learning disabilities

Basic cognitive ability refers to a data processing ability that is not affected by the level of learning, but which affects academic achievement (Kim, Lee, & Shin, 2009). Variables known at present as being related to mathematics learning and mathematics learning disabilities are working memory, short-term memory, rapid automatized naming (RAN) of objects, letters, or digits, and the subcomponents of the standardized intelligence test or cognitive test (Lee, 2007). Geary (2004) suggested that mathematical skills represent different areas of knowledge based on general cognitive or neuropsychological systems, such as a central executive function that produces and employs a linguistic system, a visual-spatial system, and a system of attentive behavior that restricts unrelated information. He added that difficulty in learning mathematics is caused by one of these cognitive systems or by their interaction, and this leads to varying mathematical skills, or different patterns of deficits in carrying out various mathematical assignments. Fletcher et al. (2007) stated that a successful performance of mathematical ability requires a working ability which is speedy enough to avoid the overload of the working memory storage that not only attends to, organizes, and changes information, but also maintains information in order to access different information when demanded while working. The present study shall search out precedent studies on working memory/executive ability and cognitive ability of language, themes that have been most frequently researched in relation to mathematics learning abilities, and shall handle also, cognition-related variables, including processing speed (rapid automatized naming).

Regardless of numerical ability, deficits in ability in solving assignments dependent on working memory (Bull & Johnston, 1997; Geary, Hoard, Byrd-Craven, & DeSoto, 2004) and executive function (Sikora, Haley, Edwards, & Butler, 2002) are commonly found among students with mathematics learning disabilities. Swanson and his colleagues' studies (Keeler & Swanson, 2001; Swanson & Sachse-Lee, 2001; Swanson & Siegel, 2001) have considered deficiencies in linguistic and visual-spatial working memory, the executive process of strategic knowledge, and an ability to make efficient use of working memory, as root causes for diminished academic performance in students with specific mathematical disabilities and also in students with both reading and mathematics learning disabilities.

Several studies have proposed that while students with mathematics learning disabilities are characterized by their problem with visual-spatial working memory (Siegel & Ryan, 1989), children with both reading and mathematics learning disabilities have difficulty with general language and linguistic working memory. Yet Keeler and Swanson (2001) demonstrated that the mathematical computational skills of students with specific mathematics learning abilities can be predicted better by linguistic working memory than by visual-spatial working memory. There is some evidence that difficulty with domain-specific working memory, as well as problems with executive control, cause more general problems. In contrast, Swanson and Sachse-Lee (2001) argued that a domain-general system, not in direct relation to reading problems, has crucial effects on the working memory of children with reading disabilities. Wilson and Swanson (2001) suggested that various features of domain-general and domain-specific working memory have bearing on an ability to obtain a strategy for mathematical computation. As Geary and his colleagues (Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Byrd-Craven, 2004) pointed out, working memory is in complex links with mathematics learning disabilities, and that additional studies are necessary to decipher how this relationship works, and whether it depends on whether mathematics disabilities involve reading disabilities. Clearly, children with both reading and mathematics disabilities tend to have more serious problems than children with one of the two disabilities (Fletcher, Morris, & Lyon, 2003). Furthermore, since working memory in its different aspects can be related to different mathematical skills, the relationships between working memory and mathematics disabilities are complicated. For children with typical development, for instance, numerical computation is linked with short-term linguistic memory or linguistic working memory according to whether it uses borrowing in calculations, but visual-spatial working memory has to do with a numerical presumption (Khemani & Barnes, 2005).

However, Carey (2004) asserts that language is significant not only for formal mathematics learning, but also for the development of the mathematics area, such as geometry, that seems to be affected by language only to a relatively insignificant degree (Spelke & Tsivkin, 2001). Language has no original meaning, but it provides a symbol that defines a stage for bridging different representation systems of the past, such as a quantitative or linguistic system. As a corollary, the combined representations may form a new and strong mental structure. Some evidence for the importance of language in the development of early mathematical skills is found in studies of young children and kindergarten students These studies note the development of computation ability using low numbers that can vary according to the linguistic quantitative signal of each language (Hodent, Bryant, & Houdé, 2005). Although the role of language in mathematical ability has been explained in relation to its typical development, the development of an abnormal linguistic system can be expected to result in deficits in specific aspects of mathematical abilities for very young children.

From a different perspective, other studies pointed out that children with reading and mathematics disabilities typically display more serious and comprehensive speech disorder than children with word recognition disabilities. This difficulty manifests itself in problems where a mathematical fact that is essential for learning, maintenance, and accurate computation is withheld. This results in an overall range of difficulty with mathematics. In accord with these findings, Jordan and Hanich (2000) made it clear that children with difficulty both in reading and mathematics displayed problems in various areas of mathematical thinking, which led to the conclusion that linguistic disabilities definitely cause difficulty in acquiring mathematical skills.

Furthermore, various cognitive skills have been pointed out by studies on learning disabilities of mathematics. Fuchs et al. (2006)

conducted a survey of the relationship between the characteristics of different children and their different mathematical disabilities. They reported that computational skills determine processing speed, phonological process, and attention. . Working memory is represented as a predictor, in the event that reading and reading-related skills have been deleted from the model. Evidence for computation-related working memory was mentioned above. Processing speed manifests itself as numerical speed in mathematical problems involving computation. That is, speedy processing releases cognitive resources, enabling more efficient use of working memory. Bull and Johnston (1997) found out that processing speed was a strong predictor of computational skills of 7-year-old children, and Hecht, Torsgesen, Wagner, and Rashotte (2001) discovered that processing speed is related to computational skills even when linguistic ability remains the control variable.. Geary (1993) argued that phonological process is important for computation, because successful computation needs an ability to create and maintain phonological representations. Evidence for this is mixed. Whereas Fuch et al. (2005) found out in their study of first-grade students that phonological process is the single predictor of computational skills, Swanson & Beebe-Frankenberger (2004) refuted this view and held that phonological process does not determine computational ability. Recent studies have accentuated the role of attention as a strong predictor of mathematical skills. For instance, Fuch et al. (2005) noted that teacher's rating of attention predicts computational skills even when several cognitive abilities are under control. Even so, teachers would probably have brought simplistic estimation of students' attention according to their learning abilities.

Fuch et al. (2006) examined four characteristics that can predict the ability of students to complete computation, which is concerned with "algorism computation" that displays the application of procedural knowledge through computation involving multiple stages. Fuch et al. (2005) said that teacher's rating of attention/distraction is the single most dominant predictor of computation. This result has been reinforced by other studies of children with ADHD (Ackerman, Anhalt, & Dykman, 1986; Lindsay, Tomazic, Levine, & Accardo, 1999). Other researchers pointed to working memory and phonological process for solving the matter of algorism (Hecht, Torgesen, Wagner, & Rashotte, 2001).

Miller and Mercer (1997) demonstrated that students with learning disabilities exhibit problems, such as attention deficit, memory problems, deficiencies in audio process, difficulty with visual-spatial abilities, behavioral disabilities, and deficiencies in information process. Busse, Berninger, Smith, and Hildebrand (2001) spoke of more varied and wider- ranging variables, proposing seven dimensions of cognitive ability variables that are necessary for mathematical thinking and problem solving: a strategy, memory (short-term, long-term, and working memory), declarative and procedural knowledge, temporal and visual-spatial process, the balance of input-output, finger skills, and a level of cognitive development (specific and operative representations as opposed to formal and manipulative representations).

In South Korea, scholarly concern has rarely been directed to the relationship between cognitive ability and mathematics learning. With a focus on under-achieving students in basic mathematics learning, whose scores were lower than the standard scores for normal students in the national test for basic competency, Lee, Choi, Jeon, and Kim's study (2007) analyzed working memory, a group of cognitive ability variables like processing speed and quantity discrimination, and the extent to which basic learning ability of mathematics accounts for the standardized academic achievement in mathematics. The result was that they found that there was a statistically meaningful difference in each cognitive ability variable between normal students and under-achieving students, and that working memory accounts significantly for the difference between them. Song, Kang, Kwon, and Suh (1998) surveyed children with arithmetic disabilities, analyzing their mathematical abilities and characteristics in data processing. In this study, children with arithmetic disabilities showed a lower performance compared to middle and high level achieving students, in terms of data processing ability, such as short-term memory, automation of processing, and basic knowledge.

2. Cognitive abilities involved in word problem solving

Over the past few years, many researchers have studied the word problem performance of students with learning disabilities (Bryant & Dix, 1999; Desoete, Roeyers, & Buysse, 2001; Jintendra, DiPipi, & Perron-Jones, 2002; Xin, Jitendra, Deatline-Buchman, Hickman, & Post, 2002). Problem solving, particularly word problem solving, includes computation, language, reasoning, reading skills, and probably visual-spatial skills (Geary, 1993). Many studies have been carried out in relation to the use of language to solve mathematical problems. Several studies have linked deficiencies in solving word problem questions with reading deficiencies (Bender, 2005). To complete word problems that require computation for solution students need not only cognitive ability related to text understanding, but computational skills also. One essential difference between computation and word problem questions is that the latter consists of additional linguistic information which makes it necessary for children to decipher in order to configure it into a non-word problem model. While computational questions are already prepared to be solved, word problem questions use the text to identify missing information, form a numerical formula, and require students to be able to create a computational question to find missing information. Studies, which handle word problem questions, focus on various executive functions that are involved in working memory, linguistic and reading abilities, problem solving, and concept forming. Regarding this, Desoete and Roeyers (2005) articulated that a linguistic factor of meaning process and a non-linguistic factor of executive function differentiate the levels of problem solving ability in mathematics. They clarified the issue as to whether students' mathematical cognition is composed of one or more components, investigating whether children with mathematics learning disabilities show the full nine dimensions of profiles, similar to what young children with the standard mathematics performance display. Through this investigation they aimed to define the theory of developmental delay. The nine types of cognitive sub-skills used here for mathematical problem solving are as follows: Nonsemantic (numeral reading and production, operation symbol reading and production, procedural calculation), semantic (number system knowledge, number sense) and semi-semantic (language comprehension, context information, mental representation, selecting relevant information) skills. As a corollary to this research, two components, semantic and non-semantic components, were required to meet the appropriateness of data requirement. Students with learning disabilities have less developed cognitive skills than students of the same age without learning disabilities. Yet out of the nine cognitive skills seven of them were not different in both groups of students. This has shed light on the fact that mathematical problem solving ability is not determined as a general component, but specifically by two more complex cognitive skills within the mathematical components. Given the fact that children with mathematics learning disabilities had a lower average score in comparison with the scores of students without learning disabilities, researchers pointed out the importance of assessment of various cognitive skills, particularly as they pertain to number system knowledge. It is to be noted however, that the results of their studies indicated that the developmental delay may have had some effect on the findings.

Although a number of researchers in educational psychology have addressed the issue of learning and solving word problem questions (Kintsch & Greeno, 1985; Mayer & Hegarty, 1996), the number of studies on the effects of working memory on ability in word problems in students is low, and most of those studies have focused on arithmetic word problem questions. In earlier studies, Johnson (1984) demonstrated that spatial ability created by an assignment of mental rotation which is positively related to an ability to solve arithmetic word problem questions. In more recent studies, Swanson, Cooney, and Brock (1993) conducted assignments of spatial knowledge (knowledge of problem classification and operationalization), cognition (executive function and word recalling), and reading comprehension, all of which are sufficiently present among 9-year-old children. Knowledge of reading comprehension and operationalization turned out to be the best predictor for accuracy in word problem solving. The amount of executive function determined accuracy in word problem solving, but failed to have a distinctive effect on it when other variables were considered. Kail and Hall (1999) confirmed these research results. They made clear that estimations of phonological and executive functions are root causes in the number of errors made. Again, these estimations have failed to have unique effect on accuracy in word problem solving when other variables-processing speed and reading skills-are under consideration. In contrast, Passolunghi and his colleagues (Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2001) articulated that a low level of problem solvers showed lower executive function scores by their lower inhibition abilities than outstanding problem solvers. Lee, Ng, Ng, and Lim (2004) applied the findings of precedent research in the solving of algebraic word problems. These precedent studies of differences in personal mathematical ability identified the contribution of working memory to the earlier algebraic performance attempting to examine the relationship among working memory, reading ability, and children's abilities for solving arithmetic word problem questions. The core outcome of this research is that the central executive function played a more significant role than was earlier expected. In view of both direct and indirect effects, the central executive span has as much an effect on mathematics performance as reading comprehension does. In agreement with the precedent studies, Lee et al.'s study demonstrated that the storage factors of working memory - phonological loop and visual-spatial sketchpad - had no direct effect on mathematics performance. Also, it is in conformity with the result of arithmetic-related studies that recognized evaluation of reading comprehension components as having stronger effects than executive function ability. Furthermore, it displayed, as a result of route analysis, that the executive function does affect reading comprehension. From an educational viewpoint, this result accentuates the significance of reading comprehension. Despite the relatively small amount of variability that can make a distinctive contribution to reading comprehension, the result is thought-provoking in clarifying the fact that reading comprehension is not only the best predictor of outcomes in word problem solving in mathematics, but also embodies an ability that can receive educational arbitration. This does not mean that particular knowledge of mathematics is not important. Through a study of mathematical knowledge, it was found that knowledge plays a role, which is at least equal, to the role of linguistic ability (Sakamoto, 1998; Swanson, Cooney, & Brock, 1993).

In an attempt to establish in the form of a formula the relationship of number ability to arithmetic, algorithmic computation, and arithmetic world problem, Fuch et al. (2006) conducted a large-scale survey of 3rd grade children, evaluating the three mathematical abilities in measuring language, non-linguistic problem solving, concept forming, processing speed, long-term memory, working memory, phonological interpretation, and sight word efficiency. A series of route analyses were used to evaluate a model that relates to the three mathematical abilities according to various cognitive abilities. The single factor that independently anticipated the three aspects of mathematical performance was the teacher's rating of inattention. Alongside inattention, only phonological interpretation and processing speed anticipated computational ability, and non-linguistic problem solving, concept forming, word reading efficiency, and linguistic skills anticipated the ability for arithmetic word problems.

3. Prospective approach to identification of learning disabilities: The intrinsic processing deficits model

From a historical perspective, a central concept in the definition of learning disabilities has been regarded as unexpected underachievement (Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012). Yet this unexpected underachievement has been conceptualized in a different way. Since Samuel Kirk’s definition in 1963, the U.S. Office of Education (1968, 1977) has defined learning disabilities officially in exclusionary terms. A discrepancy between IQ and achievement, where IQ is high and achievement is low, has been used as an indicator of unexpected underachievement. Yet this approach was criticized as a "wait-to-fail model (Fletcher, Lyon, Fuchs, & Barnes, 2007)," and the response-to-intervention (RTI) model has been selected as the model for identification of learning disabilities (IDEA, 2004). The RTI approach identifies learning disabilities based on a response to effective interventions, and is, as anticipated, a more practical and problem solving-oriented method than the existing discrepancy models. According to the RTI approach, unexpected underachievement is operationalized as exhibiting low achievement in spite of effectiveness-proven instruction. The biggest difficulty which the discrepancy and RTI models have as a diagnosis of learning disabilities and screening measures, consists in their failure to provide a decisive source or clue to explain why learning disabilities have occurred, apart from low grades, and whether learning disabilities are indeed present even after a diagnosis of difficulty in learning has been rendered according to each standard.

