공학석사학위논문

# 3D-Positining System with a Single DME Station for the Next Generation Air Transport

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**지도교수 기 창 돈**

**이 논문을 공학석사 학위논문으로 제출함**

**2013 년 1 월**


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**김오종의 공학석사 학위논문을 인준함**

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# ABSTRACT

## 3D-Positining System with a Single DME Station for the Next Generation Air Transport

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Next Generation Air Transport System (NextGen) is a program ready for the future aircraft traffic developed by US Federal Aviation Administration (FAA). Alternate Positioning Navigation and Timing (APNT), a part of NextGen, is a developing solution for the continuity and safety of air services when Global Navigation Satellite System (GNSS) is unavailable.

As a solution for APNT, there are candidates being researched. Some of them utilize the Distance Measuring Equipment (DME) which is a conventional terrestrial system at the present time. DME Passive Ranging (DMPR) using pseudolite-like signals is one of them, which is the most actively researched. DMPR is, however, at least two or three stations are required for the positioning, and time synchronization between stations are essential because of one-way continuous signals. Nowadays, additional research and costs for time-sync are considered.

MOSAIC/DME system presented in this paper is a single station based 3D positioning system. Additional multiple pseudolite-like MOSAIC antennas broadcasting one-way continuous signal are installed together with conventional DME. This one station based system makes it possible for time synchronization between signals because all signal generators can share the clock source. And it is also possible for 3D positioning using only one station to extend the usable coverage. As DME antenna itself operates without any change with current system, compatibility is proved. And two types of signals, one-way and two-way, are both used, so it is expected that aircraft traffic capacity also can be improved.

Positioning from one spot has generally bad geometrical feature, or bad Dilution of Precision, expecting that navigation performance is low. To cover this problem, MOSAIC/DME system uses the measurement of carrier phase whose noise is mm-level. But using carrier phase cause the problem of integer cycle ambiguity, the measurement has unknown integer-term multiplied by wavelength. To solve ambiguity problem, a concept named MOSAIC is utilized. Standard MOSAIC concept is generally applied to solve the ambiguity problem directly when the antenna separation is smaller than half of the wavelength. But it is expected that larger antenna separation is required for aircraft application. Therefore, extended MOSAIC concept, whose cycle ambiguities are geometrically bounded, is used for easy ambiguity resolution.

Positioning algorithm is based on Least Square Ambiguity Search Technique (LSAST), and gets its position using Weighted Least Square Solution every epoch. Ambiguity resolution methods include not only residual threshold test and ratio of LSAST but matrix singularity check, convergence check, altitude bound, and AHRS test which utilize the special features of MOSAIC/DME system.

Monte-Carlo simulation, generating measurements, are executed for the verification of accuracy properties and ambiguity resolution of MOSAIC/DME system. Simulation's results show that the accuracy performance of MOSAIC/DME satisfies the requirement of APNT. And most of ambiguity problems are solved within 20 seconds, and ambiguities of all simulation cases are solved in a minute.

This paper presented the conceptual outline of MOSAIC/DME system, and accuracy properties and ambiguity resolution are studied in priority. For the future research, ambiguity resolution improvement, simulation with various error sources, capacity study, signal analysis, message structure, and integrity for the safety of air services are considered for the implementation of this system.

Keywords : NextGen, APNT, MOSAIC, MOSAIC/DME, Extended MOSAIC, DME, Ambiguity Resolution

Student Number : 2011-20694

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# I. INTRODUCTION

## 1. Motivation and Purpose

Next Generation Air Transport System (NextGen) is an air traffic modernization program being developed by Federal Aviation Administration (FAA) for future aircraft transportation. According to FAA, “NextGen is a comprehensive overhaul of our National Airspace System to make air travel more convenient and dependable, while ensuring your flight is as safe, secure and hassle-free as possible” [1]. In NextGen program, Global Navigation Satellite System (GNSS)-based system generally performs the main role for positioning, navigation and surveillance. However, GNSS-based system is very vulnerable to Radio Frequency Interference (RFI) such as jamming and spoofing. In case of RFI or any other GNSS unavailable situations, Alternate Positioning Navigation and Timing (APNT) system becomes an essential issue to maintain the safety of air service and reduce GNSS unavailable economic impacts.

APNT is Non-GNSS architecture for aviation users to operate when GNSS services are unavailable, so this system should be not satellite-based (or space-based) but terrestrial-based system. In 2025, it is predicted that air traffic is 2-3 times more than current level [2], which means that the capacity, how many aircraft can use APNT system at the same time, and efficiency, how many aircraft can operate in a limited airspace, cannot be overlooked. The second factor ‘efficiency’ has a strong relationship with accuracy. If the navigation system provides better accuracy, the safe distance between aircrafts can be smaller, meaning that more aircraft can operate in a limited space. Cost-effectiveness, include avionics and ground station upgrading costs, is also important issue for APNT. And APNT is also required to have a compatibility with conventional aircraft navigation system, such as GNSS, Distance Measuring Equipment (DME), VHF Omni-Directional

Ranging (VOR) or Non-Directional Beacon (NDB).

Conventional terrestrial aircraft navigation systems are VOR, DME, and NDB for civil application and Tactical Air Navigation (TACAN) for military use. And these systems are generally combined for positioning, such as VOR/DME, DME/DME, VOR/TACAN and DME/DME/Inertial. However, legacy APNT systems, mentioned above, are not compatible with Performance-based Navigation (PBN) operations for Area Navigation (RNAV) and Required Navigation Performance (RNP), preparing for future aircraft traffic levels [2].

APNT working group, therefore, is search for the solution of backup navigation system. One option is to create a new signal optimized for APNT requirements. But this can cause severe disadvantages, for example, high cost for the updates of user and station equipment, difficulty to avoid interference with existing signals, and compatibility with existing navigation system. Another option is to utilize existing signals. If the performance of this option is enough for APNT requirements, this utilized system is preferred, as it can solve the problems of high cost and signal interference with existing infrastructure [3].

There are candidates for APNT utilizing existing signals. Optimized DME Networks (DME-DME), Wide Area Multi-Lateration (WAM), and DME Passive Ranging (DMPR) are those things. Those systems are generally, at least, two or three stations are required for 2D-positioning, and clock synchronization between stations without GNSS is a considerable issue which cannot be solved easily. In recent papers, therefore, time synchronization between terrestrial stations has been importantly considered [4,5].

With regard to above problems, a new concept ‘MOSAIC/DME: 3D-positioing using a single DME station’ was already presented in international conference at the beginning of the year 2012 [6]. As only a single station can provide the user for positioning, time synchronization between stations is no more severe issue, and this concept can service not only 2D-position but 3D-position include altitude. For one station positioning, a DME station with multiple antenna concept is considered. In the past, four additional pseudolite-like antennas are added to DME station [4], but this paper describes the system with five additional antennas for better performance and stability.

The key feature of MOSAIC/DME is its method of integer cycle ambiguity resolution. To accomplish the accuracy requirement, carrier phase measurements which have mm-level noise are utilized for positioning. Using carrier phase measurements, however, generally cause the problem of cycle ambiguity. And the approach to solve this ambiguity problem is a special feature of MOSAIC concept. Therefore, this paper introduces the system design of MOSAIC/DME, focusing on its ambiguity resolution.

## 2. Literature Survey

APNT working group presented their study with Leo Eldredge, belong to FAA, at the head of the list in 2010 [2]. This document describes the purpose, mission statement, background, current APNT infrastructure, APNT assumptions, APNT Analysis Objectives, and three APNT alternatives: DME/DME Network Optimization, Passive Wide-area Multi-lateration, and Pseudolite-based Multi-lateration. These alternatives have been kept on studying. In recent conference of Institute of Navigation (ION 2012, Nashville), Dr. Sherman Lo from Stanford University also examines that 1) Two way ranging (DME/DME), 2) Wide Area Multilateration, 3) Passive Ranging are main three candidate architectures. Brief positioning figures of DME/DME and DME Passive Ranging are described in Figure I-1. And conceptual design of Wide Area Multilateration is like as Figure I-2.

Stanford APNT Workgroup generally has been performing the analysis of APNT solutions. In 2010 preliminary analysis of APNT was presented [7]. And analysis of APNT using DME and UAT signals were also presented in 2012 [3,8]. In 2012, the study of signal structure for DME Passive Ranging was performed. After that the study of time synchronization method between terrestrial stations was researched recently [4].

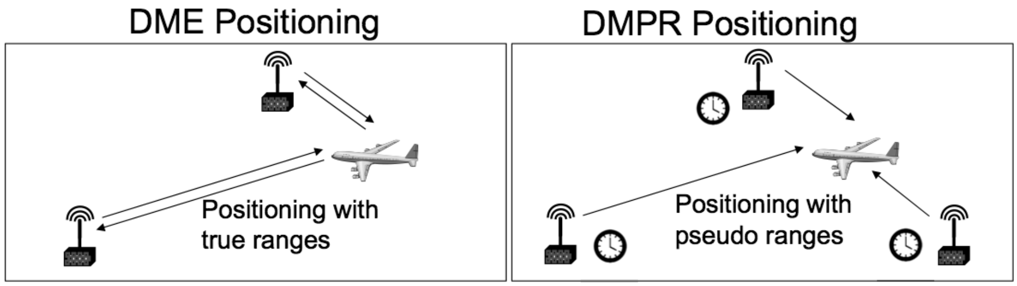


Figure I-1    Positioning Comparison of Nominal DME/DME and DMPR [9]

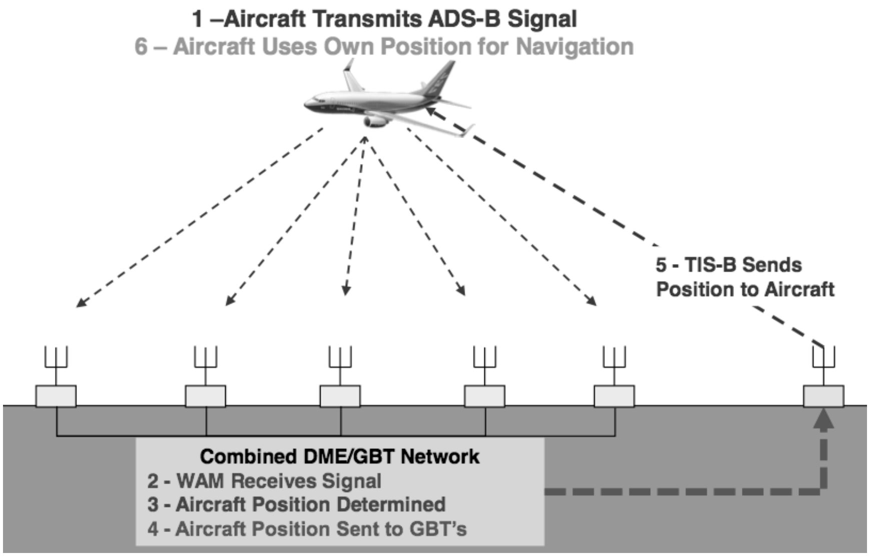


Figure I-2    Conceptual Design of Wide Area Multilateration [2]

A study of NextGen APNT has a short history, so that suitable and perfect solution of this system has not yet determined. Performance, safety and cost-effectiveness of this system are not easy to meet its requirements. So, this paper tries to suggest a new concept of APNT utilizing conventional DME station.

## 3. Outline

This paper suggests MOSAIC/DME, a single station-based 3D positioning system for APNT. Accuracy and ambiguity resolution properties of MOSIAC/DME are mainly discussed using Monte-Carlo simulation. And other properties such as traffic capacity, signal structure, cost-effectiveness and compatibility with legacy DME are conceptually described.

The first part of the text is focused on APNT requirement and the performance of legacy terrestrial navigation system: DME, VOR. It proves that current equipment cannot be applied to future backup navigation system without any change, so that finding solution for APNT is an important issue to support safe and robust air services.

In the second part, MOSIAC/DME concept is described in detail. Derivations of standard MOSIAC and Extended MOSAIC for long-ranged application are described. And components of MOSAIC/DME systems are introduced, and properties of this system are explained minutely. After that positioning algorithm using MOSAIC/DME system is derived.

The third part is about integer cycle ambiguity resolution of MOSAIC/DME system. As this system has different properties from ambiguity resolution of conventional Carrier-phase Differential GPS (CDGPS), some ambiguity resolution method needs to be transformed suitably, and new kinds of method are also suggested for MOSAIC/DME.

After that, simulation results using Monte-Carlo method are described. There are two main categories of simulation results: accuracy properties and ambiguity resolution performance.

For the last time, summary and future suggestions are mentioned in conclusion.

## 4. Contributions

MOSAIC/DME concept is a single station-based positioning for aircraft which has never been presented before. And the simulation result shows that the accuracy of this system is enough for the requirement of APNT and uncertainty of integer cycle ambiguity can be solved easily with enough time. It means that this system can be a potential solution for APNT with its own advantages: cost-effectiveness using a single station without time synchronization, full-compatibility with conventional DME system, improved traffic capacity.

So, this paper introduces the single station-based positioning system for APNT. And the ambiguity resolution methods for positioning are also described.

For the implementation of this system, there are still various issues to be solved thoughtfully, such as capacity study, integrity, signal structure, timing, and so on. But these issues will be addressed by the future work.

# II. APNT AND TERRESTRIAL NAVIGATION

## 1. APNT Requirement

APNT is a backup navigation system preparing for the future aircraft traffic which will be at least 2 times more than current level by 2025. According to APNT study document accomplished by FAA, some of assumptions are as follows [2].

- In 2025, there will be "RNAV everywhere and RNP where beneficial". It is recognized that there will likely be many different variants of RNAV and RNP that are yet to be defined.
- APNT is a means to continue RNAV and RNP operations to a safe landing during periods when it is discovered that GNSS services are unavailable, due to interference.
- Users equipped for APNT will be able to continue conducting RNAV and RNP operations (dispatch, departure, cruise, arrival) during the GNSS outage after the transition to APNT
- APNT must provide RNAV or RNP 2 en route, between RNAV or RNP 1.0 to 0.3 for terminal class B and C airspace, LNAV or RNP 0.3 for approaches, and RNAV or RNP 1 for missed approach, where economically beneficial or required for safety.
- APNT service volume consists of the conterminous 48 states. Altitude of coverage includes FL 600 down to 5,000 feet Above Ground Level (AGL), and sufficient coverage to support RNP-0.3 approaches wherever required for safety or economically justified.
- APNT services will provide backup positioning to support 3NM separation in terminal area operations for dependent surveillance, wherever required for safety

or economically justified.

- APNT will provide backup timing services for Communication, Navigation and Surveillance (CNS) and other aviation applications.
- APNT will ensure backward compatibility for existing DME and DME/DME users. Based on current plans, DME will be provided RNP 2 above FL 180 and RNP 1 at all OEP airports.
- APNT supports position reporting for conformance monitoring for security.

