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The Role of Attention in the Generation of Human and Robot Arm Movements

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ABSTRACT

The Role of Attention in the Generation of Human and Robot Arm Movements

by

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Even as robots are being asked to perform increasingly complex tasks simultaneously, current robot motion planning and control laws for the most part fail to consider limitations on the available computation, communication, and memory resources. Humans on the other hand are adept at multi-tasking, continuously shifting their attention from one task to another. In this thesis we investigate the role of control attention in the general of both robot and human and arm movements. We examine robustness to spatiotemporal quantization as a means of evaluating the performance of arm trajectory generation and control laws from the perspective of control attention. We then develop a stochastic control strategy for generating and controlling arm movements that attempts to minimize the required computation and communication resources, based on the minimum intervention principle put forth by Todorov and Jordan. We perform numerical experiments for planar arm reaching motions with respect to these spatiotemporal quantization metrics. Experimental results show that our control laws share many of the distingushing features of human reaching motions. We also examine the effectiveness of our measures in explaining human arm movement data. Specifically, we examine if the directional bias in arm movements reported in [1] can be explained with our attention-based principles for trajecotory generation and control.

Keywords: Human arm movement, attention, optimal feedback control, minimum intervention control.

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Introduction

1.1 Background and Motivation

1.1.1 Theories for Human Movement Generation and Control

Due to kinematic redundancies in the human arm, there exist an infinite number of trajectories that connect a given and final arm configuration. As is well known, humans do not use the full repertoire of possible trajectories on a consistent basis, but produce movements with certain invariant properties. For example, human arm movements are characterized by nearly straight line Cartesian hand paths, and bell-shaped velocity profiles. Indeed, one of the main goals of human motor control research has been to understand how the central nervous system (CNS) generates movements, by discovering general principles that govern trajectory generation and movement control strategies in humans and animals.

One of the earliest theories put forth to explain human movement control is the Equilibrium Point Hypothesis (EPH) [2], [3], [4]. EPH postulates that the human musculoskeletal system can be informally viewed as an elastic system connected by pairs of antagonistic springs (muscles). In the case of an arm, the equilibrium point is the configuration that the arm would tend to settle from wherever it was released, like two rubber bands pulling the joints to a stable equilibrium. Given a desired trajectory, the CNS then continually adjusts the equilibrium point to be along this trajectory, thereby generating the arm movement.

Another general paradigm for understanding motor control is the optimal control hypothesis. The premise behind this hypothesis is that movements are generated so as to optimize some criterion, e.g., kinematic criteria like acceleration or jerk, or energy-like criteria like muscle activation energy or joint torques, or criteria that correspond to cognitive load. Of the many criteria that have been proposed to explain human movements, some criteria that have received attention as being more plausible than others include the following:

(i) **Minimum jerk model**: This model, proposed by Flash and Hogan [5], asserts that human arm movements are generated so as to minimize the derivative of the Cartesian hand acceleration, or jerk. In the case of planar arm movements, the criterion is given by

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2 dt, \tag{1.1.1}$$

where (x, y) denotes the plnar coordinates for the Cartesian hand position. The predicted trajectory has nearly straight-line hand paths and bell-shaped velocity profiles. Because the minimum jerk model is based purely on the kinematics of movement, and small discrepancies are observed in predicted and observed human arm trajectories, the minimum jerk criterion is now generally regarded as an overly simplistic principle for human motor control.

(ii) **Minimum torque change model**: In the minimum torque change model proposed by Uno et al [6], the premise is that human arm movements are generated so as to minimize the changes in the vector of input joint torque τ:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \|\dot{\tau}\|^2 \, dt \tag{1.1.2}$$

Implicit in this assumption is the belief that the simplest control signal is a constant one, and that the dynamics is accounted for in the generation of movements. The performance measure is a sum of squares of the torque change rate at the joints. Although the objective functions for the minimum jerk model (which is purely kinematic) and minimum torque change model (which reflects the arm dynamics) look quite different, they are in fact related, since the rate of change of the torque is locally proportional to the jerk—observe that in the dynamic equations, the product of the mass matrix and joint accelerations is proportional to the joint torque vector. Experimental observations show that the minimum torque change model produces arm movements that are more closely correlated with actual human arm movements than the minimum jerk model, particularly with respect to movements in the vertical plane working against gravity, mirror-reflected via-point movements, movements emanating from the side of the body to in front, and movements between two points while resisting a spring-generated force.

(iii) **Minimum variance model**: The minimum variance model proposed by Harris et al. [7] starts from the assumption that neural control signals have a noise whose variance increases with the magnitude of the signals. According to this model, the goal of motor planning is to minimize the variance of the final position in the presence of such signal-dependent neuronal noise; the criterion is of the form

$$J = \sum_{i=T+1}^{T+R} Tr(Cov[x_i]), \tag{1.1.3}$$

where, for example, in the case of human arm movements, x_i represents the Cartesian position of the hand, and the noise enters into the arm's dynamic state equations. One of the compelling features of the minimum variance model is that not only does the model accurately predict human arm movements, but also offers a plausible neuro-cognitive explanation of how the motor system creates movements, in particular the most salient geometrical features of human arm trajectories as well as the inherent speed-accuracy tradeoff observed in typical human arm movements.

(iv) **Minimal intervention principle**: Models such as minimum jerk, torque change, and minimum variance initially postulate a cost criterion, and a standard variational analysis then produces a set of Euler-Lagrange equations that can then be integrated to generate predicted movements. On the other hand, the role of sensory feedback is widely acknowledged to be important in actual human movements, and yet is not explicitly addressed in sufficent detail in many of the preceding works. The variability between movements with the same task-goal is also not always adequately explained. The minimal intervention principle proposed by Todorov and Jordan [8], [9] is an attempt to overcome these and other shortcomings of existing human motor control theories. It proceeds with the assumption that the cost function should be dependent on the task-goal rather than the shapes of trajectories or the control input. A stochastic optimal feedback control framework is proposed, in which movements are governed by an optimal feedback control law that specifies how feedback gains are adjusted for performing selective error correction in lieu of heavily modulating the entirety of a movement.

1.1.2 The Role of Attention and Perception in Human Movements

Although a number of the optimal control models successfully replicate various geometric and other invariant features of human movement, there is only limited experimental basis as to why these criteria should be minimized in generating human movements. Recently several experimental studies on human motor control have attempted to uncover the role of attention and biomechanics during voluntary arm movement tasks [10], [1]. In these experiments, it is revealed that human arm movements are characterized by directional biases that may not be the result of purely physical energy-like considerations alone, and that become more pronounced as the subject is more distracted cognitively. The experimental results support the idea that in addition to physical energy-based criteria, cognitive load, or attention, is also another factor to be considered when humans generate arm movements.

Other experimental studies on the contribution of perceptual distortion in human reaching movements are reported by Wolpert [11], [12]. In these experiments, subjects are asked to reach target locations by observing not their actual hand trajectory in physical space, but a distorted version of the trajectory projected to a some plane—straight lines now appear as curved lines, and vice versa. The experiments suggest that subjects' tended to generate nearly linear hand trajectories in the projected virtual image rather than physical space. These results imply that the process underlying human trajectory formation is not based only on physical considerations like energy, but may also take into account the perceptual process by which trajectories are evaluated.

Today, robots are becoming increasingly more complex in structure and are asked to perform increasingly complex tasks, many of them simultaneously. Despite this trend, current robot motion planning and control laws for the most part do not take into account physical limitations on computation, communication, and memory. For example, trajectory generation is often framed as a dynamics-based optimization problem with respect to some physical criterion like energy. This is usually decoupled from the problem of tracking control law design, which is often done without regard to the cost of implementation, e.g., the amount of computation required to compute model-based feed-forward control inputs, and the amount of communication that takes place between the controller and actuators and sensors, usually over a communication bus of limited bandwidth.

