工學碩士學位論文

Joint CDF-based Scheduling and Power Control in Wireless Networks

무선네트워크에서 CDF 기반 스케줄링 및 전력 제어 기법

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Abstract

This thesis studies joint scheduling and power control in wireless networks. It employs opportunistic scheduling in order to exploit multiuser diversity and adopts CDF-based scheduling (CS). The CS has the advantage of enabling tractable mathematical analysis by predicting scheduling performance based on a control parameter called user weight, which can be adjusted to meet the desired quality of service (QoS). The thesis formulates a problem of jointly deciding user weights and power that minimizes overall uplink transmit power while satisfying the data rate requirement. The formulated problem is not convex but can be decomposed into two subproblems which are convex. According to simulations the proposed scheme turns out to significantly reduce uplink transmit power, when compared with the schemes that perform user scheduling and power control separately.

Keywords: opportunistic scheduling, power control, convex optimization, wireless resource management.

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Contents

Chapter 1

Introduction

Recent rapid growth of demand for high data services has driven intense research and development efforts towards the next-generation wireless communication systems. One of the key technologies commonly perceived as a throughput enhancement technology is the opportunistic scheduling which increases system throughput by exploiting the so-called multiuser diversity, i.e., by allocating resource to the user with the most favorable channel state [1]. In particular, The CDF-based scheduling (CS) proposed by Park *et al.* [2], has drawn a lot of attention due to its analytical tractability [3]-[5]. The CS scheduler schedules a user whose channel state is least likely to be better, thereby naturally taking into account fairness among users. One of the distinct features of CS is that it has a control parameter called user weight, which determines scheduling time fraction and can be adjusted to meet various user-specific quality of service (QoS) such as minimum data rate requirement or maximum delay constraint. The most remarkable property of CS is that the probability of selecting a user depends only on the channel statistic of the user irrespective of that of other users. This decoupling effect is what enables an exact estimation of various performance values, such as throughput or transmit power consumption. Due to such analytical advantages, CS takes a prominent position to work with other resource management mechanisms, e.g., power control which is an important issue in the next-generation wireless communication system. When dealing with scheduling, it is important to implement scheduling and power control jointly especially when the scheduling being considered takes fairness or QoS into consideration. The reason is that in such scheduling schemes, it may be possible that a user whose channel is not absolutely good is scheduled, as is the case in the CS algorithm. It may cause large transmit power if power control is done carelessly to meet the required QoS. For example, if QoS is given in terms of minimum data rate requirement, power control can be done in a way that meets the required data rate at any time, which may result in large transmit power. It gives rise to severe interference to its neighboring cells and causes a waste of limited battery of mobile station (MS). Therefore, it is important to manage power control in a clever manner along with user scheduling.

Liu *et al.* [6] considered an interference-based joint scheduling and power control scheme for a multi-celluar network. They presented an optimal solution under general setting but the convergence and optimality of the iterative method used to find the solution were not addressed. Bhorkar *et al.* [7] treated a joint scheduling and power control scheme under formulation similar to [6] and offered a stochastic algorithm to find the optimal solution. However the algorithm was based on the assumption that channel statistics of users are independent and identically distributed (i.i.d.), which is unrealistic in many circumstances.

This thesis presents a joint CS and power control scheme exploiting decoupling property of the CS algorithm. The proposed scheme minimizes overall uplink transmit power under the constraint of each user's data rate requirement. The formulated problem is not convex but can be decomposed into two subproblems, which can be called user problem and network problem, respectively, which can be proved to be convex. The numerical algorithm to solve them is valid even when the channel statistics of users are not i.i.d.

This thesis is organized as follows. First Chapter 2 describes the system model and briefly summarize several results associated with CS. Chapter 3 formalizes our main problem and decompose it into two subproblems. Chapter 4 and 5 describe our proposed scheme by providing solutions of subproblems. Chapter 6 examines the performance of the proposed scheme by simulation and confirms that the gain of the proposed joint scheme over the exiting separate shemes is significant. Chapter 7 concludes the thesis.

Chapter 2

System Model

We consider a single cell in a time-slotted system in which time is resource to be shared among users and at each time slot, exactly one user is allowed for transmission. Time-slotted code division multiple access (CDMA) or time division multiple access (TDMA) is an example of such a system. We consider the uplink transmission of a cell where K users are competing to get service. We assume that the BS measures channel gain from each user and quantizes it by a finite number of modulation and coding scheme (MCS) levels, $m = 1, \cdots, M$. We denote by G_m the corresponding quantized channel gain and by $m_k(n)$ the MCS level of a user k at time slot n. Thus $m_k(n)$ is a discrete random process taking one of a value in $\{1, \cdots, M\}$. We assume that each $m_k(n)$, $k = 1, \cdots, K$, is assumed to be stationary and independent of each other for analytical convenience.

As a background of the joint algorithm to be proposed, we briefly describe the parameters and properties of the CS algorithm. We denote by $q_{k,m}$ the cumulative mass function (CMF) of MCS level m_k. That is, $q_{k,m} = \sum_{i=1}^{m} p_{k,i}$ where $p_{k,i}$ denotes the probability mass function of MCS level m_k [1]. We assume that BS has perfect knowledge on $q_{k,m}$ by measuring channel for a certain period of time as described in [2]. In case $m_k(n)$ is stationary, BS estimation gets arbitrarily close to actual $q_{k,m}$ by the law of large numbers.

In the CS algorithm, to determine a user to be scheduled at slot n, the CS scheduler collects the current MCS level for each user $m_k(n)$ and generates a uniform random variable $U_k(n)$ in the interval $[q_{k,m_k(n)-1}, q_{k,m_k(n)})$. Then the scheduler selects a user whose scheduling metric $U_k(n)^{1/w_k}$ is the largest, where w_k is a control parameter called user weight. That is, the user to be scheduled at slot n, $k^*(n)$, is

$$k^*(n) = \arg\max_k U_k(n)^{1/w_k} \tag{2.1}$$

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[1] Since $m_k(n)$ is stationary and the CS scheduler is *memoryless*, all the relevant parameters are time-independent. So we drop off time index unless time is explicitly considered.