And APNT analysis Objectives are as follows [2].

- Provide a Cost Effective Alternative PNT service that Enables PBN RNAV and RNP for en-route, terminal, and non-precision approach operations equivalent to RNP-0.3
- Provide service for all users (General Aviation, Business, Regional, Air Carrier)
- Minimize impact on user avionic equipage by leveraging existing or planned equipage upgrades as much as possible
- Ensure backward compatibility for legacy DME-DME users
- Provide long lead transition time
- Avoids recapitalization costs for VORs ~ $ 1.0 billion
- Disestablish all VORs and NDBs by 2025

According to above statements, accuracy requirements for APNT are mainly RNP 1.0 for en-route and RNP 0.3 for terminal class B and C. And PNT performance area is displayed in Figure II-1. RNP means horizontal 95% total system error which includes navigation system error and flight technical error. The unit of RNP is Nautical Miles (NM). APNT also should guarantee availability and continuity when GNSS is degraded. APNT candidate also needs to ensure the compatibility with legacy DME and DME/DME. And it is also required to provide backup timing services.

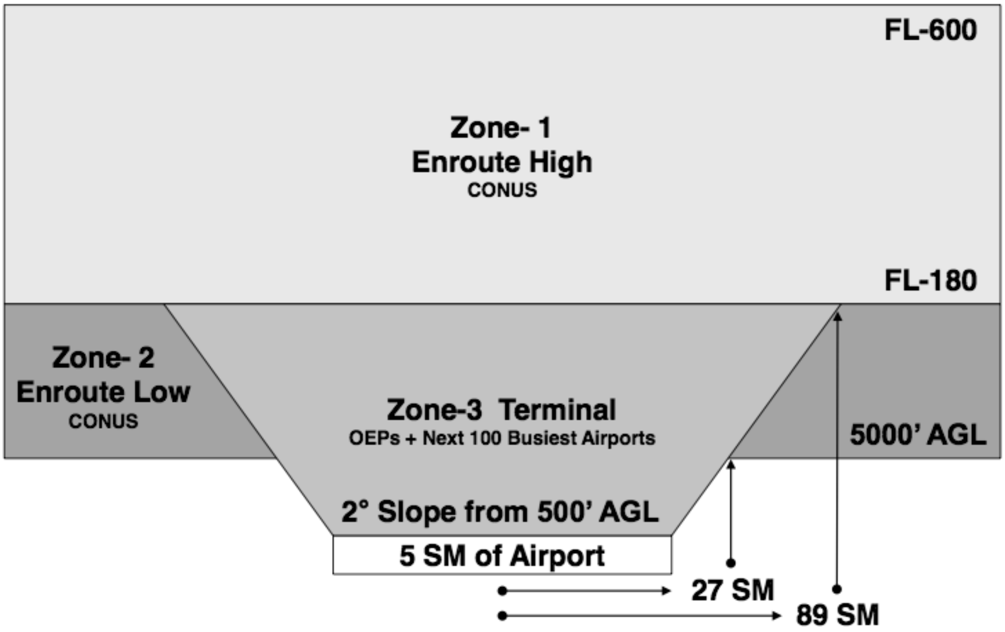


Figure II-1 PNT Performance Zones for Aircraft [10]

## 2. Distance Measuring Equipment (DME)

DME is an internationally standardized two-way pulse-ranging system for aircraft, operating in the 960-1215 MHz band. DME is widely utilized for the solution of APNT. Most of APNT candidates, DMPR, DME/DME and MOSAIC/DME, consider DME as its base system. The aircraft interrogator transmits pulses on one of 126 frequencies, spaced 1 MHz apart in the 1025-1150MHz. The pulses are in pairs, 12µsec apart, and each pulse lasting 3.5µsec as Figure II-3**.** The ground beacon receives these pulses and, after a 50µsec delay, retransmits back to the aircraft on a frequency 63 MHz below or above. Each station is designed to handle at least 50 aircrafts at a time, with 100 being a more typical number [11].

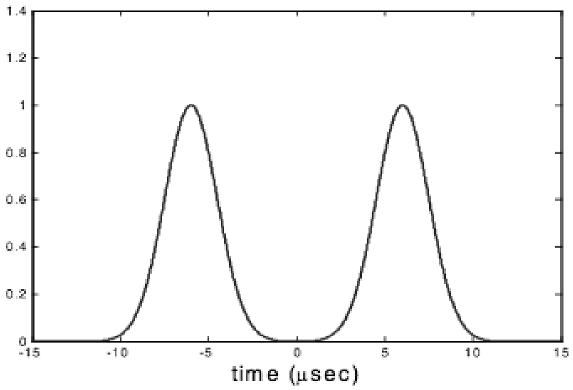


Figure II-2  Ideal DME Pulse Pair [8]

According to '2010 FEDERAL RADIONAVIGATION PLAN', DME service volumes are described in Table II-1 that maximum range is up to 130NM (~240km). The general accuracy of DME is $\pm$0.1NM (~185m) of 2σ, and the overall system error is not greater than $\pm$0.5NM or 3% of distance, whichever greater [12]. Above accuracy mentioned in 'FEDERAL RADIONAVIGATION PLAN' is the performance of legacy DME station, old-versioned. As DME station has its own potential to improve its accuracy, it is expected that it will perform with standard deviation of 35 to 70ns (10 to 20m) in case of DMPR system [3]. For active two-way signal case, with simple mathematical consideration, its accuracy can be square root two of above standard deviation (about 14m to 28m).

Table II-1 VOR/DME/TACAN Standard Service Volumes (SSV) [12]

| SSV CLASS | ALTITUDE AND RANGE BOUNDARIES |
|---|---|
| **T (Terminal)** | From 1,000ft AGL up to and including 12,000ft AGL at radial distances out to 25NM |
| **L (Low Altitude)** | From 1,000ft AGL up to and including 18,000ft AGL at radial distances out to 40NM |
| **H (High Altitude)** | From 1,000ft AGL up to and including 14,500ft AGL at radial distances out to 40NM. From 14,500 AGL up to and including 60,000ft at radial distances out to 100NM. From 18,000ft AGL up to and including 45,000ft AGL at radial distances out to 130NM. |

## 3. VOR/DME system

VOR are assigned frequencies in the 108 to 117.795 MHz (VHF) frequency band, separated by 50 kHz. VOR transmits two 30Hz modulations resulting in a relative electrical phase angle equal to the azimuth angle of the receiving aircraft providing the azimuth information from station to aircraft. And performance of VOR is about $\pm 1.4°$ of $2\sigma$ [12], which means that it becomes inaccurate as the aircraft is farther from VOR station. A single station combined VOR with DME can also provide short-range navigation [11]. It is possible for the aircraft to provide 2D-positioning using VOR and DME together providing azimuth angle and distance respectively. VOR/DME station has the following form as in Figure II-3.

In case of terminal class B, however, the farthest distance from the airport is 30NM. At this area, axial-direction position accuracy using VOR/DME is about 0.7M ($2\sigma$) greater than APNT requirement 0.3NM (<95%). VOR/DME station, even more, has a limitation in traffic capacity, because DME uses two-way active signals for measuring distance. Therefore it can provide range measurement to one aircraft at a time, not simultaneously. All these consequences that legacy VOR/DME is not suitable as a solution for APNT.

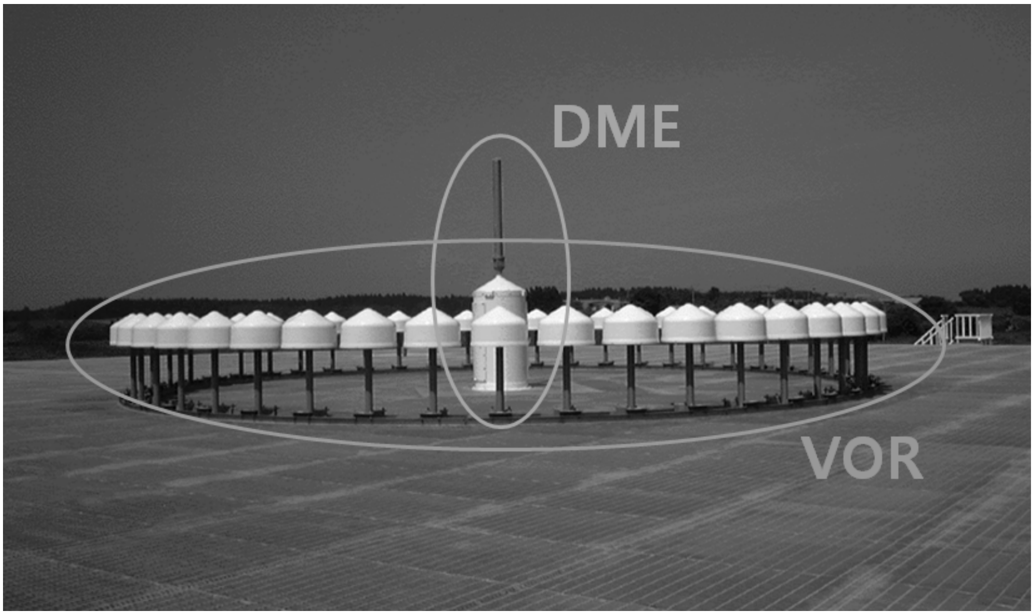


Figure II-3 A Picture of VOR/DME station

# III. MOSAIC/DME SYSTEM

## 1. MOSAIC Concept

### 1) Standard MOSAIC Concept

A Standard MOSAIC is an integer cycle ambiguity-free concept using carrier-phase measurements with limited antenna baseline: half of the wavelength. Single Differenced Carrier Phase (SDCP) is used for positioning to eliminate the clock offset between antenna and receiver. The ambiguity-free condition can be derived geometrically.

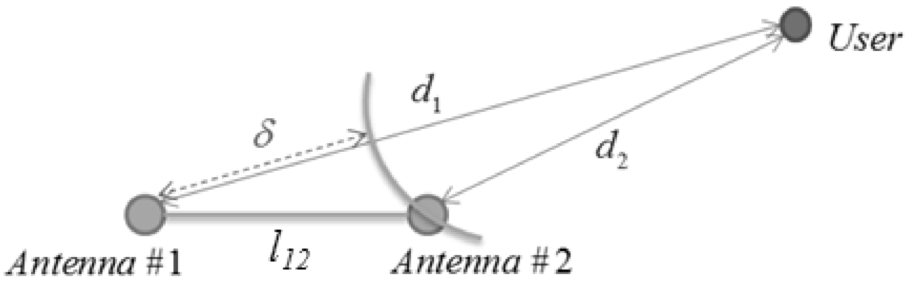


Figure III-1 Geometric Concept of Standard MOSAIC with 2 Antennas

To simplify the derivation of standard MOSAIC concept, two antennas for 2-Dimensional case are considered as in Figure III-1. Two antennas broadcast signals, which are time synchronized, with certain baseline ($l_{12}$). Ranges from each antenna to user are $d_1$ and $d_2$

respectively. And the difference between $d_1$ and $d_2$ is $\delta$ ($=\Delta d_{12}$).

The model of carrier phase measurements for GPS (or GNSS) is generally described as in equation (III-1) [13].

$$\phi^j = d^j + N^j \lambda + B - b^j + i^j + t^j + m^j + \varepsilon^j \qquad \text{(III-1)}$$

In equation (III-1), $\phi$ is measured carrier phase, $N$ is integer cycle ambiguity, $\lambda$ is wavelength, $B$ is receiver clock offset, $b$ is station clock offset, $i$ is ionospheric delay, $t$ is tropospheric delay, $m$ is multipath error, $\varepsilon$ is other noise, and superscript $j$ means the measurement from j-th satellite. This model is only suitable for GPS or GNSS, so it needs to be modified to terrestrial navigation as in equation (III-2).

$$\phi_j = d_j + N_j \lambda + B - b + t_j + m_j + \varepsilon_j \qquad \text{(III-2)}$$

As the signal comes from the ground, the index j is subscript. The signals from different antennas share its clock source (MOSAIC is a single station system), station clock offset is also shared for all measurements. And as signals don't pass the ionosphere, $i$-term can be ignored.

$$\Delta\phi_{12} \equiv \phi_1 - \phi_2 = \Delta d_{12} + \Delta N_{12} \lambda + \varepsilon'_{12} \qquad \text{(III-3)}$$

Equation (III-3) describes the SDCP measurement between antenna #1 and #2. Receiver clock offset (B) and station clock offset (b) are common between antennas, so these terms are eliminated by single difference. Tropospheric error (t) and multipath error (m) also cause slight effect with single difference, because the range ($d_j$) is much longer than antenna baseline ($l_{12}$), so that tropospheric and multipath errors are mostly common factors. Therefore these two terms (t & m) are combined with noise ($\varepsilon$) to $\varepsilon'$.

$$\frac{\Delta\phi_{12}}{\lambda} = \frac{\Delta d_{12} + \varepsilon'_{12}}{\lambda} + N_{12} \tag{III-4}$$

After dividing by wavelength (λ) as in equation (III-4), rounding both sides makes equation (III-5).

$$round\left(\frac{\Delta\phi_{12}}{\lambda}\right) = round\left(\frac{\Delta d_{12} + \varepsilon'_{12}}{\lambda}\right) + N_{12} \tag{III-5}$$

Geometrically $\Delta d_{12}$ is always smaller than $l_{12}$. If an antenna baseline ($l_{12}$) is smaller than half of the wavelength (0.5λ), then the round of $\Delta d_{12}$ term could be zero with assumption that the noise term (ε') is not big enough. As a result, cycle ambiguity ($N_{12}$) can be obtained directly using equation (III-6).

$$\Delta N_{12} = round\left(\frac{\Delta\phi_{12}}{\lambda}\right) \tag{III-6}$$

As derived above, the standard MOSAIC concept provides the simple method to solve the integer cycle ambiguity problem without time and calculation loss if the antenna baseline is smaller than half of its signal carrier's wavelength. However, half of the signal's wavelength is generally very small. For example, if the system uses L1 (1575.42MHz) as a carrier frequency, then its wavelength is about 19cm. To apply this to MOSAIC, the antenna baseline should be smaller than 9.5cm. When this system is applied for not indoor but outdoor aircraft application, whose operation range is up to about 100NM, short antenna baseline can cause bad geometry, that is to say, bad Dilution of Precision (DOP). Even though this system uses carrier phase measurements whose noise are mm-level, its navigation performance cannot be good. To resolve above problem, MOSAIC concept with longer antenna baseline is considered.

### 2) Extended MOSAIC Concept

Even though standard MOSAIC concept uses the carrier phase measurements, this is not suitable for APNT. If DME frequency (962-1215MHz) is used for carrier frequency, its wavelength is around 28cm. It means the baseline between transmitter antennas should be smaller than 14cm for standard MOSAIC. In this condition, although carrier phase noise is mm-level, the accuracy cannot meet its requirement if the range is tens or hundreds of nautical miles.