1.2 Quantifying Attention in a Control-Theoretic Framework

Practical implementations of control laws ultimately require spatio-temporal quantization of the control and measurement signals at some stage. One way in which control performance can be measured is with respect to the resolution of the underlying quantization. There is in fact a growing body of literature on the control of quantized systems, and more generally, on the control of systems subject to data rate constraints (see, e.g., [13],[14] [15],[16]). Most of these works make various simplifying assumptions about the underlying system that are often too restrictive for typical robots (e.g., all systems are linear), or focus on a particular aspect of control (e.g., feedback stabilization).

As an approach to formulate the problem of scheduling communication subject to data rate constraints, Hristu proposed the idea of limited communication control [14]. Using the general notion of a communication bus to represent data transfer between a controller and plant(sensors and actuators) at each time step, limited communication system can be embodied in the discrete time system with restriction on the dimension of the communication bus. When the communication bus is narrow, one possibility is to choose a sequence of operations for the switches that select which inputs/outputs are to be updated/sampled at a particular time.

Another work that attempts to capture control implementation costs in a continuous setting is the minimum attention control framework first proposed by Brockett [17], [18]. There are two basic premises behind this paradigm: (i) the simplest control is a constant one, and (ii) the cost of control implementation can be directly captured by the rate of the change of the control with respect to both time and state. Specifically, for a system described in state-space form by $\dot{x} = f(x, u)$, where x and u respectively denote the state and control vectors, u can be prescribed as an explicit function of both t and x; Brockett's minimum attention functional is then defined as

$$J(u) = \int\int \left\| \frac{\partial u}{\partial t} \right\|^2 + \left\| \frac{\partial u}{\partial x} \right\|^2 \, dx \, dt. \tag{1.2.4}$$

Note that the integral is taken over both state and time. A control that minimizes (1.2.4) is one that is least sensitive to changes in both the state x and time t, and thus can be regarded as the most robust from the point of view of spatiotemporal quantization. As discussed later, practical implementations of this paradigm are make difficulty by the multi-dimensional variational problem, for whom even the existence of solutions cannot be guaranteed.

1.3 Goals and Contributions of this Thesis

Whereas much of the optimal control-related human motor control literature has focused on physical criteria like energy efficiency as the optimality principle governing human motions, some of the more recent work suggests that cognitive load, or more generally the amoung of attention required by a control law, also plays a greater role than once thought. This recent perspective is particularly relevant from the point of robotics: as robotic structures become more complex and are asked to do more things, the inherent limitations on computation, communication, and memory now need to be accounted for when generating trajectories and control laws for robot motions.

This thesis has two main goals. First, we take some of the more recent control theoretic work on control subjected to limited communication and computation constraints, and also the robustness of control laws with respect to spatiotemporal quantization, and use these tools to examine the robustness of a set of robot control laws with respect to quantization. We consider the following control laws: (i) Brockett's minimum attention control law derived under a set of reasonable assumptions (i.e., the control law consists of an feedforward term added to a linear feedback term); (ii) a kinematic version of a minimum variance optimal feedback control law recently derived in [19]; (iii) a PD tracking control law, and also a computed torque control law, designed to track a minimum torque change reference trajectory that is obtained a priori. We also develop an extension of Todorov's minimum intervention control law [8], [9], in which the cost is set to the minimum torque change criterion rather than the torque as done in [8].

Taking a planar robot model as our benchmark system, we determine which of these control laws is the most robust to spatiotemporal quantization, in the sense of which control law achieves the best end-point positioning accuracy when the control laws are subject to more coarse levels of quantization. Results of our experiments suggest that control laws that take into account communication and computation limitations do in fact perform better than other control laws that don't account for such limitations. We also find that the newly developed optimal stochastic feedback control law associated with the minimum torque change criterion produces control laws that mimic certain features of humans, i.e., the transition from open-loop to closed-loop control during certain movements.

The second goal of this thesis is to re-examine the results reported in [1] on directional biases observed in human arm movements. We first point out and correct a discrepancy in the choice of directional bias index formulated in [1]. We then use this new index to determine to what extent some of the representative principles from optimal control-based human motor control theory can explain the results reported in [1]. We find that the reasons for the directional bias as reported in [1] are in fact more complicated and subtle, and that both energy efficiency and control attention together explain some of the directional bias.

It is hoped that the results of this thesis will offer insight on how to develop control laws for robots that require less communication and computational resources. At the same time, some of the control laws, and more generally the methodologies developed for quantitatively measuring the robustness of control laws to spatiotemporal quantization, and the amount of control attention required, can serve as a useful tool for comparatively examining competing theories on human motor control.

The thesis is organized as follows. Chapter 2 discusses the mathematical formulations for capturing control attention, beginning with the notion of robustness to spatiotemporal quantization, and Brockett's minimum attention functional. Then we compares the performance of various robot control laws with respect to control attention. Chapter 3 suggests an extension of minimal intervention principle that can produce the control laws that are robust to the quantization while Chapter 4 re-examines the directional bias of human arm movements using the perspective of attention developed earlier in this thesis and well-known optimization models.

2

Attention Analysis of Robot Control Laws in Deterministic system

From the perspective of attention, control laws that are robust to spatiotemporal quantization are desirable. Based on this idea, we will discuss the mathematical formulations for capturing control attention, beginning with the notion of robustness to spatiotemporal quantization, and, Brockett's minimum attention functional. We then formulate and solve the problem of minimum attention control for planar arm reaching motions and evaluate the performance compared to various control laws, in the sense of which control law achieves the best end-point positioning accuracy when the control laws are subject to more coarse levels of quantization.

2.1 Attention and Robustness to Quantization

A large class of related problems involves the control of multiple subsystems over a single communication channel of finite bandwidth (see, e.g., [14], [20], [16]). For such systems the controller must distribute its attention among the subsystems in some appropriate fashion so as to achieve the control goal, e.g., collective stabilization of all the systems subject to the communication bandwidth limits, and communication with only one subsystem is allowable at any given time. Many of the previous works typically make simplifying assumptions about the subsystems (e.g., discrete-time linear systems with quantized controls) and communication sequence among the subsystems (e.g., periodic).

Another way to measure a robot controller's attention is checking robustness to quantization. For a given continuous control $u(x, t)$, we can consider three kinds of quantization: time quantization, space quantization, and quantization of both time and space.

Time quantization is as follows. Given the system $\dot{x} = f(x)+G(x)u$, we consider its discretized version: the state equation is of the form

$$x_{k+1} = \Phi(x_k, u_k, t_k) \tag{2.1.1}$$

The control can only be updated at a finite set of N ordered times $\{t_0, \ldots, t_{N-1}\}$, and over each interval $[t_i, t_{i+1}]$, the control u_i is assumed constant.

We can also consider space quantization as follows. Shown in Figure 2.1, consider the state space divided into the grid, where the spacing between grids are sensor resolution. For a given state x, we can only predict the state by averaging the values of grid cell where the state is located:

$$\tilde{x} = \frac{x_l + x_h}{2}, \quad \text{for} \quad x_l < x < x_h, \tag{2.1.2}$$

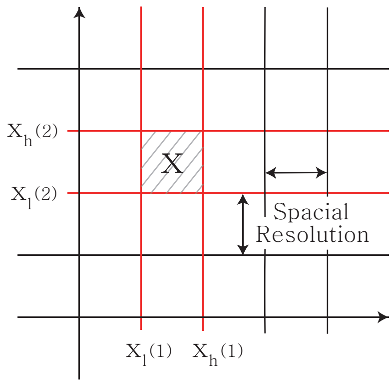


Figure 2.1: State space divided into the grid

where x_l, x_h is each side value of the grid cell in which the state is placed.