If we normalize the user weights such that $\sum_{k=1}^{K} w_k = 1$, then the probability for user k whose MCS level is $m_k = m$ to be scheduled at slot n is

$$\Pr\left(k^*(n) = k, m_k = m\right) = w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right), \tag{2.2}$$

where $q_{k,0}$ is defined to be 0 for notational convenience (see the derivation in [3]). Once scheduling probability is determined, the average transmit power P_k^{avg} and the average rate, R_k^{avg} may be calculated in a straightforward manner.

$$P_k^{avg} = \sum_{m=1}^{M} P_{k,m} \Pr\left(k^* = k,\ m_k = m\right) = \sum_{m=1}^{M} P_{k,m} w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right)$$

$$R_k^{avg} = \sum_{m=1}^{M} \log(1 + P_{k,m} G_m) \Pr(k^* = k, m_k = m) = \sum_{m=1}^{M} \log(1 + P_{k,m} G_m) w_k (q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k})$$

where $P_{k,m}$ is the transmit power allocated to a scheduled user k whose MCS level is $m_k = m$.

Note that the scheduling probability is irrelevant to other users' parameters, such as weights or channel statistics, and so are the average transmit power and the average rate. We may refer to this result as decoupling property. This is the very characteristic of CS which, as illuminated later, greatly simplifies the analysis of the problem.

Chapter 3

Problem Formulation

In this chapter, we formulate the problem where the objective is to minimize the overall transmit power while guaranteeing the required data rate. In terms of the performance parameters introduced in chapter 2, the problem may be formally stated as

$$\begin{aligned}
&\underset{\mathbf{P},\mathbf{w}}{\text{minimize}} \ \sum_{k=1}^{K}\sum_{m=1}^{M} P_{k,m} w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k} \right) \\
&\text{subject to} \ \sum_{m=1}^{M} \log\left(1 + G_m P_{k,m}\right) w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k} \right) \geq R_k, \quad k=1,\cdots,K \\
&\qquad \mathbf{w}^T \ \mathbf{1} = 1 \\
&\qquad \mathbf{w} \geq 0 \\
&\qquad P_{k,m} \geq 0, \ \ k=1,\cdots,K, \ \ m=1,\cdots,M \ ,
\end{aligned} \tag{3.1}$$

where $\mathbf{P} = (P_{k,m})$ is a $K \times M$ power allocation matrix, $\mathbf{w} = (w_1, \cdots, w_K)^{\mathrm{T}}$ is a weight allocation vector, and R_k is the required data rate for user k.

Note that the objective is convex in $\mathbf{P}$ but not in $\mathbf{w}$. This observation motivates the following approach: We treat $\mathbf{w}$ as arbitrarily given parameter and solve for $\mathbf{P}$ which minimizes the objective. That is,

$$\begin{aligned}
&\underset{\mathbf{P}=(\mathbf{P}_k,k=1,\cdots,K)}{\text{minimize}} \sum_{k=1}^{K} P_k^{avg}(\mathbf{P}_k) \\
&\text{subject to} \sum_{m=1}^{M} \log\left(1+G_m P_{k,m}\right) w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right) \ge R_k, \quad k=1,\cdots,K \\
&\qquad \mathbf{P}_k \ge 0, \qquad k=1,\cdots,K \\
&\qquad P_k^{avg}(\mathbf{P}_k) = \sum_{m=1}^{M} P_{k,m} w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right)
\end{aligned} \tag{3.2}$$

Where $\mathbf{P}_k$ denotes the k^{th} row of $\mathbf{P}$, i.e., the power allocation vector for user k, $\mathbf{P}_k = (P_{k,1}, P_{k,2}, \cdots, P_{k,M})$. Observing that $P_k^{avg}(\mathbf{P}_k)$ and constraints are dependent only on $\mathbf{P}_k$, problem (3.2) is equivalent to a set of subproblems in which each P_k^{avg} is minimized over $\mathbf{P}_k$.

$$\begin{aligned}
&\underset{\mathbf{P}_k}{\text{minimize}}\ P_k^{avg}(\mathbf{P}_k) \\
&\text{subject to} \sum_{m=1}^{M} \log\left(1+G_m P_{k,m}\right) w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right) \ge R_k, \\
&\qquad \mathbf{P}_k \ge 0, \\
&\qquad P_k^{avg}(\mathbf{P}_k) = \sum_{m=1}^{M} P_{k,m} w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right)
\end{aligned} \tag{3.3}$$

We refer to problem (3.3) as the *user problem*, since it is relevant for a given user k and is independent of other users' characteristics, such as weights or channel statistics.

Once the user problem is solved, we obtain an optimal power allocation $\mathbf{P}^*$ as a function of weights, $\mathbf{P}^* = \mathbf{P}^*(\mathbf{w}) = (\mathbf{P}_1^*(w_1), \cdots, \mathbf{P}_K^*(w_K))$. We then proceed to find $\mathbf{w}$ which yields a minimum overall transmit power.

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \ \sum_{k=1}^{K} \phi_k(w_k) \\ &\text{subject to} \ \mathbf{w}^T \ \mathbf{1} = 1, \\ &\qquad\qquad\quad \mathbf{w} \geq 0, \end{aligned} \tag{3.4}$$

where $\phi_k(w_k)$ denotes the minimum average transmit power of user k for a given $\mathbf{w}$ as obtained in the user problem. That is, $\phi_k(w_k) = P_k^{avg}(\mathbf{P}_k^*(w_k))$. We call the problem (3.4) as the *network problem*, since it determines weights so that the overall transmit power is minimized.

We provide a solution of each subproblem in the subsequent two chapters.