Considering the DOP, it is expected that position accuracy can be improved if the antenna baseline of the station becomes longer, as MOSAIC is a single station-based positioning system with multiple antenna. If the antenna baseline is longer than half of the wavelength, however, the cycle ambiguity resolution becomes a critical issue again. But one thing that cannot be overlooked is that the integer ambiguity (N) can be geometrically bounded with similar procedure described in standard MOSAIC concept.

Similar with last section, two antennas for 2-dimensional case is considered as in Figure III-1. Procedure of Extended MOSAIC concept is same with standard one until equation (III-5). After that one different thing is the antenna baseline ($l_{12}$) is larger than half of the wavelength. The distance between antennas always can be described that it is below a certain number (M) and half of the wavelength like equation (III-7).

$$\Delta d_{12} < l_{12} < M + \frac{\lambda}{2} \qquad (if\ \Delta d_{12} > 0) \tag{III-7}$$

The rounded equation (III-5) becomes as in equation (III-8).

$$\begin{aligned} &-round(\frac{\Delta d_{12}}{\lambda}) \le round(\frac{\Delta \phi_{12}}{\lambda} - \Delta N_{12}) \le round(\frac{\Delta d_{12}}{\lambda}) \\ &-M + round(\frac{\Delta \phi_{12}}{\lambda}) \le \Delta N_{12} \le round(\frac{\Delta \phi_{12}}{\lambda}) + M \end{aligned} \tag{III-8}$$

Equation (III-8) shows that its integer ambiguity is spans from *-M* to *+M,* which means that the ambiguity of Extended MOSAIC is geometrically bounded. This term 'geometrically' guarantees that the true solution must be existed between *-M* to *+M* boundary.

For example, if *M* is five, then its boundary is from -5 to 5 (-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5), so the number of ambiguity in this two-antenna case is eleven. If the system has three integer ambiguities like MOSAIC/DME, then the total number of them is $11^3$=1331. The relationship between integer (M), antenna baseline (l), and the number of total ambiguity is described in Table III-1.

Table III-1 Relationship between antenna baseline and number of total ambiguity for MOSAIC/DME system when its wave length is 0.2754m

| ***M*** | ***Antenna Baseline (m)*** | ***No. of Total Ambiguity*** |
|---|---|---|
| 0 | 0.1365 (= 0.5 $\lambda$) | 1 |
| 1 | 0.4121 (= 1.5 $\lambda$) | 27 |
| 2 | 0.6878 (= 2.5 $\lambda$) | 125 |
| 3 | 0.9634 (= 3.5 $\lambda$) | 343 |
| 4 | 1.2391 (= 4.5 $\lambda$) | 729 |
| 5 | 1.5150 (= 5.5 $\lambda$) | 1331 |

## 2. MOSAIC/DME SYSTEM

MOSAIC/DME is a DME station with multiple pseudolite-like antennas using extended MOSAIC concept. The conceptual design of this system is as below in Figure III-2.

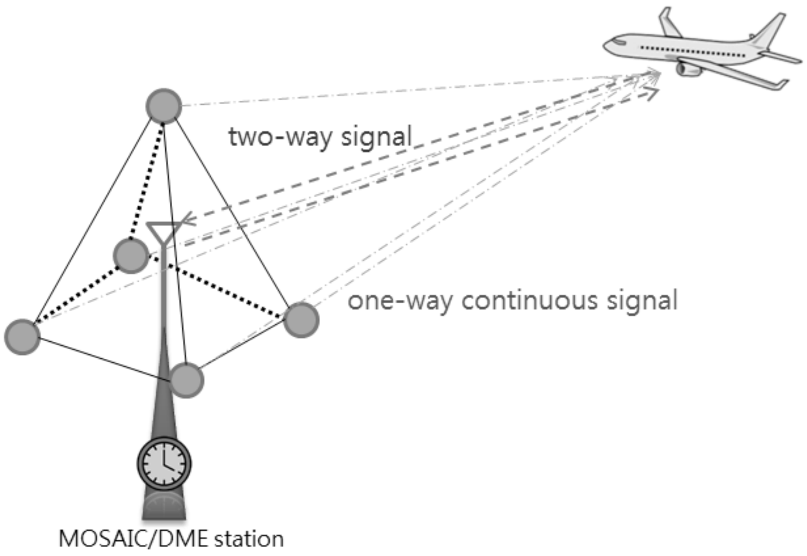


Figure III-2 Conceptual Design of MOSAIC/DME

DME station (inverse triangle) operates using two-way round signals to measure the range between station and aircraft similar with legacy DME system, and five additional antennas (circles), broadcasting one-way continuous signals, are attached to this station for MOSAIC. Why this system has a design like Figure III-2 is described below.

### 1) Geometrical Analysis

① Trajectory of MOSAIC Solutions

MOSAIC system uses single differenced carrier phase measurements for positioning, so that measurements from two antennas draw the trajectory of hyperbolic. Therefore, the solution is the intersection of multiple hyperbolic trajectories from multi antennas. This feature for 2-D is displayed in Figure III-3. The station position is (0, 0) and user position is (100, 100). The station is consists of three antennas, so two hyperbolic curves are drown after single difference of three carrier phase measurements.

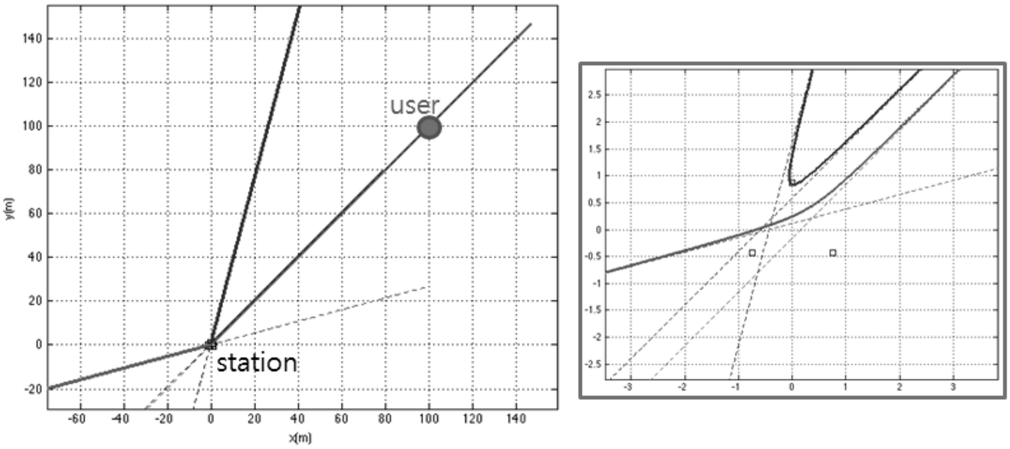


Figure III-3 Hyperbolic Curves from Single Differenced Carrier Phase [left]
and Enlargement around MOSAIC Station [right]

In Figure III-3, the hyperbolic curves are calculated analytically, and it displays that intersection of two curves are almost line (or sharp ellipse). It is expected that the covariance of the solution has a shape of sharp ellipse, because of its bad geometry. These sharp ellipse-shaped solutions are described in Figure III-4. In Figure III-4, the width of sharp ellipse is too short that it even looks like a line. With regard of bad DOP, when the user position is farther from the station, the ellipse is more widely spread through radial direction. Even though MOSAIC system provides 2-D position results, it looks like providing not position but accurate bearing information.

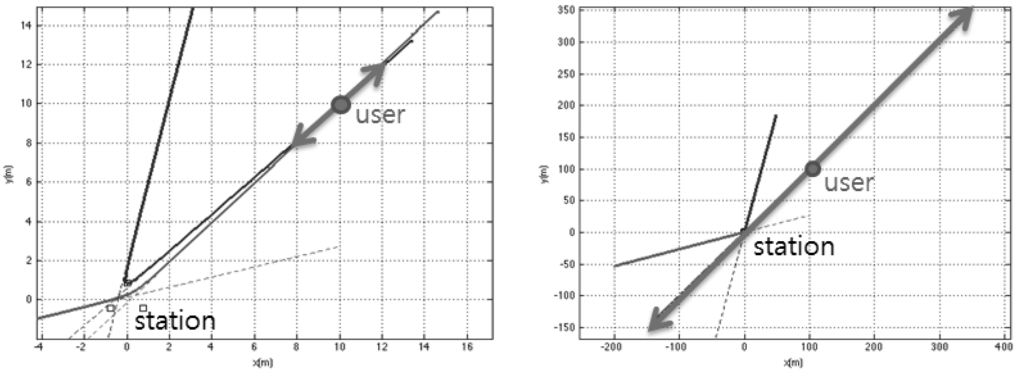


Figure III-4 One-sigma Distribution of Position when User Position is (10,10) [left] and (100,100) [right]

In 3-dimentional case, this tendency is not much different from 2-D case, except the covariance distribution of position is ellipsoid instead of ellipse.

These features describe that the information provided by MOSAIC station can be regarded as accurate bearing. Therefore if the range from station to user is known, then the position of user can be pointed. That’s why MOSAIC system is needed to combine with DME, distance measuring equipment.

② Potential Solutions of MOSAIC/DME

A single station combined with MOSAIC and DME can provide positioning. To meet the accuracy requirement for APNT, not standard MOSAIC but extended MOSAIC concept is required to be applied. Extended MOSAIC concept, however, inherently has integer cycle ambiguity problem. Ambiguity problem for MOSAIC/DME is different from conventional Carrier-phase Differential GPS (CDGPS) cases. One is that the range from station to aircraft can be fixed using two-way round DME signal. And the transmitters of signal are concentrated near station, in other words, the baseline between antennas is quite small (m-level) compared to the range (tens or hundreds of NM) between station and aircraft.

With above properties, the potential solutions spread by integer cycle ambiguity are as below. The first case is two antenna cases for 2-D positioning as in Figure III-5.

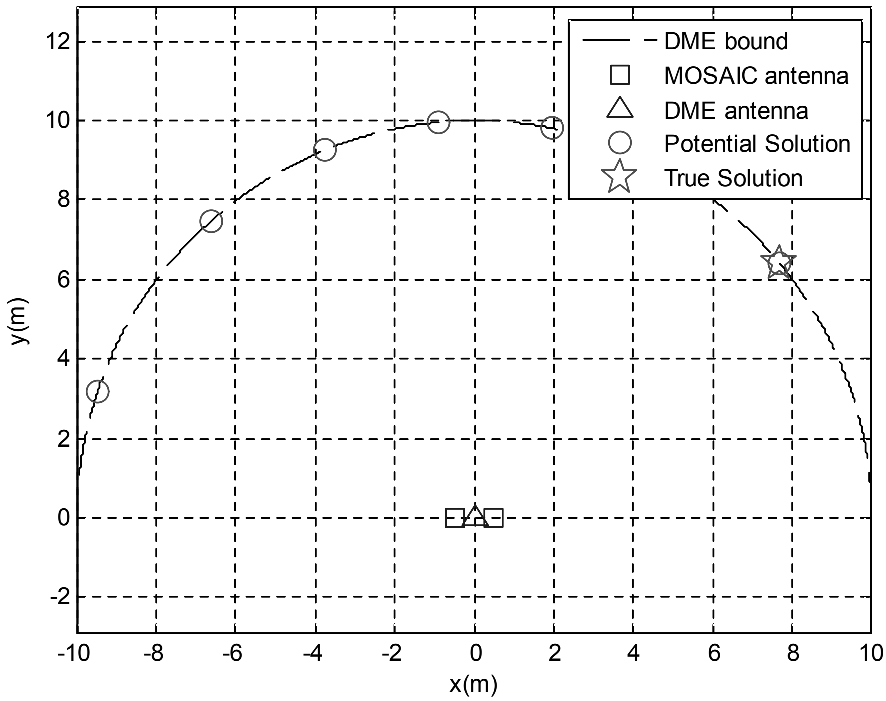


Figure III-5 Integer Ambiguity Potential Solutions of MOSAC/DME for 2-D case

Figure III-5 describes its potential solutions which can be explained by extended MOSAIC concept when the integer (M) is three. It means that the bounded integer can be from -3 to 3. Therefore, total 7 solutions include true one are plotted in Figure III-5. DME station is located at the center of 2-D coordinate, and two MOSAIC antennas are set around DME station with its antenna separation is smaller than 3.5λ.

Potential solutions are spread through the circle of DME bound. As MOSAIC system can provide bearing information, seven different bearings are calculated with extended MOSAIC concept, and the range between station and user is only decided by DME measurement. So that seven potential solutions, whose distance between station and potential solutions are all same, are displayed in Figure III-5.

If MOSAIC/DME system is applied to 3-dimensional case, then its potential solutions are calculated as below in Figure III-6 .

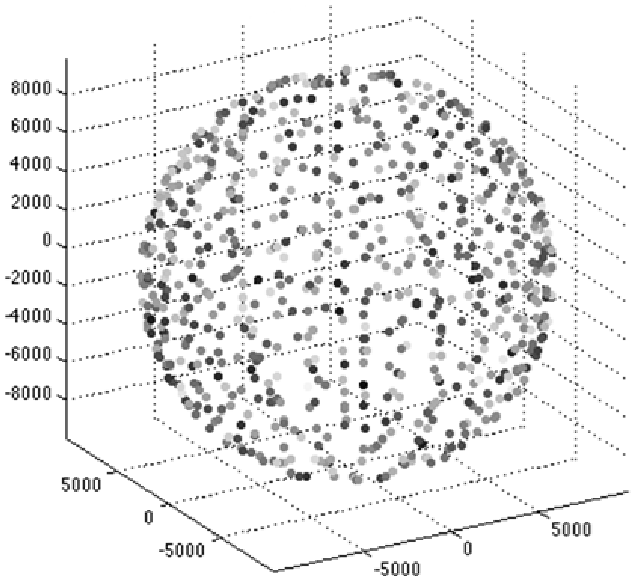


Figure III-6 Integer Ambiguity Potential Solutions of MOSAC/DME for 3-D case

For Figure III-6, one DME antenna and five MOSAIC antennas are considered for positioning, and its antenna separation of MOSAIC is smaller than 3.5λ. Potential solutions are spread on the surface of the sphere whose radius is DME measurement as similar with Figure III-5. The difference is that Figure III-5 is for 2-D and Figure III-6 is for 3-D, in other words, for circle and for sphere respectively.

One of special features of MOSAIC/DME is that its potential solutions locate surface of sphere, which is quite different from conventional CDGPS case. It also means that the distance between solutions are quite far from each other. And these features are important considerations for the ambiguity resolution of this system.