In our later evaluational studies, we consider the problem of reaching motion under time and space quantization, in which the evaluation standard is given by sum of square of final error. For a given continuous control $u(x,t)$, we check the robustness of controls when the actual inputs to the system are quantized as follows:

(i) **Time quantization** : over the interval $[t_{i-1}, t_i]$ the actual control input is set to the following average value of $u(x,t)$:

$$u_i = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} u(x(t_{i-1}), t)\, dt. \tag{2.1.3}$$

(ii) **Space quantization** : with the resolution of state measurement $x_l < x(t) < x_h$, the actual control input is set to:

$$u(t) = u(\frac{x_l + x_h}{2}, t). \tag{2.1.4}$$

(iii) **Spatio-temporal quantization** : over the interval $[t_{i-1}, t_i]$ with the resolution of state measurement $x_l(t_{i-1}) < x(t_{i-1}) < x_h(t_{i-1})$, the actual control input is set to the following average value of $u(x,t)$:

$$u_i = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} u(\frac{x_l(t_{i-1}) + x_h(t_{i-1})}{2}, t)\, dt. \tag{2.1.5}$$

2.2 Minimum Attention Control

Another work that attempts to capture control implementation costs in a continuous setting is the minimum attention control framework first proposed by Brockett [17], [18]. He proposed a measure of complexity in implementing the control law, which is named attention functional:

$$J = \int_{t_0}^{t_f} \int_{\Re^n} \alpha \| \frac{\partial u}{\partial x} \|^2 + (1-\alpha) \| \frac{\partial u}{\partial t} \|^2 dx dt \tag{2.2.6}$$

It is clear that the control with the constant input is the easiest. On the contrary to this, the more frequently the control changes, the more attention would be consumed. So the cost of implementation is linked to the rate at which the control changes with changing value of x and t for $u = u(x,t)$.

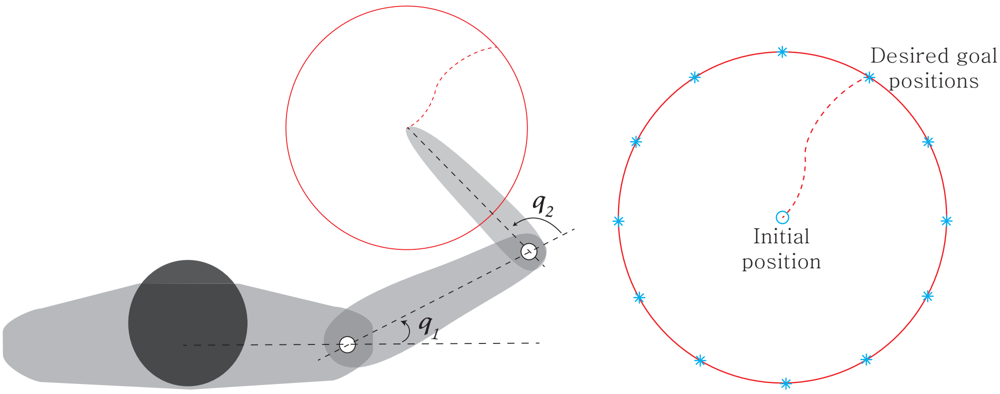
Although conceptional meaning of attention functional is simple and easily understandable, you can see that this optimal control problem is highly nonlinear and even the existence of solution can not be guaranteed for robotics application. To make this problem solvable, S Lee [21] suggested some assumptions; (i) split the control input into closed-loop term and open-loop term, (ii) consider an admissible class of feedback controls in the form of a simple PD control law. Then problem can be solved by parametrizing the trajectory through splines with the control modeled in the form of

$$u(x,t) = -K(x(t) - x_f) + v(t). \tag{2.2.7}$$

where x_f is desired final state, K is a linear feedback gain matrix and $v(t)$ is feed-forward control term. With this assumption, we can manage this problem and get the solution using spline method, but there is still the problem of local minima.

2.3 Radial Reaching Motions

Figure 2.2 describes the experimental set-up. The initial posture of the arm is given by shoulder and elbow joint values of $30°, 100°$ respectively. The goal points for the radial reaching motions are taken to be a set of 12 equally spaced points on the circle of radius 18cm and they are shown with blue star(*) in Figure 2.2(b). The duration for each movement is set to one second.



(a) Experimental set-up (b) Goal points in workspace

Figure 2.2: Radial reaching motion experiments set-up

Human-like arm can be simply modeled as 2-link manipulator moving in the horizontal plane, with shoulder and elbow joints. The dynamics of the arm model can be driven as:

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q}, \tag{2.3.8}$$

where $q \in \mathbb{R}^2$ denote the joint angles and $\tau \in \mathbb{R}^2$ the input joint torques. The kinematic and inertial parameter values are taken from [22] and chosen to closely match those of typical human arms.

We first numerically determine the minimum attention control law that achieves the desired point-to-point motions. The minimum attention control is assumed to have a feedforward term $v(t)$, and proportional and derivative feedback terms:

$$\tau = -K_v\dot{q} - K_p(q - q_d) + v(t), \tag{2.3.9}$$

where q_d denotes the joint space configuration corresponding to the hand goal point, and K_v and K_p are assumed constant. $K_p,\ K_v$, and $v(t)$ are then chosen to minimize the following attention functional:

$$\min_{K_p,K_v,v(t)} \int_{t_0}^{t_f} \alpha(\|K_p\|^2 + \|K_v\|^2) + (1-\alpha)\|\dot{v}\|^2\, dt. \tag{2.3.10}$$

For comparison purposes we consider three control laws: minimum variance kinematic feedback control law [19], and minimum torque change control laws combined with two kinds of feedback controller. As we discussed in the introduction, minimum torque change model is formulated to generate only open-loop control so sensory feedback can not be considered in this point. So we use two general approach to produce feedback term for robotics application; the standard form of the computed torque control law, i.e.,

$$\tau = M(q)(\ddot{q}_d - K_v(\dot{q} - \dot{q}_d) - K_p(q - q_d)) + C(q,\dot{q})\dot{q}, \tag{2.3.11}$$

and the augmented PD control law, i.e.,

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d - K_v(\dot{q} - \dot{q}_d) - K_p(q - q_d). \tag{2.3.12}$$

Here $C(q,\dot{q})$ denotes the Coriolis terms. For both control laws the feedback gain matrices K_v and K_p are chosen to be diagonal, with the gain values chosen to achieve critical damping.

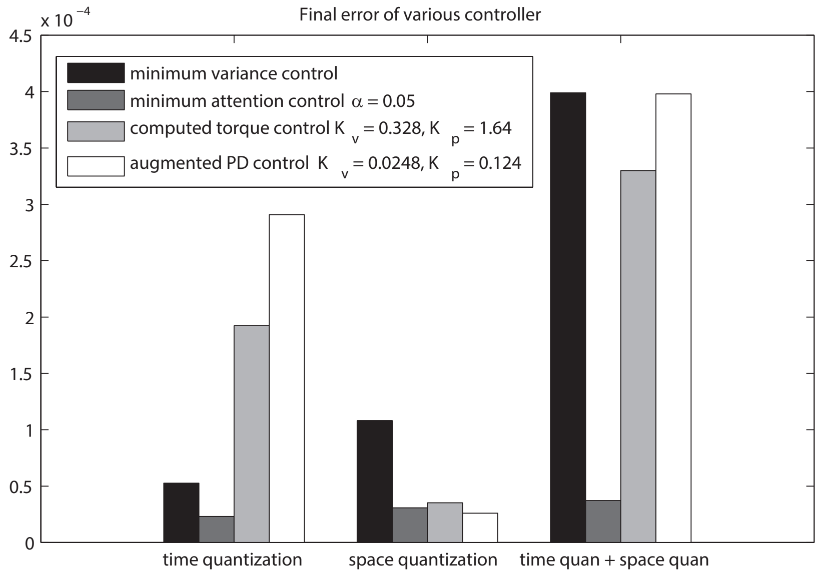
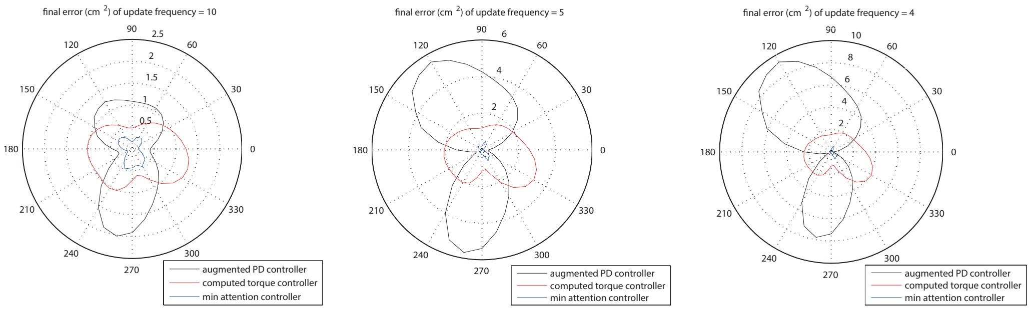


Figure 2.3: Mean values of final errors for the four controllers under time and space quantization

For the four control laws, we now conduct the following set of numerical experiments. Time is uniformly discretized into N equal intervals over the movement duration, with a 5Hz update rate. Joint position and velocity measurements are quantized to $10°$ and $10°/sec$ increments, respectively. The final position error is measured in Cartesian hand space, i.e., as the Euclidean distance between the desired and actual coordinates of the final hand position.