An essential or decisive feature in learning disabilities should first be figured out to solve this difficulty. For this, it is necessary to examine variables that are traditionally known, or are theoretically regarded, as being in relation to learning disabilities (Lee, 2007). For a diagnosis and discrimination of mathematics learning disabilities, for instance, it is essential to grasp cognitive function working crucially in mathematics learning, and to identify who possesses the function.

This approach, which identifies learning disabilities through examining both the cognitive ability related closely to academic achievement in coursework, and the extent of basic learning function, is called an intrinsic processing deficits model. In other words, the intrinsic processing deficits approach is a method that identifies what type of difference exists within or among individuals, and articulating the extent to which this difference accounts for a difference in academic achievement in coursework. Process, a concept from the perspective of the information process theory, refers here to a series of mental behavior, or operationalization, that transforms, operationalizes, and shows a specific form of response to information entered into as a stimulus over time.

As an alternative to the existing models for learning disabilities diagnosis, the intrinsic processing deficits model has various advantages. Firstly, it is proactive rather than reactive and does not rely on the incidence of failure in school prior to a diagnosis of learning disabilities. This allows for a relatively early intervention which may obviate or reduce the possibility of failure in students who display some form of learning disability. Secondly, it conducts direct assessment of intrinsic processing deficits, providing information for appropriate instructional design that can be applied to all children with general learning problems, as well as to children with specific learning disabilities. Thirdly, it is helpful to focus on intensive instruction for a field that most needs a diagnosis of intrinsic processing deficits of students with learning disabilities. That is, it is likely to make an innovative contribution to the development of an effective intervention program that fits well with an individual's pattern of intrinsic processing deficits.

Despite these advantages, however, it is difficult in the current scenario to define this model as a learning disabilities discrimination model, inasmuch as sufficient evidence, both theoretical and practical, has not yet been collected and established for the intrinsic processing deficits approach (Torgesen, 2002). To put it another way, a conceptual problem with the meaning of intrinsic processing deficits, and a technical problem with the measurements of these deficits still need to be researched, addressed, and settled.

III. METHODOLOGY

This study focused primarily on the possibility of being able to identify students with mathematics difficulties based on the growth trajectories of learning, the relationship between growth patterns and cognitive abilities, and the differences in cognitive abilities, as well as growth patterns in word problem solving depending on subtypes among students with mathematics difficulties. For this purpose, the following research questions were established: 1) Are there any identifiable groups in students' growth patterns (intercept and slope) of mathematical word problem solving ability? 2) Given that there are multiple growth patterns, what are the effects of students' cognitive abilities on their growth patterns of mathematical word problem solving ability? 3) How do the cognitive abilities differ among students with mathematical difficulties identified by their growth patterns, depending on whether accompanying problem in computation, in reading, in both computation and reading, or in neither computation nor reading?

1. Subjects

The data were collected from July to November, 2011. The participants consisted of 476 students from Incheon, South Korea. They were sampled from 20 classes in 4 elementary schools which accepted the request of research participation through an official document. After obtaining principals' approvals, the purpose and brief description of research methodology including a data collection plan were explained to the co-operating teachers who took charge of each class. Parental consent and child assent to the administration of data collection and the security of data thus collected were received through relevant and appropriate school correspondence conducted by teachers.

Of these 476 students, 14 cases, which did not complete any of the exercises in cognitive measurements, were excluded from the research. The data analysis methods employed in this study are based on maximum-likelihood as an estimator, thus, variables in the data should meet the assumption of multivariate normality to produce better parameters (Muthén, 2002; Preacher & Hayes, 2008). Yet, outliers can violate assumptions of normality (Osborne & Overbay, 2004). To detect outliers, a criterion based on *z*-scores (absolute *z*-score>3.29, p<.001, two-tailed test) and squared Mahalanobis distance (D^2, p<.001) was used (Tabachnick & Fidell, 2007). Some outliers detected by *z*-scores were modified by converting the raw score for the outlier to one unit more extreme than the next most extreme score following the recommendations of Tabachnick & Fidell (2007). Multivariate outliers were subsequently removed from the analysis. Ultimately, 441 cases were used for analysis in the present study.

All subjects were selected from third grade in elementary school, and most of them were from 8 (196, 44.4%) to 9 (242, 54.9%) years of age. Of the 441 students, 210 were males (47.6%) and 231 were females (52.4%).

The demographic information for the 441 students is described in Table Ⅲ-1.

Table Ⅲ-1. Demographic information of the sample

		School A	School B	School C	School D	Total
Age 7	Male	1	0	0	0	2
	Female	0	1	0	0	
Age 8	Male	23	22	18	28	196
	Female	36	33	16	20	
Age 9	Male	37	28	15	37	242
	Female	43	35	21	26	
Age 10	Male	1	0	0	0	1
	Female	0	0	0	0	
Total	Male	62	50	33	65	210
	Female	79	69	37	46	231
		141	119	70	111	441

2. Measures

Two dimensions of measures were administered for this study. One is for examining cognitive abilities, the other one is academic achievement. The cognitive abilities were measured in five domains – language, nonverbal reasoning, working memory, processing speed, and attention. Academic achievement data were collected from word problem solving scores which is one of the major skills in mathematics.

The third purpose of this study involves the development of a classification schema for at-risk students according to their particular academic deficits in computation or reading skills. Two kinds of curriculum based measurement tools in mathematics and reading developed in South Korea were utilized for screening in order to achieve this purpose.

The measurement tools used in this study were as follows.

1) Cognitive measures

Language was measured with K-WISC-IV Vocabulary (Kwak, Oh, & Kim, 2011). It is one of the major subtests of the verbal comprehension index (VCI) in K-WISC-IV. It assesses expressive vocabulary, verbal knowledge, and foundation of information using student exposure to 40 items. The first four items present pictures and the student is asked to identify the object in the picture. For the remaining items, the tester says a word which the student is asked to define. Responses are given a score 0, 1, or 2 depending on the quality of the observation or definition respectively which the student has provided. For any student receiving zero scores in five consecutive questions testing was terminated following the scoring guides of K-WISC-IV Vocabulary. The score is the total number of points gained by each student . As reported by the test developers, split-half reliability of this subtest is .85 to .86 at ages 8 to 9; test-retest reliability is .93. Coefficient alpha on this sample was .84.

Nonverbal reasoning was assessed with K-WISC-IV Matrix Reasoning (Kwak, Oh, & Kim, 2011) using four types of tasks – pattern completion, classification, analogy, and serial reasoning. It is one of the major subtests of perceptual reasoning index (PRI) in K-WISC-IV. Students complete a matrix, from which a section is missing, from five response options. Students get points by identifying the correct missing piece of the matrix out of a total of questions. Testing is discontinued after four errors on five consecutive items or four consecutive errors. The score is the number of correct responses. As reported by the test developers, split-half reliability of this subtest is .85, .83 at ages 8 and 9 respectively; test-retest reliability is .75. Coefficient alpha on this sample was .74.

Working memory was assessed with K-WISC-IV Digit Span (Kwak, Oh, & Kim, 2011). Digit span task is often used to measure the working memory's number storage capacity. This subtest comprises two kinds of test: one is digit span forward task, and the other is digit span backward task. Digit span forward task requires the student to repeat numbers in the same order as read aloud by the examiner, whereas digit span backward task requires the student to repeat the numbers in the reverse order of that presented by the examiner. The shift from the digit span forward task to the digit span backward task requires cognitive flexibility and mental alertness. Each task has eight items in which each of the eight items has two trials. The score is the total number of correct responses in each trial. The digit span is typical subtest of working memory index (WMI) in K-WISC-IV. As reported by the test developers, split-half reliability of this subtest is .89 and .86 at ages 8 and 9; test-retest reliability is .84. The split-half reliability on this sample was .78.

Processing speed was measured with K-WISC-IV Symbol Search (Kwak, Oh, & Kim, 2011), which is the core subtest for evaluating the processing speed index (PSI) in K-WISC-IV. The student scans a search group and indicates whether the target symbol(s) matches any of the symbols in the search group within a specified time. Perception and recognition are the two prime requirements, in addition to speed, accuracy, attention, and concentration. The symbols are geometric forms rather than familiar letters or numbers. The total score is the number of correct responses minus the wrong responses. As reported by the test developers, split-half reliability of this subtest is .65 in equal at ages 8 and 9; test-retest reliability is .80. The split-half reliability on this sample was .65.

Attention was assessed using the Frankfurt Attention Inventory (FAIR) (Moosbrugger & Oehlschlägel, 1996; Oh, 2002). This paper–pencil test consists of 640 items (two sheets each with 320 items) with four kinds of similar stimuli: squares with two points, squares with three points, circles with two points or circles with three points. Each sheet consists of 16 rows each containing 20 items. Within 6 minutes, the students have to select as many two target configurations as possible – circles with three points and squares with two points – from the left side to the right side of each row. The FAIR test yields four performance measures, but in this study, the score for total performance capacity (FAIR-P), which informs about the total number of items processed with attention during the test period, was used as a score of attention. This test provides excellent test–retest reliability scores between .85 and .91 (Cronbach's α) (Oh, 2002). The reliability coefficient was .85 on this sample.

2) Curriculum-based mathematical word problem solving measurement

The curriculum-based measurement was developed to assess the students' basic academic skills in certain academic areas, to prescribe the appropriate intervention, and to evaluate the effectiveness of intervention prescribed for students (Shinn, Shinn, Hamilton, & Clarke, 2002). There is no standard measurement instrument that focuses solely on mathematical word problems in South Korea. Some researchers, thus far, have utilized researcher-developed tests for evaluating the problem solving skills in their study. Recently, Kim (in press) developed and validated a curriculum-based measurement for word problem solving called the Basic Academic Skills Assessment: Mathematical Word Problem (BASA: MWP). It was developed reflecting the semantic structure which refers to the meaning of the statements in the problem and their interrelationships (Geary, 1994). According to the classification of word problems by Jordan and Hanich (2000, adapted from Carpenter & Moser, 1984; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983) and the guidelines for analysis of word problems by Kim (1997), analysis of word problems on the third-grade curriculum was conducted. The proportion of semantic structure presented on the curriculum was considered in the development of word problem solving tests. Additionally, problem features such as the number of sentences, length of problem, and computational demands, as well as problem materials were controlled at the level of the third-grade curriculum. Three equivalent tests were developed, and each test was composed of 20 items to be completed within 10 minutes. The types of word problems are presented in TableⅢ-2.

Internal consistency reliability (Cronbach's α) of the word problem solving test was .90. The content validity was confirmed by a panel of experts consisting of one professor and three elementary school teachers with relevant master’s degrees. The assessment of the construct validity by factor analysis shows that all the items of the word problem solving test were measuring the same unidimensional construct. Concurrent validity with the Basic Academic Skills Assessment: Math (BASA: M) (Kim, 2006) and the maze test of the Basic Academic Skills Assessment: Reading (BASA: R) (Kim, 2009) were reported as .57 and .35 respectively.

Table Ⅲ-2. Types of each word problem

Item	Type of operation	Semantic structure	Type of number model
1	Addition	Combine	(three-digit)+(three-digit)
2	Addition	Combine	(four-digit)+(three-digit)
3	Addition	Combine	(four-digit)+(four-digit)
4	Subtraction	Compare	(three-digit)-(three-digit)
5	Subtraction	Change	(three-digit)-(three-digit)
6	Subtraction	Compare	(four-digit)-(four-digit)
7	Subtraction	Change	(four-digit)-(four-digit)
8	Addition & Subtraction	Change & Combine	(four-digit)-(four-digit) +(four-digit)
9	Multiplication	Repeated addition	(two-digit)×(one-digit)
10	Multiplication	Repeated addition	(two-digit)×(one-digit)
11	Multiplication	Repeated addition	(three-digit)×(two-digit)
12	Multiplication	Repeated addition	(two-digit)×(two-digit)
13	Multiplication	Repeated addition	(two-digit)×(two-digit)
14	Multiplication	Repeated addition	(two-digit)×(two-digit)
15	Division	Division into equal parts	(two-digit)÷(one-digit)
16	Division	Division into equal parts	(two-digit)÷(one-digit)
17	Division	Division by equal part	(two-digit)÷(one-digit)
18	Division	Division into equal parts	(two-digit)÷(one-digit)
19	Division	Division by equal part	(two-digit)÷(one-digit)
20	Division	Division by equal part	(two-digit)÷(one-digit)

When scoring the word problem solving test, two points were given for the correct number model and one point for incomplete number model; but in cases where the student was recognized as being understood the algorithm for solution was assigned. Also, it was scored by counting the number of correct digits (CD) in each answer. This alternative approach is grounded in good academic-assessment research and practice. By separately scoring each digit in the answer of a computation problem, the examiner is better able to recognize and to give credit for a student's partial math competencies. The possible total score for this word problem solving test was 150.

3) Screening tools for academic deficits among students at-risk with learning disabilities

To identify students with deficits in *computation skills*, one of the baseline tests in Basic Academic Skills Assessment: Mathematics (BASA: M) (Kim, 2006) was used. It was developed on the basis of principles of curriculum-based measurement, which could assess the students' computational fluency and their progress in mathematics. BASA: M was standardized for first- to third-grade students, and it was composed of four-level subtests – Level-I, II, III, and integration level

– that were designed to measure students' computation skills based on the standard curriculum in mathematics. Each level includes 3 baseline tests and 10 equivalent-form tests for repeated measuring. As reported by the test developer, the BASA: M reliability (Cronbach's α) ranges from .79 to .93. Concurrent and criterion related validity with the BASA: M and Achievement-Cognitive Ability Endorsement Test (ACCENT) ranges from r=.45 to .68. Coefficient *alpha* on this sample was .86.

Maze test in Basic Academic Skills Assessment: Reading (BASA: R) (Kim, 2009) was utilized for screening students with *reading skills* problems. The maze test was developed to measure students' reading comprehension, which consisted of 23 items in 6 paragraphs. Total number of syllables is 812, and each item offered three choices from which students should take one pick one by looking at the context of the passage. Students have 3 minutes for completion. As reported by the test developer, the factor loadings of the maze test in BASA: R range from .33 to .76 and its reliability (Cronbach's α) is .89. Coefficient alpha on this sample was .88.

3. Procedure

The entire procedure of this study is illustrated in Figure Ⅲ-1. Development of the word problem solving test and the organization of cognitive testing battery proceeded between December, 2010 and April, 2011. The test manual was developed in April, 2011. The pilot test for manual revision was administered to 27 third-grade students in May, 2011.

December, 2010 - April, 2011	Development of word problem solving and organization of cognitive measurement
	↓
April, 2011	Development a test manual
	↓
May, 2011	Pilot test
	↓
July, 2011	Examiner training
	↓
July - November, 2011	Data collection
	↓
July - November, 2011	Scorer training and data coding
	↓
December, 2011 -	Result analysis

Figure Ⅲ-1. Procedure of this study

With completed test sets and the manual, the 11 examiners were trained in early July, 2011. All examiners were graduate students with backgrounds in special education and school counseling. The training session proceeded in two stages. In the first stage, implementation guidelines for testing were explained in detail. In the second stage each examiner gave a demonstration following the standardized directions. The trained examiners demonstrated 100% accuracy during mock administrations. The session lasted for one hour and a half.