### 2) System Configuration

Conceptual design of MOSAIC/DME is described in Figure III-2. A DME station with five MOSAIC antennas is defined as MOSAIC/DME. In MOSAIC/DME system, DME antenna operates using two-way round signals which is same with current DME operation for full compatibility. And this operation occupies two DME channels (round trip) without any change. Five additional MOSAIC antennas transmit one-way continuous signals such like pseudolite, and they occupy one more channel to avoid the signal interference with installed DME; it means that total three channels are required for MOSAIC/DME. In added DME channel, five MOSAIC antennas use different PRN to distinguish each other using Code Division Multiple Access (CDMA) like GPS satellites. Because of CDMA, the bandwidth of MOSAIC signals could over the allocated DME channel bandwidth in some condition of chipping rate. With regard to this situation, MOSAIC signals might occupy more DME channels to avoid interference.

In this paper, the antenna placement has a shape of pyramid. DME ranging antenna is set at the center of pyramid, and other five MOSAIC antennas are vertexes of pyramid. Pyramid shape is expected that it can provide uniform performance through azimuth angles and it can also provide better performance for upper hemisphere area.

Signals from MOSAIC may contain the data such as station or antenna positions, phase correction, time correction, health status, or other necessary information.

All signals from MOSAIC/DME should be time synchronized. As MOSAIC/DME is one station, it can provide time-sync operation inherently sharing clock source. It means this new system doesn't have to set high-cost accurate clock. For time service, however, the station is also required to set accurate clock.

### 3) System Performance

For APNT application, there are two main issues about performance to be satisfied: accuracy and capacity. RNP is generally considered as the accuracy requirement of APNT, and RNP means horizontal 95% Total System Error (TSE) which is the sum of Navigation System Error (NSE) and Flight Technical Error (FTE). FTE has a relationship with airframe and flight system, which can be different from airplane to airplane. Therefore, only NSE is considered for initial analysis of accuracy performance.

Accuracy of MOSAIC/DME is performed using Monte-Carlo simulation using generated measurements. And the simulation result proves that MOSAIC/DME system can offer the enough accuracy for APNT requirement. Details are discussed in chapter V.2.

It is expected that the capacity of DME is also required to improve for future aircraft traffic. Even though capacity of the system should be treated thoughtfully, it is not performed sufficient in this paper. However, improved capacity of MOSAIC/DME can be explained conceptually as below.

Both two-way round signal and one-way continuous signal are used for MOSAIC/DME.

At first, both signals are used for the calculation of aircraft position for some times (a few second). After that the clock bias and clock drift between station and aircraft can be estimated. Using estimated parameters, positioning can be possible using only one-way continuous signals, because pseudorange can be regarded as real range measurement with clock offset information. As a result, the number of acquiring of DME measurement can be decreased using clock offset estimation, meaning that the capacity of DME, limited by the occupation of round signals, can be improved.

## 3. Positioning Algorithm for MOSAIC/DME

Simple positioning algorithm for MOSIAC/DME is least square solution with weighting using variance of each measurement. For positioning include resolving ambiguity problem, Least Square Ambiguity Search Technique (LSAST) is considered.

### 1) Weighted Least Squares (WLS) Solution

In this paper, Weighted Least Square (WLS) approach is generally used for every epoch to solve user's position [14]. General WLS equation can be expressed as in equation (III-9).

$$\begin{aligned} \overline{z} &= H \cdot \overline{x} + \overline{v} \\ \overline{x} &= (H^T V^{-1} H)^{-1} H^T V^{-1} \overline{z} \end{aligned} \tag{III-9}$$

The system is expressed linearly, and $\overline{x}$ is the position what we want calculate, $\overline{z}$ is measurement vector, and H matrix is composed of line-of-sight unit vector which is directed from user position to transmit antenna. Vector $\overline{v}$ is the noise of measurement vector $\overline{z}$ and matrix V is the covariance matrix of $\overline{z}$ .

Single differenced carrier phase measurement as in Equation (III-3) and DME measurement, described in equation (III-10), are constitute the vector $\overline{z}$ .

$$r = d + v \tag{III-10}$$

In equation (III-10), 'r' term is the measurement from DME, 'd' is the true distance from DME antenna to user, and 'v' is the DME noise.

Equation (III-3) can be expressed as below equations (III-11) and (III-12). Equation (III-10) also can be explained as (III-13) and (III-14).

$$\begin{aligned}\Delta\phi_{ij} &= \Delta d_{ij} + \Delta N_{ij}\lambda + \varepsilon'_{ij} \\ &= (\overline{R}_i - \overline{R}_u)\cdot\hat{e}_i - (\overline{R}_j - \overline{R}_u)\cdot\hat{e}_j + \Delta N_{ij}\lambda + \varepsilon'_{ij}\end{aligned} \tag{III-11}$$

$$\Delta\phi_{ij} - \overline{R}_i\cdot\hat{e}_i + \overline{R}_j\cdot\hat{e}_j - \Delta N_{ij}\lambda = (\hat{e}_j - \hat{e}_i)\cdot\overline{R}_u + \varepsilon'_{ij} \tag{III-12}$$

$$r = d + v = (\overline{R}_r - \overline{R}_u)\cdot\hat{e}_r + v \tag{III-13}$$

$$r - \overline{R}_r\cdot\hat{e}_r = -\hat{e}_r\cdot\overline{R}_u + v \tag{III-14}$$

$\overline{R}_i, \overline{R}_j$ are i-th and j-th MOSAIC antenna's positions respectively, and $\overline{R}_r$ is the position of DME antenna. $\overline{R}_u$ is the aircraft (user) position. $\hat{e}_i$ means the line-of-sight unit vector from user to i-th antenna (subscript 'r' means to DME antenna).

Using equation (III-12) and (III-14), WLS equation (III-9) can be written as equation (III-15) for five MOSAIC antennas and one DME antenna.

$$\begin{bmatrix} \Delta\phi_{12} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_2 \cdot \hat{e}_2 - \Delta N_{12}\lambda \\ \Delta\phi_{13} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_3 \cdot \hat{e}_3 - \Delta N_{13}\lambda \\ \Delta\phi_{14} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_4 \cdot \hat{e}_4 - \Delta N_{14}\lambda \\ \Delta\phi_{15} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_5 \cdot \hat{e}_5 - \Delta N_{15}\lambda \\ r - \overline{R}_r \cdot \hat{e}_r \end{bmatrix} = \begin{bmatrix} \hat{e}_2 - \hat{e}_1 \\ \hat{e}_3 - \hat{e}_1 \\ \hat{e}_4 - \hat{e}_1 \\ \hat{e}_5 - \hat{e}_1 \\ -\hat{e}_r \end{bmatrix} \overline{R}_u + \begin{pmatrix} \varepsilon'_{12} \\ \varepsilon'_{13} \\ \varepsilon'_{14} \\ \varepsilon'_{15} \\ \nu \end{pmatrix} \qquad \text{(III-15)}$$

And its covariance matrix is as in (III-16). The variance of MOSAIC carrier phase ( $\sigma_\phi^2$ ) and DME range ( $\sigma_r^2$ ) are independent, but variances between carrier phases are correlated because single-differenced measurements are used for positioning as in (III-15).

$$V' \equiv \operatorname{cov}[\overline{v}] = \begin{bmatrix} \sigma_\phi^2 & \sigma_\phi^2/2 & \sigma_\phi^2/2 & \sigma_\phi^2/2 & 0 \\ \sigma_\phi^2/2 & \sigma_\phi^2 & \sigma_\phi^2/2 & \sigma_\phi^2/2 & 0 \\ \sigma_\phi^2/2 & \sigma_\phi^2/2 & \sigma_\phi^2 & \sigma_\phi^2/2 & 0 \\ \sigma_\phi^2/2 & \sigma_\phi^2/2 & \sigma_\phi^2/2 & \sigma_\phi^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_r^2 \end{bmatrix} \qquad \text{(III-16)}$$

For positioning with equation (III-15), line-of-sight unit vector ( $\hat{e}_i$ ) is usually unknown parameter. Therefore, iteration method checking convergence is generally used for both position and line-of-sight unit vector. In iteration process, avoiding the matrix singular case with bad condition number, equation (III-15) and (III-16) can be modified to (III-17) and (III-18), respectively.

$$\begin{bmatrix} (\Delta\phi_{12} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_2 \cdot \hat{e}_2 - \Delta N_{12}\lambda)/\sqrt{2}\sigma_\phi \\ (\Delta\phi_{13} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_3 \cdot \hat{e}_3 - \Delta N_{13}\lambda)/\sqrt{2}\sigma_\phi \\ (\Delta\phi_{14} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_4 \cdot \hat{e}_4 - \Delta N_{14}\lambda)/\sqrt{2}\sigma_\phi \\ (\Delta\phi_{15} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_5 \cdot \hat{e}_5 - \Delta N_{15}\lambda)/\sqrt{2}\sigma_\phi \\ (r - \overline{R}_r \cdot \hat{e}_r)/\sigma_r \end{bmatrix} = \begin{bmatrix} (\hat{e}_2 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ (\hat{e}_3 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ (\hat{e}_4 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ (\hat{e}_5 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ -\hat{e}_r/\sigma_r \end{bmatrix} \overline{R}_u + \overline{v} \qquad \text{(III-17)}$$

$$V = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 & 0 \\ 0.5 & 1 & 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 1 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(III-18)}$$

Positioning equation of MOSAIC/DME system can be singular easily, because most of line-of-sight unit vector are similar, and two kinds of measurements which have different order are used. DME measurement can be up to 200km, but single differenced carrier phase is generally sub-meter levels. Divided by its standard deviation term as in (III-17) can improve the condition number of its measurement equation, because noise levels of carrier phase and DME range are also different order; mm-level for carrier phase and tens of meters for DME range.

In equation (III-17), there are four ambiguity terms; $\Delta N_{12}$, $\Delta N_{13}$, $\Delta N_{14}$, and $\Delta N_{15}$. These ambiguity terms are bounded according to extended MOSAIC concept. But one of them can be easily find without searching process of ambiguity candidates.

## 2) Least Square Ambiguity Search Technique (LSAST)

According to LSAST algorithm, all satellites are classified by two groups; primary satellite set and secondary satellite set [15]. Primary set is generally composed of the least required number of satellite for positioning, and its ambiguity candidates are needed to be resolved with some process. Secondary satellite set is the remainder except for primary set. And ambiguity candidates of secondary set can be analytically calculated using the result only performed by primary satellite sets.

Similar with above description, antennas of MOSAIC/DME system also can be divided to primary and secondary antenna sets. Primary antenna set is consists of total five antennas, DME antenna and four MOSAIC antennas, and secondary set is composed of one MOSAIC antenna. In equation (III-17), there are four ambiguities; $\Delta N_{12}$, $\Delta N_{13}$, $\Delta N_{14}$, and $\Delta N_{15}$. If the fifth MOSAIC antenna is secondary set, $\Delta N_{15}$ can be calculated immediately with the equation (III-19), using the result from primary antenna set, explained in (III-20).

$$\Delta N_{15} = round\left( \frac{\Delta\phi_{15} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_5 \cdot \hat{e}_5 + (\hat{e}_1 - \hat{e}_5) \cdot \hat{R}_p}{\lambda} \right) \tag{III-19}$$

$\hat{R}_p$ is the calculated position using only primary antenna set, DME antenna and four MOSAIC antennas as in (III-20).

$$\begin{aligned} \overline{z}_p &= H_p \cdot \overline{R}_p + \overline{v}_p \\ \hat{R}_p &= (H_p{}^T V_p{}^{-1} H_p)^{-1} H_p{}^T V_p{}^{-1} \overline{z}_p \end{aligned} \tag{III-20}$$

Each term in equation (III-20) has subscript ‘p’ which means ‘primary’. And these terms can be described as below equations; (III-21), (III-22), (III-23), and (III-24).

$$\overline{z}_p = \begin{bmatrix} (\Delta\phi_{12} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_2 \cdot \hat{e}_2 - \Delta N_{12}\lambda) / \sqrt{2}\sigma_\phi \\ (\Delta\phi_{13} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_3 \cdot \hat{e}_3 - \Delta N_{13}\lambda) / \sqrt{2}\sigma_\phi \\ (\Delta\phi_{14} - \overline{R}_1 \cdot \hat{e}_1 + \overline{R}_4 \cdot \hat{e}_4 - \Delta N_{14}\lambda) / \sqrt{2}\sigma_\phi \\ (r - \overline{R}_r \cdot \hat{e}_r) / \sigma_r \end{bmatrix} \tag{III-21}$$

$$H_p = \begin{bmatrix} (\hat{e}_2 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ (\hat{e}_3 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ (\hat{e}_4 - \hat{e}_1)/\sqrt{2}\sigma_\phi \\ -\hat{e}_r / \sigma_r \end{bmatrix} \tag{III-22}$$

$$\overline{v} = \begin{bmatrix} \varepsilon'_{12} \\ \varepsilon'_{13} \\ \varepsilon'_{14} \\ v \end{bmatrix} \tag{III-23}$$

$$V_p = \begin{bmatrix} 1 & 0.5 & 0.5 & 0 \\ 0.5 & 1 & 0.5 & 0 \\ 0.5 & 0.5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{III-24}$$

LSAST method can reduce the ambiguity candidates considering only primary antennas. If all the antenna separations between MOSAIC antennas are smaller than 5.5λ, then M is five which means number of ambiguity is 11. In this case, using LSAST, its total number of ambiguity is reduced from $11^4$=14641 to $11^3$=1331. Therefore it is only required to select the true one from 1331 ambiguities.

# IV. AMBIGUITY RESOULTION METHOD

As mentioned in Chapter III, extended MOSAIC concept can be applied to MOSAIC/DME system with bounded ambiguity whose total number is depending on the separation between MOSAIC antennas. And it is essential to resolve the carrier phase integer ambiguity for getting user's position. There are conventional methods to eliminate the false position such as residual threshold test and ratio test using Weighted Square Sum of Error (WSSE) as described in LSAST [14]. In addition to these methods, some particular approach to solve ambiguity problem is considered to MOSAIC/DME system; singularity check, convergence check, altitude bound, and Attitude and Heading Reference System (AHRS) test.

## 1. Residual Threshold Test

Residual threshold test consider the square sum of residual errors has forms of chi-square distribution. The chi-squared distribution with k degrees of freedom is the distribution of sum of the squares of k independent standard normal random variables [16]. The probability density function of this distribution is as below in Figure IV-1 with different degrees of freedom.

WSSE of MOSAIC/DME system can be calculated as in equation (IV-1).

$$\begin{aligned} &\delta\overline{z} \equiv \hat{z} - \overline{z} = H \cdot \hat{R}_u - \overline{z} \\ &WSSE:\ \Omega \equiv \delta\overline{z}^T V^{-1} \delta\overline{z} = \overline{z}^T V^{-1}(I-P)\overline{z} \\ &(P \equiv H(H^T V^{-1} H)^{-1} H^T V^{-1}) \end{aligned} \qquad \text{(IV-1)}$$

Vectors and matrixes used in equation (IV-1) are already defined in III.3. According to residual threshold test, WSSE has a distribution of chi-square distribution, so that it is possible to set some specific percentage for threshold. And the degree of freedom for MOSAIC/DME system is two as described in Table IV-1.

