



**M.S Thesis** 

## A Flexible Method for Assessing and Monitoring Static Voltage Stability

using The Saddle Node Bifurcation Set in Two

**Dimensional Power Parameter Space** 

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**Graduate School of Seoul National University** 

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Department

Nguyen Van Thang

# A Flexible Method for Assessing and Monitoring Static Voltage Stability

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**Dimensional Power Parameter Space** 

指導教授 尹 容 兌

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서울大學校 大學院

電氣·컴퓨터 工學部

Nguyen Van Thang

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委	員	長	(퉈	<u>)</u>
副	委	員	(印	)
委		員	(印	)

#### Abstract

This thesis proposes a new method for assessing and monitoring static voltage stability using the saddle node bifurcation set or P-Q curve in two dimensional power parameter space. The method includes three main stages. First stage is to determine the critical buses and the second stage is building the static voltage stability boundary or the saddle node bifurcation set. Final stage is assessing the voltage stability through the distance from current operating point to the boundary. Critical buses are defined through the right eigenvector by direct method. The boundary of the static voltage stability region is a quadratic curve that can be obtained by the proposed method that combines a variation of standard direct method and Thevenin equivalent two bus model of electric power system. And finally the distance is computed through the Euclid norm of normal vector of the boundary at the closest saddle node bifurcation point. The advantage of the proposed method is that it gets the main advantages of both robust methods, the accuracy of the direct method and the simplicity of Thevenin Equivalent model. Thus, the proposed method holds some promises in terms of performing the real-time voltage stability assessment of power system. In addition this thesis gives new method to calculate Thevenin parameters that is suitable for voltage stability analysis and power flow applications. Test results of New England 39 bus system are presented to show the effectiveness of the proposed method.

**Keywords**: Direct Method, Loadability Boundary, Saddle Node Bifurcation, Thevenin Equivalent, Voltage Stability Assessment.

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#### **Chapter 1: Introduction**

In recent years, several blackouts related to voltage instability problems have occurred in many countries. In particularly on August 14<sup>th</sup>, 2003 The North American blackout affected approximately 50 million people in eight U.S. states and two Canadian provinces. One of causes is lack of real time voltage stability assessment and control tools [29].

In addition now a day, the deregulation of electric power industry which has resulted in a significant increase in loading level of inter-area ties, which just designed for operating conditions in regulation environment. That makes power systems operate closer to their limits. Therefore stability problems, especially is voltage stability assessment and control problem, although not a new issue, is now receiving a special attention again. With state of the art the approach ways of recent researches focus on how to make effective and flexible tools that support strongly operators in real time operation in general, real time voltage stability assessment in special. Those tools try to build a security region that contains the current operating point in multi-dimensional power parameters space. Then voltage stability assessment and control based on calculating distance and monitoring movement of the current operating point versus the boundary of the security region in real time [4-5]. It is also a part in works of this thesis.

As defined in [6], voltage stability is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance. The study of voltage stability can be analyzed under different approaches, but specially, the assessment of how close the system is to voltage collapse can be very useful for operators. Most of existing online or offline tools that support the operators in assessment of voltage stability use performance indices to predict proximity to voltage collapse point from the current operating point. Besides using the performance indices, offline planning of electric power system usually use bifurcation diagrams that have well-known as the nose curves such as P-V curves and Q-V curves. P-V curves represent the relationship between the load and the voltage of a selected bus. The load is defined as the bus load or the total load in an area or the system. P-V curves typically are calculated using the continuous power flow solutions by step-by-step increasing the loads. The "nose point" of the curves corresponds to the maximum power which can be delivered to the load. Q-V curves that represent the bus voltage vs. reactive power of the same bus typically obtained by a series of power flow simulations for various values of the bus voltage and the corresponding needed reactive power injection [4]. However, these curves are just typically used in off-line analysis; they support not much for operators in assessing and monitoring voltage stability on line. Therefore we should use the other tools such as the voltage stability boundary or parameter indices defined in the power parameter space. They will give visual interface for the operators to can assess and monitor voltage stability on line. Especially the boundary can easy to directly answer questions i.e.: "Can the system withstand a 100 MVA increase on an arbitrary bus without encountering voltage collapse?" and provide more information that helps the operators in collapse preventive controls.

The main objective of this thesis is to give a new method for assessing and monitoring static voltage stability through using the saddle node bifurcation set or the loadability boundary or can be called as the static voltage stability boundary due to only considering voltage instability phenomenon caused by increasing load gradually. The boundary is P-Q curve in two dimensional power parameter space. In addition The Thevenin parameters that are obtained in step 2 of this method are calculated via load flow information with some advantages.

This thesis proposes a new method for assessing and monitoring static voltage stability based on the saddle node bifurcation set (loadability boundary) in two dimensional power parameter space. The method includes three main steps. Step 1 we identify the critical buses using the right eigenvector corresponding to the zero eigenvalue obtained from solving directly the saddle node bifurcation condition equations (or direct method) that apply for power flow model and a given instability scenario such as increasing load at all buses in system through a single loading parameter  $\lambda$ . In step 2 corresponding to the critical buses that obtained from step 1, we build the "actual" shape saddle node bifurcation set or the loadability boundary for each bus. The boundary is built based on three sub-step process. Sub-step 2.1 builds the rough boundary from the new equivalent model seen from the critical bus. Sub-step 2.2 finds exactly the closest point of collapse (or turning point) of system corresponding to the worst load increasing scenario at the critical bus (both real and reactive power increase independently). Due to having some errors from estimating equivalent parameters, therefore the rough boundary has to be displaced to build the "actual" boundary. Sub-step 2.3 displaces the rough boundary into the "actual" boundary along a displacing vector that

connects two turning points, one is obtained from direct method (exact point), and the other (approximated point) is obtained from the new equivalent model.

Advantage of the proposed method is it takes the main advantages of both two robust methods; the accuracy of the SNB theory based the direct method and the simplicity of the new equivalent model method. Beside the proposed method is Fast, Simple, Flexible. The volatile of load can be monitored online through the boundary that is likely "actual" boundary.

The structure of the thesis is as followings:

Chapter 1 (this chapter) give introduction and objectives of the thesis

Chapter 2 recalls the background of voltage stability problem. In this chapter, basic definitions, classification and nature analysis of voltage stability are given.

Chapter 3 deals with the background of bifurcation theory and its applications in electric power system.

Chapter 4 is also main section of this thesis. It focuses on the three steps of the proposed method for assessing voltage stability through using the saddle node bifurcation set in two dimensional power parameter space.

Chapter 5 shows simulation results of the proposed method for a standard test system.

Finally, conclusions as well as future research directions are presented in chapter

6.

## Chapter 2. Voltage Stability Problem: Definition, Classification and Nature Analysis

#### 2.1. Voltage Stability Definition

According to the definition proposed by [6] voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition. It depends on the ability to maintain/restore equilibrium between load demand and load supply from the power system. Instability may result in the form of a progressive fall of voltages of some buses. A possible outcome of voltage instability is loss of load in an area, or tripping of transmission lines and other elements by their protective systems leading to cascading outages.

#### 2.2. Voltage Stability Classification

Depend on the time scales of phenomina and the size of disturbances, the voltage stability can be classified into large-disturbances voltage stability and small-disturbances voltage stability or long-term voltage stability and short-term voltage stability as figure 1.