Table Ⅲ-3. Timing of administration

Construct	Measure	Administered			
		Jul.	Sep.	Oct.	Nov.
Cognitive ability					
Language	K-WISC-IV Vocabulary	X			
Nonverbal reasoning	K-WISC-IV Matrix Reasoning	X			
Working memory	K-WISC-IV Digit Span	X			
Processing speed	K-WISC-IV Symbol Search	X			
Attention	Frankfurt Attention Inventory (FAIR)	X			
Mathematics achievement					
Word problems	Word problem solving tests (BASA: MWP)		X	X	X
Computation skills	BASA: M	X			
Reading skills	Maze test in BASA: R	X			

Data on the five cognitive dimensions were collected from third-grade students in July, 2011. Word problem solving tests were implemented three times –in September, in October, and in November, 2011 (See TableⅢ-3 for timing of test administration). Tests were administered using morning study periods and mathematics lessons times.

Collected data was scored and coded by 11 examiners. To check the inter-rater agreement, 20% of all subjects, distributed equally across examiners, were selected randomly. Inter-rater agreement was 96.8%. The formula to calculate the inter-rater agreement (Barlow & Herson, 1984) was as follows:

$$\text{Inter-rater agreement} = \frac{\text{number of agreements}}{\text{number of agreements} + \text{number of disagreements}}$$

4. Data Analysis

The data were analyzed in different ways according to the research questions. For research questions 1 and 2, the growth mixture modeling (GMM) approach was used. First of all, the purpose was to explore if there were distinct growth patterns of mathematical word problem solving ability among students by latent class growth analysis (LCGA), which is the longitudinal mixture model that does not contain any predictors. Then, some cognitive predictors of attention, working memory, processing speed, language, and nonverbal reasoning were added to the model to test the effects of these variables on determining the likelihood that each student follows a specific growth trajectory. Lastly, multivariate analysis of variance (MANOVA) was applied to examine the differences among subtypes of students with mathematics difficulty. The PASW Statistics 18.0 and Mplus 5.1 software were utilized for analysis.

1) Research Question 1: identifiable subgroups based on growth patterns

Research question 1 arose from the assumption that each student is an individual with unique characteristics from a heterogeneous population, and that there might exist latent classes to represent unobserved heterogeneity as a specific set of parameter values. Here, research question 1 was employed to analyze the third-grade students' growth trajectories of word problem solving ability, and to capture the differences in the growth trajectories over time. In this study, latent class growth analysis was employed to explore the growth patterns based on estimation of differences among students and to assign latent class membership to them. Each latent class represents a different growth model; that is, each group has its own intercept and shape.

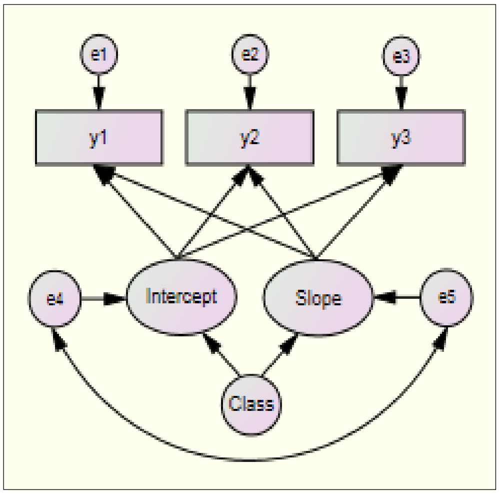


Figure Ⅲ-2. Latent class growth model

Latent class growth analysis is the simplest longitudinal mixture model, which refers to modeling with categorical latent variables that represent subgroups where group membership is not known *a priori*, but is inferred from the data (Muthén & Muthén, 2010). This analysis utilizes a semi-parametric technique which is able to capture unobserved heterogeneity and identify distinct subgroups that follow different patterns which change over time. Unlike conventional latent growth modeling, latent class growth analysis fixes the slope and the intercept to equality across all individuals within a certain trajectory, and individual differences can be captured by the multiple trajectories.

The latent class growth analysis model (Figure Ⅲ-2) is defined as,

$$y_{ti} = \eta_{k0i} + \eta_{k1i} a_t + \varepsilon_{ti} \tag{1}$$

$$\eta_{k0i} = \alpha_{k0} \tag{2}$$

$$\eta_{k1i} = \alpha_{k1} \tag{3}$$

where k represents unobserved latent subgroups that differ only in their growth trajectory means, y_{ti} (t=1, 2, ..., T; i=1, 2, ..., n) are repeated measures on the outcome, a relates the t observed repeated measures from i individuals to latent intercept (η_{k0i}) and slope (η_{k1i}) variables, and ε_{ti} are constrained to be homogeneous across the k-specific latent growth trajectories. In equations (2) and (3), α_{k0} and α_{k1} represent fixed effect mean intercept and slope estimate, respectively. Latent class growth analysis is essentially a fixed effects analysis model; no random effects on intercept or slope are estimated, as shown in equations (2) and (3). Moreover, all off-diagonal model-reproduced covariances are constrained to zero within each of the *k* growth trajectories. Thus, in latent class growth analysis, the mixture corresponds to different latent trajectory classes, and variation across individuals is not allowed within classes (Nagin, 1999; Roeder, Lynch, & Nagin, 1999; Kreuter & Muthén, 2008). Latent class growth analysis utilizes a person-centered approach to define trajectories, and allows for grouping of individuals into categories for which individuals in a given category are similar to one another yet different from individuals in other categories (Muthén & Muthén, 2000).

In order to determine the number of trajectory classes that best describe the data, a series of latent class growth models were specified in which each successive model added one additional class. Determining the number of classes depends on a combination of factors in addition to fit indices, including research question, parsimony, theoretical justification, and interpretability (Jung & Wickrama, 2008). Nylund et al. (2007) suggest that one could use the Bayesian information criterion (BIC) and the Lo-Mendell-Rubin (LMR) adjusted LRT *p*-values as guides to get close to possible solutions and then once a few plausible models have been identified, reanalyze these models requesting the bootstrapped likelihood ratio test (BLRT). Following their recommendation, the model with the low BIC, adjusted BIC values and a significant LMR *P*-value (<.05) comparing the *k* and the $k - 1$ class model was initially adopted. According to criteria suggested by Muthén & Muthén (2000), posterior probabilities were examined including Akaike information criterion (AIC), loglikelihood, entropy, and BLRT. Entropy is also used as an indicator of how well the model classifies people, with values closer to 1 or exactly 1 indicating better classification. There are no set cut-off criteria for deciding whether the entropy is reasonably high, therefore entropy should always be examined in conjunction with other model fit indices.

2) Research Question 2: relationship between growth patterns and cognitive abilities

In order to investigate the differences in cognitive correlation with word problem solving ability among subgroups by growth patterns, the growth mixture modeling (GMM; Muthén, 2004; Muthén & Asparouhov, 2008; Muthén et al., 2002; Muthén & Shedden, 1999) approach was employed, which is another type of longitudinal mixture model. The growth mixture model used in this study is presented in Figure Ⅲ-3.

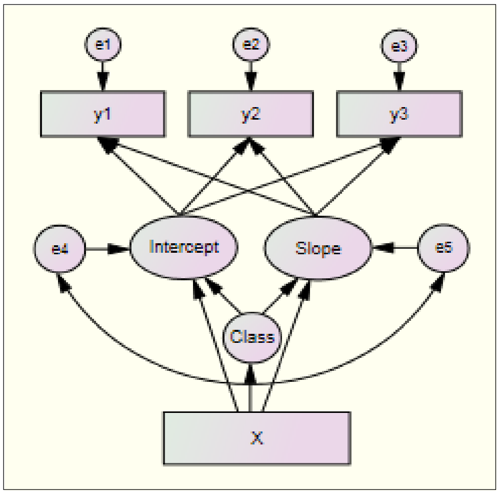


Figure Ⅲ-3. Growth mixture model

Assume for individual *i* in class *k* (*k*=1, 2, ..., *K*),

$$y_{ti} = \eta_{k0i} + \eta_{k1i} a_t + \varepsilon_{kti} \tag{4}$$

$$\eta_{k0i} = \alpha_{k0} + \gamma_{k0} x_i + \zeta_{k0i} \tag{5}$$

$$\eta_{k1i} = \alpha_{k1} + \gamma_{k1} x_i + \zeta_{k1i} \tag{6}$$

where y_{ti} (t=1, 2, ..., T; i=1, 2, ..., n) are repeated measures on the outcome, which indicates students' achievement in mathematical word problem solving, influenced by the random effects, η_{k0i}, η_{k1i}, a relates the t observed repeated measures from i individuals to latent intercept (η_{k0i}) and slope (η_{k1i}) variables, and the residuals ε_{ti} have a T X T covariance matrix Θ_k, possibly varying across the trajectory classes

(k=1, 2, ..., K). In equations (5) and (6), x_i represents time-invariant covariates, which are attention, working memory, processing speed, language, and nonverbal reasoning in this study. α_{k0} and α_{k1} represent fixed effect mean intercept and slope estimate respectively, γ_{k0} and γ_{k1} describe the effects of cognitive abilities as a change in average growth rate that can be different for different classes.

Let c denote a latent categorical variable with K classes, $c_i = (c_{i1}, c_{i2}, \dots, c_{ik})'$, where $c_{ik} = 1$ if individual i belongs to class k and zero if said individual does not. The covariate x influences c and has direct effects on the growth factors γ_{k0} and γ_{k1}. Consider the prediction of the latent class variable by covariate x using a multinomial logistic regression model for K classes,

$$\mathrm{P}\,(c_i = k \mid x_i) = \frac{e^{\gamma_{k0}+\gamma_{k1}x_i}}{\sum_{s=1}^{K} e^{\gamma_{s0}+\gamma_{s1}x_i}} \tag{7}$$

with the standardization $\gamma_{k0}=0$, $\gamma_{k1}=0$.

Unlike the latent class growth model, however, growth mixture model is a random effects analysis model. Within-class variation of individuals is allowed for the latent trajectory classes as being captured by random effects which represent continuous variation across individuals in growth parameters (Muthén, 2001a). The k-specific off-diagonal model-reproduced covariance matrix elements are not constrained to zero as in LCGA; all model-reproduced covariance matrix elements within each k subgroup are freely estimated. Variation about the k-specific growth trajectory means (i.e., VAR[ζ_{k0i}, ζ_{k1i}]) is not considered random error, but is considered to be naturally occurring variation within subgroups, and is estimated. However, because population heterogeneity is typically unobserved, researchers are faced with the question of how many k subgroup growth trajectories (i.e., k=1, 2, ..., K) need to be identified before sample data heterogeneity has been modeled accurately.

3) Research Question 3: cognitive characteristics of students with difficulty in word problem solving

In order to investigate the differences in cognitive abilities among all subtypes classified by students' academic skills, multivariate analysis of variance (MANOVA) was used. Multivariate analysis of variance (MANOVA) is similar to analysis of variance (ANOVA), but with multiple dependent outcome variables. MANOVA is a statistical test procedure for comparing multivariate (population) means of several groups. Unlike ANOVA, it uses the variance-covariance between variables in testing the statistical significance of the mean differences. It helps to answer the following questions: (a) do changes in the independent variable(s) have significant effects on the dependent variables, and (b) what are the interactions among the dependent variables and among the independent variables (Stevens, 2002). For an analysis using MANOVA, one has to check multivariate normality and homogeneity of the variance-covariance matrix.

IV. RESULTS

The results of the analyses conducted to evaluate each of the research questions are presented in this chapter. Above all, descriptive statistics of the sample were reported in order to illuminate the overall understanding of the sample with respect to the variables of interest. Next, the findings from the analyses were denoted according to each of the research questions.

1. Descriptive Statistics

Means, standard deviations, and correlations for the sample of 441 students on word problem solving and five cognitive dimensions (language, nonverbal reasoning, verbal working memory, processing speed, and attention) are reported in Table Ⅳ-1. Word problem solving skills of third graders were measured three times: in September, in October, and in November, of 2011. The raw scores of each of the five cognitive variables were transformed to z-score (M=.00, SD=1.00) for a following profile analysis.

Table Ⅳ-1. Means, Standard Deviations on the sample (n=441)

Variables	N	Raw scores		*Correlations*							
		M	SD	1	2	3	4	5	6	7	8
1. WPS1 [a] (September)	437	72.76	29.810	-							
2. WPS2 (October)	422	87.08	33.778	.815**	-						
3. WPS3 (November)	425	104.45	32.255	.742**	.824**	-					
4. Attention [b]	441	224.42	64.036	.317**	.243**	.255**	-				
5. Working memory [c]	441	18.53	4.484	.293**	.269**	.253**	.213**	-			
6. Processing speed [d]	441	24.62	6.419	.313**	.254**	.278**	.413**	.157**	-		
7. Language [e]	441	23.22	8.102	.392**	.320**	.343**	.313**	.418**	.263**	-	
8. Nonverbal reasoning [f]	441	23.76	3.062	.124**	.138**	.150**	.250**	.283**	.221**	.275**	-

** p<.01

Note. WPS=Word problem solving.

[a] Test of Word Problem Solving. [b] Score for total performance capacity on Frankfurt Attention Inventory (FAIR-P). [c] Wechsler Intelligence Scale for Children-Forth Edition (WISC- IV) Digit Span. [d] WISC- IV Symbol Search. [e] WISC- IV Vocabulary. [f] WISC- IV Matrix Reasoning.

Table Ⅳ-1 indicates that subjects' scores improved from September to November. All cognitive variables are correlated significantly ($p<.01$, 2-tailed) with the scores on the test of word problem solving.

2. Preliminary analyses: Normality test and analysis of overall growth pattern of word problem solving skills

Normality test

As mentioned in Chapter 3, outliers were already handled at the stage of data cleaning. To assess the normality of data used in analysis, skewness and kurtosis were examined. In general, if skewness and kurtosis are not between -2 and +2, the data is too far away from a normal distribution (Hair, Anderson, Tatham, & Black, 1995). No variables had an absolute value of more than 2 in kurtosis as well as in skewness as a result of descriptive statistics. Moreover, Mahalanobis distance was used to identify multivariate outliers which impact normality, with alpha set at .001. No violations of multivariate normality were found..

Analysis of overall growth pattern of word problem solving skills

Prior to the main analyses, longitudinal growth curve modeling was used to identify the overall growth pattern of all subjects in this study. Figure Ⅳ-1 was the unconditional model, which does not include additional variables to predict outcomes to explore the developmental pattern over time. Estimates of parameters and the model fit indices for word problem solving are presented in Table Ⅳ-2 and Table Ⅳ-3.