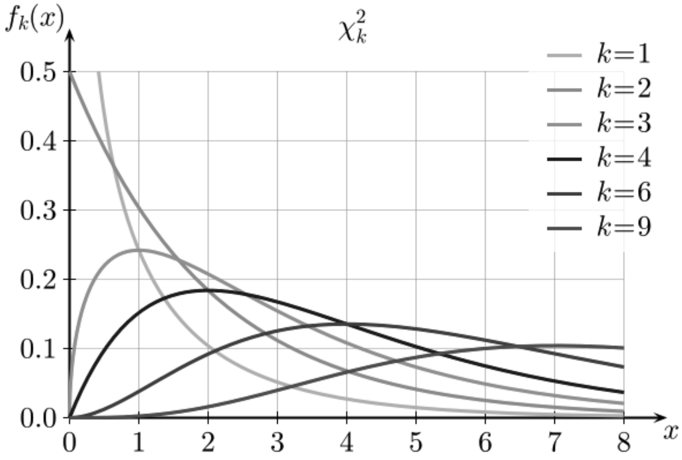


Figure IV-1 Probability Density Function of Chi-squared distribution [13]

Table IV-1 Number of Measurements and Unknowns in MOSAIC/DME System

| **Measurement (n=5)** | **Unknown (u=3)** |
|---|---|
| $\Delta\Phi_{12}$, $\Delta\Phi_{13}$, $\Delta\Phi_{14}$, $\Delta\Phi_{15}$, r | position (x, y, z) |

For example, if the percentage is 99% with degree of freedom 2, then its threshold is as in (IV-2).

$$\chi^2_{P\%,k} = \chi^2_{99\%,k=2} = 9.2103 \qquad \text{(IV-2)}$$

It means that the value of WSSE of MOSAIC/DME (degree of freedom: 2) is smaller than 9.2103 with probability 99%. It is expected that false positions have large WSSE compared to the true position, so that proper threshold value can eliminate the position calculated by false ambiguities. In Figure IV-2, only WSSE of false positions are expected over threshold (filled area).

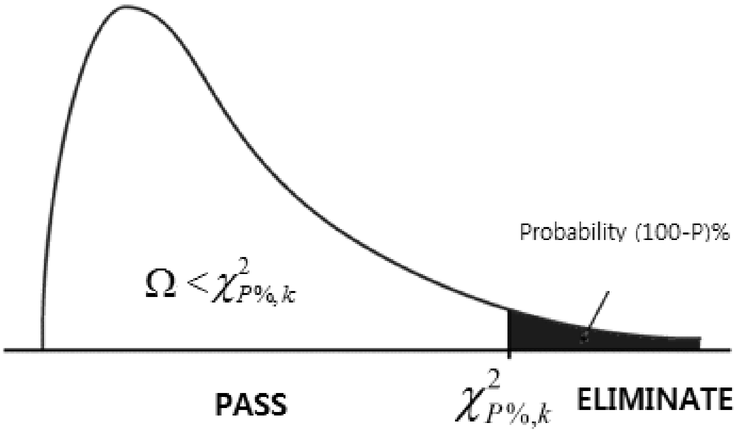


Figure IV-2 Chi-squared Distribution and its Threshold

## 2. Ratio Test

The base concept of ratio test is a ratio of two chi-square distribution has a form of f-distribution. The ratio equation can be expressed as in equation (IV-3).

$$F_{n,m} \equiv \frac{\chi_n^2 / n}{\chi_m^2 / m} \tag{IV-3}$$

$\chi_n^2$ and $\chi_m^2$ are independent chi-square distribution with n, m degrees of freedom respectively. And f-distribution is expressed by the numerator Chi-square with its degree of freedom and the denominator Chi-square with its degree of freedom. And f-distribution has a shape of as in Figure IV-3 with different degrees of freedom.

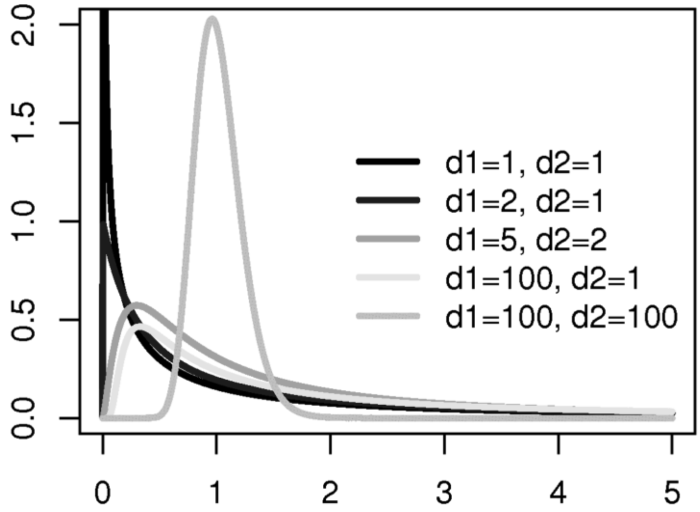


Figure IV-3 Probability Density Function of F-distribution [17]

In MOSAIC/DME, the ratio test can be performed in two steps. The definition for ratio test is as equation (IV-4). Degree of freedom is not considered because the denominator and numerator have the save value.

$$\begin{aligned}
&\Omega_0 \equiv \text{first minimum } \Omega \\
&\Omega_1 \equiv \text{second minimum } \Omega \\
&\Omega_2 \equiv \text{third minimum } \Omega \\
&\qquad\qquad \vdots \\
&\text{Ratio: } \Omega_1 / \Omega_0, \Omega_2 / \Omega_0, \cdots
\end{aligned} \tag{IV-4}$$

In the first step, only ratio in a certain epoch is considered. At this epoch, if the ratio $\Omega_1/\Omega_0$ is bigger than the threshold, which is the value in f-distribution with certain probability, then the ambiguity set of $\Omega_1$ can be eliminated. Similarly if $\Omega_2/ \Omega_0$ is large compared to the threshold, then $\Omega_2$ can be ignored.

Instead of WSSE at a certain epoch, accumulated WSSE are also used for ratio test as equation (IV-5) in the second step.

$$\begin{aligned}
&\sum \Omega_0 \equiv \text{accumulated first minimum } \Omega \\
&\sum \Omega_1 \equiv \text{accumulated second minimum } \Omega \\
&\sum \Omega_2 \equiv \text{accumulated third minimum } \Omega \\
&\qquad\qquad \vdots \\
&\text{Ratio: } \sum \Omega_1 / \sum \Omega_0, \sum \Omega_2 / \sum \Omega_0, \cdots
\end{aligned} \tag{IV-5}$$

Accumulated WSSE is expected that it has the form of cumulative density function of f-distribution as in Figure IV-4.

Ratio test using accumulated WSSE of certain period is performed by different threshold compared to the ratio test using one epoch WSSE. The degree of freedom of threshold need to be multiplied how many epochs are accumulated, because two WSSE form one ambiguity with different epochs are independent. Therefore, if the ratio test is performed sequentially, then the degree of freedom of the threshold is also incremented.

If $\Sigma\Omega_1/\Sigma\Omega_0$ is large compared to the threshold, then the solution of $\Omega_1$ is eliminated. And same process for $\Sigma\Omega_2/\Sigma\Omega_0$.

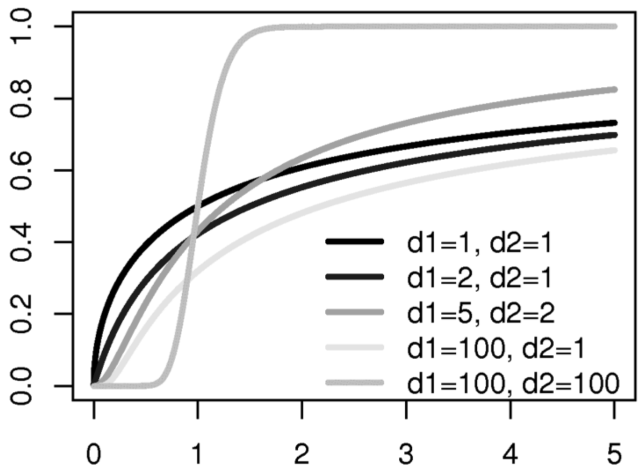


Figure IV-4 Cumulative Density Function of F-distribution [14]

## 3. Singularity Check

In equation (III-9), the inverse process of $H^T V^{-1} H$ is cannot be avoid and repeated for iteration. In some false ambiguity cases, this inverse process cannot be performed because of numerical matrix singularity. Therefore, checking the condition number of matrix $H^T V^{-1} H$ before inversing matrix can eliminate the false ambiguity without unnecessary process.

## 4. Convergence Check

Similar with singularity check, some solutions by false ambiguity sets are hard to converge in a certain number of iteration during WLS process (III-9). Therefore, it is possible to limit the iteration number for checking over counted iteration. Over iteration means that this ambiguity set can be regarded as a false one.

## 5. Altitude Bound

As described in Figure III-6, the potential solutions, positions by all ambiguity sets, are distributed on the surface of the sphere, whose radius is DME range measurement (= r). It also means that potential solutions have widely distributed altitudes from –r to +r. Therefore, if the aircraft has a reliable altimeter, then altitudes around the value which altimeter indicates are enough to consider. This altitude area is expressed inside of the rectangular box on the left side of Figure IV-5. In this case, most of ambiguities can be eliminated rapidly because of this particular feature of MOSAIC/DME system.

For general case, aircraft without altimeter, it is also possible to use this altitude bound approach. It is possible to exclude the ambiguity sets obviously whose altitude is underground level. About half of the ambiguities are lower than station altitude. In addition, the altitude over 60,000ft (18.29km) is also out of consideration, because conventional aircraft has the altitude from 0 to 60,000ft as described on the right side of Figure IV-5. With upper limit of altitude, if the range from station to aircraft is larger, more ambiguities can be excluded, because more false solutions have the altitude over 60,000ft.

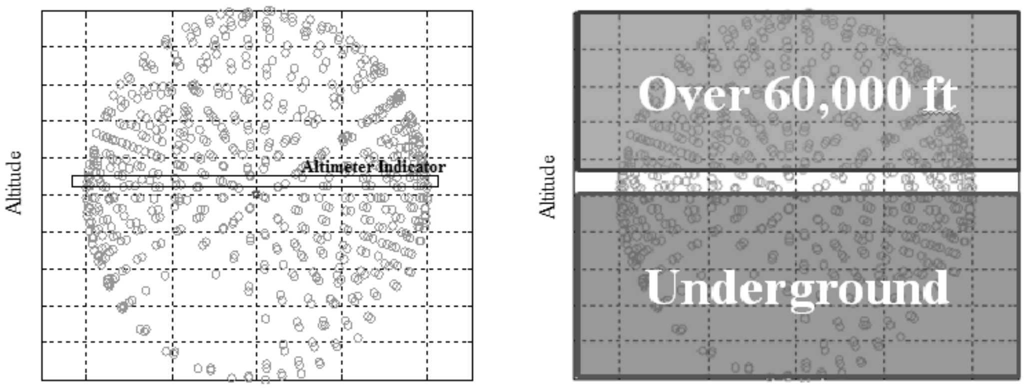


Figure IV-5 Bounded Ambiguities by Altimeter (left) and Bounded by Airspace (right) when the Range from Station to Aircraft is 54NM (100km)

## 6. AHRS Test

Different from conventional CDGPS, the potential solutions by cycle integer ambiguity sets are widely distributed, the largest gap between solutions could be twice of DME range measurement (up to 260NM). And these positions have the shape of sphere where the radius of sphere is range measurement by DME. So, after some time, these positions make another sphere whose range measurement is different from before one. As the distances between station and aircraft are fixed for all potential solutions, it is expected that each solution have different directions of trajectory. This result is displayed in Figure IV-6 when the aircraft is approach to the station.

In this case, heading to the station, each potential solution has different bearings which can be distinguished if the attitude (or heading) of the aircraft is known. When the aircraft equip the AHRS device, then its bearing can be a known parameter. With AHRS result, it is possible to exclude the false solutions comparing the bearing of each solution's trajectories with AHRS result. This process can be displayed simply as in Figure IV-7.

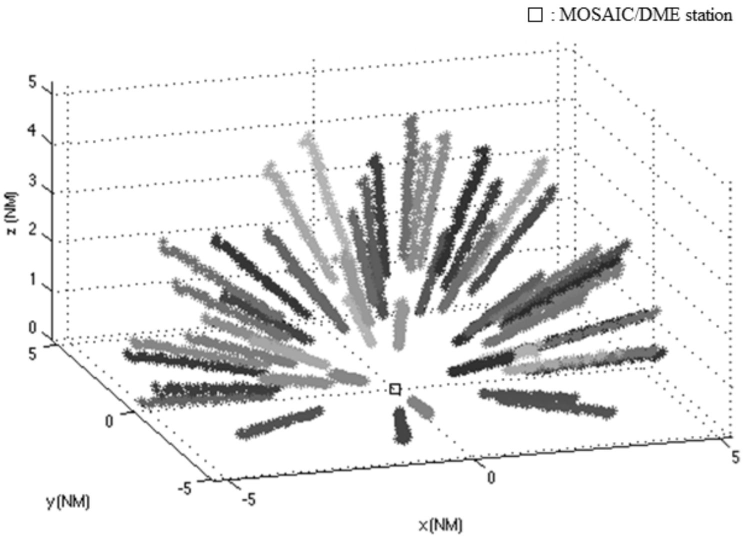


Figure IV-6 Trajectories of Potential Solutions by MOSAIC/DME whose Antenna Separation is 3.5 λ.

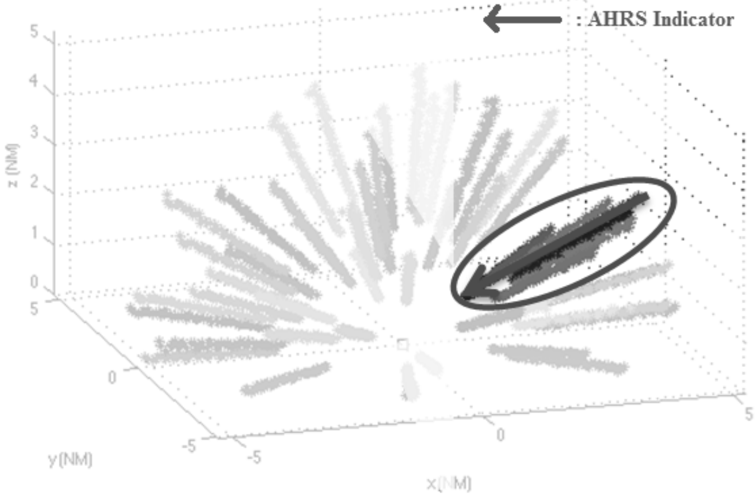


Figure IV-7 Ambiguity Resolution by AHRS test (inside of the ellipse)

The red arrow is the indicated value by AHRS, and the solutions inside the ellipse can be regarded as having similar bearing compared to AHRS. And potential solutions outside the ellipse can be eliminated as false trajectories.