#### Fig.1. Classification of voltage stability [6]

The above definition and classification based on the physical viewpoint. This thesis looks into the voltage instability problem under the mathematical perspective, especially based on bifurcation theory. Therefore the voltage instability analysis will be defined and classified according to the bifurcation theory as following. According to the bifurcation theory one common way in which stability is lost in voltage collapse is that the change causes the stable operating point to "disappear" due to a bifurcation. The lack of a stable operating point results in a system transient characterized by a dynamic fall of voltages, which can be identified as a voltage collapse problem. The transient collapse can be complex, with an initially slow decline in voltages, punctuated by further changes in the system followed by a faster decline in voltages. Thus the transient collapse can include dynamics at either or both of the transient and long-term time scales defined above. Voltage instability analysis can be classified into two problems, they are static (steady state) voltage instability analysis and dynamic voltage instability analysis. The static voltage instability analysis is typically associated with the small disturbances such as increasing of load and outage of transmission lines; these cause the overload of Total Transfer Capacity (TTC) of transmission lines and the local lack of reactive power of load site. Then a cascade collapse of voltage at load site occurs (due to the reactive power of load is not matched, so the voltage drop through the critical value (nose point)). This phenomenon is associated with the static saddle-node bifurcation of equilibrium and its mathematical model is algebraic equations that represent the equilibrium state of power system. In other

hand, dynamic voltage instability analysis is usually used for voltage instability caused by large disturbances such as system faults, loss of load, or loss of generation. The mathematical model of power system in dynamic analysis is usually Ordinary Different Equations (ODEs) that represent the dynamic behavior of generators, dynamic loads, control and protection devices.

In addition the time scale of static voltage instability analysis is usually long-term time scale whereas the dynamic voltage instability analysis can include both of the transient and long-term time scales.

#### 2.3. Nature Analysis of the Voltage Stability

Because the thesis just focuses on the static voltage stability that relates to the loadbility of transmission system or the maximum power delivered to the load. Therefore a simple two bus system gives nature insight of the static voltage instability phenomenon. Consider the simple two bus system as shown in Figure. 2. The generator at bus 1 transfers power through a transmission line having an impedance of Z = R+jX to a load at bus 2. Bus 1 is considered as the swing bus where both the voltage magnitude E and angle  $\delta_1 = 0$  are kept constant. From the simple two bus system in Figure 2, the voltage equation and voltage stability boundary in power parameter space at bus 2 can be defined as follows:

$$V \angle -\delta = E \angle 0 - (R + jX) \frac{P - jQ}{V \angle \delta}$$
(1)  

$$E \angle 0 \qquad V \angle -\delta$$
  
bus 1 R jX bus 2  
P + jQ

Fig. 2. A simple two bus system

After some mathematical manipulations we arrive at the voltage equation at bus 2 as follows:

$$V^{4} + (2RP + 2XQ - E^{2})V^{2} + (R^{2} + X^{2})(P^{2} + Q^{2}) = 0$$
(2)

The Equation (2) is a biquadratic equation that represents the dependence of the voltage at load bus on the power parameters at that bus. When the load increases to the maximum value, the voltage reaches to the corresponding critical value or voltage collapse point. The critical point is obtained by finding the singularity condition of Equation (2) and is also voltage stability constraint being  $\Delta$ =0. Since

$$\Delta = (2RP + 2XQ - E^2)^2 - 4(R^2 + X^2)(P^2 + Q^2)$$
  
=  $E^4 - 4(RP + QX)E^2 - 4X^2P^2 - 4R^2Q^2 + 8RXPQ$ 

 $\Delta = 0$  can be written as

$$X^{2}P^{2} + R^{2}Q^{2} - 2RXPQ + RE^{2}P + XE^{2}Q - \frac{E^{4}}{4} = 0$$
(3)

where, Equation (3) is a quadratic equation that represents the boundary of static voltage stability region in power parameter space at bus 2 of the system. Each point on the boundary is power injection limit for load at that bus. The critical voltage corresponding to the critical power is then calculated as follows:

$$V_{cr} = \sqrt{\frac{E^2}{2} - \left(RP_{cr} + XQ_{cr}\right)} \tag{4}$$

where,  $(P_{cr}, Q_{cr})$  represents the critical point on the boundary and satisfies Equation (3), and  $V_{cr}$  is the critical voltage corresponding to this critical point.

Because of the changing of the power factor of load, the critical voltage is also changed to follow Equation (4). The Equality constraint in Equation (3) corresponds to the static voltage stability boundary for the simple two-bus system. Inside the boundary, the voltage equation has two solutions; one is the solution corresponding to the normal operating condition of system while the other one is the unacceptable low voltage solution. On the boundary, two solutions merge and this point represents the voltage collapse point, i.e. the nose point in P-V curve. There are no solutions outside the boundary. The envelope of operating condition becomes the loci of voltage collapse points where the quadratic curve with two axes correspond to the real and reactive power injections at the node. The distance from operating point to the boundary is called security margin. If the resistance of transmission line can be ignored, i.e., R = 0, Equation (3) reduces to:

$$Q = \frac{E^2}{4X} - \frac{X}{E^2} P^2$$
(5)

where, Equation (5) is parabolic with the top on the vertical axis, i.e., reactive power axis. The intersection point between the parabolic and the vertical axis is the point that has the active power P = 0 or power factor is equal to zero and achieves the maximum reactive power  $Q_{max} = E^2/4X$ . This value is the reactive power limit of the load bus. The other point with coordinates  $P = E^2/2X$  and Q =0 corresponds to the peak of P-V curve when the power factor is unity.

## Chapter 3.Background: Bifurcation Theory in Power System

#### **3.1. Bifurcation Theory**

Bifurcation theory is the mathematical study associated with Ordinary Differential Equations (ODEs) where a small smooth change of parameters cause the qualitative and topological structure of solutions to change suddenly. Therefore study the change of solutions corresponding to the change of the parameters that is the main task of bifurcation theory. The mathematical representation of non-linear phenomenon of dynamic systems such as power systems typically is ODEs. Therefore Bifurcation theory applied widely in power system stability analysis, especially in voltage stability problem. To have deep insight the later problems, firstly we briefly summarize some theoretical backgrounds about the bifurcation theory.

#### 3.1.1. Equilibia and Branches

Consider the system of Different Algebraic Equations (DAEs) that represent the mathematical model of dynamic systems as following:

$$\stackrel{*}{y = f(x, y, \lambda)} 0 = g(x, y, \lambda)$$
(6)

Or compact form in n – dimensional state space.

$$\stackrel{*}{X} = F(X,\lambda), X \in \mathbb{R}^{n}, \lambda \in \mathbb{R}^{k}$$
(7)

In where *X* denotes state variables, *y* denotes the dynamic state variables, for power system models these are rotor angles, rotor angular speeds, electromotive forces ..., *x* denotes algebraic state variables, for power system models these are voltage magnitudes and angles at buses,  $\lambda$  is vector of parameters, for power system model these are power injections at buses. At an equilibrium point ( $X_{0}$ ,  $\lambda_{0}$ ), the left hand term of Eq.7 becomes zero, the steady state solution of Eq.7 satisfies the set of nonlinear algebraic equations

$$F(X_0, \lambda_0) = 0. \tag{8}$$

The equilibrium point is stable or unstable depend on the eigenvalues of Jacobian  $\partial F/\partial X$ .

If the eigenvalues of the Jacobian  $\partial F / \partial X$  become non-zero, then according to implicit function theorem the equilibria of Eq.8 can be expressed as the smooth function  $X = X(\lambda)$ . The function  $X(\lambda)$  is called the branch of equilibria [2]. The intersection point of different branches of equilibria is called a *bifurcation point*. And the parameter set  $\lambda_c$  where the system loses its stability is called a bifurcation set.

#### **3.1.2. Saddle Node Bifurcation (SNB)**

Stability problems usually associated with the local bifurcations. In other hand, chaos, attractors and tori phenomena associated with the global bifurcations. In this work, we just concern about voltage stability problem associated with local bifurcations, especially saddle node bifurcation. Local bifurcations refer to qualitative changes of state variables in the neighborhood of equilibrium when parameters change. In the next part the definition and features of saddle node bifurcation are briefly summarized.