Chapter 4

User Problem

User problem is stated in (3.3) and is repeated for convenience.

$$\begin{aligned} & \underset{\mathbf{P}_k}{\text{minimize}} \; P_k^{avg}(\mathbf{P}_k) \\ & \text{subject to} \; \sum_{m=1}^{M} \log\left(1+G_m P_{k,m}\right) w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right) \geq R_k, \\ & \qquad\qquad \mathbf{P}_k \geq 0, \end{aligned} \tag{4.1}$$

The problem is convex and is solved by standard convex optimization technique [6]. Noting that $\mathbf{P}_k^*$ will satisfy the rate constraint with equality, its Lagrangian function takes the expression

$$L_k = \sum_{m=1}^{M} P_{k,m} w_k (q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}) - \lambda_k \left(w_k \sum_{m=1}^{M} \log(1+G_m P_{k,m})(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}) - R_k \right) - \sum_{m=1}^{M} \sigma_{k,m} P_{k,m} \; ,$$

where λ_k is an arbitrary real number and $\sigma_{k,m} \geq 0$ for $m = 1, \cdots, M$.

From optimality condition [8]

$$\frac{\partial L_k}{\partial P_{k,m}} = w_k (q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}) \left(1 - \lambda_k \frac{G_m}{1+G_m P_m}\right) - \sigma_{k,m} = 0 \qquad m = 1,\cdots,M$$

$$\sum_{m=1}^{M} w_k \log(1+G_m P_{k,m})(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}) = R_k \qquad (4.2)$$

$$\sigma_{k,m} P_{k,m} = 0 \quad m = 1,\cdots,M$$

We get

$$P_{k,m} = \max\left(\lambda_k - \frac{1}{G_m},\ 0\right) = \left(\lambda_k - \frac{1}{G_m},\ 0\right)^{+} \qquad m = 1,\cdots,M \qquad (4.3)$$

where λ_k is set such that it satisfies the rate constraint,

$$\sum_{m=1}^{M} \left(\log\left(\lambda_k G_{k,m}\right)\right)^{+} w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right) = R_k . \qquad (4.4)$$

This is a solution of the user problem for user k. If we solve (4.3)-(4.4) for each user k, then we get a collection of optimal power allocation matrix $\mathbf{P}^*(\mathbf{w})$ for each weight allocation vector $\mathbf{w}$.

Note that optimal power takes the water-filling form. We refer to this as water-filling power control (WFPC). This is in agreement with [6] with λ_k being given by (4.4). One can think of another, simple power control that makes the receive power equal at every channel realization. Assuming

$G_1 = 0$,

$$\sum_{m=1}^{M} \log\left(1+G_m P_{k,m}\right) w_k \left(q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}\right) = R_k \ ,$$

$$P_{k,m} = \frac{1}{G_m} 2^{\frac{R_k}{w_k (1-q_{k,1}^{1/w_k})}} \ , \tag{4.5}$$

We refer to power control according to (4.5) as equal receive power control (ERPC).

Note that ERPC allocates more power to a worse channel state, while WFPC allocates more power to a better channel state.

Chapter 5

Network Problem

Now that we have solved the user problem according to (4.4), the network problem may be formally stated as follows.

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \ \sum_{k=1}^{K} \phi_k(w_k) \\ &\text{subject to} \ \mathbf{w}^T \ \mathbf{1} = 1 \\ &\qquad\qquad\quad \mathbf{w} \geq 0 \\ &\qquad\qquad\quad \phi_k(w_k) = \sum_{m=1}^{M} \left(\lambda_k - \frac{1}{G_{k,m}} \right)^{+} w_k (q_{k,m}^{1/w_k} - q_{k,m-1}^{1/w_k}). \end{aligned} \tag{5.1}$$

If $\phi_k(w_k)$ is convex, then (5.1) is convex optimization. This is what truly happens and the proof is presented in Appendix. The proof uses only the elementary techniques of differentiation and algebraic manipulation but we put it in Appendix because it is lengthy due to some technical points.

Lagrangian function for (5.1) takes the form

$$L(\mathbf{w},\ \mu) = \sum_{k=1}^{K} \phi_k(w_k) - \mu\left(\sum_{k=1}^{K} w_k - 1\right), \tag{5.2}$$

where we drop off the Lagrange multipliers for nonnegativity constraints since w_k must be strictly positive[2]. Thus optimality condition reduces to

$$\begin{cases} \dfrac{d\phi_k}{dw_k}(w_k^*) = \mu, & k = 1, \cdots, K. \\ \mathbf{w}^{*\,\mathrm{T}}\,\mathbf{1} = 1. \end{cases} \tag{5.3}$$

We can interpret the above equation as follows. As shown in Appendix, $d\phi_k / dw_k$ is always negative. Then $|d\phi_k / dw_k|\Delta w$ is interpreted as an amount of transmit power saving arising from allocating additional small (positive) weight Δw to user k. Thus we call $u_k \equiv |d\phi_k / dw_k|$ marginal utility. Now we consider two users, user 1 and user 2, assuming that, without loss of generality, marginal utility of user 1 is larger than that of user 2, i.e., $u_1 > u_2$. Then by increasing weight of user 1 by Δw at the

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[2] If $w_k = 0$, then user k is never selected and the rate requirement is not satisfied.

sacrifice of deducting the same amount from that of user 2, the overall transmit power changes by $(u_1 - u_2)\Delta w$. Since $u_1 > u_2$, the overall transmit

Method of bisection for solving (5.3)

Initialize

$up = \max_k \phi_k(1/K)$, $down = \min_k \phi_k(1/K)$

$\mu = \frac{up + down}{2}$

$w_k \leftarrow (\phi_k^{'})^{-1}(\mu)$, $\forall k$

while $\left|\sum_k w_k - 1\right| \geq \varepsilon$

  if $\sum_k w_k > 1$, **then** $up \leftarrow \mu$

  if $\sum_k w_k \leq 1$, **then** $down \leftarrow \mu$

  Update

    $\mu \leftarrow (up + down)/2$

    $w_k \leftarrow (\phi_k^{'})^{-1}(\mu)$, $\forall k$

End while

Figure 5.1. Method of bisection for solving (5.3).

power reduces. This is why the optimal set of user weights must give the same marginal utility across users, as dictated in the first line of (5.3).