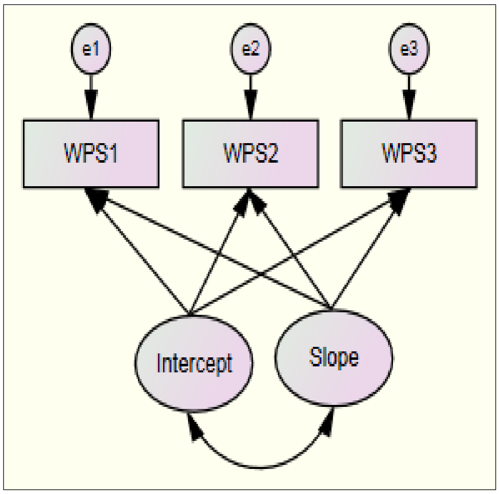


Figure Ⅳ-1. Unconditional latent growth model

In word problem solving, the initial mean score of all participants in September, 2011 was 72.8, and the mean growth rate (slope) across three time points was 15.778. This signifies that the scores of participants increased by 15.778 points on average per month. Importantly, the variances of the intercept and the slope were statistically significant (<.001); that is, there were individual differences in the intercept and the slope. Therefore, the growth pattern shown by the intercept and slope should not be interpreted as being homogeneous. The model fit indices presented in Table Ⅳ-4 met the criteria (CFI and TLI >.90, RMSEA<.05), thus the results from this latent growth model could be interpreted reliably.

Table Ⅳ-2. Estimate of parameters for word problem solving (all subjects)

	Estimate	S.E.	Est./S.E.	*p*-value
<u>Means</u>				
Intercept	72.800	1.419	51.319	0.000
Slope	15.778	0.544	29.000	0.000
<u>Covariances</u>				
Slope<->Intercept	-87.956	16.727	-5.258	0.000
<u>Variances</u>				
Intercept	884.300	59.664	14.821	0.000
Slope	125.647	8.650	14.526	0.000

Table Ⅳ-3. Model fit indices for latent growth model

CFI	TLI	RMSEA
0.997	0.997	0.049

Overall growth trajectory through the estimated means in this model was depicted in Figure Ⅳ-2.

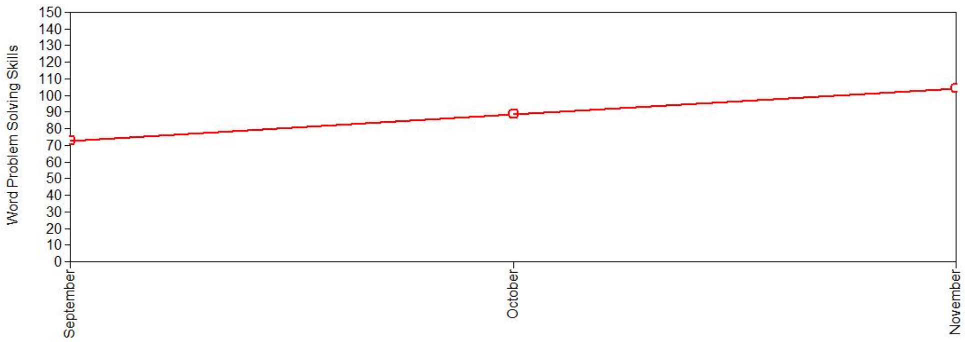


Figure Ⅳ-2. Growth trajectory for all participant

3. Research question 1: Identifiable subgroups based on growth patterns

Although response-to-intervention model is significant, there is still some debate with respect to its implementation, and whether an adequate definition of the degree of adequate response has been agreed upon (Yeo & Hong, 2011). The most common method in use for identifying learning disabilities is a dual discrepancy approach which is the method used for regarding students as non-responders to certain intervention when both their initial performance level and progress measured by curriculum-based measurement are significantly lower than those of other students (Fuchs, 2003). There are several analytic methods to identify students at-risk and students with learning disabilities, and to classify them into subgroups under response-to-intervention measures. The most notable statistical method to analyze the progress monitoring data is latent class growth analysis (LCGA). Even though the latent growth modeling (LGM) is definitely useful as an evaluation tool for examining change over time, it “cannot capture heterogeneity that corresponds to qualitatively different development” (Muthén, 2001b, p. 296). Therefore, in order to explore the heterogeneity in growth trajectories of students’ data repeatedly measured, latent class growth analysis was conducted. In this exercise, an unconditional model without any predictors was utilized for the entire sample. The model for analysis is presented in Figure Ⅳ-3.

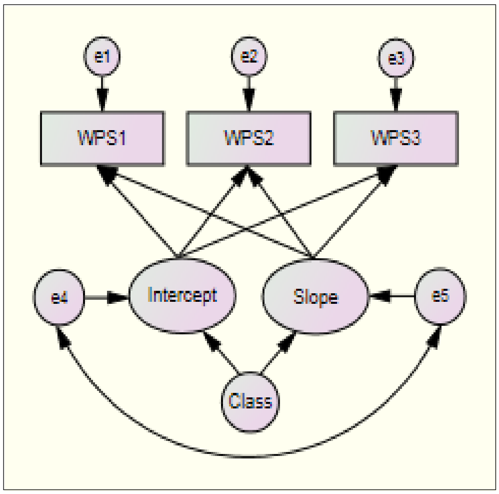


Figure Ⅳ-3. Model for latent class growth analysis

Determining the number of classes

As a first step, in order to determine the numbers of trajectory classes that optimally describe the data set, a series of models were specified in which each successive model added one additional class. The model testing began with a single class model and continued until the addition of a new class accorded with fit indices, including parsimony, theoretical justification, and interpretability. As a result, up to five classes were evaluated and a four- class model emerged as the optimal fit for the data, as evidenced by the BIC and adjusted BIC values and by the LMR-LRT *p*-value and BLRT *p*-value. The details of some indicators are presented in Table Ⅳ-4.

Table Ⅳ-4. Model fit indices for one- to five-class solutions of LCGA

Number of Classes	Log-likelihood	AIC	BIC (aBIC)	Entropy	aLMR-LRT *p*-value	BLRT *p*-value
1	-5788.260	11592.521	11625.233 (11599.845)	-	-	-
2	-5748.245	11518.490	11563.470 (11528.561)	0.851	0.0000	0.0000
3	-5725.403	11478.805	11536.052 (11491.622)	0.689	0.0006	0.0000
4	**-5698.158**	**11430.316**	**11499.830 (11445.880)**	**0.787**	**0.0087**	**0.0000**
5	-5690.458	11420.917	11502.698 (11439.227)	0.763	0.5052	0.0128

Note. Bold indicates best fit. AIC=Akaike information criterion, BIC=Bayesian information criterion, aBIC=adjusted Bayesian information criterion, aLMR-LRT=Lo-Mendell-Rubin adjusted LRT test, BLRT=Bootstrapped likelihood ratio test.

The model fitting resulted in a log-likelihood value of -5788.26 and a BIC (an adjusted BIC) of 11625.233 (11599.845) for the one-class model, a log-likelihood value of -5748.245, a BIC (an adjusted BIC) of 11563.47 (11528.561), an adjusted LMR-LRT *p*-value of less than . 001, and a BLRT *p*-value of less than . 001 for the two-class model, and a log-likelihood value of -5725.403, a BIC (an adjusted BIC) of 11536.052 (11491.622), an adjusted LMR-LRT *p*-value of less than .001, and a BLRT *p*-value of less than .001, likewise, for the three-class model. The four-class model (a log-likelihood of -5698.158) yielded a BIC of 11499.830, which was the lowest value (see Figure Ⅳ-4), and an adjusted BIC values of 11445.88. Additionally, its adjusted LMR-LRT *p*-value of less than .01 suggest that it means the model has a better fit than the situation with one less class. Its BLRT *p*-value also confirmed that the four-class model was fitted more than the three-class model ($p<.001$). In five-class model, the model fitting indicated a log-likelihood of -5690.458, a BIC (an adjusted BIC) of 11502.698 (11439.227), an adjusted LMR-LRT *p*-value of more than .05, and a BLRT *p*-value of less than .05. Although the value of adjusted BIC for five-class model is the lowest, the drop in adjusted BIC from model 4 to model 5 is not as large as the drop from model 3 to model 4 (see Figure Ⅳ-4). Furthermore, its BLRT *p*-value of less than .05 but its adjusted LMR-LRT *p*-value did not support that the five-class model was a better solution than its *k*-1 model, the four-class model. Therefore, four-class model is the best model and fits the data more adequately.

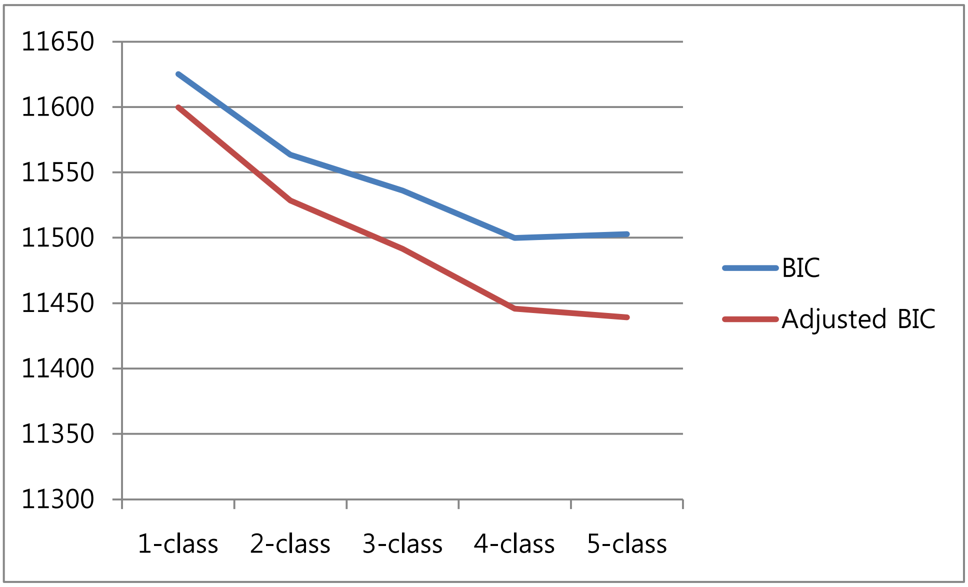


Figure Ⅳ-4. BIC and adjusted BIC estimates for model 1 to model 5

The number of students classified by the four-class solution and the posterior probabilities which indicate the likelihood that the individual belongs to a given class are reported in Table Ⅳ-5 and Table Ⅳ-6 respectively. All the proportions for the latent classes are above .01 (1%). The largest proportion (42.2%) of the total sample was assigned in class 3, followed by class 2 (28.6%) and then class 4 (15.0%) and finally class 1 (14.3%). The closer the diagonal values of the estimated posterior probabilities are to 1, the cleaner the classification. The range of posterior probabilities (diagonal values) for the four-class model was between .846 and .907, which was a reasonably good average.

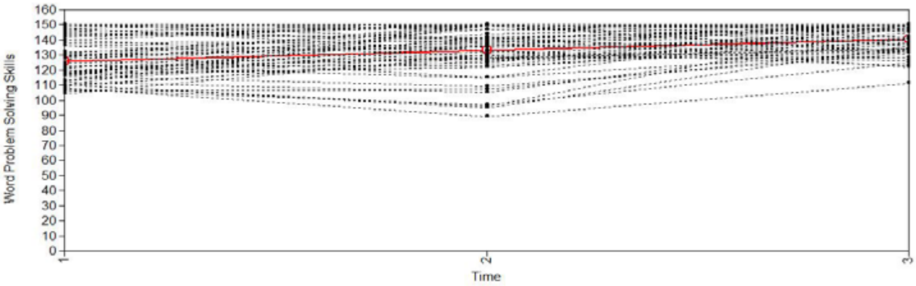
Results above support the four-class model. The estimated means and observed individual growth trajectories for each latent class are presented in Figure Ⅳ-5 allows the adequacy of classes to be visually observed.

Table Ⅳ-5. Class counts and proportions in the four-class model

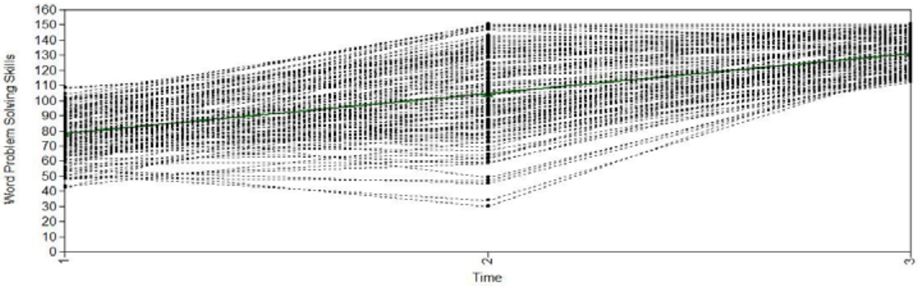
Latent Classes	Class 1	Class 2	Class 3	Class 4
Class Counts	63	126	186	66
Proportion of Total Sample	14.3%	28.6%	42.2%	15.0%

Table Ⅳ-6. Posterior probabilities for the unconditional four-class LCGA

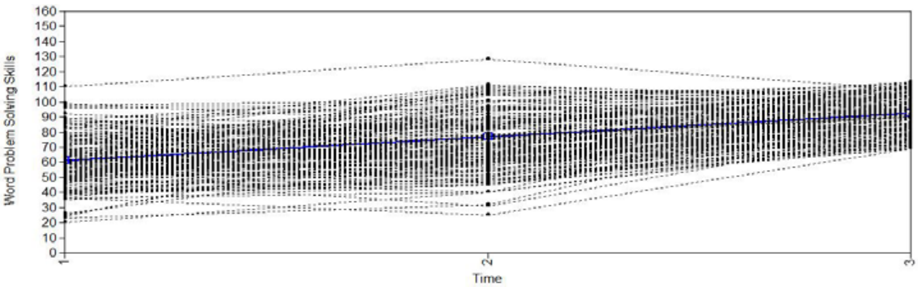
Latent Classes	Class 1	Class 2	Class 3	Class 4
Class 1	**0.907**	0.088	0.005	0.000
Class 2	0.063	**0.846**	0.090	0.000
Class 3	0.005	0.072	**0.874**	0.049
Class 4	0.000	0.001	0.101	**0.899**



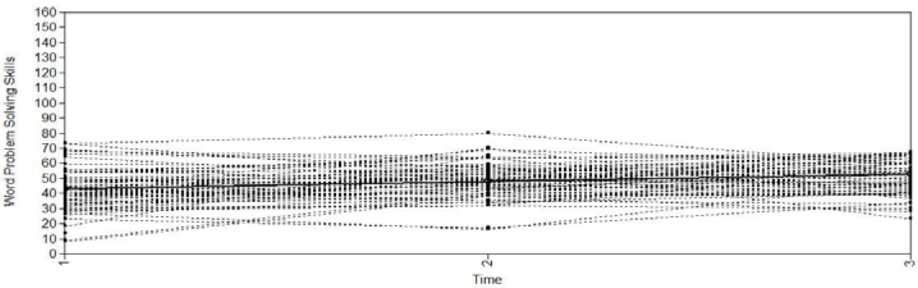
Class 1



Class 2



Class 3



Class 4

Figure Ⅳ-5. Estimated means and observed individual growth trajectories for each latent class

Characteristics of subtypes classified by LCGA

The estimated trajectories of each of latent subtypes identified by the four-class model are depicted in Figure Ⅳ-6 and the standardized estimates of each class are reported in Table Ⅳ-7.