#### Saddle node bifurcation (or fold)

*Definition*: Saddle node bifurcation occurs when a stable equilibrium point and another equilibrium point coalesce and disappear as parameter varies slowly

The Transversality conditions:  $\frac{\partial F}{\partial \lambda} \neq 0; \frac{\partial F}{\partial X^2} \neq 0$ 

*The bifurcation conditions*:  $\begin{cases} F(X_*, \lambda_*) = 0\\ \det(F_X((X_*, \lambda_*))) = 0 \end{cases}$ 

The feature of eigenvalues: Jacobian has a simple zero eigenvalue

**Bifurcation diagram** 



**Fig. 3. The Bifurcation Diagram** 

**Analysis**: when parameter  $\lambda$  varies slowly and reach the critical value ( $\lambda_c$ ) then two equibria (green and black points) coalesce, and after that parameter  $\lambda$ continues to increase to pass through the critical value, the coalesced equilibrium (red point) disappear, the collapse occurs initially.

# 3.1.3. Saddle Node Bifurcation Set in Multi-dimensional Parameter Space

Reconsider the system

$$\overset{*}{X} = F(X,\lambda), X \in \mathbb{R}^{n}, \lambda \in \mathbb{R}^{k}$$
(9)

X is n dimensional state variable vector,  $\lambda$  is k dimensional parameter vector. A saddle node bifurcation point lies in the (n+k) – dimensional state and parameter space. If the SNB points are projected onto the k-dimensional parameter space, a smooth hyper-surface  $\Sigma$  of dimension (k-1) is defined, called bifurcation set or bifurcation surface or the boundary of feasibility region. (In next section, it also is called as static voltage stability boundary). Generally the determining exactly the analysis expression or geometry shape of the bifurcation set is very complicated, even impossible because of its non-convex properties [16-18]. However, the bifurcation set is a very effective tool for preventive control of collapse of system at saddle node bifurcation points. Therefore there are many efforts to approximate the saddle node bifurcation set. This thesis also tries to build the saddle node bifurcation set in static voltage stability assessment of power system.

## **3.2. Saddle Node Bifurcation and Voltage Collapse Problem** in Electric Power System

#### 3.2.1. Power System Model for Saddle Node Bifurcation

As definition of [1], the voltage collapse phenomena are originated by loss of stability caused by the lack of reactive power and saddle node bifurcation. Therefore the features and properties have a direct implication in voltage instability and collapse. In range of this work, we just focus on voltage instability corresponding to slow varying of load. That means static voltage instability associated with static saddle node bifurcation occurs near maximum power transfer. Therefore the electrical power system model for saddle node bifurcation follows some assumptions:

- Ignore reactive power limit of devices (synchronous generators, compensate devices...)
- Using power flow model for saddle node bifurcation analysis.

#### **3.2.2.** Eigenvectors, the Weakest Bus and the Normal Vector

As pointed out above, at the saddle node bifurcation point, Jacobian of system has a simple zero eigenvalue. The corresponding eigenvector contains valuable information on the nature of the bifurcation. The right eigenvector represents the response of the system and shows the direction in state space along which voltage instability will evolve. The components of the right eigenvector are proportional to bus sensitivities that indicate how weak a particular bus is near the critical point and help determine the areas close to voltage instability. The greater the bus sensitivity value, the weaker the bus is [2]. Therefore the largest components in magnitude of the right eigenvector are useful in identifying the weak area of the power system, especially a bus in that area in which the voltage collapse is occurs initially, that bus is called as the weakest bus or critical bus.

Whereas the left eigenvector is very useful to determine the normal vector of the saddle node bifurcation set (or the boundary), the sensitivity of the boundary to parameters and the most effectively preventive control direction in parameter space. Geometrically, the left eigenvector parallels with the normal vector to the bifurcation set at  $\lambda_*$ , which is very important in the calculation of the closest SNB [7].

$$N(\lambda_*) = \alpha.w.f_{\lambda} \tag{10}$$

In where  $N(\lambda_*)$  is the normal vector,  $\alpha$  is scaling factor, w is the left eigenvector,  $f_{\lambda}$  is the Jacobian of f with respect to  $\lambda$ . Note that  $f_{\lambda}$  is the constant matrix.



Fig. 4. The saddle node bifurcation set in parameter space.

#### 3.2.3. Saddle Node Bifurcation and Voltage Collapse

A saddle-node bifurcation is the disappearance of system equilibrium as parameters change slowly. The saddle node bifurcation point is the intersection point where different branches of equilibria (stable or unstable) meet. At this point the Jacobian matrix of the system becomes singular or the Jacobian has a simple zero eigenvalue. In addition the system loses stability and voltage collapse initially occurs at that point. Therefore in monitoring and assessment of voltage stability it is necessary to study saddle node bifurcation of power system in order to understand and avoid the voltage collapse. The most important work is finding exactly the SNB point where voltage collapse initially occurs. This thesis contributes one in many efforts to solve this problem.

## Chapter 4: Formulation: A New Method for Static Voltage Stability Monitoring and Assessment

All efforts to assess Voltage Stability have to answer the question: "How are scenarios?" That means

1) Increase the load at a single bus or all buses or a combination of some buses (local or global)?

2) Direction of increase? (Bus loads increased proportionally or in some other ways?)

3) Power factor at loads is constant or vary?

Typically the scenarios depend on the historical and forecast information of load. In case of lacking of those information, most existing applications focus on the following scenario increase load at all buses, load increases proportionally to normal operating level and power factor of each load is constant. Result of simulation following the scenario is P-V curve at each bus. Voltage Stability Assessment based on those P-V curves or some performance indices that associated with the scenario. Most of those performance indices are defined in the state space of power system models [28]. Therefore they cannot directly answer question such as: "Can the system withstand a 100 MVA increase on an arbitrary bus without encountering voltage collapse?" In order to answer directly the question the performance indices should be defined in the parameter space of power system model. In this thesis, a parameter space based performance index is defined through the distance from the current operating point (at the critical bus) to the saddle node bifurcation set.

In addition in real time operation of power system, voltage stability monitoring and assessment based on the performance indices are insufficient. Those indices just provide information about the distance from the current state of system to critical point or proximity to voltage instability, the changing of the value of the indices represents the operating point coming towards or going away the critical point. The system loses stability when the operating point approaches and passes through the critical point. However they cannot provide visual monitoring about the changing of system. Therefore it supports not much for system operators in real time. A visual boundary separates the feasibility operating region and unstable region is necessary to monitor and assess online voltage stability. It can provide more information that relate to voltage instability. Therefore the boundary together parameter space based the performance indices are useful tools to support the operators in collapse preventive control.

In addition voltage instability is a local phenomenon and it originates at buses within an area with high loads and low voltage profile. Those buses are called is critical buses or the weakest buses. Therefore the voltage stability assessment typically associates with the monitoring the changing of load at the critical buses. From that, it is necessary to have a voltage stability monitoring method that can handle the whole loading level scenarios of the critical buses for operating power system, especially in real-time. This approach is flexible and visual in assessing and monitoring online the voltage stability of power system.