It is challenging to solve (5.3) for $\mathbf{w}^*$ in an explicit form, since (5.3) ϕ_k takes very complex form in w_k. We adopt a numerical method to find $\mathbf{w}^*$, noting that $d\phi_k / dw_k$ is monotonically increasing function of w_k due to convexity of ϕ_k. Once this point is observed, (5.3) can be solved numerically as follows. We solve (5.3) numerically by the method of bisection with $\max_k d\phi_k / dw_k|_{1/K}$, and $\min_k d\phi_k / dw_k|_{1/K}$ for its upper bound and lower bound, respectively. It is noteworthy that its effectiveness stems from the decoupling property of the CS algorithm discussed in Chapter 2. CS makes user's performance values dependent only on that specific user's parameters. It is this decoupling property that enables this simple numerical technique. We describe its pseudocode in Figure 5.1 where ε is a prescribed tolerance. We refer to our proposed scheme as *CS and water-filling power control* (CS-WFPC).

Chapter 6

Numerical Results

In this chapter, we report the numerical results obtained out of simulations to verify the performance of the proposed scheme. For the simulations we assumed a single cell in which five users are served. Each user experiences Rayleigh-fading channel, average SNR of which is as given in Table 1. The SNR thresholds and the associated MCS level are set according to the CDMA2000 1xEv-Do system [9] and are given in Table 2. Finally we assume that $R_1 = \cdots = R_K = R$, that is, the required data rate is the same for all users.

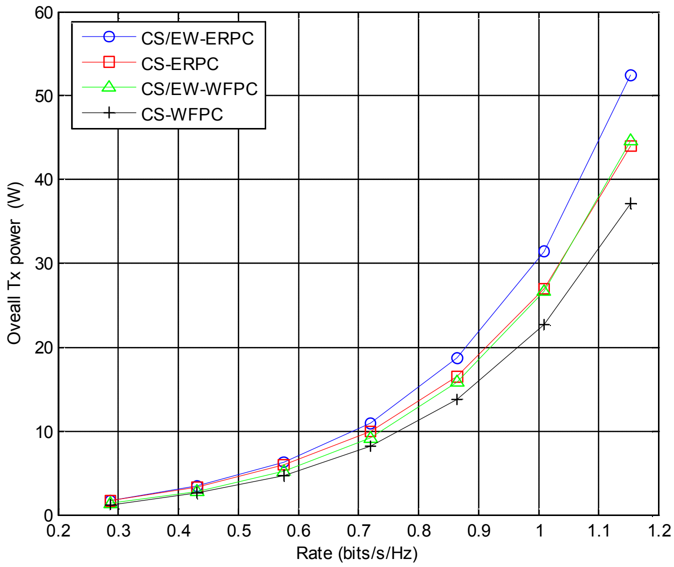
For the purpose of comparison, we also simulated the following three schemes along with the proposed scheme, CS-WFPC. First, in the scheme that we refer to as *CS with equal weight and equal recieve power control* (CS/EW-ERPC), user weights are all set to be equal and power control is

Table 6.1. Average SNR of each user.

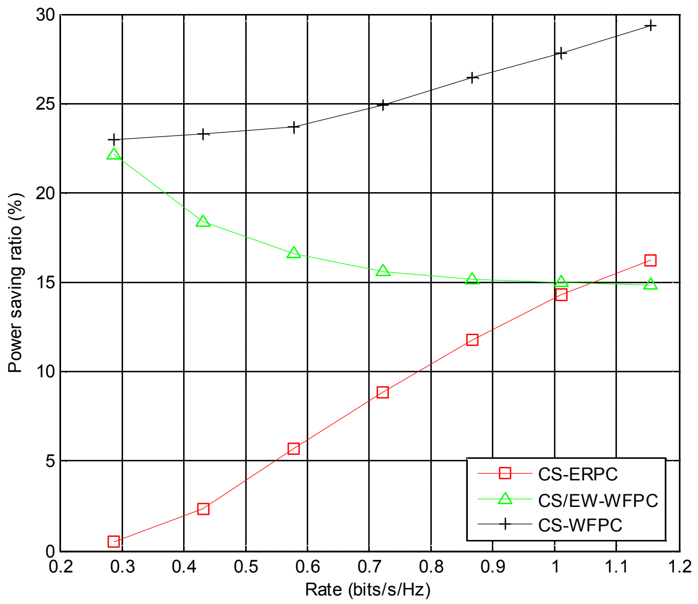
User	Average SNR(dB)
1	-5
2	-2
3	1
4	4
5	7

Table 6.2. Relationship between MCS levels and SNR thresholds in CDMA2000 1xEv-Do system [9].

MCS level	SNR threshold(dB)
1	-∞
2	-12.5
3	-9.5
4	-8.5
5	-6.5
6	-5.7
7	-4.0
8	-1.0
9	1.3
10	3.0
11	7.2
12	9.5



(a)



(b)

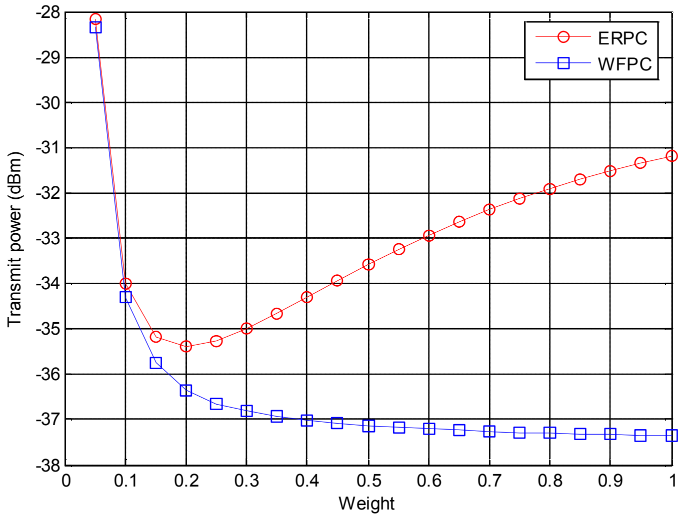
Figure 6.1. Performance comparison with respect to the required data rate: (a) Overall transmit power and (b) power saving ratio.

done by ERPC. We improve this scheme either by optimizing power control or user weights. The former is called *CS with equal weight and water-filling power control* (CS/EW-WFPC) and the latter, *CS and equal receive power control,* CS-ERPC.