Figure 2.3 shows the mean values of the final positioning errors for the four controllers under various time, space, and spatiotemporal quantization. The minimum attention controller is seen to be the most robust to quantization in time and space, while the minimum variance feedback control is the most sensitive. These results confirm our intuition that feedforward control terms are important to achieving robustness to quantization effects.



(a) Update rate = 10 Hz (b) Update rate=5Hz (c) Update rate=4Hz

Figure 2.4: Position errors with respect to heading angle direction for the minimum attention, computed torque, and augmented PD control laws

We also compare the final positioning errors for the minimum attention, computed torque, and augmented PD controllers with respect to the goal point direction. As shown in Figure 2.4, we can see the minimum attention controller is the most robust to the time quantization with respect to the all goal points.

3

Attention Analysis of Robot Control Laws in Stochastic system

In this chapter, we analyze control laws in stochastic system using the similar evaluation method suggested in the previous chapter. To achieve our goal of producing the control laws that take into account communication and computation limitations in stochastic system, we suggest newly developed optimal stochastic feedback control law associated with the minimum torque change criterion. We find that this control laws do in fact perform better than the one in which the cost is set to the torque as done in [8],[23], and mimic certain features of humans, i.e., the transition from open-loop to closed-loop control during certain movements.

3.1 Optimal Feedback Control Laws Considering Attention

Although many optimality models are suggested for the generation of human-like motion in human motor control, most of them are only associated with the shape of trajectories or magnitude of control input. These models only offer an open-loop control and do not consider the implementation of control. At an intuitive level, it seems that biological motor control involves not only pure open-loop control but also a gradation of modalities spanning a range between open-loop and closed-loop operation.

There are two main approach to find a control law with both open-loop and closed-loop control term based on the optimization. One is the minimum attention control [17] discussed in the previous chapter. We can find balanced terms of open-loop and closed-loop control reducing a measure of complexity in the implementation of the control law with this approach. The other is minimal intervention principle [23] that suggests a sensory feedback gains optimized to achieve task goal. Reminding our purpose to find control laws that take into account attention, we consider an extension version of minimal intervention principle with new task goals considering attention.

From the point of view of the attention, the control laws for multi-tasking should be robust to the low data rate and actuator update frequency. One way to generate the simplest control law is minimizing attention functional introduced in the previous sections. But controller minimizing only attention functional is hard to be applied in practical system, because this control laws have difficulty modeling noise together. In order to solve this problem, it is worth considering control change alternatively which can be considered as the particular case of attention functional($\alpha = 0$) but more manageable. With the purpose of producing the control laws for multi-tasking which provide reliable feedback together, we suggest control laws that have following two task goals:(i) minimizing torque change and (ii) minimizing final state error.

Then our cost function to be minimized can be formulated as

$$J = \frac{1}{2}\left((x(t_f) - x_f)^T Q(x(t_f) - x_f) + \int_{t_0}^{t_f} \rho \|\dot{\tau}\|^2 dt\right) \tag{3.1.1}$$

where $t_0, t_f, \in \Re$ is given initial and final time, $(x(t_f)-x_f)^T Q(x(t_f)-x_f)$ is final state error in quadratic form with desired target state x_f and ρ is the weight parameter for sum of square of torque change integrated over time.

To obtain the optimal feedback gains based on the minimum intervention principle, iterative LQG algorithm [23] will be used. In this algorithm, cost function and state equation should be able to represent as a function of state and input variables. In this thesis, we suggest to set up state and input vector of the problem as $x = [q, \dot{q}, \tau]^T$ and, $u = \dot{\tau}$ to manage the problem. Under this setting, dynamic system with noise modeling can be described by nonlinear stochastic differential equation in general form of

$$dx = f(x, u)dt + F(x, u)d\omega \tag{3.1.2}$$

with standard Brownian motion noise ω.

Iterative Linear Quadratic Gaussian Algorithm

Iterative LQG algorithm was firstly suggested by Todorv [23]. It provides the control laws with locally-optimal feedback term in nonlinear stochastic system. In this section, we will introduce the algorithm slightly modified for our problem. We first linearize the system around nominal state and input trajectory $\bar{x} \in \Re^n, \bar{u} \in \Re^m$. Then the discrete time dynamics and the cost function can be described by derivate variables of the state and input $\delta x, \delta u$:

$$\begin{aligned}
\delta x_{k+1} &= A_k \delta x_k + B_k \delta u_k + C_k(\delta x_k)\xi_k \\
C_k(\delta x_k) &\triangleq [c_{1,k} + C_{1,k}\delta x_k, c_{2,k} + C_{2,k}\delta x_k] \\
\delta J &= \delta x_T^T Q(\bar{x}_T - x_f) + \frac{1}{2}\delta x_T^T Q \delta x_T^T \\
&\quad + \sum_{k=1}^{T} \delta u_k^T (\rho \triangle t)\bar{u}_k + \frac{1}{2}\sum_{k=1}^{T} \delta u_k^T (\rho \triangle t)\delta u_k \\
\text{immediate cost}_k &= \delta u_k^T (\rho \triangle t)\bar{u}_k + \frac{1}{2}\delta u_k^T (\rho \triangle t)\delta u_k
\end{aligned}$$

where,

$$A_k = I + \frac{\partial f}{\partial x}(\bar{x}_k, \bar{u}_k)\triangle t, B_k = \frac{\partial f}{\partial u}(\bar{x}_k, \bar{u}_k)\triangle t,$$

$$c_{i,k} = F^{[i]}\sqrt{\triangle t},\ C_{i,k} = \frac{\partial F^{[i]}}{\partial x}\sqrt{\triangle t},$$

and the independent random variables $\xi_k \in \Re^m$ with zero-mean Gaussian white noises with the covariance matrix of $diag(\sigma_1, \sigma_2, ..., \sigma_m)$.
And then, we can compute the optimal input policy backwards in time by minimizing cost-to-go function at the each step. In this algorithm, cost-to-go function remains in the quadratic form with linearly designed optimal input policy from Bellman equation.

$$\begin{aligned}
v_k(\delta x_k) &= s_k + \delta x_k^T \mathbf{s}_k + \frac{1}{2}\delta x_k^T S_k \delta x_k \\
&= \text{immediate cost}_k + \ E[v_{k+1}(A_k \delta x_k + B_k \delta u_k + C_k(\delta x_k)\xi_k)] \\
v_T(\delta x_T) &= \delta x_T^T Q(\bar{x}_T - x_f) + \frac{1}{2}\delta x_T^T Q \delta x_T
\end{aligned}$$