With above mentioned reasons, this thesis proposes a new method for assessing and monitoring static voltage stability based on the saddle node bifurcation set (loadability boundary) in two dimensional power parameter space. The method includes three main steps. Step 1 we identify the critical buses using the right eigenvector corresponding to the zero eigenvalue obtained from solving directly the saddle node bifurcation condition equations (or direct method) that apply for
power flow model and a given instability scenario such as increasing load at all buses in system through a single loading parameter  $\lambda$ . In step 2 corresponding to the critical buses that obtained from step 1, we build the "actual" shape saddle node bifurcation set or the loadability boundary for each bus. The boundary is built based on three sub-step process. Sub-step 2.1 builds the rough boundary from the new equivalent model seen from the critical bus. Sub-step 2.2 finds exactly the closest point of collapse (or turning point) of system corresponding to the worst load increasing scenario at the critical bus (both real and reactive power increase independently). Due to having some errors from estimating equivalent parameters, therefore the rough boundary has to be displaced to build the "actual" boundary. Sub-step 2.3 displaces the rough boundary into the "actual" boundary along a displacing vector that connects two turning points, one is obtained from direct method (exact point), and the other (approximated point) is obtained from the new equivalent model.

Advantage of the proposed method is it takes the main advantages of both two robust methods; the accuracy of the SNB theory based the direct method and the simplicity of the new equivalent model method. From the new equivalent model, the rough shape of the boundary is easy to be obtained. This shape is natural shape; it is either convex or non-convex quadratic curve depend on the value of equivalent parameters. In addition the drawback of direct method is overcome in the proposed method. The main drawback of direct method is the high computational cost as the number of equations increases two fold with respect to the system steady state equations, especially in case of multi parameters. In the proposed method both two steps that apply the direct method just use a single parameter in step 1 or two parameters in step 2.2. Therefore the number of additional equations is negligible. However in this work we assume that ignoring the reactive power limit of generators and the other reactive power compensate devices to the result from saddle node bifurcation calculation is correct.



Fig.5. The new method for assessing and monitoring static voltage stability

# 4.1. Identify the Critical Buses through the Right Eigenvector

The purpose of step 1 is identifying the critical buses (or the weakest buses). There are some methods to do that [1-2], [19-20]. In [2] Ajjarapu and in [19] Souza propose to use the tangent vector to identify the weakest bus. The tangent vector at proximity of SNB point can be obtained by the continuation power flow method. In [1], Van Cutsem recommends to use the right eigenvector of Jacobian corresponding to zero eigenvalue at the SNB point to identify the weakest bus. The right eigenvector can be obtained by direct method. At the SNB point, the tangent vector is the right eigenvector [2]. But the time of direct method is less than that of continuation power method in case of a single parameter system such as in here. Because of its' computational efficiency in this thesis we use the right eigenvector corresponding to the zero eigenvalue of Jacobian matrix of power flow model to identify the critical buses.

The static saddle node bifurcation conditions of power flow model of power system are represented through the following equations:

$$\int f(x^*, \lambda^*) = 0 \tag{11a}$$

$$\begin{cases} f_x(x^*,\lambda^*).v = 0 \tag{11b} \end{cases}$$

$$\|v\| = 1 \tag{11c}$$

Equation (11a) represents a set of power flow equations,  $\mathbf{x}$  is a vector of system algebraic state variables, such as bus voltage magnitudes and angles,  $\lambda \in \mathbb{R}^k$  is parameter vector such as real and reactive powers at buses. Equation (11b) represents the saddle node bifurcation at voltage collapse point, at that point, the power flow Jacobian matrix  $f_x$  is singular and is vanished by a right eigenvector  $\mathbf{v}$ corresponding to the zero eigenvalue. (11c) is a normalization condition that shows the right eigenvector  $\mathbf{v}$  is not a zero vector. The whole equations characterize the SNB conditions of the generic static voltage collapse point. The information from the right eigenvector can be used to identify the critical buses. In this thesis, we use equation system (11) in step 1 to define the critical buses in case of single parameter, that mean k=1. Calculating scenario for step 1 is load at all buses increase proportionally to the initial loading level.

After that in Step 2.2, we use a variation of equation system (11) in case of k=2 to find exactly the closest collapse point corresponding to the worst load level at the critical bus. Note equation system (11) can be obtained by KKT conditions in

constraint Optimization problems with objective function is maximization of the Distance to Voltage Collapse such as [24-26].

Due to equation system (11) is non-linear system; we use Newton Raphson – Sydel to solve it.

$$\begin{bmatrix} f_x & 0 & f_\lambda \\ f_{xx} \cdot v & f_x & 0 \\ 0 & \frac{\partial \|v\|}{\partial v} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta v \\ \Delta \lambda \end{bmatrix} = -\begin{bmatrix} f(x,\lambda) \\ f_x \cdot v \\ \|v\| - 1 \end{bmatrix}$$
(12)

In equation (12) Hessian  $f_{xx}$  is a N×N×N tensor and that  $f_{xx}$  is a N×N matrix, where N is the dimension of Jacobian matrix of power flow problem. Solving the equation (12) requires good initial conditions, particularly for v. An efficient mean is to initiate the eigenvector of the power flow Jacobian matrix corresponding to the minimal eigenvalue by using inverse power method [8] at the current operating point.

# 4.2. The New Equivalent Model and Build the Rough Boundary

In power system, The Thevenin equivalent theory is used for short circuit calculation. And now a day with the deploying the Phasor Measurement Unit (PMU) in power system, Thevenin equivalent model is receiving some attentions related to the local voltage stability assessment [21-23].Based on the equivalent model of the original system, voltage stability indices (VSI) are defined such as

$$VSI_{i} = 1 - \frac{|Z_{Load,equi,i}|}{|Z_{The,i}|} [22][32]$$
(13a)

Or 
$$VSI_i = \frac{(E_{th} - Z_{th} \cdot I_{th})^2}{4Z_{th}}$$
 [31] (13b)

Or 
$$VSI_i = \frac{P_{i,\max} - P_{i,0}}{P_{i,\max}}$$
 [22] (13c)

Voltage instability occurs when the indices approach the critical value (such as in case of (13a)). The voltage stability assessment based on those indices has some drawbacks. The firstly, the critical criterion of voltage instability based on those indices is that the absolute value of equivalent load impedance becomes equal to the absolute value of the Thevenin equivalent impedance [1][22][32]. That just

considers the maximum deliverable real power condition that means it just mentions the role of real power in voltage instability problem. That is not directly applicable to power systems because in power systems both real and reactive powers affect to the voltage instability, especially the role of reactive power is more clearly and overwhelmed that of real power. The secondly, they are inconsistent in model of load. They use the impedance model to represent load in calculating, but in the final criterion and conclusion (the definition of indices), they use the power load model such as in (13c). Those drawbacks will be overcome thoroughly by the proposed method in this thesis.

There are many methods to calculate the Thevenin equivalent parameters. In [23] Khoi Vu applied the traditional curve-fitting technique that use data in two consecutive measurements; Thevenin equivalent parameters are calculated by using data from at least two consecutive measurements of voltage and current. If more than two sets of voltage and current measurements are obtained, the Thevenin equivalent parameters can be estimated by using the least square method. [21-22] reviewed almost existing methods to calculate the Thevenin equivalent parameters, most of them use impedance load model. Again, remind that the methods use the impedance load model that is not suitable for static

voltage instability [22].

This thesis proposes a new method to define Thevenin parameters based on the power model and information of load flow calculations for the critical bus in two base cases. No-load case is used to calculate the equivalent voltage. The other case is currently consumed load case that calculates the equivalent impedance. After that the maximum deliverable apparent powers can be obtained by the extrapolation and the singularity condition of voltage equation at the critical bus. The more accuracy the result will be if the larger the currently consumed load is.



Fig.6. The original system

In the equivalent two-bus system shown in Fig. 7, the critical bus of the original system is kept intact but the rest of the system is replaced by its Thevenin equivalent circuit. We claim that when the system is represented by such an equivalent circuit, a generator can be modeled by a terminal with constant

voltage source having zero reactance (X=0) and given real power, i.e., PV bus, while a load can be modeled as having constant real and reactive powers, i.e., PQ bus. That means we use load flow model for calculating Thevenin parameters.