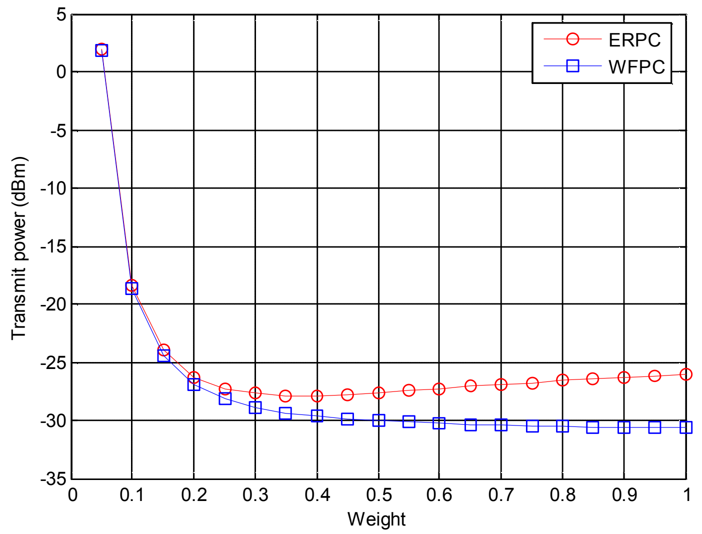
Figure 6.1 (a) shows the overall transmit power of all the schemes and Figure 6.1 (b) exhibits power saving ratio of all schemes other than CS/EW-ERPC which is defined by the ratio of the difference between the transmit power of CS/EW-ERPC and that of the scheme to the transmit power of CS/EW-ERPC.

We can make three observations. First, the overall transmit power increases in all the schemes as the data rate requirement gets higher. Second, when the rate requirement is low, CS/EW-WFPC saves more transmit power in terms of power saving ratio and, as the rate requirement gets higher, the optimal allocation of user weights becomes a more important issue. Third, the proposed scheme, CS-WFPC, saves more power compared with both CS/EW-WFPC and CS-ERPC. Furthermore, its percentage saving increases with the required data rate. In particular the second observation is noteworthy and can be appreciated by investigating the transmit power of ERPC and WFPC.

Figure 6.2 depicts the transmit power (in dB) of each power control



(a)



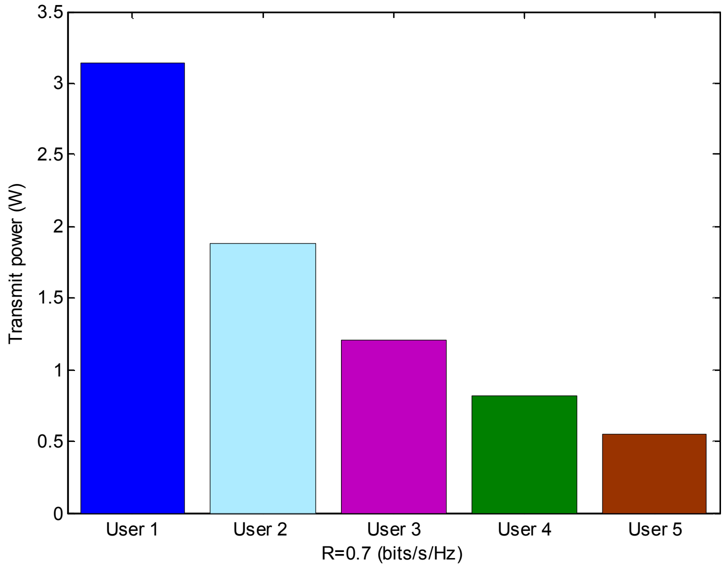
(b)

Figure 6.2. The transmit power of WFPC and ERPC. The average SNR of a user is 0 (dB) : (a) $R = 0.3$ bits/s/Hz and (b) $R = 0.7$ bits/s/Hz.

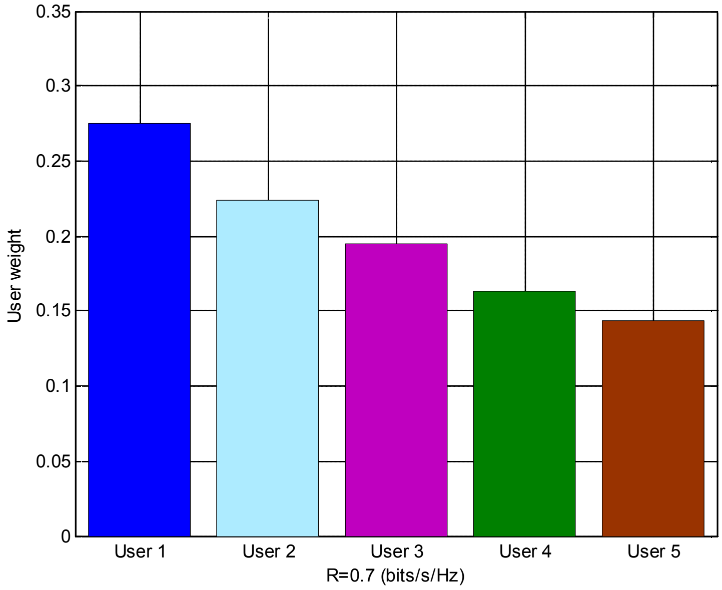
scheme when applied to a user with an average SNR 0 dB under the two different required data rates. The difference between the transmit power of ERPC and that of WFPC increases as the required data rate decreases. Therefore, when the required rate is low, it is important to select an *appropriate* power control curve but, as the required rate gets higher, the gap between the two curves gets narrower and thus clever allocation of weights among users plays a dominant role in reducing the overall transmit power.