# V. SIMULATION RESULTS

Monte-Carlo Simulations using MATLAB software are performed with consideration of contents mentioned in previous chapters. Two main kinds of simulations are executed; accuracy properties and ambiguity resolution. In the first simulation, accuracy properties of MOSAIC/DME system and performance verification for requirements of APNT are main issues. In the simulation of ambiguity resolution, methods explained in chapter IV are applied and displayed how it works.

## 1. Simulation Settings

### 1) Antenna Array

For simulations, it is assumed that user knows the positions of all antennas.

Total six antennas are considered for simulations. One conventional DME antenna with five additional MOSAIC antennas are set as described in Figure V-1. In simulations the position of the DME antenna is regarded as the origin of the coordinate system (0, 0, 0). And five MOSAIC antennas have their positions with a form of pyramid, whose center of mass is the DME antenna. Pyramid form is selected for the primary stage of simulations, because, it is expected that pyramid shape can service uniform performance around the station, and our main concern is not under the station but the air over the station.

For extended MOSAIC concept, the important concern is the separation between antennas. In this paper, antenna separation 1.446m ($<5.5\ \lambda$) is mainly considered. For instances, in Figure V-1, the separations between antenna #1 and antenna #2, antenna #1 and antenna

#3, antenna #1 and antenna #4, and antenna #1 and antenna #5 are 1.446m.

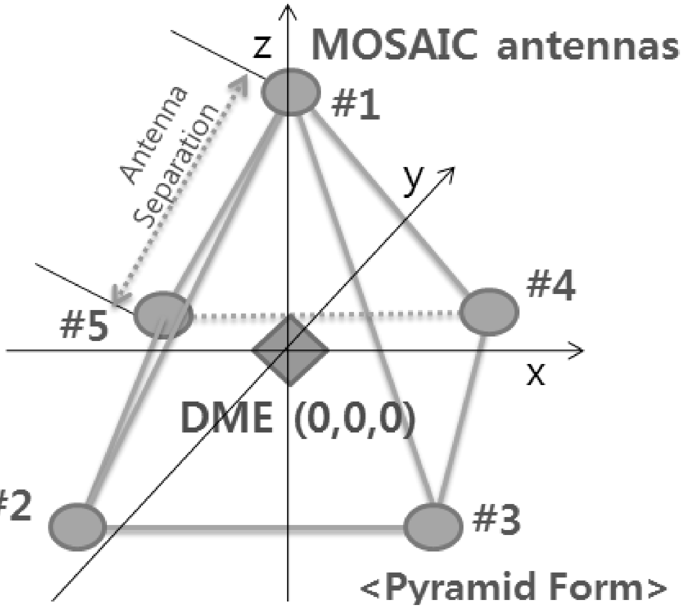


Figure V-1 Antenna Array of MOSAIC/DME system

### 2) Signals

Two types of signals are transmitted from MOSAIC/DME system. DME operates without any change from conventional system using two-way round trip signals. And five MOSAIC antennas broadcast one-way continuous signals as pseudolite. In simulation, MOSAIC signals have the carrier frequency of 1088.5MHz, the center of DME operational frequency 962~1215MHz. With this frequency, the wavelength is about 27.5cm.

### 3) Measurements

MOSAIC/DME system services two types of signals, therefore, two types of measurements are generated for Monte-Carlo simulation.

First one is carrier-phase measurements for MOSAIC antennas. The general noise of carrier-phase measurement is 1% of wavelength for standard deviation [18]. For this system, the standard deviation of noise is 2.8mm because its wavelength is 27.5cm.

Second one is DME range measurement. DME antenna can service the real range (not pseudorange) using round trip signals. According to 2010 Federal Radio Navigation Plan [12], DME station is required to offer the range accuracy better than 0.1NM (~185m) of 95% error. This standard can be regarded as the conventional performance of DME station at the present day. Most of current DME systems, however, are old-versioned, so that there are still potentialities to improve their performance. And it can reach the performance of 30m of 95% error [3]. Therefore, for the simulations of this paper, both noises 185m and 30m of 95% error ($2\sigma$) are considered together.

Simulation settings are summarized in Table V-1.

Table V-1 Simulation Settings

| | Carrier Frequency | Wavelength | NO. of Antennas | Measurement Type | Measurement Noise ($2\sigma$) |
|---|---|---|---|---|---|
| MOSAIC | 1088.5MHz | 27.5cm | 5 | carrier phase | 5.6mm |
| DME | conventional channel | not care | 1 | range | 30m, 185m |

## 2. Accuracy Properties

In these simulations, understandings of accuracy properties of MOSAIC/DME system and performance verification for APNT both are executed. For the first time, the relationship between range from station to aircraft and accuracy is verified. After that the accuracy properties with antenna separation and relative bearing from station are performed in order. As a last, performance verification in Terminal Class B area is confirmed.

It is assumed that cycle ambiguities of MOSAIC system are solved, only to confirm the position accuracy. And 1000 times of Monte-Carlo simulations are executed for the statistical results.

### 1) Range and Accuracy

Simulation results display the relationship between slant range from station to aircraft and accuracy through the glide slope of 3° as in Figure V-2 with MOSAIC antenna separation setting is 1.45m.

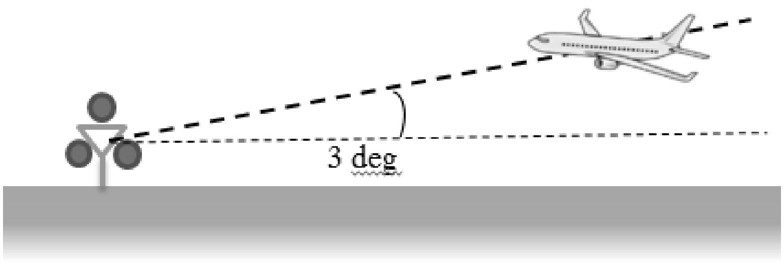


Figure V-2 Glide Slope of 3°

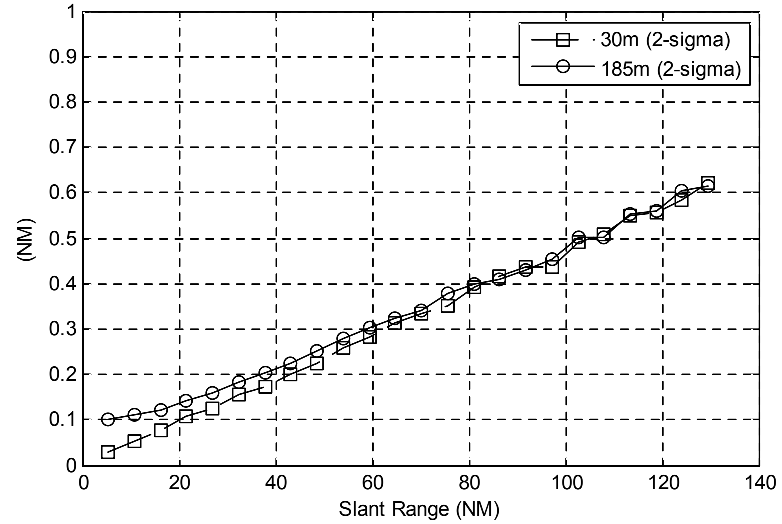


Figure V-3 Horizontal Position Error 2drms (<95%)
through the Glide Slope of 3 degrees with different DME noise levels

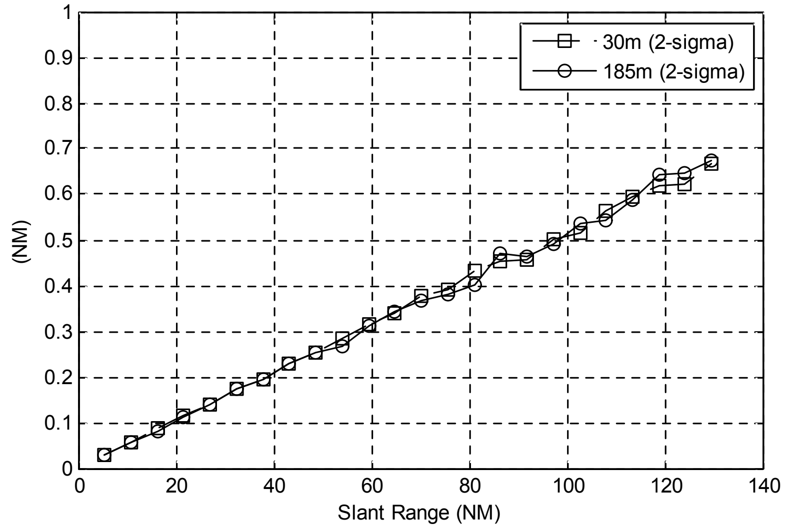


Figure V-4 Vertical Position Error 2σ (<95%)
through the Glide Slope of 3 degrees with different DME noise levels

Figure V-3 displays the horizontal error, and Figure V-4 displays the vertical error. In both figures, as the range increased, the position error is also increased almost linearly. It seems there is a linear relation between range and accuracy.

For APNT requirement, RNP 1.0 is required for en-route. In Figure V-3 displays the horizontal 95% error by this system up to 130NM. And its maximum error is 60% of the requirement. Therefore, MOSAIC/DME system with antenna separation 1.45m can perform enough accuracy for en-route APNT requirement.

### 2) Antenna Separation and Accuracy

It is expected that large antenna separation can make better accuracy. And the simulation results are also displayed in Figure V-5 and Figure V-6 as expected.

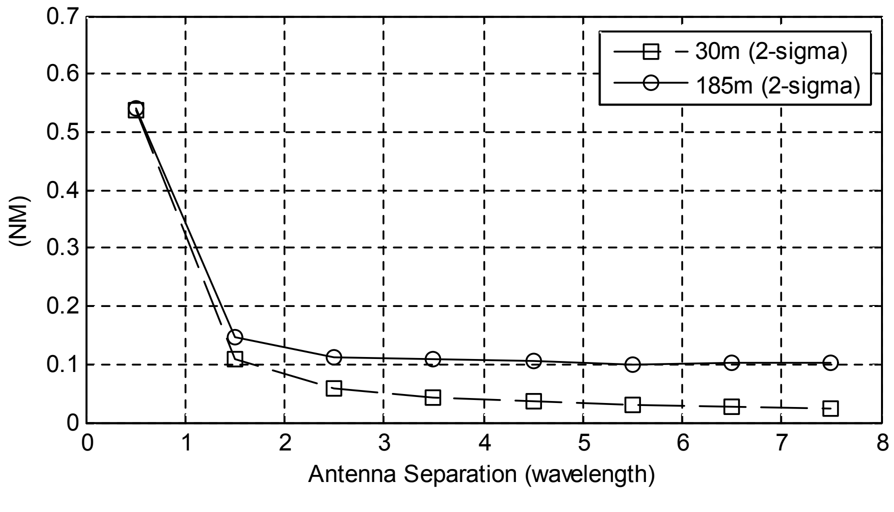


Figure V-5 Horizontal Position Error 2drms (<95%) with Range 5.4NM (=10km)

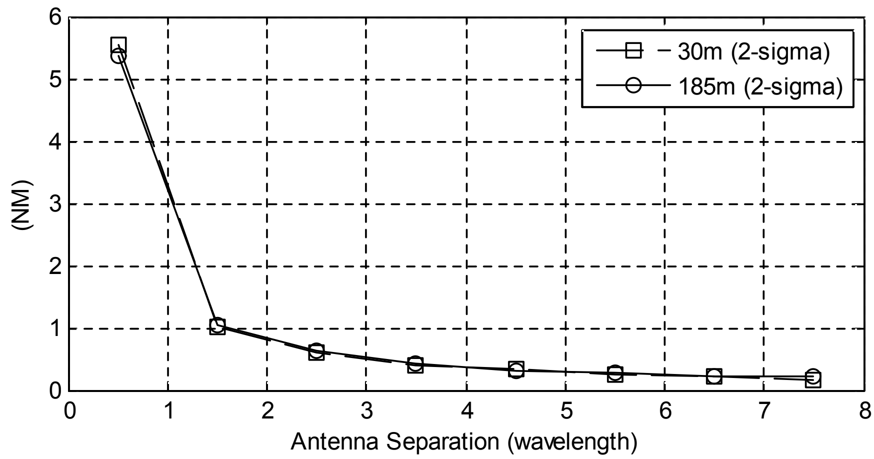


Figure V-6 Horizontal Position Error 2drms (<95%) with Range 54NM (=100km)

The x-axis in Figure V-5 and Figure V-6 is MOSAIC antenna separation whose unit is wavelength (=27.5cm); from 0.5 λ to 7.5 λ (from 13.8cm to 206cm). And two kinds of DME noises are displayed together.

These results prove that the larger antenna separation shows better accuracy. The tendencies in Figure V-5 and Figure V-6, however, are different. In Figure V-5, the effect of DME noise is existed so that the case of DME noise of 30m (2-sigma) shows better accuracy. In contrast, the effect by DME noises is mostly neglected in Figure V-6 when the antenna separation is 54NM.

This phenomenon can be explained with the fact that two types of signals are combined for positioning. DME range noise is considered as same with different ranges from station to aircraft. On the other hand, error by MOSAIC is expected it has bigger error if the range is larger.

In Figure V-5, the range is 5.4NM. If the range from station to aircraft is relatively close, the position error by MOSAIC can be smaller than the error noise of DME (185m). It means that the accuracy of MOSAIC/DME is dominant by DME noise within certain range. Therefore different DME noises show different accuracy.

In contrast to Figure V-5, the range of Figure V-6 is 54NM, which is enough far that the dominant error source is MOSAIC system. Therefore two lines by different DME noises are overlapped.

### 3) Relative Position and Accuracy

Accuracy of different relative positions from MOSAIC/DME station is performed. These relative positions are set with fixed range 5.4NM (=10km), azimuth angles from 5° to 360° with 5° resolution, and elevation angles from 5° to 90° with 5° resolution as displayed in Figure V-7. Antenna separation is 1.45m (<5.5λ) and only 30m of DME noise is considered.