A naming convention was adopted for each of subtypes depending on the characteristics of growth pattern. Class 1 was labeled as ‘high achieving students (HAS).’ Class 2 and Class 3 were given the labels of ‘average and fast growing students (AFG)’ and ‘low but steadily growing students (LSG)’ respectively. Class 4, the focus group in this study, was named after ‘students with dual discrepancy (SDD).’

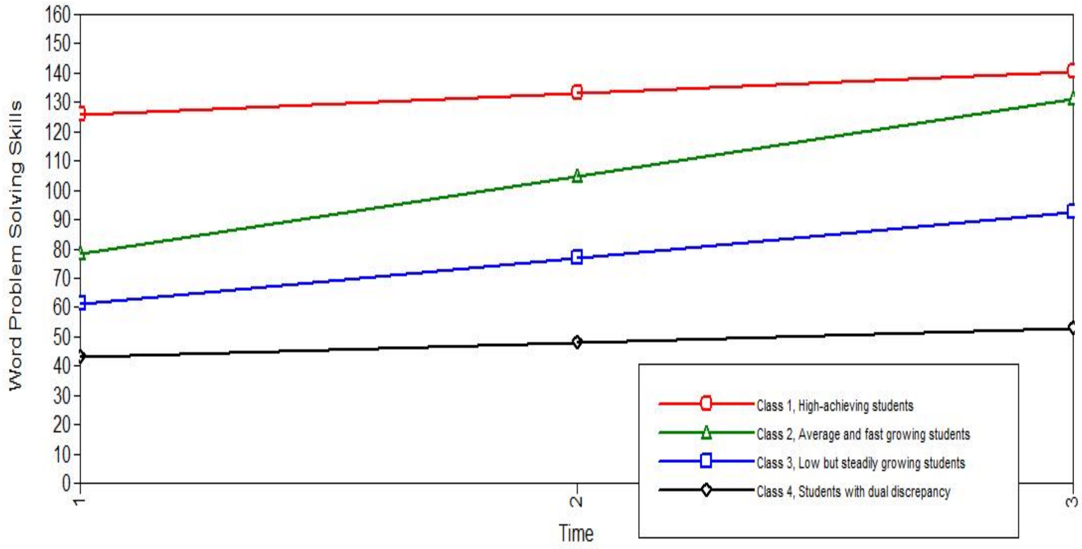


Figure Ⅳ-6. Estimated means trajectories for the unconditional four-class LCGA

Table Ⅳ-7. Mean estimates (standardized estimates) of the four classes

Latent Classes	Parameter	Estimate	S.E.	Est./S.E.	*p*-value
Class 1	Intercept	125.775 (7.358)	3.615 (0.538)	34.794 (13.667)	0.000 (0.000)
	Slope	7.341 (0.798)	1.342 (0.214)	5.469 (3.731)	0.000 (0.000)
Class 2	Intercept	78.149 (4.572)	2.572 (0.335)	30.382 (13.654)	0.000 (0.000)
	Slope	26.525 (2.882)	1.805 (0.583)	14.696 (4.946)	0.000 (0.000)
Class 3	Intercept	61.226 (3.582)	1.927 (0.282)	31.769 (12.716)	0.000 (0.000)
	Slope	15.666 (1.702)	0.906 (0.315)	17.300 (5.404)	0.000 (0.000)
Class 4	Intercept	42.982 (2.515)	2.954 (0.266)	14.552 (9.455)	0.000 (0.000)
	Slope	4.826 (0.524)	1.053 (0.145)	4.583 (3.626)	0.000 (0.000)

Note. The numbers in parentheses refer to the standardized estimates.

As shown in Table Ⅳ-7, on the linear growth of each group, Class 1 is, a group of high achieving students reporting a remarkably high intercept of 125.775 ($p<.001$) compared to the other groups. The average slope was 7.341, which was also statistically significant, as it depicts a slightly low rate of progress. It can be interpreted that students in the high achieving group are almost reaching the required level in the third grade curriculum of mathematics.

The second group, which indicates the group of average and fast growing students, shows the intercept of 78.149 and the slope of

26.525. Both values were statistically significant at the p –value level of .001. The slope was the highest value among the four groups. It means students in this group make an improvement of 26.525 points on average every month.

The linear growth of the group of low but steadily growing students has a low initial value (61.226, $p<.001$) but shows progress slightly below the average (15.666, $p<.001$). The students in this group respond to instruction in the general education settings (i.e. tier-1 settings) but its pace is slower than the second group. These students may not achieve the curriculum goals by the end of the school year. They have already shown a lower initial score than the average and when this is coupled with a relatively slow rate of progress, there is reason for concern about their academic achievement and progress. Therefore, intervention and remediation processes could be initiated to ensure that they attain some acceptable standard of academic competence.

The focus group of students with dual discrepancy is characterized as having the lowest intercept starting point and also the most gradual progress. Their initial score was only 42.982 ($p<.001$) and relatively little progress of 4.826 ($p<.001$) was made reflecting a near stagnancy in academic growth. Without intervention it is to be expected that the students of class 4 will continue to fail in mathematics and never reach the goal of the standard curriculum. Teachers may already have some trouble in teaching them in tier-1 settings and might have considered referring them to after school programs or to specially designed education plans or initiatives.

4. Research question 2: Relationship between growth patterns and cognitive abilities

Ever since the term ‘learning disabilities’ began to be used officially, cognitive deficits were regarded as the primary determinant of the unexpected underachievement of students with learning disabilities. It can be hypothesized, therefore, that students who have experienced difficulties in learning persistently would show distinctive profiles of strengths and weaknesses across cognitive dimensions. Testing of this hypothesis may verify a correlation between learning difficulties and cognitive function deficits. In the case where word problem solving in mathematics is the area where students may experience difficulty, the hypothesis to be tested may be specified accordingly. This study has attempted to show that there is a correlation between these entities.

This section examines the relationship between students' learning progress in word problem solving and their cognitive abilities, using the growth mixture modeling approach. The cognitive variables such as attention, working memory, processing speed, language, and nonverbal reasoning were added to the four-class model drawn from research question 1. The effects of these variables on determining the likelihood that each student follows a specific growth trajectory, that is, the effects of the covariates on the latent classes were explored. These estimates are represented as multinomial logit values because the outcome variables are discrete latent classes. The model used for these analyses is presented in Figure Ⅳ-7.

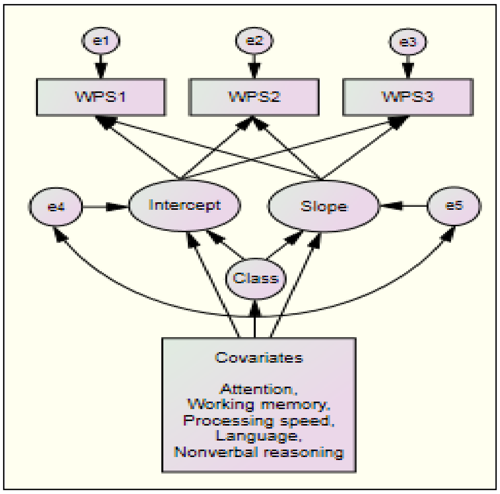


Figure Ⅳ-7. Growth mixture model added cognitive variables as covariates

Descriptive statistics of cognitive variables by subgroups

Based on the growth patterns, the total sample was sorted into four groups of "high achieving students (HAS)," "average and fast growing students (AFG)," "low but steadily growing students (LSG)," and "students with dual discrepancy (SDD)." The raw score and *z*-score means and standard deviations on the five cognitive variables, which are attention, working memory, processing speed, language, and nonverbal reasoning, by each subgroup are presented in Table Ⅳ-8, Table Ⅳ-9.

Table Ⅳ-8. Descriptive statistics of five cognitive factors by subgroups (raw score)

Variables	Subtypes by growth pattern							
	HAS (n=63)		AFG (n=126)		LSG (n=186)		SDD (n=66)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Attention	250.10	57.00	235.47	65.18	219.42	63.11	192.94	56.58
Working memory	20.92	4.32	18.92	4.66	18.00	4.24	16.97	4.02
Processing speed	27.11	5.39	25.84	6.09	24.28	6.41	20.88	6.30
Language	28.02	7.34	24.63	8.34	22.36	7.47	18.33	6.92
Nonverbal reasoning	24.25	2.91	23.93	3.11	23.72	3.05	23.05	3.07

Note. HAS=High achieving students, AFG=Average and fast growing students, LSG=Low but steadily growing students, SDD= Students with dual discrepancy.

Table Ⅳ-9. Descriptive statistics of five cognitive factors by subgroups (z-score)

Variables	Subtypes by growth pattern							
	HAS(n=63)		AFG (n=126)		LSG (n=186)		SDD (n=66)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Attention	.49	.85	.27	.97	.03	.94	-.37	.85
Working memory	.55	.92	.12	1.00	-.08	.91	-.30	.86
Processing speed	.43	.78	.25	.89	.02	.93	-.47	.92
Language	.61	.88	.20	1.00	-.07	.90	-.56	.83
Nonverbal reasoning	.22	.88	.12	.95	.06	.93	-.15	.93

Note. HAS=High achieving students, AFG=Average and fast growing students, LSG=Low but steadily growing students, SDD=Students with dual discrepancy.

The representative profiles of each group based on z-score means are tabulated in Figure Ⅳ-8. As shown in Figure Ⅳ-8, the SDD group is at the lowest level throughout all cognitive domains, while the HAS group presents the highest level. The LSG group shows higher level than the SDD group, but their cognitive abilities slightly lower than the AFG. A relatively significant difference among four groups emerged in language ability, while there was little difference in nonverbal reasoning.

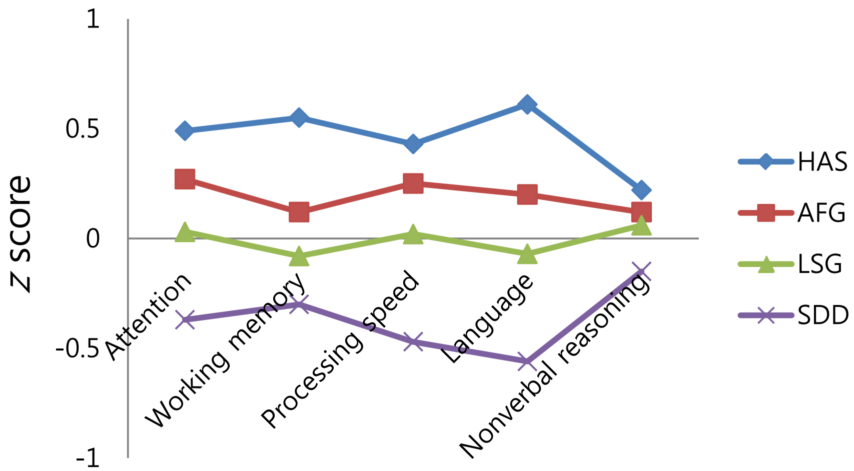


Figure Ⅳ-8. Mean *z*-scores on five cognitive dimensions by subgroups

Note. HAS=High achieving students, AFG=Average and fast growing students, LSG=Low but steadily growing students, SDD= Students with dual discrepancy.

Effects of cognitive abilities on the subgroups

The modeling of the influence of covariates, which are attention, working memory, processing speed, language, and nonverbal reasoning, on class membership was expressed as in multinomial logistic regression, because the class membership as an outcome variable is nominal, representing several independent categorical outcomes. The results from multinomial logistic regression are presented in Table Ⅳ-10 below. The estimated parameters have been converted into odds ratios for interpretation.

Table Ⅳ-10. Multinomial logistic regression: factors influence to latent classes

Variables	Reference group: HAS (n=63, 14.3%)		
	AFG (n=126, 28.6%)	LSG (n=186, 42.2%)	SDD (n=66, 15.0%)
Attention	1.183	0.987	0.955
Working memory	0.596*	0.624**	0.721
Processing speed	0.969	0.643*	0.343***
Language	0.824	0.717	0.453**
Nonverbal reasoning	1.029	1.192	1.245
Variables	Reference group: AFG		
	HAS	LSG	SDD
Attention	0.845	0.834	0.807
Working memory	1.677*	1.047	1.209
Processing speed	1.031	0.663*	0.353***
Language	1.214	0.869	0.550**
Nonverbal reasoning	0.971	1.158	1.209
Variables	Reference group: LSG		
	HAS	AFG	SDD
Attention	1.013	1.198	0.968

Working memory	1.602**	0.955	1.156
Processing speed	1.556*	1.508*	0.533**
Language	1.395	1.150	0.633*
Nonverbal reasoning	0.839	0.863	1.044
Variables	Reference group: SDD		
	HAS	AFG	LSG
Attention	1.047	1.238	1.033
Working memory	1.386	0.827	0.865
Processing speed	2.920***	2.830***	1.877**
Language	2.205**	1.817**	1.580*
Nonverbal reasoning	0.803	0.827	0.958

*P<.05, **P<.01, ***P<.001.

Note. Estimates are reported in odds ratios. HAS=High achieving students, AFG=Average and fast growing students, LSG=Low but steadily growing students, SDD= Students with dual discrepancy.

Overall, processing speed, language, and working memory emerged as significant variables in determining which group students might belong to. When a HAS group is used as a reference, the lower the students' working memory ability is, the greater is the possibility for students to be assorted into an AFG group (odds ratio=0.596). The probability of being in a HAS group increases as a function of increasing students' working memory (odds ratio=0.624) and processing speed (odds ratio=0.643) in contrast with a LSG group. The probability of being in a HAS group increases as a function of increasing students' processing speed (odds ratio=0.343) and language (odds ratio=0.453) in contrast with a SDD group.

When an AFG group is used as a reference, the higher the students' working memory is, the possibility for students to be assorted into an HAS group is approximately 1.7 times greater. The lower the students' processing speed is (odds ratio=0.663), the greater the possibility for students to be classified into a LSG group. The estimates of the multinomial logistic regression show that the probability of being in a SDD group increases as the values of processing speed and language decreases (odds ratio=0.353 and 0.550, respectively).

When a LSG group is used a reference group, the higher the students' working memory (odds ratio=1.602) and processing speed (odds ratio=1.556) are, the greater the possibility for students to be classified into a HAS group. The higher the students' processing speed is, the greater the possibility for students to be assorted into an AFG group by approximately 1.5 times. Having lower processing speed (odds ratio=0.533) and language (odds ratio=0.633) increased the estimated odds of being in a SDD group.

When a SDD group is used as a reference, the higher the processing speed (odds ratio=2.920) and language (odds ratio=2.205) are, the higher the probability of a student being in a HAS group by the approximately two to three times. Similarly, the probability of being in an AFG group increases by approximately three times when students have higher processing speed (odds ratio=2.830), and the probability of being in an AFG group increases when students have higher language ability (odds ratio=1.817). Also, when the values of processing speed (odds ratio=1.877) and language (odds ratio=1.562) increase, the probability that students belong to a LSG group increases. Of significant note, attention and nonverbal reasoning are not related to group contrasts.