Figure. 7. The Thevenin Equivalent System seen from Bus k

"Equivalent" term represents the calculated objective values have to equal in both of models, the original model and the equivalent model. Therefore before giving the method to calculate the equivalent parameters, it is necessary to determine the results that are the same as in the both models. In here we choose two cases to hold the same as objective values in both models, those are no-load case and the current consumed load case. No-load case is to calculate Thevenin voltage and the currently consumed load case to calculate Thevenin impedance.

As the definition of Thevenin voltage is open-circuit voltage at the candidate bus. In power flow applications of power system, open-circuit at one bus means that no-load (or no-power injection) at that bus. Therefore Thevenin voltage is the voltage at the candidate bus obtained by the solution of power flow in base case without load at that bus.

$$V_{k,Th} = V_k^0 \tag{14}$$

To improve the accuracy of calculating of Thevenin impedance, its value should change following the changing of system conditions and load at that bus. Therefore The Thevenin impedance will be calculated as following.

From Fig. 7, we have

$$\begin{split} \frac{V_{k}^{0} \angle \delta^{0} - V_{k} \angle \delta}{\left(R_{k,Th} + jX_{k,Th}\right)} &= \frac{P_{k} - jQ_{k}}{V_{k} \angle -\delta} \\ \rightarrow R_{k,Th} + jX_{k,Th} &= \frac{V_{k} \angle -\delta.(V_{k}^{0} \angle \delta^{0} - V_{k} \angle \delta)}{P_{k} - jQ_{k}} \\ &= \frac{V_{k}.V_{k}^{0} \angle (\delta^{0} - \delta) - V_{k}^{2}}{P_{k} - jQ_{k}} \\ &= \frac{V_{k}.V_{k}^{0}.S_{k} \angle (\delta^{0} - \delta + \phi)}{P_{k}^{2} + Q_{k}^{2}} - \frac{V_{k}^{2}.(P_{k} + jQ_{k})}{P_{k}^{2} + Q_{k}^{2}} \\ &= \frac{V_{k}.V_{k}^{0}.S_{k}.\cos(\delta^{0} - \delta + \phi) - V_{k}^{2}.P_{k}}{P_{k}^{2} + Q_{k}^{2}} + j\frac{V_{k}.V_{k}^{0}.S_{k}.\sin(\delta^{0} - \delta + \phi) - V_{k}^{2}.Q_{k}}{P_{k}^{2} + Q_{k}^{2}} \end{split}$$

Therefore

$$R_{k,Th} = \frac{V_k . V_k^0 . S_k . \cos(\delta^0 - \delta + \phi) - V_k^2 . P_k}{P_k^2 + Q_k^2}$$

$$X_{k,Th} = \frac{V_k . V_k^0 . S_k . \sin(\delta^0 - \delta + \phi) - V_k^2 . Q_k}{P_k^2 + Q_k^2}$$
(15)

In where  $S_k \angle \phi = P_k + jQ_k$  is load at bus k

 $V_k^0 \angle \delta^0$  is no-load (open-circuit) voltage at bus k

### $V_k \angle \delta$ is voltage at bus k when consumed load is $P_k + jQ_k$

 $V_k \angle \delta$  is obtained by load flow calculation in case the consumed load at bus k is  $P_k+jQ_k$ . Therefore, in general Thevenin impedance is a function of load and system conditions. In addition, since the shape of voltage stability boundary depends on the exact of Thevenin parameters, the accurate calculation of Thevenin parameters becomes an important key to the proposed method. There are some advantages of the new equivalent model. It guarantees the consistent of load model in voltage instability problem (power load model). Therefore it preserves the non-linear native of voltage instability phenomena. Beside that it represents both the role of real and reactive powers on the voltage instability phenomena. It is easy to update the equivalent impedance when knowing the currently consumed load, the more accuracy the result will be if the larger the currently consumed load, because the extrapolation point will be closer to the

SNB point (the maximum deliverable apparent power point) especially when the consumed load is maximum, the result is the most accuracy for applying to build the loadability boundary (or the static voltage stability boundary).

After getting equivalent parameters, apply them into Eq. (3) in session 2.3 (the native analysis of voltage stability phenomena) of the two bus system, we get the approximated voltage stability boundary that gives the native "shape" of the boundary but not yet exactly in analytical equation. The boundary equation will become more exactly, more nearly actual by displacing process that will be presented in next sections.

### 4.3. Find the Closest SNB Point of Collapse



#### 4.3.1. Direct Method

Fig.8. The closest SNP point

Because the static voltage stability boundary is saddle node bifurcation set, each point on the boundary is a saddle node bifurcation point corresponding to a given instability scenario (that means power factor vary), The question is which two collapse points (each point corresponding to each method, direct method and Thevenin equivalent model) we should choose to create the displacing vector  $\vec{D}$ ? The answer is we should choose the closest SNB point because direction along normal vector at the closest SNB point is the worst case load power margin for voltage collapse.

In section 3.2.2, we mentioned about the relationship between the left eigenvector and the normal vector of the SNB set at the SNB point (Eq. 10). In addition the closest SNB point of collapse is intersection point between the line perpendicular with the boundary and the boundary. Therefore that line is parallel with the normal vector at the closest SNB point. The equation of that line is

$$(\lambda^* - \lambda^0) - \alpha f_{\lambda}^T . w^T = 0$$
<sup>(16)</sup>

To find exactly the closest SNB point, the Eq.16 has to be added to the modified Eq.11 (SNB conditions are written with the left eigenvector), as following:

$$\int f(x,\lambda) = 0 \tag{17a}$$

$$\left[w.f_{x}(x^{*},\lambda^{*})\right]^{T} = 0$$
(17b)

$$\left(\lambda^* - \lambda^0\right) - \alpha f_{\lambda}^T w^T = 0$$
(17c)

$$\|w\| = 1 \tag{17d}$$

where, *w* is left eigenvector,  $f_{\lambda}^{T}$ ,  $w^{T}$  is the normal vector at saddle node bifurcation point,  $\alpha$  is scaling parameter. Instead of using the right eigenvector *v* we use the left eigenvector *w* corresponding to the zero eigenvalue in Eq.17b to represent the singular condition at the saddle node bifurcation point. The left eigenvector provides valuable information regarding the geometry of the bifurcation. Geometrically, it parallels with the normal vector of the saddle node bifurcation set at  $(x^*, \lambda^*)$  in parameter space, this is represent in Eq.17c. In step 2.2.of the proposed method in this thesis we claim that the actual boundary obtained from displacing the rough boundary along a displacing vector should contain the closest saddle node bifurcation point. Therefore to compute the closest SNB point equation system (17) is solved by Newton Raphson – Sydel method similar to the above one.

### 4.3.2. The New Equivalent Method

The static voltage stability boundary is quadratic curve in two dimensional power parameter space, the general quadratic equation is

$$\mathbf{Q}(\mathbf{z}) = \mathbf{z}^{\mathrm{T}} \mathbf{A} \mathbf{z} + \mathbf{b}^{\mathrm{T}} \mathbf{z} + \mathbf{c} = 0$$
(18)

where **A** is a symmetric k×k matrix, **b** is an k×1 vector and **c** is scalar. The parameter **z** is an k×1 vector, in this thesis, k=2.Given the curve  $Q(\mathbf{z}) = 0$  and an operating point **y**, find the distance from **y** to the boundary curve and compute a closest point **z**. Geometrically, the closest point **z** on the curve to **y** must satisfy the condition that  $||\mathbf{y} - \mathbf{z}||$  is normal to the curve. Since the curve gradient  $\nabla Q(\mathbf{z})$  is normal to the curve, the algebraic condition for the closest point is:

$$\mathbf{z} = (\mathbf{I} + 2t\mathbf{A})^{-1}(\mathbf{y} - t\mathbf{b}) \tag{19}$$

And the distance from a given operating point to the boundary curve can be calculated as following:

$$\|\mathbf{y} - \mathbf{z}\| = t \nabla \mathbf{Q}(\mathbf{z}) = t (2\mathbf{A}\mathbf{z} + \mathbf{b})$$
(20)

in where *t* is scaling parameter (similar  $\alpha$  in section 4.3.1).