Figure 6.3 shows the allocated weights and the corresponding transmit power of each user when CS-WFPC is applied at the rate requirement 0.7 bits/s/Hz. In Figure 6.3 (a), we observe that more transmit power is consumed for a user with lower average SNR, as expected. However this does not mean a user with a lower average SNR is scheduled for a less fraction of time. From Figure 6.3 (b), we observe that CS scheduler allocates more weight to a user with lower average SNR.

Figure 6.4 depicts a marginal utility curve of each user to provide an insight into this result. We observe that the user with a lower average SNR has its marginal utility curve located above that of the user with a higher average SNR. According to an argument presented in Chapter 5, this implies that instead of allocating weights equally across all users, one



(a)



(b)

Figure 6.3. Per user performance of the proposed scheme at $R = 0.7$ bits/s/Hz: (a) transmit power and (b) user weights.

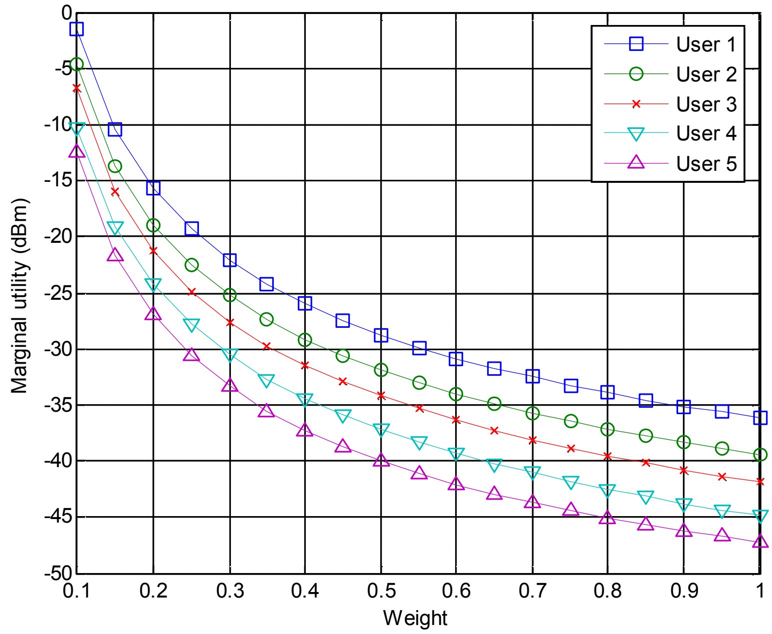


Figure 6.4. Marginal utility of each user versus weight at data rate 0.7 bits/s/Hz.

can expect lower overall transmit power by allocating more weight to a lower average SNR user. In this specific example, the universal value of marginal utility is −27.9 dBm.

Chapter 7

Conclusion

In this thesis, we have proposed a joint CDF-based scheduling and power control scheme that efficiently manages the transmit power in the uplink wireless networks. Taking advantage of performance analyzability of CS, we formulated a problem that minimizes overall transmit power under the user-specific data rate requirement. The problem is convex in power but not in user weights. We decomposed the problem into two subproblems: The first subproblem, called the user problem deals with the power allocation for a given weight. It turns out that the optimal power allocation takes the form of water-filling. The second subproblem called the network problem is formulated based on a collection of solutions of the user problems. We proved that the network problem is convex and solved the problem numerically by adopting the method of bisection. We were able to make numerical method remarkably simple by taking advantage of the decoupling property of the CS algorithm. Simulation results revealed that the proposed scheme that jointly optimizes weight and power allocation consumes lower overall transmit power than that of any other schemes that do not optimize user weights and power allocation jointly. We observed that the power saving increases as the data rate requirement gets higher and leads to up to 29% saving when the data rate requirement is 1.15bits/s/Hz per user. The simulation results also revealed the property that the lower is the average SNR of the channel that a user experiences, the more weight is allocated to reduce overall transmit power.

Appendix

Proof of Convexity of the Network Problem

In this appendix we prove that the network problem (5.1) is convex. To prove convexity, it is sufficient to prove that the objective $\phi_k(w_k)$ is convex. To this end, we proceed as follows. First, we partition interval (0,1] and prove that $\phi_k(w_k)$ is piecewise convex. i.e., $\phi_k(w_k)$ is convex in each element of partition. Then we conclude that $\phi_k(w_k)$ is convex by showing that its first derivative is continuous

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Lemma A.1: ϕ_k is piecewise convex.

Proof: We define $N_k \equiv \min\{m \mid \lambda_k G_{k,m} \geq 1\}$. Then $N_k(w_k)$ is a monotonically increasing step function of w_k that is left-continuous. We denote the interval where $N_k(w_k)$ is constant at $N_k(w_k) = m$ by $(w_k^{(m)}, w_k^{(m+1)}]$. Then a collection of $(w_k^{(m)}, w_k^{(m+1)}]$ for each possible m forms a partition of (0,1]. We prove that $\phi_k(w_k)$ is convex in each interval $(w_k^{(m)}, w_k^{(m+1)}]$.

In the sequel, we drop off the user index k for notational convenience. Also we restrict the interval being considered to one of the $(w^{(m)}, w^{(m+1)}]$.

Before proceeding to prove **Lemma A.1**, we need several auxiliary results.

Proposition A.1: Let $f_m(w) \equiv w(q_m^{1/w} - q_{m-1}^{1/w})$. Then $f_m^{'}(w) > 0$, for all $m = N_k, N_k + 1, \cdots, M$.

Proof: Let $F_m(w\,; q_m) \equiv \frac{d}{dw} w q_m^{1/w}$. Then the statement is equivalent to $F(w\,; q_m) - F(w\,; q_{m-1}) > 0$. Since $q_m > q_{m-1}$, it is sufficient to show that $F_m(w\,; x) = (1 - \frac{\log x}{w}) x^{1/w}$ is monotonic increasing in x at each w. This is easily shown from its partial derivative.

$$\frac{\partial}{\partial x} F_m(w\,; x) = -\frac{1}{xw} x^{(1/w)} + (1 - \frac{\log x}{w}) \frac{1}{w} x^{(1/w)-1} = -\frac{\log x}{w^2} x^{(1/w)-1} > 0 \qquad \blacksquare$$

Proposition A.2: $\lambda^{'}(w) \le 0$ in each $(w_k^{(m)}, w_k^{(m+1)}]$.