5. Research question 3: Cognitive characteristics of students with difficulties in word problem solving

In an attempt to understand the heterogeneity of learning disabilities, research on mathematical disabilities has attempted to classify students with mathematics learning disabilities into specific subtypes based on their cognitive abilities and academic skills (e.g. Badian, 1983; Geary, 1993; Rourke & Conway, 1997). Many of these studies have focused primarily on the interplay between mathematics and reading in terms of crossover areas of learning disabilities, and this perspective has been continued as comorbidity studies (e.g., Jordan & Hanich, 2000; Robinon, Menchetti, & Torgesen, 2002; Vukovic, Lesaux, & Siegel, 2010). Those studies have focused on the frequent coexistence between mathematics disabilities and reading disabilities, and suggest that the comorbidity of mathematics disabilities and reading disabilities should be viewed as one of the subtypes.

According to these studies, students with difficulties in both mathematics and reading experience more pervasive deficits in problem solving (e.g., Fuchs & Fuchs, 2002; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). Some research argues that there are, in fact, two distinct groups in most cognitive skills: one where the mathematics learning disabilities are concurrent with reading disabilities and one where mathematics learning disabilities occur without reading disabilities (Fuchs & Fuchs, 2002; Jordan, Hanich, & Kaplan, 2003; Swanson, Jerman, & Zheng, 2008).

This type of research, which deals with the issue of reading difficulty, is important for generating hypotheses about the different nature of subtypes among mathematics learning disabilities. In this section, therefore, the different growth patterns and cognitive abilities among subtypes of students with difficulties in word problem solving were explored. Based on the hypothesis that students would have different cognitive characteristics depending on whether they have difficulties in computation, in reading, in both computation and reading, or in neither computation nor reading, multivariate analysis of variance (MANOVA) was used to explore their differences in cognitive abilities.

Difficulty status group formation

To subdivide the group that has difficulties in word problem solving, which was identified by latent class growth analysis above, into subgroups according to a specific impairments in computation, reading, or in both computation and reading, the following criterion was applied.

The focus centered around the SDD and LSG groups, which were selected as subjects for analysis in this part. The reason why the LSG group was included is that not only did they show word problem solving scores below average in all time points, but also that they are not expected to reach the curriculum goal at the end of the school year.

The 40th percentile is used commonly in the research literature as a cut-off point for designating lack of difficulty. Cutoffs for designating difficulty of disability vary more in the literature. In this study, 15th percentile was selected because it is useful for understanding disability as practiced in the school (Fuchs et al., 2008).

Table Ⅳ-11. Difficulty group status formation

	Computation	Reading	Subtypes	*n*
	≥ 40th percentile	≥ 40th percentile	PD	75
	≤ 15th percentile	≥ 40th percentile	CPD	26
	≥ 40th percentile	≤ 15th percentile	RPD	14
	≤ 15th percentile	≤ 15th percentile	CRPD	23
Total				138

Note. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading skills difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

From among 252 students of the SDD and LSG groups, any student who scored above the 40th percentile on the computation test score and above the 40th percentile on the reading skills test score was designated as having only word problem solving difficulty (PD). Any student who scored below the 15th percentile on the computation test score but above the 40th percentile on the reading skills test score was designated as having computational difficulty (CPD). Any student who scored below the 15th percentile on the reading skills test score but above the 40th percentile on the computation test score was designated as having reading difficulty (RPD). Any student who scored below the 15th percentile on the computation test score and below the 15th percentile on the reading skills test score was designated as having computation and reading skills difficulty (CRPD). This placed 114 students in the buffer zone (i.e., scoring between the 16th and 39th percentiles on either or both computation and reading skills test) and resulted in 75 PD students, 26 CPD students, 14 RPD students, and 23 CRPD students (see Table Ⅳ-11).

Table Ⅳ-12. Subgroup by growth pattern × difficulty group status formation

	PD	CPD	RPD	CRPD	Total
LSG	71	16	9	5	101
SDD	4	10	5	18	37
Total	75	26	14	23	138

Note. LSG=Low but steadily growing students, SDD= Students with dual discrepancy, PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading skills difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

Table Ⅳ-12 above shows which subgroups by growth patterns in word problem solving that students from difficulty group status formation belong to. 101 (73.19%) of 138 cases used for analysis are in LSG and 37 (26.81%) are in SDD. 75 (54.35%) of total are PD, 26 (18.84%) are CPD, 14 (10.14%) are RPD, and 23 (16.67%) are CRPD.

Table Ⅳ-13 below presents the means and standard deviations of a computation, a reading skill, and three word problem solving tests by each subtype. The observed means trajectories of word problem solving by difficulty status are plotted in Figure Ⅳ-9 below.

Table Ⅳ-13. Means, Standard Deviations by subtypes

Subtypes	*N*	Computation	Reading	WPS 1	WPS 2	WPS 3
		M (SD)	*M (SD)*	*M (SD)*	*M (SD)*	*M (SD)*
PD	75	46.61 (3.904)	20.47 (1.934)	66.97 (14.722)	79.00 (20.327)	92.99 (13.982)
CPD	26	25.08 (2.897)	20.04 (1.886)	49.20 (14.180)	58.76 (17.929)	72.12 (23.314)
RPD	14	43.93 (3.626)	11.29 (1.684)	52.86 (16.090)	62.14 (15.575)	84.71 (16.429)
CRPD	23	22.13 (5.128)	10.35 (2.014)	37.13 (13.123)	48.81 (13.600)	58.27 (18.586)

Note. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading skills difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

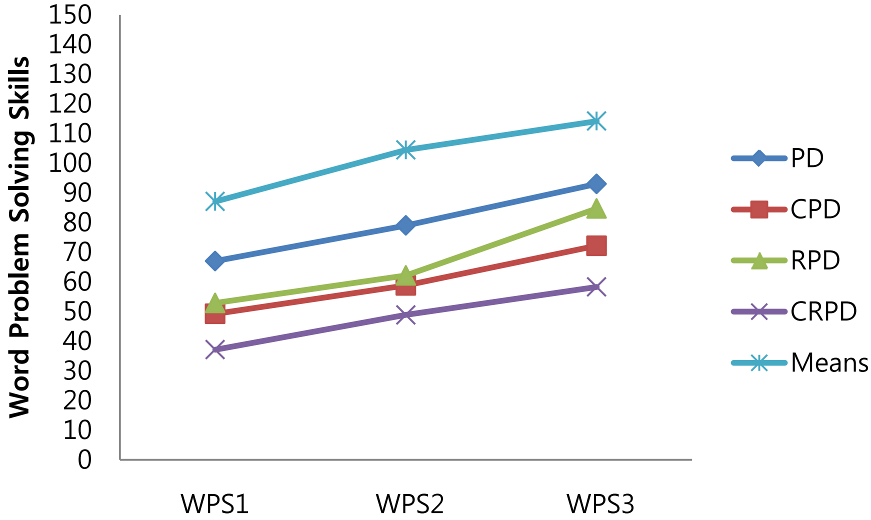


Figure Ⅳ-9. Observed means trajectories by difficulty status

Note. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading skills difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

Group comparisons on cognitive variables

Table Ⅳ-14 presents *z*-score means and standard deviations, by difficulty status, on the computation and reading skills and on attention, working memory, processing speed, language, and nonverbal reasoning.

In order to help interpret the interaction between difficulty status and cognitive dimension, *z* scores on the five cognitive variables for each of the four difficulty status groups were plotted. As shown in Figure Ⅳ-10, the PD and CPD groups mostly tend to perform at a higher level than did the RPD and CRPD groups. PD and CPD showed similar shape profiles, and RPD and CRPD also reported analogous except nonverbal reasoning.

Table Ⅳ-14. Cognitive abilities by difficulty status

Variables	Difficulty status (n=138)							
	PD (n=75)		CPD (n=26)		RPD (n=14)		CRPD (n=23)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Attention	0.19	0.91	0.07	0.91	-0.19	1.15	-0.25	0.76
Working memory	0.14	0.91	0.07	0.73	-0.75	100.83	-0.58	0.87
Processing speed	0.29	0.90	0.01	0.77	-0.57	1.25	-0.68	0.76
Language	0.21	0.80	0.03	0.98	-0.78	0.83	-0.63	0.96
Nonverbal reasoning	0.21	0.93	-0.02	0.82	-0.81	0.76	-0.21	0.74

Note. Performance is expressed as z scores in relation to the representative sample of 441. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

In order to examine whether significant differences existed across four subtypes by difficulty status, MANOVA with Scheffé's *post hoc* comparison tests was utilized. An assumption of the MANOVA is that the covariance matrices of the dependent variables are the same across subtypes (determined by levels of the independent variable) in the population. In this analysis, the *p*-value of Box's M was .779 which suggests that the hypothesis of equal covariance matrices cannot be rejected (i.e. the assumption of MANOVA was not violated).

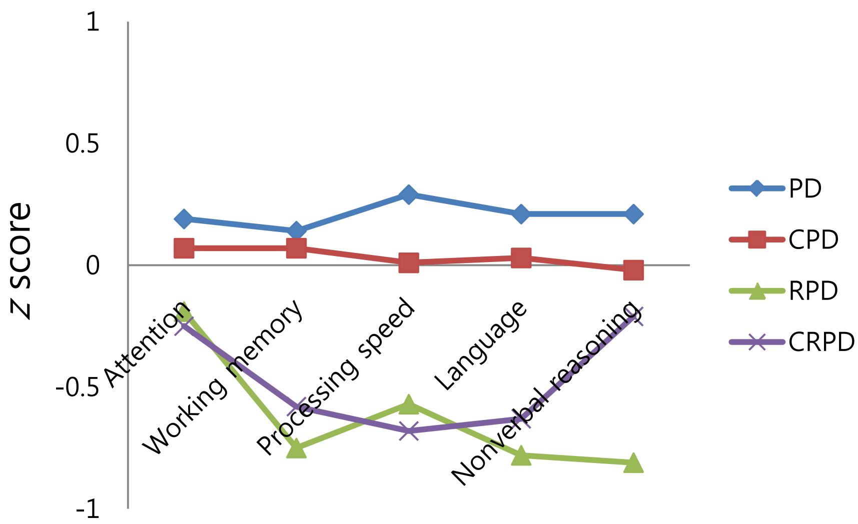


Figure Ⅳ-10. Mean *z*-scores on five cognitive dimensions by difficulty status

Note. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

As a result of MANOVA, Wilks' lambda was .648 (p=.000), which indicates that the means on the composite variables are not the same across subtypes. Before interpreting the univariate F testing for each of the dependent variables, the standard Levene's test was examined for the assumption of equal variances for each of dependent variables as this is an assumption of the ANOVA. For all the five cognitive dimensions, the test produces a nonsignificant *p* value, so the null hypotheses regarding equal variances cannot be rejected for either dependent variable, thus ANOVA is acceptable. Significant univariate F ratios show that there are meaningful group differences in most cognitive abilities (attention, working memory, processing speed, and language) (p<.001), but no significant distinction of attention was found. Following the η^2 value that presents the explanatory power of an independent variable for dependent one, 17% of language factor's total variance was explained by subtypes. Also, the group differences accounted for 12 to 17% of each of the total variances of processing speed, working memory, and nonverbal reasoning.

Table Ⅳ-15. Results of MANOVA on cognitive dimensions by difficulty status

Variables	Wilks' Lambda	F	*df*	Sig.	η^2
Attention	.648 (p=.000)	1.753	3/138	.159	.04
Working memory		7.152	3/138	.000	.14
Processing speed		8.877	3/138	.000	.17
Language		9.100	3/138	.000	.17
Nonverbal reasoning		6.011	3/138	.001	.12

Because the differences of cognitive variables by difficulty status were showed up, Scheffé's follow-up test was conducted. As shown in Table Ⅳ-16, RPD and CRPD report significantly lower working memory, processing speed, and language than PD. RPD is also lower in a nonverbal reasoning than PD. In comparison with CPD, RPD presents lower working memory, language, and nonverbal reasoning and CRPD is significantly lower only in processing speed. At the .1 significance level, which is more lenient, working memory and language are lower than CPD. No significant difference is found between PD and CPD as well as between RPD and CRPD. In addition, no significant difference in attention is revealed in any of contrasts.

Table Ⅳ-16. Scheffé's post hoc comparisons results

Variables	Contrast					
	PD vs.			CPD vs.		RPD vs.
	CPD	RPD	CRPD	RPD	CRPD	CRPD
Attention	.12	.39	.44	.27	.32	.05
Working memory	.07	.89**	.72*	.82*	.65	-.17
Processing speed	.27	.86**	.97***	.58	.70*	.11
Language	.18	.99**	.85*	.81*	.67	-.14
Nonverbal reasoning	.23	1.02***	.42	.79*	.19	-.60

$^{*}P<.05$, $^{**}P<.01$, $^{***}P<.001$.

Note. PD=Word problem solving difficulty, CPD=Computational difficulty with word problem solving difficulty, RPD= Reading difficulty with word problem solving difficulty, CRPD= Computational and reading difficulty with word problem solving difficulty.

V. DISCUSSION

Two main issues are dealt with in this study. One issue deals with the evaluation of the empirically used response-to-intervention model as a diagnostic tool for identifying learning disabilities, and the other issue deals with the examination of the effects of and relationship between cognitive abilities and mathematical word problem solving. In order to explore these issues, the study was aimed at using the response-to-intervention model, which is being actively discussed in academia, to identify and verify the existence of the group with dual discrepancy i.e. mathematics disability and reading disability, and their size relative to the student population. Cognitive correlates of inadequate response to the word problem solving were then examined, after which cognitive differences were investigated among the skill-based subtypes of students with mathematical difficulties.

For purposes of this study, the following research questions were established: (1) Are there any identifiable groups in students' growth patterns (intercept and slope) of mathematical word problem solving ability? (2) Given that there are multiple growth patterns, what are the effects of students' cognitive abilities on their growth patterns of mathematical word problem solving ability? And (3) how do the cognitive abilities differ among students with mathematical difficulties identified by their growth patterns, depending on whether accompanying problem in computation, in reading, in both computation and reading, or in neither computation nor reading?

A significant aspect of this study is its attempt to construct the foundation upon which to apply a response-to-intervention (RTI) model for the purpose of identifying learning disabilities. Although the RTI model has been recognized as one of the key theoretical frameworks for evaluating and classifying learning disabilities, the validity of the model has not yet been empirically evaluated for the Korean context. Also, this study could provide the fundamental research for an intrinsic processing deficits approach to evaluate learning disabilities by identifying the cognitive correlates to word problem solving in mathematics. There is an insufficient knowledge base about students' mathematical word problem solving ability and its cognitive correlates in terms of learning disabilities. The importance of examining students' cognitive abilities related to the mathematical competence is not only to verify the core nature of learning disabilities, but also to provide the appropriate and relevant instructional applications through grasping the fundamental cause of learning problems in mathematical word problem solving.

1. General Discussion

1) Research question 1: Identifiable subgroups based on growth patterns

Research question 1 was designed to examine identifiable subgroups based on growth patterns of word problem solving. In order to explore the heterogeneity in growth trajectories of students' repeatedly measured data, latent class growth analysis was conducted. As a result of latent class growth analysis, four distinct classes emerged based on characteristics of growth patterns (initial level and growth rate): Class 1 (14.3%) characterized as high intercept and slow progress, Class 2 (28.6%) characterized as average intercept and fast progress, Class 3 (42.2%) characterized as low intercept and slow progress, and Class 4 (15.0%) characterized as lowest intercept and little progress. Four categories classified by exploratory methods were labeled as high-achieving students (HAS), average and fast growing students (AFG), low but steadily growing students (LSG), and students with dual discrepancy (SDD) respectively.