### **4.4. Displace the Boundary (Improve Approximation)**

Expression (3) is represented in P-Q coordinate. Two collapse points, one is obtained from direct method (exact point), and the other (approximated point) is

obtained from the new equivalent model. The vector that links two collapse points called is displacing vector to move the rough boundary (expression (3)) obtained from the equivalent two bus model to become the actual static voltage stability boundary of the critical bus.

$$\vec{D} = \left(P_1^* - P_2^*\right)\vec{i_P} + \left(Q_1^* - Q_2^*\right)\vec{i_Q}$$
(21)

In where  $\vec{i_p}$  and  $\vec{i_Q}$  are the unit vectors corresponding to real power axis and reactive power axis.

Therefore the horizontal and vertical components of the displacing vector are:

$$\begin{cases} D_P = P_1^* - P_2^* \\ D_Q = Q_1^* - Q_2^* \end{cases}$$
(22)

where  $(P_1^*, Q_1^*)$  is the exact closest collapse point obtained by direct method,  $(P_2^*, Q_2^*)$  is the closest collapse point obtained by Thevenin equivalent model of the critical bus. Substitute (22) into (3), we get the expression of actual boundary in two dimensional parameter space as following:

$$X^{2}(P-D_{P})^{2} + R^{2}(Q-D_{Q})^{2} - 2RX(P-D_{P})(Q-D_{Q}) + RE^{2}(P-D_{P}) + XE^{2}(Q-D_{Q}) - \frac{E^{4}}{4} = 0$$
(23)

or 
$$X^2 P^2 + R^2 Q^2 - 2RXPQ + d.P + e.Q + f = 0$$
 (24)

In where  $d = -2X^2 \cdot D_P + 2RX \cdot D_Q + RE^2$ 

$$e = -2R^{2}.D_{Q} + 2RX.D_{P} + XE^{2}$$
$$f = X^{2}.D_{P}^{2} + R^{2}.D_{Q}^{2} - 2RX.D_{P}.D_{Q} - RE^{2}.D_{P} - XE^{2}.D_{Q} - E^{4}/4$$

Equation (24) is the most nearly actual boundary of the static voltage boundary in two dimensional power parameter spaces corresponding to the critical bus



Fig.9. The Displacing vector

In figure 9, the dashed curve is the rough boundary obtained by Thevenin Equivalent model. The solid curve is the "actual" boundary obtained by moving

the rough boundary along the displacing vector. The green point is the approximated closest SNB point obtained by Thevenin Equivalent model. The blue point is the exact closest SNB point obtained by direct method. The red vector is the displacing vector.

# 4.5. Monitoring and Assessing Voltage Stability via the Boundary and Performance Index (Margin Distance) (Step 3)

The distance from a given operating point to the voltage stability boundary in parameter space gives a security margin regarding voltage collapse. Thus, in order to preserve a safe power system operation, the distance to the boundary from the current loads ( $P_0$ ,  $Q_0$ ) can be monitored; from that operators can make preventive control decisions to avoid a possible collapse of the system.

In this thesis the distance from a given operating point to the boundary curve considered as performance index to assess static voltage stability and can be calculated as in part 4.3

Performance index = 
$$\|\lambda^* - \lambda^0\|$$
 (25)

In addition we can monitor the changing of load at the critical bus in real time through the visual boundary in interface screen such as in figure 10.



Fig.10. The monitoring and assessing of voltage stability via the boundary.

In figure 10, the red point is the closest SNB point, the blue points are other SNB points, the red vector measures the distance from the current operating point to the actual boundary, it can be considered as a performance index for assessing voltage stability.

# Chapter 5. Simulation and Analysis

The proposed method is tested on New England 39 bus system



Fig.11. New England 39 bus system

Step 1 with the scenario is the loads at buses are increased proportional to their initial load levels to define the critical buses. In the New England 39-bus test system, there are 29 load buses, 9 generation buses and bus 31 is chosen as the slack bus. Therefore the dimension of right eigenvector is  $67 \times 1$ . The simulation result for step 1 is showed in table 1 and figure 12. From table 1 and figure 4, we can see that  $d\delta_8 = -0.2130$  (index in the right eigenvector is 8th) and  $dV_8 = -0.1181$  (index in the right eigenvector is 46th) are the biggest components in magnitude in comparison with the corresponding others of the right eigenvector. Therefore bus number 8 is considered as the critical bus

No	Components of	Value	Magnitude
	the right eigenvector		
1	$d\delta_1$	-0.1900	0.1900
2	$d\delta_2$	-0.1719	0.1719
3	$d\delta_3$	-0.1826	0.1826
4	$d\delta_4$	-0.1928	0.1928
5	$d\delta_5$	-0.1432	0.1432

Table 1. The right eigenvector and the weakest bus

$d\delta_6$	-0.1199	0.1199
$d\delta_7$	-0.1948	0.1948
dõ <sub>8</sub>	-0.2130	0.2130
dδ9	-0.1998	0.1998
$d\delta_{10}$	-0.0706	0.0706
$d\delta_{11}$	-0.0850	0.0850
$d\delta_{12}$	-0.0886	0.0886
$d\delta_{13}$	-0.0883	0.0883
$d\delta_{14}$	-0.1383	0.1383
$d\delta_{15}$	-0.1712	0.1712
$d\delta_{16}$	-0.1565	0.1565
$d\delta_{17}$	-0.1683	0.1683
$d\delta_{18}$	-0.1736	0.1736
$d\delta_{19}$	-0.1311	0.1311
$d\delta_{20}$	-0.1344	0.1344
$d\delta_{21}$	-0.1329	0.1329
$d\delta_{22}$	-0.1063	0.1063
$d\delta_{23}$	-0.1072	0.1072
	$d\delta_6$ $d\delta_7$ $d\delta_8$ $d\delta_9$ $d\delta_{10}$ $d\delta_{10}$ $d\delta_{11}$ $d\delta_{12}$ $d\delta_{12}$ $d\delta_{13}$ $d\delta_{13}$ $d\delta_{14}$ $d\delta_{15}$ $d\delta_{16}$ $d\delta_{16}$ $d\delta_{16}$ $d\delta_{17}$ $d\delta_{18}$ $d\delta_{18}$ $d\delta_{19}$ $d\delta_{20}$ $d\delta_{21}$ $d\delta_{22}$ $d\delta_{23}$	$d\delta_6$ -0.1199 $d\delta_7$ -0.1948 $d\delta_8$ -0.2130 $d\delta_8$ -0.2130 $d\delta_9$ -0.1998 $d\delta_{10}$ -0.0706 $d\delta_{11}$ -0.0850 $d\delta_{12}$ -0.0886 $d\delta_{13}$ -0.0883 $d\delta_{14}$ -0.1383 $d\delta_{15}$ -0.1712 $d\delta_{16}$ -0.1565 $d\delta_{17}$ -0.1683 $d\delta_{18}$ -0.1736 $d\delta_{19}$ -0.1311 $d\delta_{20}$ -0.1329 $d\delta_{21}$ -0.1063 $d\delta_{23}$ -0.1072