By induction from final time T and applying input policy minimizing cost-to-go function at each time step, we can compute backward recursion for $s, \mathbf{s}, S$ and design optimal input policy as:

$$
\begin{aligned}
s_k &= s_{k+1} + \frac{1}{2}\sum_{i=1}^{2} c_{i,k}^T S_{k+1} c_{i,k} - \frac{1}{2} g_k^T H_k^{-1} g_k \\
\mathbf{s}_k &= A_k \mathbf{s}_{k+1} + \frac{1}{2}\sum_{i=1}^{2} C_{i,k}^T S_{k+1} c_{i,k} - G_k^T H_k^{-1} g_k \\
S_k &= A_k^T S_{k+1} A_k + \frac{1}{2}\sum_{i=1}^{2} C_{i,k}^T S_{k+1} C_{i,k} - G_k^T H_k^{-1} G_k \\
\delta u_k &= \pi_k^\delta(\delta x_k) = -H_k^{-1} g_k - H_k^{-1} G_k \delta x_k
\end{aligned}
$$

where,

$$
\begin{aligned}
g_k &= \rho \bar{u}_k + B_k^T \mathbf{s}_{k+1} \\
H_k &= B_k^T S_{k+1} B_k + \frac{1}{2}\rho I_{m\times m} \\
G_k &= B_k^T S_{k+1} A_k
\end{aligned}
$$

Finally, apply the input policy $\pi_k^\delta(\delta x_k)$ to the deterministic system $\delta x_{k+1} = A_k \delta x_k + B_k \delta u_k$ in a forward pass starting from δx_1. We can take optimal δx_1 from solving the following problem,

$$
\delta x_1 = \arg\min_{\delta x_1} v_1(\delta x_1) \quad \text{Subject to } [\delta q, \delta \dot{q}]_1 = 0
$$

And then, computing the new trajectory $\tilde{x} = \bar{x} + \delta x$, $\tilde{u} = \bar{u} + \delta u$ along the nominal state and input variables. If the sequences $\tilde{u}$ and $\bar{u}$ are sufficiently close, end the iteration. Otherwise set $\bar{u} = \tilde{u}$ and go to the first step of iteration. To implement line search, use $\pi_k^\delta(\delta x_k) = -(\alpha H_k^{-1} g_k + H_k^{-1} G_k \delta x_k)$ in the forward pass, where α is the line search parameter. In order to guarantee the convergence over the linearized approximation of non-linear system, we use backtracking line search: start with $\alpha = 1$, and decrease it by a factor of $n_\alpha < 1$ until expected cost of the open-loop control law becomes smaller than the old one. We also need to apply this parameter to the initial state derivative term, δx_1 for the convergence. From the last iteration optimal input policy input can be rewritten as:

$$u_k = \bar{u}_k + \pi_k^{\delta}(\delta x_k) = \bar{u}_k - H_k^{-1} g_k - H_k^{-1} G_k \delta x_k$$

We now, design locally optimized feedback control policy for the torque as a function of time and observable sensing values ($y = [q, \dot{q}]^T$: joint positions and joint velocities). Following is one possible method:

$$\tau_k = \pi_k(y_k) = K_k(y_k - y_d) + v_k$$

where $y_d = [q^{goal}, \dot{q}^{goal}]^T$ is desired goal joint positions and joint velocities and we have following gains for $n' = n - m$,

$$\begin{aligned}
K_k &= -H_k^{-1} G_k \begin{pmatrix} I_{n' \times n'} \\ O_{m \times n'} \end{pmatrix} \triangle t \\
v_k &= (I_{m \times m} - H_k^{-1} G_k \begin{pmatrix} O_{n' \times m} \\ I_{m \times m} \end{pmatrix} \triangle t) \bar{\tau}_k + \ (-H_k^{-1} G_k \begin{pmatrix} I_{n' \times n'} \\ O_{m \times n'} \end{pmatrix} \triangle t) y_d \\
&\quad + (\bar{u}_k + H_k^{-1} G_k \bar{x}_k - H_k^{-1} g_k) \triangle t.
\end{aligned}$$

3.2 Radial Reaching Motions

Experiment set up for radial reaching motions is same to the one that described in the previous chapter. Dynamics of the arm model we used here can be driven as:

$$\mathcal{M}(q)\ddot{q} + \mathcal{C}(q, \dot{q}) + \mathcal{B}\dot{q} = \tau \tag{3.2.3}$$

where $q \in \mathbb{R}^2$ is the joint angle vector(shoulder: q_1 , elbow: q_2), $\mathcal{M}(q) \in \Re^{2 \times 2}$ is a positive definite symmetric inertia matrix, $\mathcal{C}(q, \dot{q}) \in \Re^2$ is a vector of centripetal and

Coriolis forces, $\mathcal{B} \in \Re^{2\times 2}$ is the joint friction matrix, and $\tau \in \Re^2$ is the joint torque that the muscles generate. Parameters for this arm model are taken from [23].

To produce optimal feedback controller that takes into account attention, we suggest control laws that have two main task goal:(i) minimizing torque change and (ii) minimizing final state error. Following is our cost function to be minimized:

$$J = \frac{1}{2}\left(\|q(t_f) - q_f\|^2 + \|\dot{q}(t_f)\|^2 + \int_{t_0}^{t_f} \rho\|\dot{\tau}\|^2 dt \right) \tag{3.2.4}$$

where $t_0, t_f, \in \Re, q_f \in \Re^n$ is given and ρ is the weight parameter of torque change values integrated over time.

To solve the problem, we set the state vector and input vector of the system as $x = [q_1, q_2, \dot{q}_1, \dot{q}_2, \tau_1, \tau_2]^T$ and, $u = [\dot{\tau}_1, \dot{\tau}_2]^T$. We then organize system for human-like arm model described in Equation 3.2.3 by following nonlinear stochastic differential equation:

$$dx = f(x,u)dt + F(x,u)d\omega \tag{3.2.5}$$

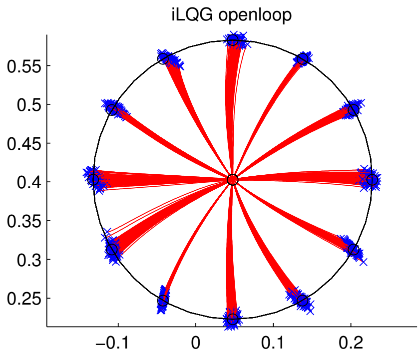
with standard Brownian motion noise $\omega \in \Re^2$. We assumed signal-dependent noise, which is general in both human motor system and robot, and here, the system supposed to put their signal with the torque. We then have,

$$f(x,u) = \begin{pmatrix} \dot{q} \\ \mathcal{M}(q)^{-1}(\tau - \mathcal{C}(q,\dot{q}) - \mathcal{B}\dot{q}) \\ u \end{pmatrix} \in \Re^6 \tag{3.2.6}$$

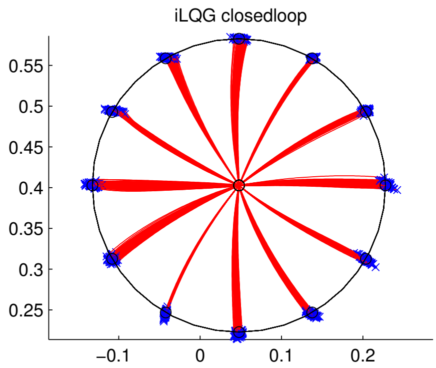
$$F(x,u) = \begin{pmatrix} O_{2\times 2} \\ \mathcal{M}(q)^{-1}\begin{pmatrix} \sigma_1|\tau_1| & 0 \\ 0 & \sigma_2|\tau_2| \end{pmatrix} \\ O_{2\times 2} \end{pmatrix} \in \Re^{6\times 2} \tag{3.2.7}$$

where $\sigma_1, \sigma_2 \in \Re$ is standard deviation of noise.