This result confirmed that a response-to-intervention model demonstrates functionality in identifying at-risk students for learning disabilities based on progress monitoring data. If the response-to-intervention model is able to diagnose learning disabilities, then researchers may be able to measure students' responsiveness in the tier-1 setting and classify students into subgroups by their responsiveness in terms of mathematical problem solving ability. The result is in congruence with preceding research (e.g., Geary et al., 2009; Jordan, Kaplan, Oláh, & Locuniak, 2006; Murphy, Mazzocco, Hanich, & Early, 2007) that has found the existence of a number of classes of students with different initial levels of mathematical achievement. While Murphy et al.'s (2007) study was unable to reveal the difference in growth between low achieving students, who are similar to students in the LSG in this study, and typically achieving students, who are similar to students in the AFG in this study, students with different initial levels of mathematical competence demonstrated different growth rates. Jordan et al. (2006) identified groups with different growth patterns, but focused on mathematical cognition tasks, not achievement measures. Geary et al. (2009) also identified different groups by growth patterns focusing on first graders' arithmetic ability, and the numbers and proportions in each tier in the stratification closely match the results of this study.

Since the RTI model has been suggested as an alternative method of LD identification, there have been few studies to evaluate the discriminant validity of the RTI model in practice; it is worthwhile exercise, therefore, to verify the value of the RTI model empirically. For South Korea, Yeo and Hong (2011), Hong and Yeo (2011), and Kim (2012) have carried out investigations into the validity of the RTI as an identification model in terms of reading fluency and computation skills. Hong and Yeo (2011) found out by measuring the computation skills at tier-1 education and the lowest 16th percentile of students who participated in tier-2 intervention that 24.1% of third-grade students could be classified as having mathematics difficulties. Kim (2012) determined that 6.4% of second-graders could be classified as at-risk students through CBM of computation skills. Whereas in Geary et al. (2009), the proportion of students with mathematical learning disabilities (corresponding to SDD in this study) was 6% of the total sample and 50% as low achieving students (corresponding to LSG in this study), in this study, 15% of students were presented as SDD. This 15% consisted of students who were not only behind the other groups in initial ability, but who continued to lag behind the progress rate for the other groups. Furthermore, approximately 42% of students were presented as LSG, which consisted of students who are slowly growing but continuously achieving at a lower level. Although their initial level was at the average level, their growth rate was not sufficiently strong to expect that they could reach the desired level as per curriculum requirements at the end of year. From an educational perspective, although the proportion of students categorized as learning disabled was excessively high if the students in the LSG were included in the at-risk group, categorizing them thus is more beneficial in terms of providing timely intervention Moreover, the LSG students should be provided the opportunity to be referred to tier-2 intervention in order to identify the latent group with learning disabilities within the LSG and avoid the false negative effect.

2) Research question 2: Relationship between growth patterns and cognitive abilities

Research question 2 was designed to examine the effect of cognitive abilities on students' growth patterns of mathematical word problem solving ability. The growth mixture model, to which cognitive variables as predictors were added, analyzed the information gathered from research question 1. Results indicated that processing speed and language factors were involved in differentiating SDD from HAS, AFG and LSG groups. Working memory emerged as a factor of significance in the classification of students into AFG or LSG compared to HAS. Only the processing speed factor emerged in contrast with the classification of students into the AFG as opposed to the LSG.

These results are congruent with many research findings, including research dealing with the specificity hypothesis of learning disabilities (e.g., Compton et al., 2012) thus far with respect to the relationship between cognitive factors and mathematics skills especially as they pertain to problem. What stood out in the results most was that language ability played an important role in achieving word problem solving competence. These findings corroborate those of previous studies, which revealed the possibility that reading skill may explain differences in competence and performance in mathematical word problem solving (e.g. Fuchs et al., 2005, 2006, 2008; Lee, Ng, Ng, & Lim, 2004; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). It is an inevitable result given the obvious need to process linguistic information when building a problem representation of a word problem.

Processing speed emerged as a strong predictor of word problem solving performance in this study. It has been considered a crucial cognitive factor with respect to mathematical achievement in prior research (Lee, 2007). Such research has shown that processing speed is dynamically related to mental capacity (Kail & Salthouse, 1994), performance and development in reading (Kail & Hall, 1994), and reasoning by the conservation of cognitive resources and the efficient use of working memory space used for higher order fluid tasks (Fry & Hale, 1996; Kail, 2000). Preceding work, in fact, has usually investigated processing speed as a predictor for computation, or has reported limited effect on computation when controlling the verbal factors (e.g. Bull & Johnston, 1997; Fuchs et al, 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001; Swanson, 2006). Fuchs et al. (2006) reported processing speed as one of the unique predictors of competence in arithmetic, and demonstrated the arithmetic-mediated effect of processing speed to arithmetic word problems. Even though processing speed has no direct relevance to word problem solving, processing speed may facilitate counting speed so that as students figure out arithmetic operations, they pair problems with answers more quickly before completing the computational sequence, and thus retain more space in working memory to focus on other things for solving word problems. It can be argued, therefore, that low achievement of SDD and LSG students is caused by a lack of acquisition of arithmetic facts or by an accumulated gap in computational skills in terms of processing speed. In addition, this finding can be explained as the influence by the time limit for completing the word problem solving test. Students had only 10 minutes per 20 questions. They were required to write the formula for solution and the answer for each of the questions. The time limit was decided upon to avoid the ceiling effect and low discrimination following the pilot study The imposition of this time limit presented a major challenge to most students. In practice, however, students face this type of time limit challenge for most of their school evaluations. Under this time constraint scenario, processing speed must be a significant factor in predicting academic achievement.

With regard to predictors of arithmetic problem solving, prior work has focused heavily on working memory. Although the nature of the relationship among the three components of working memory is still unclear, many studies about working memory, including Swanson and his colleagues' vigorous investigation into it, have reported that it is a significant determinant of performance in problem solving (e.g. Lee, Ng, Ng, & Lim, 2004; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004). Prior research has also shown that working memory is an essential component of fluid reasoning and other higher order cognitive processes, and is closely related to achievement and learning (Fry & Hale, 1996; Perlow, Jattuso, & Moore, 1997; Swanson, 1996). The result of this study that working memory emerged as a significant factor to differentiate AFG and LSG from HAS supports these previous findings. Between LSG and SDD, however, there was no significant difference that can be attributed to working memory in this study. To some extent this finding contradicts previous work showing the powerful influence of working memory to word problems (e.g., Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001). Fuchs et al. (2006, 2008) reported no significant effect of working memory on word problem solving and interpreted this to mean that the finding is possibly caused by the method used to measure working memory. Moreover, they indicated that most previous research about the relationship between working memory and mathematical achievement had studied the effect of working memory as a univariate factor without simultaneous consideration of other cognitive abilities, and reported that working memory had a significant effect on word problem solving when controlling the effect of language. In terms of working memory, further research should be pursued with valid and segmented measurement tools as has been developed for other cognitive factors. Whereas the word problem was presented as a listening task in previous research, in this study students were asked to read the word problems on paper. This procedural difference might have contributed to the results that language emerged as a powerful factor and working memory showed limited relevance.

Results from this study which conflicted with prior research findings centered around the roles of attention and nonverbal reasoning. It is presumed that the result that the attention factor was not significant in this study came from a difference in the measurement method for attention. This study attempted to accomplish the measurement of students' attention in a more direct way by allowing their teachers to provide a rating for each student. However, because the task of rating students' attention might have been unfamiliar to teachers as well as to students, especially under a large scale testing situation, some difficulty in understanding might have occurred. Also, because problem solving requires students not only to build a problem model, but also to distinguish relevant from irrelevant information, and to determine how numerical quantities fit into the slots of the problem model (Fuchs et al., 2005, 2006, 2008), nonverbal reasoning ability was regarded as a crucial factor in previous studies. However, in this study, nonverbal reasoning factor measured by matrix reasoning was determined not to influence achievement in word problem solving. It can be conjectured that one of the reasons for this result is that the problem solving tasks which students were required to complete were not complex enough, which eliminated any difference in result from nonverbal reasoning ability testing. Word problems provided to students did not include any irrelevant information, but were simple arithmetic word problems, which did not require any high order thinking for obtaining solutions.

These results revealed that students categorized on the basis of the RTI criterion also had lower levels of cognitive abilities; that is, their learning problems are hypothesized to be caused by cognitive abilities. This could provide powerful supports for the presence of learning disabilities. As mentioned above, even though the main notion of learning disabilities is unexpected underachievement from basic cognitive abilities, identification models for learning disabilities, thus far, have not truly reflected the meaning of these disabilities.

3) Research question 3: Cognitive characteristics of students with difficulties in word problem solving

Research question 3 was designed to explore the differences in cognitive profiles among subgroups of students with difficulties in word problem solving by difficulty status as it pertained to computation, reading, both computation and reading, or neither computation nor reading. For group formation, 25th percentile and 40th percentile were selected as cutoffs and four subtypes were identified by difficulty status – word problem solving difficulty (PD), computational difficulty with word problem solving difficulty (CPD), reading difficulty with word problem solving difficulty (RPD), computational and reading difficulty with word problem solving difficulty (CRPD). Observed mean trajectories revealed CPD, RPD, and CRPD showed significantly lower growth levels compared to PD. CRPD showed the lowest growth levels among the four groups. Using MANOVA, significantly difference scores in cognitive dimension among subgroups by difficulty status were presented. Significant differences between PD and RPD were found in working memory, processing speed, language, and nonverbal reasoning. These differences were as follows: working memory, processing speed, and language between PD and CRPD, working memory, language, and nonverbal reasoning between CPD and RPD, and processing speed between CPD and CRPD. No difference in attention was found in any contrast. Moreover, differences in cognitive abilities in PD versus CPD were not found.

Previous research has classified students with mathematics learning disabilities into subtypes - mathematics learning disabilities with and without reading disabilities - and investigated cognitive correlates by subtypes. From the results, the subtypes of mathematics disabilities based on deficit in basic academic skills could be identified, and it confirmed the finding from prior research which indicated that students who had mathematics disabilities with reading disabilities would experience more severe learning problems and distinctive cognitive profiles compared to those who have mathematics disabilities only. Also, the finding that cognitive characteristics of students with computational disabilities were not distinguishable from cognitive characteristics of students with computational disabilities and with problem solving difficulties was corresponded with studies done by Compton et al. (2012) which indicated that there was no significant difference of cognitive profiles between calculations LD and applied problems LD. In addition, as suggested in prior work on distinctions between mathematics disabilities and mathematics disabilities with reading disabilities (e. g. Fletcher, 2005; Fuchs & Fuchs, 2002; Geary, 1993; Jordan, Hanich, & Kaplan, 2003), mathematics disabilities with reading disabilities are more like language-based learning disabilities than mathematics disabilities (Rourke, 1993; Rourke & Finlayson, 1978). Whereas reading disabilities and reading disabilities with mathematics disabilities may be a comorbid association in understanding the reading component (i.e., two disabilities with a common origin), the relation of reading disabilities with mathematics disabilities and mathematics disabilities does not appear to reflect a comorbid association. Rather, mathematics disabilities and mathematics disabilities with reading disabilities are distinctly different types of mathematics disabilities.

This is not only theoretically important, but also has implications in terms of identifying mathematics disabilities and for designing effective methods for preventing and remediating mathematics disabilities. There has been much interest in cognitively focused instruction as an alternative to skills-based approaches (Fuchs et al., 2012). For example, because of their low working memory, low achieving students in word problem solving can be provided with some intervention via training sessions to increase the size of their working memory. However, the training of cognitive abilities is not the only way to consider the importance of cognitive abilities to instruction. A different way is to explore whether cognitive abilities may modify instruction (Frazier, Tix, & Barron, 2004; Fuchs et al., 2012). Some cognitive abilities may cause differential responses to the same or different instructional programs by interacting with features of instruction. For example, students with low processing speed can be provided prolonged time to solve the same problem, or students with low language ability can be provided word problems with scaffolding for facilitating their understanding. The point is that cognitive abilities may be important not as targets of remediation, but as indicators to tailor instruction. Or maybe an important aspect of remediation may be tailored instruction.

2. Limitations

One limitation of this study is that the data in the word problem data were collected at only three time-points. In general, to monitor a student's progress, researchers should collect data for a whole school year (e.g., Baker et al., 2002; Case, 1998; Clarke, 2002; Gestern & Chard, 1999; Lago, 2007). Moreover, when a minimum of four time-points data are secured, it facilitates the exploration as to whether the growth pattern is linear or quadratic.

The other limitation is that in this study, students' representation skills and problem solving strategies, which are critical factors in problem solving, were not considered. Although numerous studies on representation skills in terms of mathematical problem solving have been conducted, one limitation of these studies is that there has been no concrete method for the measurement of representation skills. Therefore, it is recommended that further studies be done in this area.

Another limitation is that other demographic data should have been collected for more information, such as socio-economic status (SES) and private tutoring. Achievement is often influenced by SES (e. g. income level) (Starkey, Klein, & Wakeley, 2004), and many low-income students are at risk for developing mathematics difficulties (Jordan, Levine, & Huttenlocher, 1994). In addition, many studies have been conducted whose findings state that private tutoring could affect the level of student achievement. With data on factors that could affect mathematics achievement, the unique action of cognitive abilities to word problem solving could be explored more accurately.

Additionally, in the present study, only tier-1 setting was dealt with. Since the RTI framework is based on three-tier stages, in order to identify the usefulness and application, the study should be conducted including the tier-2 and tier-3 interventions.

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국 문 요 약

1. 연구의 필요성 및 목적

수학(mathematics)은 학습과 인지 발달에 중요한 역할을 하며, 여타 학습을 위한 도구 교과이자 그 자체의 위계적 특성으로 인하여 학령 초기부터 중요하게 다루어 질 필요가 있다. 수학 능력은 학업 성적에도 밀접하게 관련되어 있을 뿐만 아니라, 졸업 이후 취업 및 연봉에도 상당한 영향을 끼친다.

학교 수학의 핵심 목표는 문제 해결(problem solving)이며, 이러한 문제 해결 능력을 기르기 위한 하나의 방법이자, 해결해야 할 문제의 형태로서 제시되는 것을 문장제(word problem 혹은 story problem)라고 한다.

학생들은 흔히 문장제 해결에 있어 어려움을 느끼는데, 이것은 문장제를 해결하기 위해 요구되는 복잡한 인지적 절차에 기인한다. 학생들이 문장제를 해결하기 위해서는 우선 문장제 내의 일상 언어 형태로 제시된 문제를 읽고 이해해야 하고, 일상 언어를 수학적 언어(수와 연산자, 식 등)로 변환해야 하며, 수학적 언어로 변환된 연산 문제를 해결하고 점검하는 등의 절차를 거쳐야 한다.

주의력, 언어 능력, 작업기억능력 등의 인지적 측면에서 낮은 특성을 지니고 있는 학습장애(learning disabilities) 학생들은 문장제를 해결하는 데에 특히 어려움을 느끼게 된다. 그렇지만 그들의 인지적 특성과 문장제 해결 능력과의 관련성에 대한

연구들은 많지 않다.