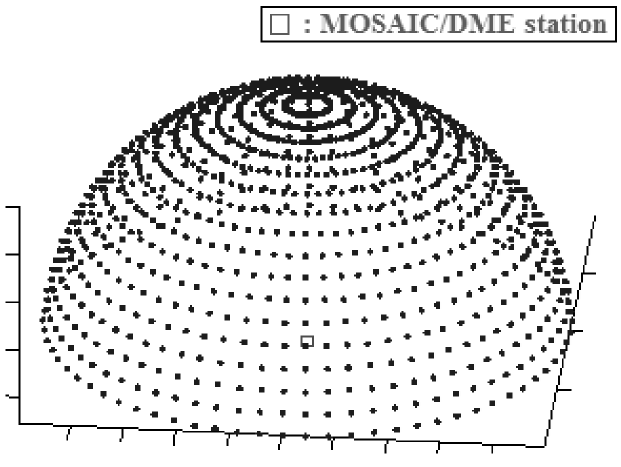


Figure V-7 Grids for the Simulation of Relative Positions

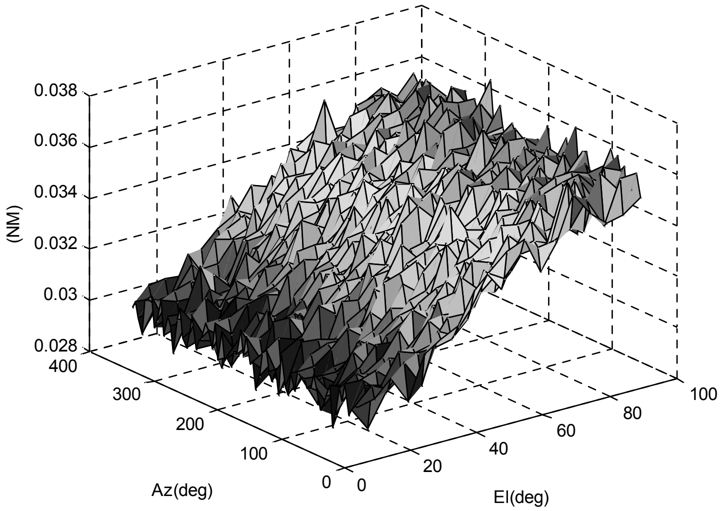


Figure V-8 Horizontal Position Error 2drms (<95%)

through the relative Azimuth and Elevation angles with Range 5.4NM

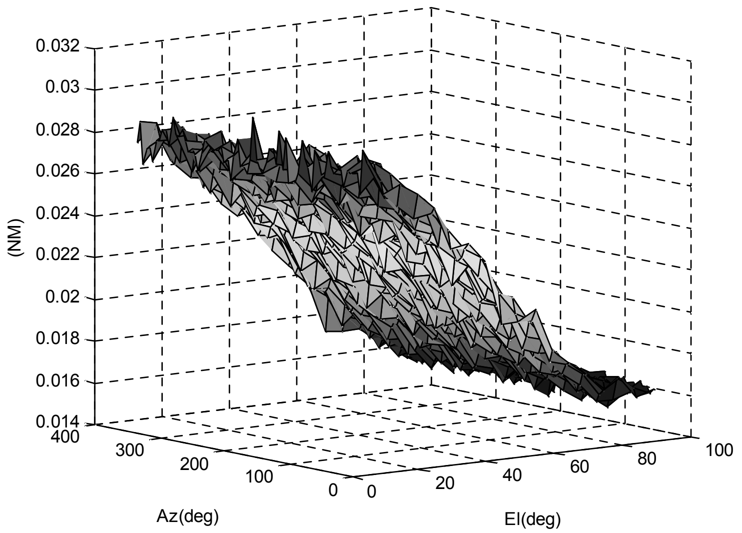


Figure V-9 Vertical Position Error 2 σ (<95%)

through the relative Azimuth and Elevation angles with Range 5.4NM

The results in Figure V-8 and Figure V-9 show that position errors through azimuth angle can be regarded as uniform, on the other hand, errors are dominantly depend on elevation angle.

And there is also a tendency that the error of this system is combined of two signals, MOSAIC and DME. DME can offer range measurement, which can be vertical error if the elevation angle is high, and be horizontal if the elevation angle is low. In contrast the MOSAIC system has error distribution through angular direction, so that in high elevation angle its measurement error is dominantly on horizontal, and could be vertical for low elevation angle. If the range is 5.4NM, the main error source is not DME but MOSAIC, so that vertical error shows better performance at high angles, and better horizontal accuracy at low angles.

### 4) Class B APNT Requirement

In the simulation of V.2.1) Range and Accuracy, the requirement of APNT for en-route area is verified. Therefore it is required to verify the RNP 0.3 for airspace class B near terminal. Airspace class B terminal area is different from each airport, because environments around airport can affect the shape of terminal area. But, in general, its shape resembles an upside-down wedding cake with three stages as in Figure V-10.

If MOSAIC/DME station is located at the center of the terminal area, then according to the accuracy properties verified above, it is enough to consider the outer edge of class B to prove the performance of MOSAIC/DME, because there is a linear relationship between range and accuracy. General outer edge of class B area has a horizontal radius of 30NM (~55.6km) and 10,000ft (~3km) altitude.

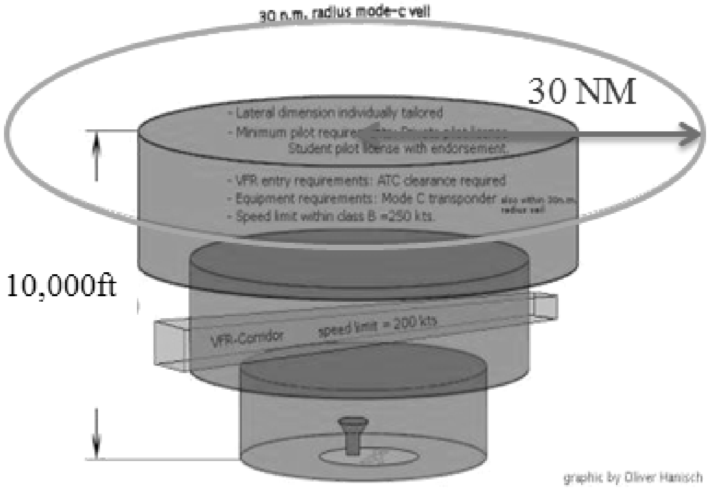


Figure V-10 General Class B Terminal Area

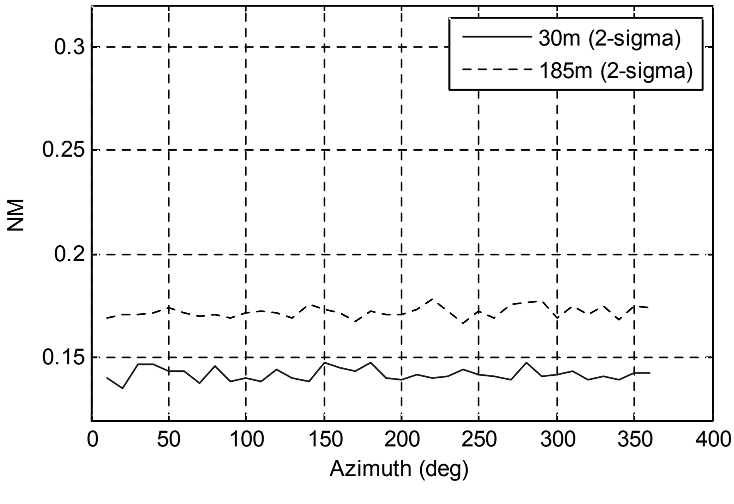


Figure V-11 Horizontal Position Error 2drms (<95%)

through the Outer Edge of Airspace Class B

The performance through outer edge of class B terminal area is displayed in Figure V-11. The required performance in this area is RNP 0.3, horizontal 95% error in Nautical Miles, and the simulation result proves that MOSAIC/DME system has enough performance for terminal area satisfying APNT requirement.

## 3. Simulation of Ambiguity Resolution

### 1) Simulation Settings for Ambiguity Resolution

Six steps of ambiguity resolution methods are explained in chapter IV. For the verification of ambiguity resolution methods mentioned above, Monte-Carlo simulations generating measurements are executed.

Basic simulation settings are already mentioned in V.1. In this simulation, antenna separation of 1.45m (<5.5λ) and DME noise of 30m are mainly concerned. And the speed of aircraft is 135kn (~250km/h), which is the landing speed of conventional jet airliner. And DME operates in 10Hz, so that positioning is also performed ten times in a second.

Ambiguity Resolution Settings are as follows. The possibility of residual threshold test is 99.99999% and ratio test is 99.9% with degree of freedom 2. These possibility values are decided experimentally.

For singularity test, if the condition number of matrix ( $H^T V^{-1} H$ ) is smaller than ten times of EPS (2.2204e-16: spacing of floating point numbers in MATLAB), then this false solution is excluded. For convergence test, max iteration number is 20 for position convergence threshold of 1e-5. And potential solutions with the altitude from 0 to 60,000ft are our concern.

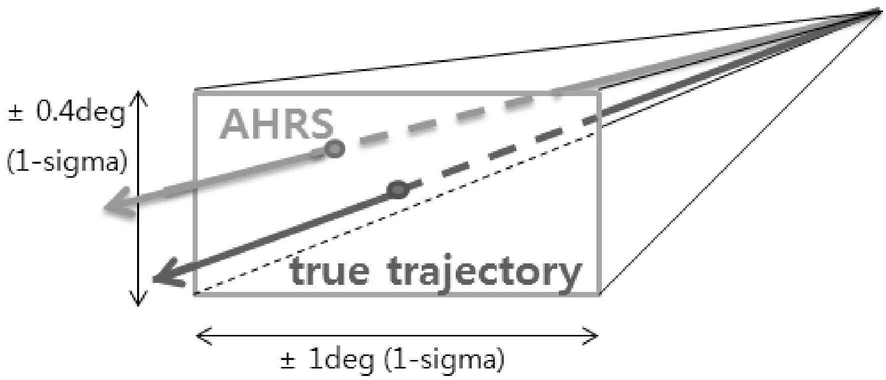


Figure V-12 AHRS Trajectory Errors used in Simulation

The threshold of AHRS test is decided by the specs of commercial MEMS-level AHRS devices; 'ANC1000' and 'NAV440' from CrossBow company, '3DM-GX3-45' from MicroStrain company, and 'LandMark 30 AHRS' from Gladiator company. The AHRS spec used in simulation is 1deg of standard deviation for heading, and 0.4 degree of standard deviation for pitch as described in Figure V-12. And threshold to eliminate the false trajectories is 4deg (4σ of heading accuracy) with assumption that high speed aircraft has a same attitude with its direction. For ambiguity resolution, the trajectories of aircraft (after 10 seconds) are compared to the AHRS indicated bearing to check with threshold.

Ambiguity resolution methods are verified with settings mentioned above for various cases. It would be perfect if all cases of ambiguity resolutions are performed. But it is practically impossible to consider all cases, including direction and speed of aircraft, relative position, range, and so on. Therefore simulation cases are generated as below in Table V-2.

Mainly three ranges are considered; 5.4NM, 10.8NM and 54NM. At each range, relative azimuth angle from station is considered for all direction with resolution 10°. For elevation angle, each range has different bounds because of the altitude of aircraft cannot be over 60,000ft. And three kinds of direction are considered as in Figure V-13. The first direction is approach to the station, the second is horizontal flight heading to the station, and the third one is the cross product of the second and the first. Therefore total 972 cases are verified for 5.4NM range, 540cases for 10.8NM, and 756 cases for 54NM. And the simulation is performed once for each case during 120 seconds of flight

Table V-2 Cases used in Ambiguity Resolution Simulation

| | **Range = 5.4NM (~10km)** | **Range = 10.8NM (~50km)** | **Range = 54NM (~100km)** |
|---|---|---|---|
| **Azimuth Angle** | 10~360 ° (step 10 °) | | |
| **Elevation Angle** | 5~85 ° (step 10 °) | 4~20 ° (step 5 °) | 3~9 ° (step 1 °) |
| **Direction** | Direction 1, 2, 3 | | |
| **Total Case** | **972** | **540** | **756** |

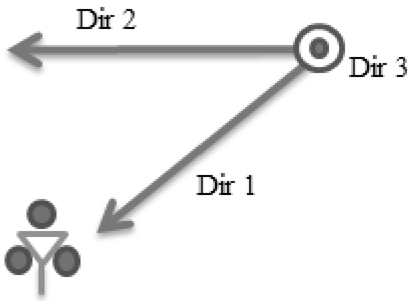


Figure V-13 Three Directions of Aircraft for Simulation

### 2) Ambiguity Resolution without AHRS Test

This simulation describes the result without AHRS device, purely MOSAIC/DME system. Ambiguity resolution times of these cases are displayed in Figure V-14, Figure V-15, and Figure V-16.

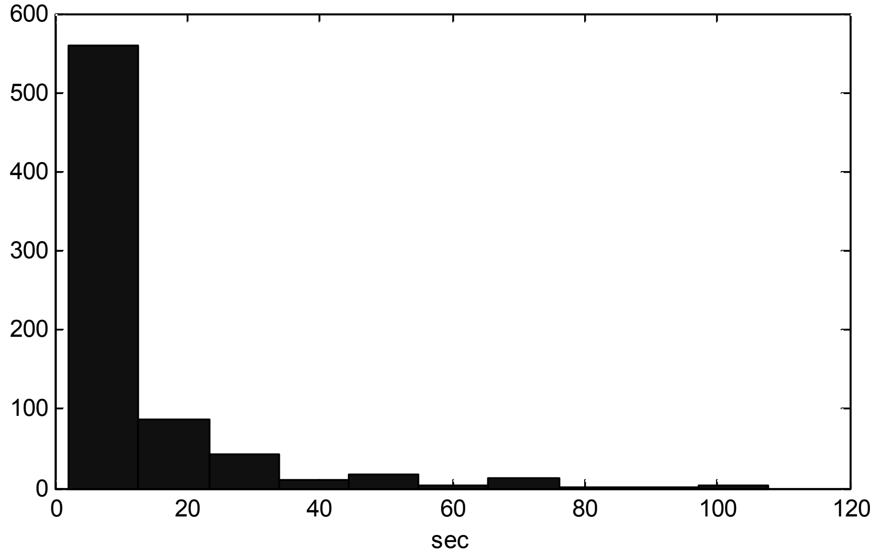


Figure V-14 Histogram of Ambiguity Resolution Time without AHRS (Range=5.4NM): 234 Unsolved Cases

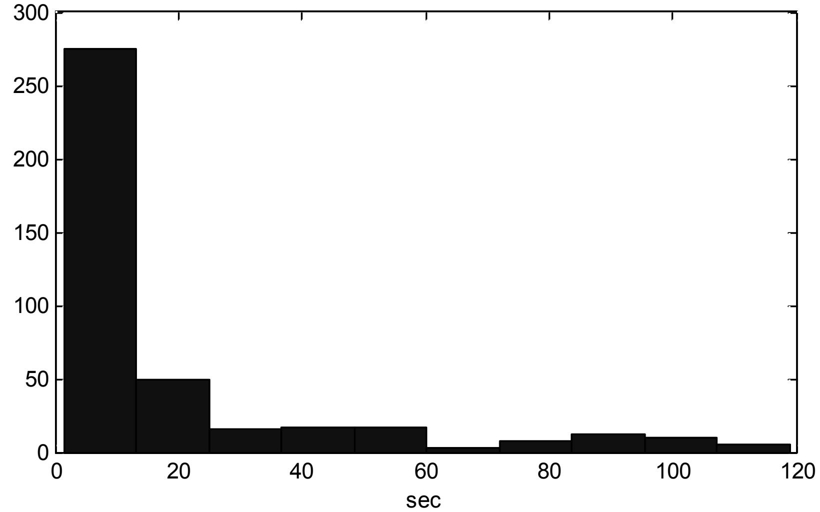


Figure V-15 Histogram of Ambiguity Resolution Time without AHRS (Range=10.8NM): 128 Unsolved Cases