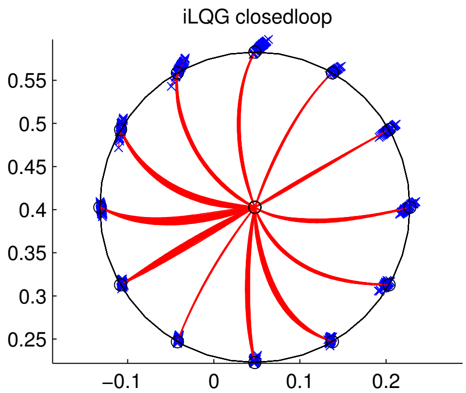
Then we can get feedback control laws optimized in discrete time domain using the algorithm described in section 3.1,



(a) feedforward control laws(torque change)



(b) feedback control laws(torque change)



(c) feedback control laws(torque)

Figure 3.1: Numerical experiments in the quantized noisy system

To evaluate the performance of optimal feedback control laws from the perspective attention, we now conduct the numerical experiments for following three controllers obtained by ILQG method: feedforward control laws and feedback control

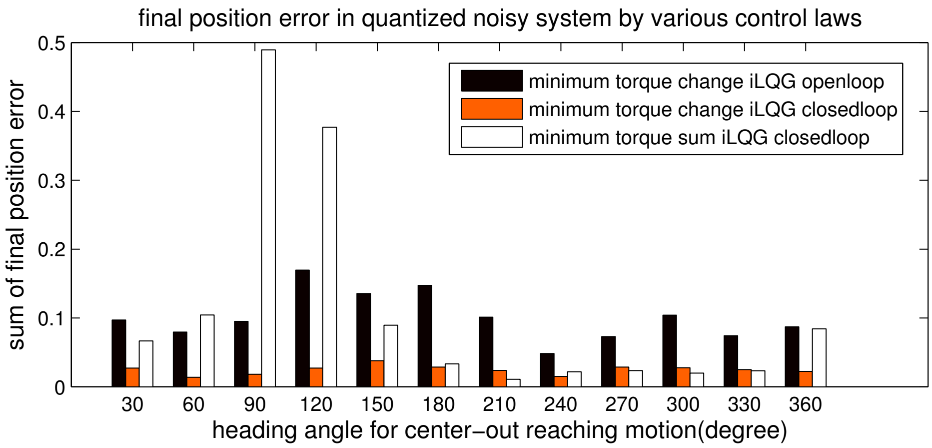


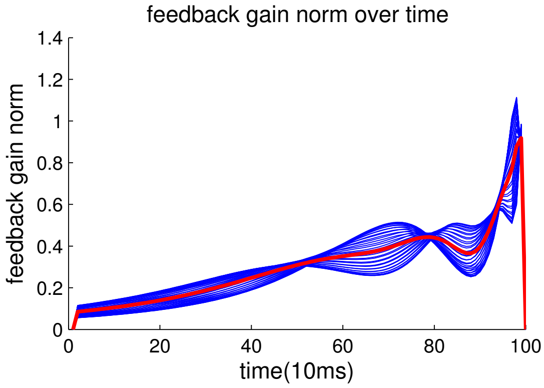
Figure 3.2: Final position error in quantized noisy system by various control laws

laws associated with the minimum torque change and feedback control laws associated with the minimum torque, which are originally suggested in [8],[23]. In the limited communication system, we assume that state sensing and actuator updating can be done with only 10Hz frequency and actuator noise is modeled in stochastic system with $\sigma = 2$ to be proportional to its torque size. Each radial reaching motion to 12 goal positions is performed 50 times with randomly produced noise.

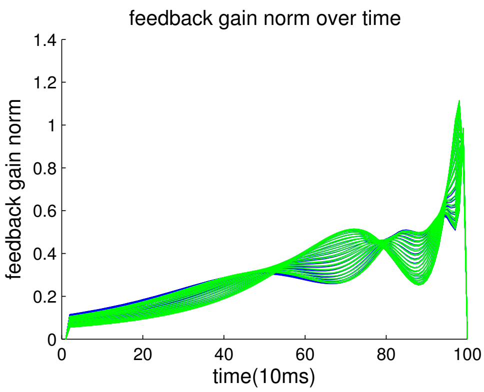
Figure 3.1 shows the hand trajectories resulting from various control laws in stochastic system. In the figure, trajectories are described by red line starting from initial position(black o) to final positions(blue x). Comparing the trajectories generated with torque change and torque, we can see more straight shape in 3.1(a), 3.1(b) than 3.1(c), which is more general in human motion. Furthermore, as shown in the endpoint distribution, feedback control minimizing torque change scheme substantially reduces the effects of both the noise and quantization. Figure 3.2 shows this result more visibly. Black bars and orange bars represent the error sum of final position produced by torque change open-loop control and closed-loop control respectively and white bars represent the error sum of final position by closed-loop control laws minimizing torque sum. For all the direction of control, minimum torque change closed-loop control makes final position errors about 2 to 6 times as small as open-loop control makes. In addition, closed-loop controls minimizing torque sum instead of torque change, which is generally used to generate control laws, are poorly performed under the low update frequency while the advantages are still existent in energy effectiveness and compensation for signal-dependent noise.

3.2.1 Features of Reaching Motion Control

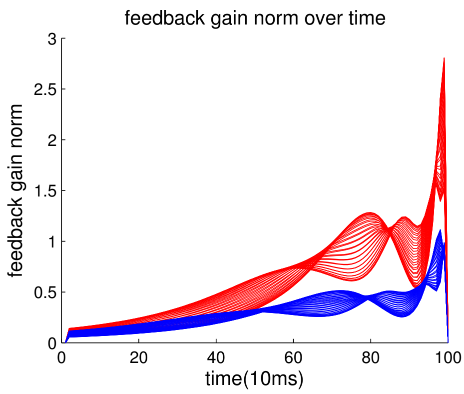
When humans do reaching motions, they usually apply open-loop control initially and gradually change it to closed-loop control near the reaching. In this section, we will discuss about these features by analyzing the feedback terms generated by our approach. Figure 3.3 describes the feedback terms over time under various conditions. To evaluate the magnitude of feedback term in matrix form, we use Frobenius norm here in the paper. As shown in Figure 3.3, feedback gain is initially small, but has large values near final times. It is clear that control needs large feedback gain near the final time for the accurate reaching motion. Furthermore, as shown in Figure 3.3(b) and 3.3(c), feedback gain does not change much with sigma but change with weight parameter ρ in cost function. Smaller weight parameter ρ tends to produce a control law taking smaller final state error and this causes larger feedback gain. Therefore, in order to generate control laws well operated in practical system, we can make it by adjusting the value of ρ.



(a) $\sigma_1 = \sigma_2 = 2, \rho = 5e - 5$



(b) varying σ



(c) varying ρ

Figure 3.3: Feedback gain over time with various parameters

3.2.2 Comparison of Human Motions and Predicted Motions

To capture the features of human reaching motions, we obtained human experimental data under same experiment setting described in the previous chapter. The initial posture of the arm is given by shoulder and elbow joint values of $30°, 100°$ respectively in planar workspace. The goal points for the center-out reaching motions are taken to be a set of 12 equally spaced points on the circle of radius 18cm and they are shown with blue star(*) in Figure 2.2(b). Real human hand-effector trajectories were captured by Wiimote controller and IRED pointer shown in Figure 3.2.2.

Figure 3.4: Wiimote controller and IRED pointer

From previous chapter, we obtained the optimal feedback control policies achieving two task goals:(i) minimizing torque change and (ii) minimizing final error. For this simulation, we set the noise parameters and weight parameters to be $\sigma_1 = \sigma_2 = 2$ and, $\rho = 5e - 5$.

To compare the motion resultant from human and our control laws, we try to give same experiment set up as much as possible. The duration for each movement is set to one second, and to make the human movement time duration constant, we used metronome in the experiment. Experimenters do the reaching motion 4 6 times to each goal point with IRED pointer at their hand and data are captured by wiimote controller. All the data are smoothen by cubic spline method to take out the sensor noise and some inaccuracy of Wiimote system.