24	$d\delta_{24}$	-0.1558	0.1558
25	$d\delta_{25}$	-0.1713	0.1713
26	$d\delta_{26}$	-0.1703	0.1703
27	$d\delta_{27}$	-0.1786	0.1786
28	$d\delta_{28}$	-0.1589	0.1589
29	$d\delta_{29}$	-0.1538	0.1538
30	$d\delta_{30}$	-0.1686	0.1686
31	$d\delta_{32}$	-0.0135	0.0135
32	$d\delta_{33}$	-0.1271	0.1271
33	$d\delta_{34}$	-0.1321	0.1321
34	$d\delta_{35}$	-0.0988	0.0988
35	$d\delta_{36}$	-0.0944	0.0944
36	$d\delta_{37}$	-0.1648	0.1648
37	$d\delta_{38}$	-0.1499	0.1499
38	$d\delta_{39}$	-0.1959	0.1959
39	$dV_1$	-0.0117	0.0117
40	dV <sub>2</sub>	-0.0270	0.0270
41	dV <sub>3</sub>	-0.0600	0.0600

42	$dV_4$	-0.1019	0.1019
43	$dV_5$	-0.1095	0.1095
44	$dV_6$	-0.1066	0.1066
45	dV <sub>7</sub>	-0.1164	0.1164
46	$dV_8$	-0.1181	0.1181
47	dV <sub>9</sub>	-0.0468	0.0468
48	$dV_{10}$	-0.0867	0.0867
49	$dV_{11}$	-0.0953	0.0953
50	$dV_{12}$	-0.1058	0.1058
51	dV <sub>13</sub>	-0.0917	0.0917
52	$dV_{14}$	-0.0950	0.0950
53	dV <sub>15</sub>	-0.0723	0.0723
54	$dV_{16}$	-0.0545	0.0545
55	$dV_{17}$	-0.0536	0.0536
56	$dV_{18}$	-0.0563	0.0563
57	dV <sub>19</sub>	-0.0200	0.0200
58	dV <sub>20</sub>	-0.0120	0.0120
59	dV <sub>21</sub>	-0.0466	0.0466

60	dV <sub>22</sub>	-0.0272	0.0272
61	dV <sub>23</sub>	-0.0288	0.0288
62	$dV_{24}$	-0.0522	0.0522
63	dV <sub>25</sub>	-0.0217	0.0217
64	dV <sub>26</sub>	-0.0333	0.0333
65	dV <sub>27</sub>	-0.0445	0.0445
66	$dV_{28}$	-0.0194	0.0194
67	dV <sub>29</sub>	-0.0135	0.0135



Fig.12. The right eigenvector corresponding to zero eigenvalue in the given scenario.

After defining the critical bus, we continue the step 2. The result for step 2 is the actual boundary expression in quadratic form of P and Q parameters.

The Thevenin voltage  $V_{Th} = 0.9809 \angle 0.0163$ , Thevenin impedance  $Z_{Th} = 0.00682 + j0.02925$ . The closest SNB points obtained by methods, the direct method and Thevenin method is showed in the table 2.

	Direct	The new	Error
	Method	equivalent method	(%)
The closest SNB point	(836.28;	(758.50;	10.53
(MW,MVAr)	602.14)	524.21)	
The collapse voltage (pu)	0.5079	0.5258	3.52

Table 2: The closest SNB points obtained by both methods

The error due to in calculation we use the constant Thevenin impedance that represent the linear native of Thevenin equivalent theory. This error is corrected apart in next process in our method. The rough boundary equation that is defined from Thevenin model is:

$$(0.8556P^2 - 0.399PQ + 0.0465Q^2) \cdot 10^{-3} + 0.0066P + 0.0281Q - 0.2314 = 0$$
(26)

The displacing vector  $\vec{D}$  is (0.7778; 0.7793). The "actual" boundary equation that is obtained by moving the above rough boundary along the displacing vector is:

$$(0.8556P^2 - 0.399PQ + 0.0465Q^2) \cdot 10^{-3} + 0.005542P + 0.028381Q - 0.25817 = 0$$
(27)

The visual interface for both boundaries is showed in figure 13.



Fig. 13. The rough and actual static voltage stability boundaries at the critical bus (bus 8)

In figure 13, the inside red curve is the rough boundary obtained from Thevenin Equivalent two bus seen from bus 8, with Thevenin voltage and Thevenin impedance parameters are calculated above, the outside pink curve is the "actual" boundary that obtained from moving the rough boundary along the displacing vector  $\vec{D}$ . From the rough and "actual" boundaries, we can see that the Thevenin equivalent model underestimates the shape of the boundary. The error is corrected by the displacing vector $\vec{D}$ .

Step 3 is assessing the voltage stability via the performance index that is the distance from the current operating point to the "actual" boundary. Simulation result for security distance from the current operating point (522 MW; 176.6 MVAr) to the actual boundary at bus 8 is 529.01 MVA, the corresponding closest SNB point is (836.28 MW, 602.14 MVAr). The worst load increasing scenario is the direction along the red bold vector. In addition we can be easy to monitor the volatile of load at bus 8, any changing direction, any changing of power factor. The changes are monitored through the visual interface screen in control center.

## **Chapter 6. Conclusion and Future Work**

## **6.1.** Conclusion

The voltage stability assessment and monitoring is one of the most important problems in the electric power system operation. It is desirable that this assessment can be performed in real-time while the boundary of voltage stability region is determined accurately. In this thesis we propose a new method for assessing static voltage stability via distance from the current operating point to the Saddle Node Bifurcation Set (or the static voltage stability boundary) in two dimensional power parameter spaces. The boundary is determined based on the combination of direct method and Thevenin equivalent two bus model of the original system seen from the critical bus. The critical bus is defined through the right eigenvector corresponding to the zero eigenvalue of Jacobian matrix obtained from Saddle Node Bifurcation condition of power system model. The proposed method is simple, fast and relatively accurate. It makes use of the quadratic equality that represents the static voltage stability boundary for the

critical bus in system and that may be easily plotted in two dimensional power parameter space. It is also shown that it is sufficient to determine the voltage stability margin at the critical bus for the given initial operating conditions. In addition the Thevenin voltage and Thevenin impedance are defined based on the constant power model of load, which keeps the integrity of model in finding voltage stability boundary. The effectiveness of the proposed method is demonstrated on New England 39 bus system.

### **6.2. Future Work**

The effect of the reactive power limits of generator and other compensate devices should be studied. Then the Transcritical Bifurcation (called as limit inducted bifurcation in power system) should be applied in voltage instability analysis. In addition combination with the historical and forecast information of loads, build the static voltage stability boundary associated with scenarios that are most suitable with those information. Improve the method to calculate the Thevenin parameters more accuracy such as using the varying Thevenin impedance model that can represent the equivalent system more similar to initial system.

## REFERENCES

 T. Van Cutsem; "Voltage Stability of Electric Power Systems", KluwerAcademic Publishers, 1998, pp214-255

[2] V. Ajjarapu; "Computational Techniques for Voltage Stability Assessment and Control", Springer, 2006, pp 124-128

[3] P. Kundur, "Power System Stability and Control", McGraw-Hill, 1994, pp969-1012.

[4] J.H. Eto, M. Parashar, Y.V.Makarov, I.Dobson, "Real Time System Operation
 2006 – 2007: Real-Time Voltage Security Assessment Report on Algorithms and
 Framework" Prepared By: Lawrence Berkeley National Laboratory

[5] Y.V.Makarov, S.Lu, X.Guo, J.Gronquist, P.Du, T. Nguyen, J. Burns, "Wide Area Security Region Final Report", Prepared By: Pacific Northwest National Laboratory, March 2010.

[6] P. Kundur, J.Paserba, V. Ajjarapu, G. Anderson, A. Bose, C.A.Canizares, N.Hatziargyriou, D. Hill, A. Stankovic, C. Taylor, T.V. Cutsem, V. Vittal,

"Definition and Classification of Power System Stability", IEEE/CIGRE Joint Task Force on Stability Terms and Definitions, *IEEE Transaction on Power Systems*, Vol. 19, No. 2, May 2004.