Proof: $\lambda(w)$ is implicitly defined by (4.4), $\sum_N^M \log\left(\lambda(w) G_m\right) w_k \left(q_m^{1/w} - q_{m-1}^{1/w}\right) = R$. By differentiating both sides and multiplying by $\lambda(w)$,

$$\sum_{m=N}^{M} \lambda^{'}(w) f_m(w) + \sum_{m=N}^{M} \lambda(w) \log(\lambda(w) G_m) f_m^{'}(w) = 0 \tag{A.1}$$

Thus

$$\lambda^{'}(w) = -\frac{\sum_{m=N}^{M} \lambda(w) \log(\lambda(w) G_m) f_m^{'}(w)}{w - w q_{N-1}^{1/w}} \le 0 \tag{A.2}$$

Since $\lambda(w) G_m \ge 1$ and $f_m^{'}(w) > 0$ by **Proposition A.1.** ■

Proposition A.3: $w q_m^{1/w}$ is convex over $w \in (0,1]$.

Proof: The second order derivative of $w q_m^{1/w}$ is

$$\frac{d}{dw}\frac{d}{dw} w q_m^{1/w} = \frac{d}{dw}\left(1 - \frac{\log q_m}{w}\right) q_m^{1/w} = \frac{\log q_m}{w^2} q_m^{1/w} + \left(1 - \frac{\log q_m}{w}\right)\left(-\frac{\log q_m}{w^2}\right) q_m^{1/w}$$

$$= \left(-\frac{\log q_m}{w}\right)\left(-\frac{\log q_m}{w^2}\right) q_m^{1/w} \ge 0,$$

which completes the proof. ■

Proposition A.4: $1-\frac{1}{x}-\log x$ is monotonic decreasing and $1-\frac{1}{x}-\log x \le 0$ when $x \ge 1$

Proof : Observe that

$$\frac{d}{dx}\left(1-\frac{1}{x}-\log x\right)=\frac{1}{x^2}-\frac{1}{x}\le 0 \text{ for } x \ge 1.$$

Therefore, its value over $x \ge 1$ is equal to or less than zero. ■

Now we are in a position to prove **Lemma A.1**. We compute ϕ'' as follows.

$$\phi(w)=\sum_{m=N}^{M}\left(\lambda(w)-\frac{1}{G_m}\right)w\left(q_m^{1/w}-q_{m-1}^{1/w}\right),$$

$$\phi'(w)=\sum_{m=N}^{M}\lambda'(w)f_m(w)+\sum_{m=N}^{M}\left(\lambda(w)-\frac{1}{G_m}\right)f_m'(w), \tag{A.3}$$

$$\phi''(w)=\sum_{m=N}^{M}\lambda''(w)f_m(w)+\sum_{m=N}^{M}2\lambda'(w)f_m'(w)+\sum_{m=N}^{M}\left(\lambda(w)-\frac{1}{G_m}\right)f_m''(w). \tag{A.4}$$

To develop further, we differentiate (A.1) to get

$$\sum_{m=N}^{M}\lambda''(w)f_m(w)+\sum_{m=N}^{M}\lambda'(w)f_m'(w)+\sum_{m=N}^{M}\lambda'(w)\log(\lambda(w)G_m)f_m'(w)$$
$$+\sum_{m=N}^{M}\lambda'(w)f_m'(w)+\sum_{m=N}^{M}\lambda(w)\log(\lambda(w)G_m)f_m''(w)=0.$$

So the term $\sum_{m=N}^{M}\lambda''(w)f_m(w)+\sum_{m=N}^{M}2\lambda'(w)f_m'(w)$ in (A.4) can be replaced to get

$$\phi''(w)=-\sum_{m=N}^{M}\lambda'(w)\log(\lambda(w)G_m)f_m'(w)-\sum_{m=N}^{M}\lambda(w)\log(\lambda(w)G_m)f_m''(w)+\sum_{m=N}^{M}\left(\lambda(w)-\frac{1}{G_m}\right)f_m''(w)$$
$$=-\sum_{m=N}^{M}\lambda'(w)\log(\lambda(w)G_m)f_m'(w)+\sum_{m=N}^{M}\left\{\left(\lambda(w)-\frac{1}{G_m}\right)-\lambda(w)\log(\lambda(w)G_m)\right\}f_m''(w)$$

(A.5)

Now we examine the sign of each summation. The first term $-\sum_{m=N}^{M}\lambda'(w)\log(\lambda(w)G_m)f_m'(w)$ is positive since $\lambda'(w)\le 0$ (by **Proposition A.2**), $\lambda(w)G_m\ge 1$ and $f_m'(w)>0$ (by **Proposition A.1)**.

For the second term $\sum_{m=N}^{M}\left\{\left(\lambda(w)-\frac{1}{G_m}\right)-\lambda(w)\log(\lambda(w)G_m)\right\}f_m''(w)$, we observe

$$\sum_{m=N}^{M}\left\{\left(\lambda(w)-\frac{1}{G_m}\right)-\lambda(w)\log(\lambda(w)G_m)\right\}f_m''(w)=\sum_{m=N}^{M}\lambda(w)\left\{\left(1-\frac{1}{\lambda(w)G_m}\right)-\log(\lambda(w)G_m)\right\}f_m''(w)$$

Since $\lambda(w)$ is obviously positive, to prove that the second term is positive it is sufficient to show the following.

Proposition A.5: $\sum_{m=N}^{M}\left\{\left(1-\frac{1}{\lambda(w)G_m}\right)-\log(\lambda(w)G_m)\right\}f_m''(w)$ is positive.