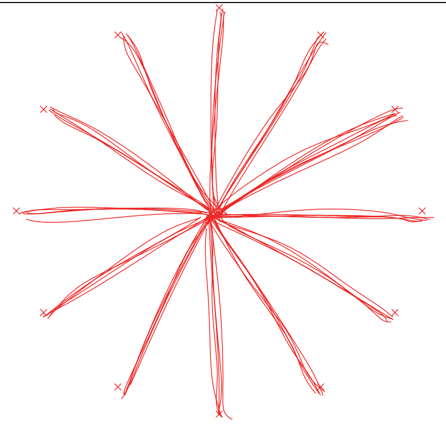
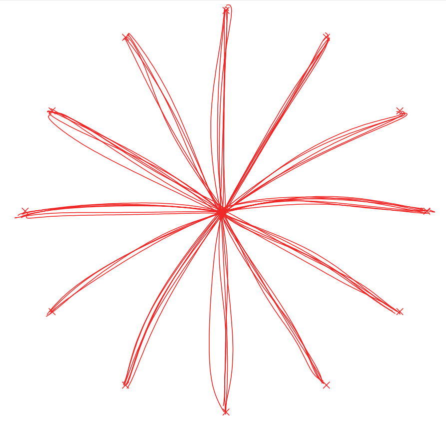
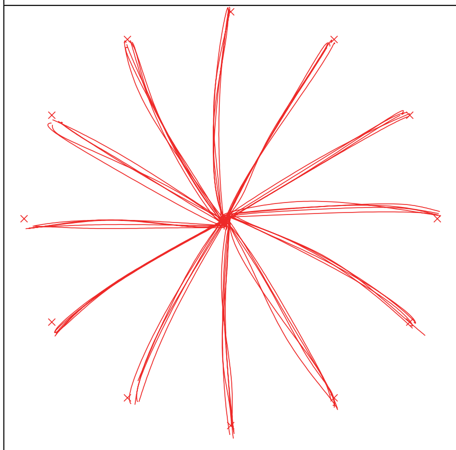
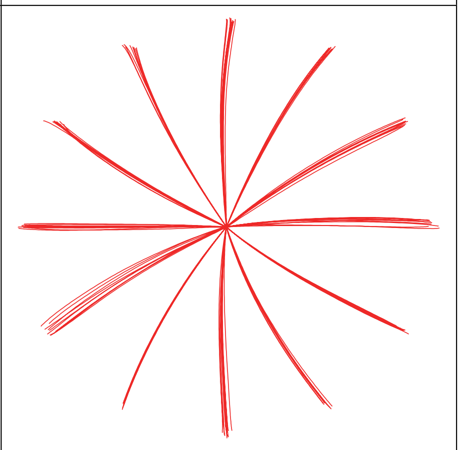
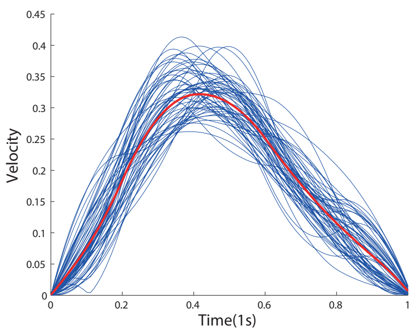
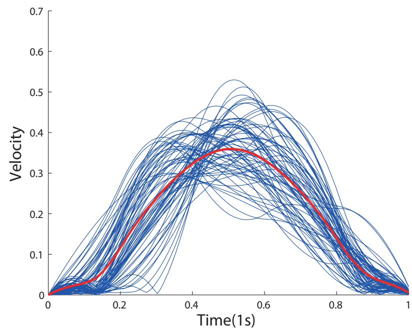
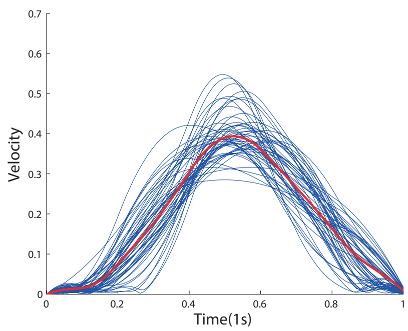
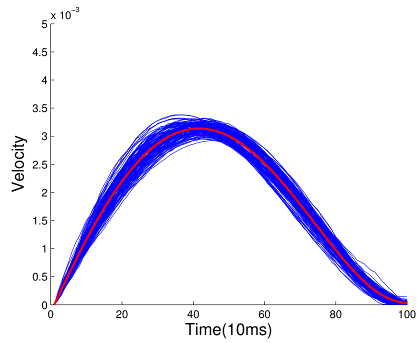
<table>
<tr><td colspan="2">Human</td></tr>
<tr><td>subject 1</td><td>subject 2</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Human
subject 3</td><td>Simulation</td></tr>
<tr><td></td><td></td></tr>
</table>

Table 3.1: Hand end-effector trajectories for center-out reaching motion

<table>
<tr><td colspan="2">Human</td></tr>
<tr><td>subject 1</td><td>subject 2</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Human
subject 3</td><td>Simulation</td></tr>
<tr><td>
</td><td>
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</table>

Table 3.2: Hand end-effector velocity profile for center-out reaching motion

As shown in Table 3.1, trajectories for the reaching motion from human and our control law seem to be similar except trajectories in the 4th quadrant. It is quite different. One possible guess to explain this result is the human joints' limit, which is existent but not considered in the model used for our simulation. In Table 3.2, both tangential velocity plot from human and our control law shows the features of reaching motion, bell-shaped profile. Because human experiments are performed with metronome to fix the movement time length as one second, our human result has a halt at the initial time and final time. Except this halt and some noise, it looks quite similar.

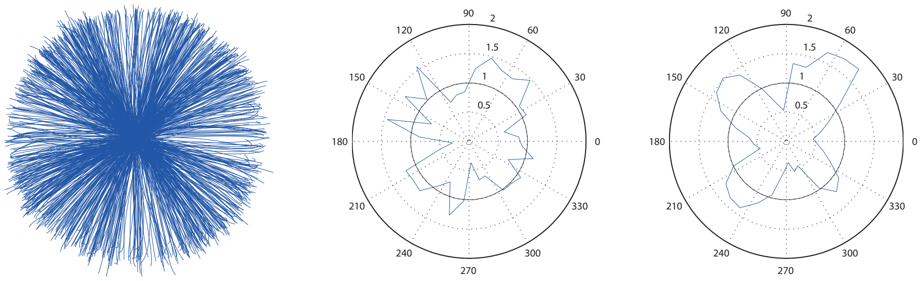
4

Human Arm movements and Attention

In recent experimental studies on the the role of attention in human voluntary arm movements [10][1], it is shown that humans have directional biases that are strongly influenced by both the biomechanical properties of the arm and cognitive load. In this section, we re-examine the experimental findings from our attention perspective, to determine whether our model is appropriate to explain human attention.

4.1 Analysis on Human Experimental Data

The experimental setup is similar to that described earlier. Each human subject is asked to repeatedly move from the center of the circle to random points distributed uniformly along the circle, but under a range of cognitive loads (e.g. hit the name of vegetables featured in the slide show). According to [1], clear directional biases are observed when humans are subject to high cognitive load and we also discovered similar result from our re-examination in Figure 4.1.



(a) Strokes from experiment (b) No secondary task (c) With secondary task

Figure 4.1: Polar histograms of strokes' heading angle

Figure 4.1(a) shows one part of strokes data, which are collected from 26 subjects(aged 20-36). We used this data after spline smoothing work to get rid of noise and nonuniformity of time sampling. The polar histograms of strokes' heading angle shown in Figure 4.1 is produced by placing heading angle data into 36, 10° bins. Heading angle data we used in this experiment were calculated from strokes data by averaging the direction angle over the final half movement of each stroke. Comparing Figure 4.1(b) and 4.1(c), with secondary task, we can see stronger directional bias in the production of strokes to the direction of $45°, 135°, 230°$ and, $315°$. In the findings reported in [1], subjects tend to show a dominant preferential bias for the 30°, 60°, and 240° directions, and also a slightly smaller preferential bias for the 315° direction. It looks slightly different but similar to the result from our experiment.