[7] I.Dobson, T.Van Cutsem, C.Vournas, C.L.DeMarco, M.Venkatasubramanian,
 T.Overbye, C.A.Canizares, "Voltage Stability Assessment: Concepts, Practices and Tools", IEEE-PES Power Systems Stability Subcommittee Special Publication SP101PSS, IEEE-PES General Meeting, Toronto 2003.

[8] J.L. Buchanan, "Numerical methods and analysis", New York, McGraw-Hill, 1992

[9] R.Seydel, "From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis", Elsevier, 1988.

[10]I. Dobson, L.Lu, "Using an Iterative Method to Compute a Closest Saddle Node Bifurcation in Load Power Parameter Space of an Electric Power System", Bulk Power System Voltage Phenomena, Voltage Stability and Security NSF Workshop, Deep Creek Lake, August 1991.

[11]. I. Dobson, L.Lu "New Methods for Computing a Closest Saddle NodeBifurcation and Worst Case Load Power Margin for Voltage Collapse", *IEEE* 

Transactions on Power Systems, Vol. 8. No. 3. August 1993. pp. 905-913.

[12] I.Dobson, "Computing a Closest Bifurcation Instability in Multidimensional Parameter Space", *Journal of Nonlinear Science*, Vol 3. pp.307-327 (1993).

[13] I. Dobson, "Observations on the geometry of saddle node bifurcation and voltage collapse in electric power systems", *IEEE Transactions on Circuits and Systems*, Part1: Fundamental Theory and Applications, Vol. 39, No. 3, March 1992, pp. 240-243.

[14] Yuri V. Makarov, David J.Hill; "Computation of Bifurcation Boundaries for Power Systems: A new  $\Delta$ -Plane Method"; *IEEE Transactions on circuits and systems*; Volume 47, Issue 4. April 2000, Pages 536-544.

[15] Y. V. Makarov and Ian A. Hiskens, "A continuation method approach to finding the closest saddle node bifurcation point," in Proc. NSF/ECC Workshop Bulk Power System Voltage Phenomena III, Davos, Switzerland, Aug. 1994

[16] Yuri V. Makarov, Zhao Yang Dong, David J. Hill, "On Convexity of Power Flow Feasibility Boundary", *IEEE Transaction on Power Systems*, Vol. 23, No. 2, May 2008, pp. 811-813.

[17] Bernard C. Lesieutre, Ian A. Hiskens, "Convexity of the Set of Feasible
Injections and Revenue Adequacy in FTR Markets", *IEEE Transaction on Power Systems*, Vol. 20, No. 4, November 2005, pp. 1790-1798

[18] Ian A. Hiskens, Robert J.Davy, "Exploring the Power Flow Solution Space Boundary", *IEEE Transaction on Power Systems*, Vol. 16, No. 3, August 2001, pp. 389-395.

[19] A. C. Z. de Souza, C. A. Canizares, and V. H. Quintana, "Critical bus and point of collapse determination using tangent vectors"; Proc. NAPS, M.I.T., November 1996, pp. 329-333.

[20] I. Musirin, T.K. Abdul Rahman, "Estimating Maximum Loadability for Weak Bus Identification Using FVSI", *IEEE Power Engineering Review*, November 2002, pp. 50-52.

[21] P.Nagendra, T.Datta, S. Halder, S.Paul, "Power System Voltage Stability Assessment Using Network Equivalents- A Review", *Journal of Applied Sciences*, 2010.

[22] J. Zhao, Y. Yang, Z. Gao, "A Review on Online Voltage Stability Monitoring Indices and Methods based on Local Phasor Measurements", 17th Power Systems Computation Conference, Stockholm Sweden, August 22-26, 2011. [23] Khoi Vu, Miroslav M. Begovic, Damir Novosel, Murari Mohan Saha, "Use of Local Measurements to Estimate Voltage-Stability Margin", *IEEE Transactions on Power Systems*, Vol. 14, No. 3, August 1999, pp. 1029-1035.

[24] C. L. Canizares, "Calculating optimal system parameters to maximize the distance to saddle node bifurcations", *IEEE Transactions on Circuits and Systems*, Vol. 45, No. 3, March 1998, pp. 225-237.

[25] C. L. Canizares, "Applications of Optimization to Voltage Collapse Analysis", IEEE/PES Summer Meeting, San Diego, July, 1998.

[26] G. D. Irisarri, X. Wang, J. Tong, and S. Moktari, "Maximum loadability of power systems using interior point non-linear optimization method " *IEEE Trans. Power Systems*, vol. 12, no. 1, February 1997, pp. 162-172.

[27] V.Venkatasubramanian, H. Schättler, J. Zaborszky, "Local Bifurcations and Feasibity Regions in Differential-Algebraic Systems". *IEEE Transactionson Automatic Control*, Vol. 40, No. 12, December 1995.

[28] Hsiao-Dong Chiang, "Application of Bifurcation Analysis to Power Systems", Lecture Notes in Control and Information Sciences, 2003, Volume 293/2003, Springer. [29] U.S. - Canada Power System Outage Task Force, "Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations", April 2004.

[30] Y.Kataoka, "A Probabilistic Nodal Loading Model and Worst Case Solutions for Electric Power System Voltage Stability Assessment", *IEEE Transactions on Power Systems*, Vol. 18, No. 4, November 2003.

[31] J.H. Eto, M. Parashar, Y.V.Makarov, I.Dobson, "Real Time System Operation 2006 – 2007, Appendix A: Phasor Technology Applications Feasibility Assessment and Research Results Report".

[32] Ivan Smon, G. Verbic, F. Gubina, "Local Voltage Stability Index using Tellegen's Theorem", *IEEE Transactions on Power Systems*, Vol. 21, No. 3, August 2006.

초록

본 논문은 새들 노드 분기점 세트 혹은 2 차원 전력 파라미터 공간에서 PQ 곡선을 사용하여 정적 전압 안정도를 평가하고 모니터링 하기 위한 새로운 방법을 제안한다. 그 방법에는 세 가지 주요 단계가 포함된다. 첫 번째 단계에서는 중요 버스를 결정하고, 두 번째 단계에서는 정적 전압 안정도 경계 또는 새들 노드 분기점 세트를 구축한다. 마지막 단계에서는 현재 운영점에서부터 경계까지의 거리를 통해서 전압 안정도를 평가한다. 여기서 중요 버스는 직접해법으로 우 고유 벡터를 통해서 정의된다. 정적 전압 안정성 영역의 경계는 표준 직접해법과 전력 시스템의 테브닌 등가 모델을 결합한 제안된 방법에 의해서 구해진 이차 곡선이다. 그리고 마지막으로 현재 운영점에서부터 경계까지의 거리는 가장 가까운 새들 노드 분기점에서 경계 법선벡터의 유클리드 기하 평균으로 계산된다. 제안된 방법은 두 가지 방법의 이점, 즉, 직접해법의 정확성과 테브닌 등가 모델의 단순성을 모두 가진다는 장점이 있다. 따라서 제안된 방법은 전력 시스템의 실시간 전압 안정도 평가에 관하여 몇 가지 약속을 보유한다. 또한 본 논문은 전압 안정도를 평가하고 조류 계산 어플리케이션에 적합한 테브닌 파라미터를 계산하기 위한 새로운 방법도 제공한다. 마지막으로 뉴잉글랜드 39 버스 시스템의 테스트 결과는 제안된 방법의 유효성을 보여주기 위해 제공된다.

주요어: 직접 방법, Loadability 경계, 안장 노드 분기, 테브닌등가, 전압 안정도 평가

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