학습장애의 핵심 개념은 기본적인 인지처리과정의 결함으로 인한 기대치 않은 저성취(unexpected underachievement)이다. 그렇지만 그간 학습장애의 정의 및 진단·판별 모형에 있어 기본적인 인지처리과정의 결함은 직접적으로 반영되어 오지 못하였다. 이렇게 인지처리과정 결함에 기반하여 학습장애를 규정하고자 하는 접근이 내재성 처리과정 결함(intrinsic processing deficits) 접근이다. 내재성 처리과정 결함 접근에 의해 학습장애를 규정하는 것은 학습장애의 본질을 다루고 있다는 점에서 중요하며, 이를 적용하려면 우선 학습의 기술이 세분화되고, 세분화된 기술에 관여하는 인지 변인들에 대한 연구들이 축적될 필요가 있다.

이러한 배경에서 본 연구는 수학 교과에서 핵심 기술로 꼽히는 문장제 해결 기술의 발달 패턴을 탐색하고, 이에 관여하는 인지 변인에 대해 수학 학습장애 위험군을 중심으로 연구하고자 하였다. 우선, 문장제 해결 능력의 발달 패턴이 계층화는 현상을 살펴봄으로써 현재 학습장애의 진단 모형으로서 활발히 논의되고 있는 중재반응모형(responsive-to-intervention; RTI)의 적용 타당성을 탐색하고, 수학 학습장애 위험군을 파악하였다. 다음으로, 학생들의 문장제 해결 능력 발달 패턴이 인지 능력과 어떠한 관계를 가지고 있는지 탐색해 보았다.

수학 학습장애의 문장제 학습에 관여하는 인지적 특성 파악을 통해, 수학 학습장애의 타당한 진단의 토대를 마련할 뿐만 아니라, 학습장애 학생들의 이질성에 기반한 효과적인 중재 설계를 위한 가능성을 제시할 수 있는 데 본 연구의 의의가 있다.

2. 연구문제

1) 수학 문장제 문제해결 능력의 발달 패턴(초기 성취 및 진전도)은 구별 가능한 집단으로 나누어 지는가?

2) 수학 문장제 해결 능력의 발달 패턴과 인지 변인 간에 어떠한 관계가 있는가?

3) 수학 학습장애 위험군 내의 하위 유형별 수학 문장제 해결 능력의 발달 및 인지 변인의 차이는 어떠한가?

3. 연구방법

연구 참여에 동의한 인천 지역의 4개 초등학교 3학년 20개 반에 소속된 학생 441명(남자 210명, 여자 231명)을 대상으로 하였다. 3학년을 대상으로 한 것은, 1) 사칙연산에 대한 기본 학습이 완료되는 시점이면서, 2) 문장제의 의미론적 구조에 고르게 익숙해 지는 시점이고, 또한, 3) 1, 2학년에서 익힌 연산 능력을 바탕으로 수학 능력 편차가 벌어지기 시작하는 시점이기 때문이다.

측정도구는 크게 인지 변인 측정도구와 수학 성취 측정도구, 그리고 학습기술의 결함을 진단하기 위한 측정도구로 구성되었다. 인지 변인 측정도구는 선행 연구들에서 문장제에 관여하고 있다고

알려진 주의력(FAIR 주의력 검사), 작업기억(K-WISC-IV 숫자), 처리속도(K-WISC-IV 동형찾기), 언어(K-WISC-IV 어휘), 비언어적 추론(K-WISC-IV 행렬추리)의 5가지를 사용하였으며, 수학 성취 측정도구는 교육과정중심 수학 문장제 해결 능력 검사로 제작된 BASA: MWP(20문항/10분, 동형검사)를 사용하였다. 학습기술의 결함을 진단하기 위한 측정도구로서는 연산 검사(BASA: M)와 빈칸 채우기 검사(BASA: R)를 사용하였다. 인지 변인 측정 및 학습기술 결함 진단 검사는 7월에 1회 실시하였고, 문장제 해결 능력 검사는 9월, 10월, 11월에 1회씩 총 3회에 걸쳐 실시하였다.

종단적으로 수집된 문장제 성취 검사 자료에서 이질적인 성취 패턴을 찾아내기 위해 잠재계층성장분석(latent class growth analysis)을 실시하였고, 수학 문장제 문제해결 능력의 발달 패턴과 인지 능력과의 관련성을 파악하기 위해 잠재계층성장분석에서 예측변인이 추가된 모형이 성장혼합모형(growth mixture modeling)을 사용하였다. 또한, 수학 학습장애 위험군 내의 하위 유형별 인지능력의 차이를 파악하기 위해 다변량분산분석을 실시하였다. 기술 통계량의 산출과 다변량분산분석을 위해 PASW Statistics 18.0을, 잠재성장모형 및 성장혼합모형의 분석을 위해 Mplus 5.1을 사용하였다.

4. 연구결과

연구결과 1에서는 중재반응모형의 개념을 바탕으로 학생들의 문장제 해결 능력 성취 패턴에 의해 수학 학습장애

위험군이 분류되는가를 탐색적으로 알아보고자 하였다. 이를 위해 잠재계층성장분석이 사용되었다. 그 결과, 수학 문장제 해결력 성취 패턴에 의해 네 집단(고성취 집단(HAS, 14.3%), 평균의 빠르게 성장하는 집단(AFG, 28.6%), 저성취이면서 평균보다 낮은 성장률을 보이는 집단(LSG, 42.2%), 그리고 이중 불일치 집단(SDD, 15.0%))으로 구분되었다. 마지막 SDD 집단은 중재반응모형의 1단계에서도 특수교육으로 직접적으로 의뢰 가능할 수 있는 집단으로, LSG 집단 중 일부 학생들은 2단계에서 특수교육으로 의뢰될 가능성이 높은 것으로 추정된다. 따라서, 본 연구에서는 LGS 및 SDD를 수학 학습에 어려움을 지닌 학생들로 보았다.

연구결과 2에서는 연구결과 1에서 도출된 수학 문장제 해결 능력 발달 패턴과 인지 변인과의 관계를 파악하기 위해 인지 변인을 예측변인으로 투입한 성장혼합모형을 분석하였다. 그 결과, HAS 집단을 참조집단으로 설정하였을 때, 낮은 작업기억을 가진 학생들은 AFG 혹은 LSG 집단으로 분류될 가능성이 높았고, 낮은 처리속도 및 언어능력을 가진 학생들은 SDD로 분류될 가능성이 높게 나타났다. AFG 집단을 참조집단으로 설정하였을 때, 낮은 처리속도는 LSG 집단으로 분류될 가능성을 높였고, 낮은 처리속도 및 언어능력은 SDD로 분류될 가능성을 높였다. LSG와 SDD 집단을 구분하는 인지변인은 마찬가지로 처리속도 및 언어능력으로 나타났다.

끝으로, 연구결과 3에서는 수학 문장제 해결에 어려움을 지닌 학생들 중에서 학습기술 결함을 중심으로 하위 유형을 구분한 뒤, 분류된 집단 간 인지 능력 차이를 알아보기 위해 다변량분산분석을 실시하였다. 그 결과, CRPD의 경우 가장 낮은

성취 패턴을 나타냈고, CPD 혹은 RPD의 경우 비슷한 성취 패턴을 나타냈으며, PD 보다는 낮은 수행을 보였다. 인지 능력에 있어서는 PD와 CPD가, 그리고 RPD와 CRPD가 비슷한 인지 능력 수준을 나타냈고, 주의력을 제외한 나머지 인지 변인들에서 네 집단의 유의미한 차이가 나타났다. Scheffé 사후검정 결과, PD와 RPD 간에는 작업기억, 처리속도, 언어능력, 그리고 비언어적 추론에서 유의미한 차이가 있었고, PD와 CRPD 간에는 작업기억, 처리속도, 그리고 언어능력에서 유의미한 차이가 나타났다. CPD와 RPD 간에는 작업기억 및 언어능력에서, CPD와 CRPD 간에는 처리속도만이 .05 수준에서 유의하게 나타났다. 하지만, 유의수준을 좀 더 관대하게 .10으로 설정하였을 때, CPD-RPD와 마찬가지로 CPD-CRPD 역시 작업기억 및 언어능력에서도 차이가 있는 것으로 해석할 수 있었다. PD-CPD 및 RPD-CRPD는 통계적으로 유의미한 차이가 없는 것으로 나타났다.

5. 논의 및 제언

연구결과 1을 통해, RTI 모형이 수학 학습 기술 중 문장제 해결 영역에 있어서 수학 학습장애 위험군의 하위 유형을 적합하게 분류해 낼 수 있음을 통해, 진단모형적 가치를 확인할 수 있었다. 초기 성취와 진전도 모두에서 낮은 수준을 보인 SDD의 경우, RTI에서 Tier 2의 소집단 집중교수 대상인 15% 비율로 나타나는 것을 확인하였고, 이러한 결과는 이중불일치(dual discrepancy)를 보이는 집단을 탐색적으로 확인한 점에서 관련 선행 연구인

홍성두와 여승수(2011) 및 김근하(2011)의 결과와 맥을 같이 한다. 약 42%의 LSG 집단으로 확인되었고, 상당수의 비율이 평균보다 낮은 초기 성취와 진전도를 나타내고 있었는데, 이러한 결과는 Geary 등(2009)의 간단한 선별도구에 의한 50%의 수학 저성취 집단 분류를 지지하는 것으로 나타났다.

연구결과 2는 성취 패턴에 의해 분류된 집단별로 인지 능력에 있어서 차이가 있음을 확인한 것은 학습장애의 특수성 가설(specificity hypothesis)를 확인한 Compton 등(2012)의 연구 결과를 비롯하여, 학습장애와 인지 변인 간의 관련성을 탐색한 선행 연구들의 결과를 지지하며, 종단적인 성취 결과를 토대로 인지 능력과의 관련성을 탐색한 점에서 의의가 있다. 언어능력이 가장 많은 차이를 설명하고 있는 점은 문장제가 언어를 기반으로 하고 있고, 기존 연구와 달리 스스로 읽어야 하는 과제로 제시된 점에서 당연한 결과라 볼 수 있다. 처리속도 역시 개인차를 만드는 주요한 인지 변인으로서, 지적 가용성, 읽기 수행과 발달, 보유하고 있는 인지적 자원으로부터의 추론 능력과 역학적 관련이 있는 변인인 바, 가장 강력한 예측변수의 하나로 나타난 점을 이해할 수 있다. 특히, 본 연구에서는 속도 검사 형태로 과제가 제시된 점에서도 파악할 수 있다. 작업기억 또한 선행연구와 마찬가지로 집단 간 유의미한 예측변인으로서 작용하고 있었지만, LSG와 SDD 간에는 유의미하게 나타나지 않았다. 이는 Fuchs 등(2006, 2008)의 연구에서도 비슷하게 발견되었는데, 그들은 작업기억 검사도구에 따라 다른 결과가 나타날 수 있음을 지적하였다. 또한, 언어능력의 중요성을 볼 때, 이를 통제하였을 때에만 작업기억이 유의미한 영향을 가지게 된 선행 연구의 결과로부터 이를 해석할 수 있을 것이다. 한편, 본 연구에서는 주의력과 비언어적 추론의 영향이 유의미하지 않게 나온 결과는 선

행 연구들과 상반되는 결과이지만, 이는 문장제가 새로운 모델을 만들어야 할 필요가 없는 과제(불필요한 정보를 포함하거나 새로운 전략의 탐색을 필요로 하는 과제가 아님)로서 반복 제시된 점, 문장제는 비언어적 추론에서 측정하는 시공간적 처리능력보다는 보다 언어에 기반한 능력을 요구한다는 점에서 근거를 찾을 수 있을 것이다.

연구결과 3은 그 간 제기되어 온 공존성에 기반한 수학 학습장애 하위 유형 간 인지 능력 차이에 대해 탐색해 보고자 한 데에서 의의를 찾을 수 있다. LSG와 SDD를 학습 위험 집단으로 분류한 데에는, 이들 모두 평균보다 낮은 성취를 나타내고, 그 성장률을 통해 미루어 볼 때, 학년 말에 적절한 표준에 다다르지 못할 것이라고 판단되기 때문이며, LSG 중에 포함되어 있을 학습장애 아동들에 대한 부정 오류(false negative)를 피하기 위해서도 이들을 적어도 Tier 1에서는 위험군으로 볼 필요가 있다고 판단되었다. 또한, 전체 아동의 5%에 해당하는 집단이 학습 기술 결함의 공존성에 의해 분류되었고(CRPD), 이들의 성취 및 인지 능력은 가장 낮은 수준으로, 수학 학습장애로 판별되어 Tier 3에 의뢰될 가능성이 가장 높은 것으로 보여진다. 주의력에서 주요한 차이를 나타내지 못한 것은 네 집단 모두 가장 낮은 수준에서 변산이 없는 주의력 수준을 보여주었기 때문으로 해석할 수 있다. 연구결과 3은 읽기 결함이 공존되면 구분되는 인지 특성을 나타낼 것이라는 선행 연구를 지지하였다. 한편, 읽기 기술의 결함이 있는 경우에는 비언어적 추론에 있어서도 유의미하게 낮은 결과를 나타냈는데, 언어능력의 주요 영향 외에 비언어적 추론이 하위 유형별 성취에 어떻게 영향을 미치는지, 좀더 장기적인 성취 자료와 아동 수를 기반으로 연구될 필요가 있을 것이다. 뿐만 아니라, CPD와 RPD는 비슷한 성취 수준을 나타내고 있

지만, 인지 변인에서 유의미한 차이가 나타나는 바, 마찬가지로 이러한 차이가 어떻게 장기적으로 영향을 미칠 수 있는지 탐색할 필요가 있겠다.

본 연구는 3학년 2학기로 제한된 시간 동안 3시점의 자료로 성장 패턴을 탐색하였는데, 보다 장기적으로 계획된 기간 동안 여러 시점의 자료를 통해 연구될 필요가 있다. 또한, 본 연구에서는 Tier 1에서의 중재반응 패턴만을 다루었는데, Tier 2 단계를 포괄적으로 다룬 연구가 수행될 필요가 있다. 다음으로, 각각의 인지 변인의 하위 항목별로 세분화하여 자료를 수집함으로써 보다 구체적인 정보를 얻을 수 있을 것이다. 예를 들면, 주의력의 경우 지속적 주의력과 선택적 주의력으로 구분하여 측정할 수 있고, 작업기억의 경우 중앙집행기, 음운루프, 시공간 잡기장과 같이 구분하여 측정함으로써 인지 변인의 하위 항목 상에서 어떠한 차이를 나타내는지 확인해 볼 수 있을 것이다. 더 나아가 성취에 주요하게 영향을 미치는 자료 예를 들면, SES나 사교육 정도 등의 자료를 수집함으로써 인지 변인의 고유한 효과를 연구할 수 있을 것이다. 뿐만 아니라, 인지 변인의 성차가 많이 연구된 바, 인구학적 변수와 관련하여 성취와 인지 능력 간의 관계가 연구될 수 있을 것이다.

주요어: 문장제 해결, 인지 능력, 수학 학습장애, 진단/판별, 잠재계층성장분석, 성장혼합모형

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