Proof: Let $A_m(w)\equiv\left(1-\frac{1}{\lambda(w)G_m}\right)-\log(\lambda(w)G_m)$ and $B_m(w)\equiv\left(wq_m^{1/w}\right)''$. Then the expression of the statement becomes $\sum_{N}^{M}A_m\left(B_m-B_{m-1}\right)$. Now observe that

$$\sum_{N}^{M}A_mB_m-\sum_{N}^{M}A_mB_{m-1}=\sum_{N}^{M}A_mB_m-\sum_{N-1}^{M-1}A_{m+1}B_m=\sum_{N}^{M-1}(A_m-A_{m+1})B_m-A_NB_{N-1}+A_MB_M$$

The first term is positive since $A_m-A_{m+1}>0$ and $B_m>0$ (by **Proposition A.4** and **Proposition A.3**, respectively.) and the second term is positive since $A_N<0$ and $B_{N-1}>0$ and $B_M=0$ $\left(\because B_M=\frac{d^2}{dw^2}wq_M^{1/w}=\frac{d^2}{dw^2}w=0,\ q_M=1\right)$.

Therefore $\sum_{m=N}^{M}\left\{\left(1-\frac{1}{\lambda(w)G_m}\right)-\log(\lambda(w)G_m)\right\}f_m''(w)$ is positive. ■

–End of the proof of **Lemma A.1**

Now we prove that ϕ is convex over each interval $w \in (w^{(i)}, w^{(i+1)}]$, we move on to the next step, proving $\phi^{'}$ is continuous over (0,1].

Lemma A.2: $\phi^{'}$ is continuous over (0,1].

Proof: We examine its continuity at $w_k^{(i)}$. Observing that $\lambda(w^{(i)})G_{N(w^{(i)})} = 1$, we get from (A.2),

$$\phi^{'}(w^{(i)}) = \sum_{m=N(w^{(i)})}^{M} \lambda^{'}(w^{(i)}) f_m(w^{(i)}) + \sum_{m=N(w^{(i)})}^{M} \left(\lambda(w^{(i)}) - \frac{1}{G_m}\right) f_m^{'}(w^{(i)})$$ [3]

However, since

$$\lambda^{'}(w^{(i)}) \sum_{m=N(w^{(i)})}^{M} f_m(w^{(i)}) = -\frac{\sum_{m=N(w^{(i)})}^{M} \lambda(w^{(i)}) \log(\lambda(w^{(i)}) G_m) f_m^{'}(w^{(i)})}{w^{(i)} - w^{(i)} q_{N(w^{(i)})-1}^{1/w^{(i)}}} \cdot (w^{(i)} - w^{(i)} q_{N(w^{(i)})-1}^{1/w^{(i)}})$$

$$= -\sum_{m=N(w^{(i)})}^{M} \lambda(w^{(i)}) \log(\lambda(w^{(i)}) G_m) f_m^{'}(w^{(i)})$$

$$= -\sum_{m=N(w^{(i)})+1}^{M} \lambda(w^{(i)}) \log(\lambda(w^{(i)}) G_m) f_m^{'}(w^{(i)}),$$

we get

$$\phi^{'}(w^{(i)}) = \sum_{m=N(w^{(i)})+1}^{M} \lambda^{'}(w^{(i)}) f_m(w^{(i)}) + \sum_{m=N(w^{(i)})+1}^{M} \left(\lambda(w^{(i)}) - \frac{1}{G_m}\right) f_m^{'}(w^{(i)}) \qquad \text{(A.6)}$$

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[3] Strictly speaking, $\phi^{'}(w^{(i)})$ calculated this way gives left-derivative of $\phi(w)$.

Now we see that this is equal to $\lim_{w \downarrow w^{(i)}} \phi^{'}(w)$, where $w \in (w^{(i)}, w^{(i+1)}]$

$$\phi^{'}(w) = \sum_{m=N(w^{(i+1)})}^{M} \lambda^{'}(w) f_m(w) + \sum_{m=N(w^{(i+1)})}^{M} \left(\lambda(w) - \frac{1}{G_m} \right) f_m^{'}(w) \quad . \tag{A.7}$$

Since $N(w^{(i+1)}) = N(w^{(i)}) + 1$ and each term in summation is continuous.

–End of the proof of **Lemma A.2**

Now we are in a position to prove our main result.

Theorem A.1: $\phi(w)$ is convex over (0,1]

Proof: The proof follows the same line as the standard proof of the second order condition for convexity [8]. Let $0 < x < y \le 1$. Then

$$\begin{aligned} 0 &\le \int_x^y \phi^{''}(w)(y-w)\, dw \\ &= \phi^{'}(w)(y-w)\Big|_x^y + \int_x^y \phi^{'}(w)\, dw \\ &= -\phi^{'}(x)(y-x) + \phi(y) - \phi(x) \end{aligned} \tag{A.8}$$

This shows that $\phi(y) \ge \phi^{'}(x)(y-x) + \phi(x)$, which proves convexity of $\phi(w)$. Note that we have used that $\phi^{'}(w)$ is continuous in order to use integration by part.[4] –End of the proof of **Theorem A.1**

[4] Continuity of $\phi(w)$ is easy to see and we omit the proof.

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國文 抄錄

본 논문에서는 무선 네트워크 환경에서 사용자 스케줄링과 전력 제어를 합동으로 하는 자원 관리 기법을 제안한다. 사용자 스케줄링 기법으로 누적 함수 기반 스케줄링(CDF-based Scheduling, CS)을 사용하는데, CS 는 스케줄링 성능을 수학적으로 예측하기 용이하고 시간 할당 계수(weight)라는 매개변수를 조절하여 사용자들의 서비스 품질을 보장할 수 있다는 장점이 있다. 본 논문은 사용자들의 요구 수율을 보장하면서 송신 전력을 최소화하는 문제를 도출하고 CS 의 사용자 시간 할당 계수와 사용자 별 송신 전력을 동시에 결정한다. 형성된 최적화 문제는 비볼록(nonconvex)인데 이를 각각 볼록(convex)인 두 개의 문제로 나누어 푼다. 모의 실험에 의하면 제안 기법이 스케줄링과 전력 할당을 독립적으로 수행하는 기존 기법과 비교하여 전력 감소 이득이 큼을 확인할 수 있었다.

주요어 : 사용자 스케줄링, 전력 제어, 볼록 최적화, 무선 자원 관리.
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