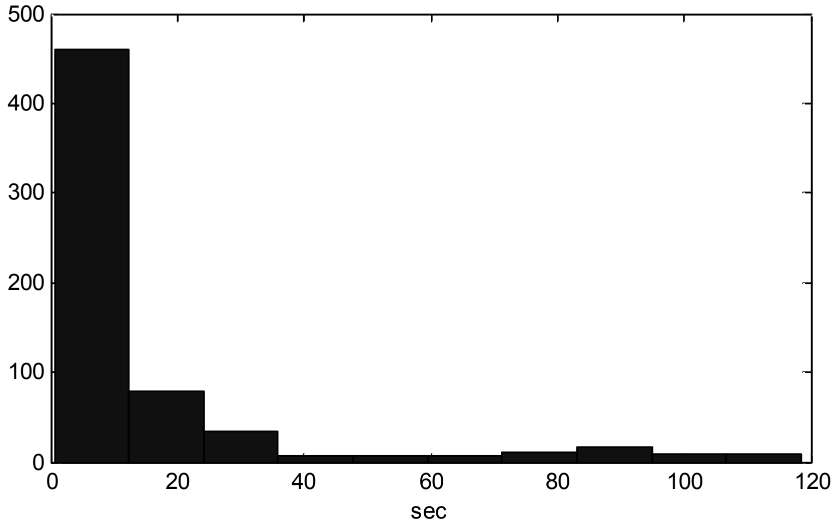


Figure V-16 Histogram of Ambiguity Resolution Time without AHRS (Range=54NM): 120 Unsolved Cases

All three simulation results show that there are unsolved cases for 120 seconds of simulation; 234/972 cases for 5.4NM range, 128/540 cases for 10.8NM, and 120/756 cases for 54NM. If simulations are executed longer than 120 seconds, it is expected that these unsolved cases would be solved in some time. For better time performance, however, if aircraft equips AHRS device, its ambiguity resolution performance is much better.

### 3) Ambiguity Resolution with AHRS Test

Ambiguity Resolutions include AHRS Test are displayed in Figure V-17, Figure V-18, Figure V-19.

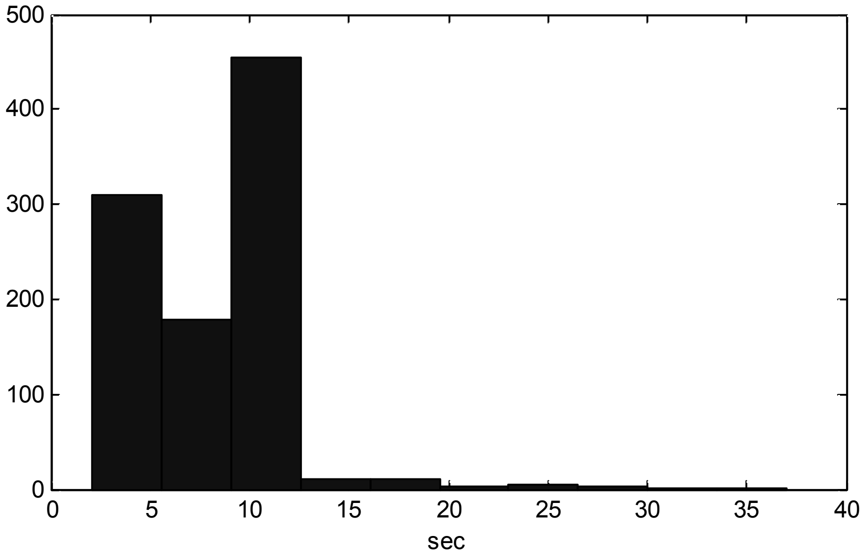


Figure V-17 Histogram of Ambiguity Resolution Time with AHRS (Range=5.4NM)

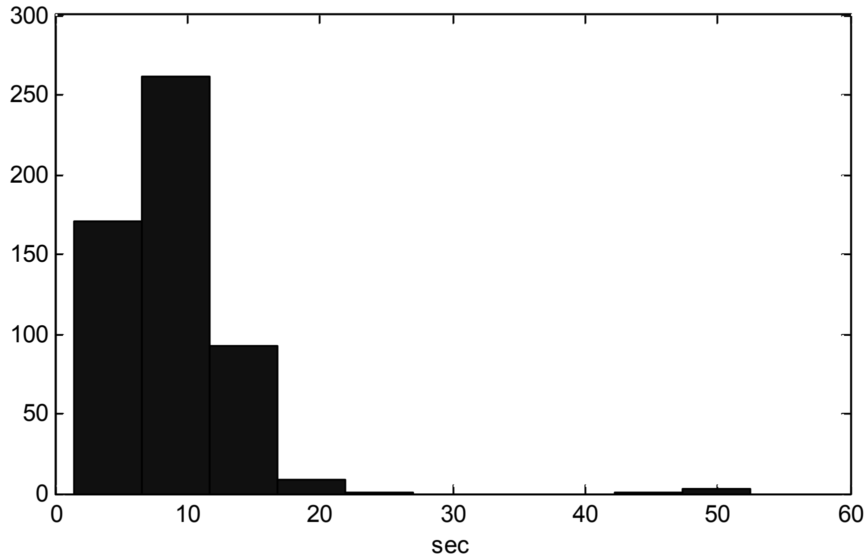


Figure V-18 Histogram of Ambiguity Resolution Time with AHRS (Range=10.8NM)

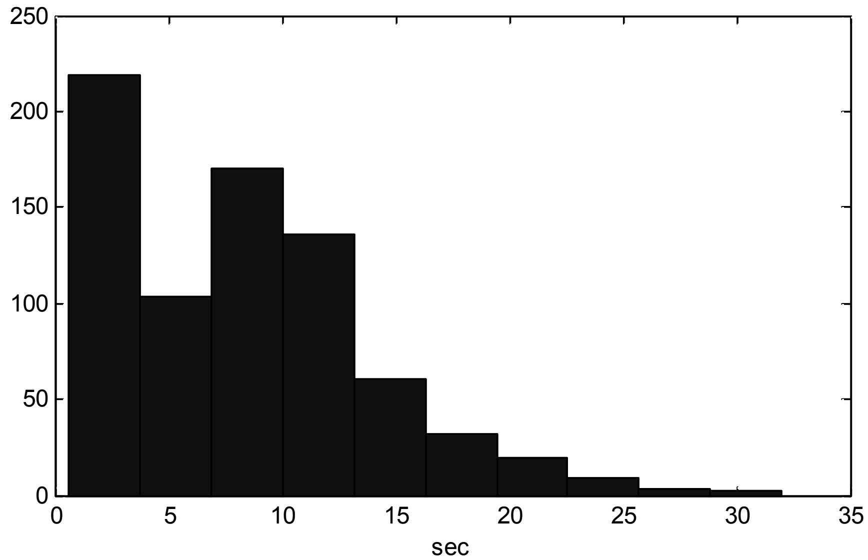


Figure V-19 Histogram of Ambiguity Resolution Time with AHRS (Range=54NM)

In Figure V-17 when the range is 5.4NM, the last ambiguity is resolved at 37 seconds. In Figure V-18, the last one is 52.5 seconds, and 32 seconds for Figure V-19.

Ambiguity Resolution with AHRS Test in these simulation cases proves that all cycle integer ambiguities are resolved within one minute. And most of them are solved in 20 seconds.

# VI. CONCLUSION

## 1. Summary

If GNSS is unavailable, because of RF interference, satellite failure, or other causes, APNT is required to perform important role for continuity and safety of air services. As a solution for APNT terrestrial navigation system, MOSAIC/DME system is presented in this paper. The MOSAIC/DME is a new concept, makes it possible for 3D positioning using only single station. Five additional MOSAIC pseudolite-like antennas are set together with DME station and transmit one-continuous signal. DME antenna operates same with conventional DME system. Carrier-phase measurements from MOSAIC antennas and range measurement from DME antenna are used for positioning algorithm. The cycle integer ambiguities of MOSAIC antennas are geometrically bounded with extended MOSAIC concept, so that ambiguity resolution of this system is relatively easy compared to conventional CDGPS. And all signals from six antennas are built-in time synchronized because they can share one clock source.

Positioning algorithm of MOSAIC/DME is based on LSAST, and its position is calculated using weighted least square solution for every epoch. Ambiguity resolution methods of MOSAIC/DME system are composed of six steps. Residual threshold test and ratio test from LSAST, singularity check, convergence check, altitude bound, and AHRS test.

Performing Monte-Carlo simulations, it is proved that MOSAIC/DME system has enough performance for the requirements of APNT, and also proves that most of cycle ambiguities are solved within 20 seconds. If the aircraft is in the multi-station coverage area, its accuracy performance can be increased, and its ambiguity resolution is also highly improved.

## 2. Suggestions for Future Work

In this paper, conceptual design of MOSAIC/DME system is presented, and only accuracy properties and ambiguity resolutions are verified using simulation. Therefore, there are still many important issues to be concerned if this system is ready for implementation.

First of all, ambiguity resolution is need to be improved. Ambiguity resolution time and its accuracy should be confirmed. It is the best if the ambiguities are solved directly without probabilistic approach.

Simulations using various error sources, such as multipath and tropospheric delay, are also considered for the verification of this system.

Signal analysis of this system is also required to be studied. Signal modulation and message structure are issues.

Integrity is an essential factor for the operation of air services, which are very sensitive to the safety.

And capacity, antenna array patterns, and multi-station performance studies are also needed to be performed for the implementation of MOSAIC/DME system.

# 요 약

본 논문에서는 위성항법시스템(Global Navigation Satellite System)을 사용할 수 없는 상황에서 항공기의 경제성과 안전성을 보장하고, 앞으로 계속 증가할 미래 항공 운송량에 대한 대비를 위한 대체 항법(Alternate Positioning Navigation and Timing)에 관한 연구를 진행하였다.

현재 대체항법은 미연방항공청(FAA)가 주도하는 NextGen이라는 차세대 항공운항 프로그램 내에 포함되어 있으며, 대체항법의 해결책으로써 몇몇 대안들에 대한 연구가 진행되고 있다. DME(Distance Measuring Equipment)를 이용한 연구들이 주를 이루며, 그 중 DMPR(DME Passive Ranging)에 대한 연구가 가장 활발하게 진행되고 있지만, 위치 결정에 2개 혹은 3개의 스테이션이 필요하고, 각 스테이션 간의 시각 동기가 필수적이기 때문에 이 부분에 대한 추가적인 연구와 비용에 대해 검토되고 있는 상황이다.

본 논문에서 제안하는 MOSAIC/DME시스템은 기존의 DME시스템에 의사위성과 유사한 다수의 MOSAIC안테나를 추가적으로 장착하여 단방향 연속신호를 방송하도록 하였다. 이 경우 장점으로는 하나의 스테이션에서 클럭을 공유하기 때문에 모든 신호가 동기 되어 있다는 것과 하나의 스테이션 만으로 3차원 위치 결정이 가능해져 실제 가용영역이 넓어질 수 있다는 점이다. 또한 DME신호는 기존의 방법과 동일하게 동작하므로, 기존의 DME와의 호환성이 보장되며, 양방향 신호와 단방향 신호를 동시에 사용하므로 스테이션의 항공기 수용능력 역시 증가하리라 기대할 수 있다.

하나의 스테이션에서 위치 결정을 수행할 경우, 기하학적으로 배치가 좋지 않기 때문에 위치 정확도가 매우 낮으리라 예상된다. MOSAIC/DME 시스템에

서는 이러한 문제점을 해결하기 위해, 측정치로서 반송파 위상을 사용하게 된다. 하지만 일반적으로 반송파 위상 측정치를 사용하는 경우 파장길이의 미지정수 항이 추가되어 실질적으로 위치계산 수행이 불가능하게 된다. 여기서 이 미지정수 항을 해결하기 위해 MOSAIC개념을 사용하게 된다. 기본 MOSAIC 개념은 안테나간 간격이 반 파장 이내일 때 미지정수를 바로 결정할 수 있게 된다. 하지만 항공기에 사용할 수 있을 정도의 정확도를 얻기 위해서는 안테나 간의 간격이 일정 범위 이상 될 필요가 있으므로, 이 때에는 확장된 MOSAIC 개념을 적용하게 된다. 확장된 MOSAIC개념에 따르면 MOSAIC/DME 시스템의 미지정수는 기하학적으로 제한되어 있기 때문에, 기존의 CDGPS(Carrier-phase Differential GPS)와는 다르게 쉽게 미지정수를 해결할 수 있다.

실제 위치를 결정하는 과정은 LSAST(Least Square Ambiguity Search Technique) 기법을 이용하여, 매 시점마다 가중최소자승법(Weighted Least Square Solution) 계산식으로 위치를 계산한다. 미지정수 결정 과정은 LSAST에서 이용하는 Residual Threshold Test와 Ratio Test를 수행하며, 여기에 MOSAIC/DME시스템의 특징을 이용한 행렬 특이성 확인, 수렴성 확인, 고도제한, AHRS(Attitude Heading Reference System)을 이용한 확인 방법들을 이용하여 미지정수를 결정하게 된다.

MOSAIC/DME시스템의 정확도 특성과 미지정수 결정 검증을 위해 몬테 카를로 시뮬레이션을 수행한 결과, 이 시스템이 대체항법에서 요구하는 정확도를 모두 만족하는 사실을 확인할 수 있었으며, 미지정수의 경우에도 대부분 20초 내에 해결되고, 현재의 시뮬레이션 환경에서는 최대 1분 내에 모두 풀리는 것을 확인하였다.

본 논문에서는 MOSAIC/DME시스템에 대해 개념적인 큰 그림을 먼저 제시하고, 이 중 정확도 부분과 미지정수 결정 부분에 대해 중점적으로 다루었다. 앞으로도 연구를 진행하면서, 실제 시스템의 구현을 위해서는 미지정수 결정

개선, 다양한 오차를 포함한 시뮬레이션, 항공기 수용능력 검증, 신호 규격, 메시지 타입, 무결성, 안테나 배치, 다중 스테이션을 이용했을 경우의 성능 검증 등을 추가적으로 진행할 필요가 있다.

주요어: 대체항법(APNT), 차세대 항공기용 위치결정 시스템(NextGen), 모자이크 개념, 확장된 모자이크 개념, 거리측정장치(DME), 미지정수 결정, MOSAIC/DME

학번 : 2011-20694

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