4.2 Directions of Minimal Cost Based on Human Data

To verify whether these directional biases can be explained by well known optimization models, Dounskaia[1] plot following optimization index definition(4.2.1) as a function of heading angle. This index definition (4.2.1) suggested to be ranging from 0 to 1 and values near 1 corresponding to the optimal value of each cost.

$$1 - \frac{J(\theta)}{\max_\theta J(\theta)} \tag{4.2.1}$$

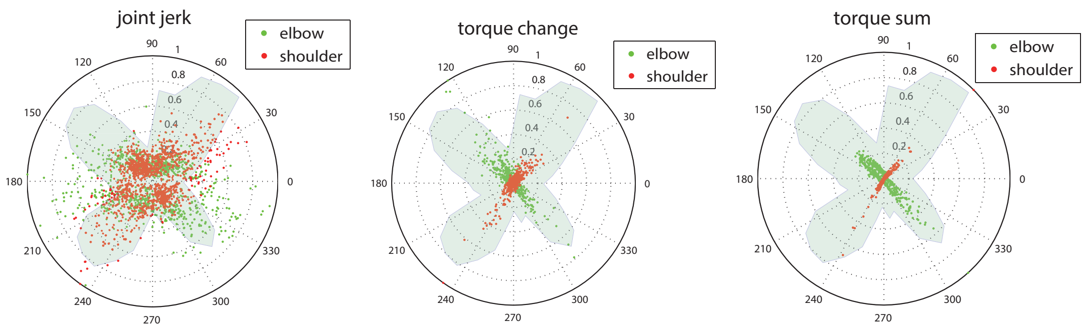
where θ is heading angle and following optimization models are used:

$$\begin{aligned}
\text{Joint jerk : } J &= \frac{1}{2}\int_{t_0}^{t_f} \left(\frac{d^3 q_1}{dt^3}\right)^2 + \left(\frac{d^3 q_2}{dt^3}\right)^2 dt \\
\text{Torque change : } J &= \frac{1}{2}\int_{t_0}^{t_f} \|\dot{\tau}\|^2 dt \\
\text{Torque square sum : } J &= \frac{1}{2}\int_{t_0}^{t_f} \|\tau\|^2 dt
\end{aligned}$$

From her paper, she concluded these models can not explain the directional bias but criteria such as interaction torque of each joint, inertial resistance and kinematic manipulability have a significant relation with the directional bias. However, we found that this result caused because suggested index cannot show the direction with low cost clearly when the costs of the model are mostly distributed near minimum value, and it is critical because the optimization costs introduced above are consist of squared form while other criteria are consist of absolute form. To avoid this result, we suggest following new optimization index definition (4.2.2), which also ranges from 0 to 1 and values near 1 corresponds to the optimal value of each cost. But this index definition can show the direction with low cost more clearly.

$$\frac{\min_\theta J(\theta)}{J(\theta)}, \tag{4.2.2}$$

For the cost functions such as joint jerk, torque sum and torque change, we calculate the optimization index(4.2.2) of each cost along the trajectory of each stroke obtained by experimenters. And then, we plot them according to the heading angle of stroke as shown in Figure 4.2. We also plot the cost separately for each joint,



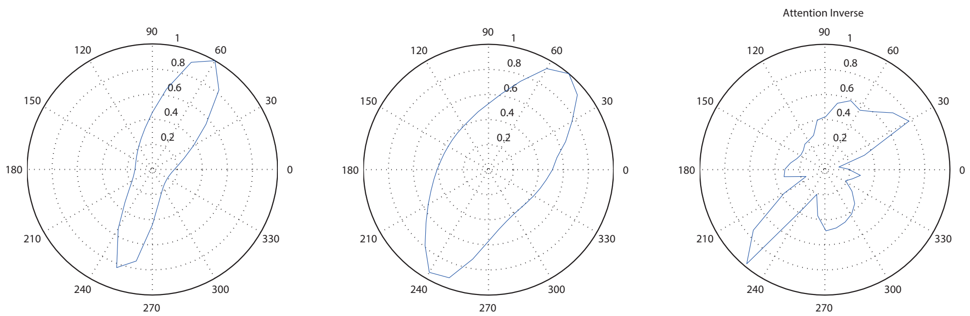
(a) Joint jerk (b) Torque change (c) Torque sum

Figure 4.2: Optimization index plot according to the heading angle of strokes

since torque values for the shoulder joint are typically three times larger than torque values for the elbow. This different scale of torque at each joint can ignore the cost generated by elbow joint if we consider only sum of the values. To visualize our result, we plot histogram in Figure 4.1(c) together. As shown in Figure 4.2, for all cases the preferred directions for the torque and torque change integral criteria are nearly identical and these directions with low cost of torque change and torque sum seem to be quite similar to the preferred direction discovered in human experiment. It is quite reasonable that the direction with minimum torque sum and torque change are similar because both values appear to be very similar in planar motion. Especially, cost of shoulder seems to be relevant to the directional bias in 45° and 135°, and cost of elbow seems to be relevant to the directional bias in 315° and, 230°. Minor differences in preference direction can be explained by differences in human arm dynamics.

4.3 Directions of Minimum Attention Functional Cost for Various Controllers

We calculate Brockett's attention functional cost for various controllers. Due to the features of attention functional cost, only feedback controller can be used for this analysis. Methodologies producing feedback control laws introduced earlier in this thesis such as minimum attention control laws and optimal feedback control laws associated with minimum torque change model will be used in the analysis. In addition, we will also calculate attention functional cost for minimum variance kinematic feedback control [19], which produces human-like motion minimizing variance. For each heading direction θ, the cost is calculated from (1.2.4), and the directional preference index (4.2.2) is plotted.



(a) minimum attention control

(b) minimum torque change stochastic feedback control

(c) minimum variance kinematic stochastic feedback control

Figure 4.3: Indexed attention cost distribution for various control laws

The results are shown in Figure 4.3. The three control laws all produce similar hand trajectories, but plot of attention functional cost distribution shows noticeable differences depending on the choice of control law. Preferred directions predicted from minimum attention control laws are 60° and 250° and from minimum torque change stochastic feedback control laws are 50° and 240°. But this result fails to explain preferred directions in 135° and, 315°. While Figure 4.3 and 4.3(b) indicate similar direction in the plot, Figure 4.3(c) looks different in shape. This result seems to be caused because attention functional has partial torque change terms in the cost while minimum variance stochastic kinematic feedback control law set its parameters forcing to generate human-like motion. It is also interesting to observe that the preferred directions for the minimum variance control law appear to be similar to the experimental results reported in [1].

To conclude, we can see torque change and attention functional values seems to have some kind of relation with human attention. Although we can not strongly claim that torque change is more appropriate than torque sum to explain the role of attention in human arm movement, it is the fact that minimizing torque change generate more human-like trajectory than minimizing torque sum. And furthermore we also compare the robustness to quantization based on the both criteria. Therefore, according to these simulation results, we can conclude that our assumption that minimizing torque change or attention functional is pertinent to reducing complexity in control seems to be right. Clearly more careful experimental studies and analysis are needed to draw meaningful conclusions.

5 Conclusion

In this thesis, we examine the various ways attention can be considered when generating trajectories and controlling movements. As robots being asked to perform increasingly complex tasks, often simultaneously, it is timely to ask whether and how robot motion planning and control laws should consider the limitations on the available computation, communication, and memory resources. From the inspiration by the way human do multi-tasking; continuously shifting their attention from one task to another, we evaluate the control laws, in the sense of which control law achieves the best end-point positioning accuracy when the control laws are subject to more coarse levels of quantization.

Minimum attention control is the one of main approach that can control this problem, so we firstly solve this optimal control problem for planar arm reaching motion, and evaluate the control laws by checking the robustness to spatio-temporal quantization. Compare to other control laws, we found that minimum attention control theory produce the laws that are robust to the quantization.

In spite of their good performance in low data rate system, we recognize the needs of noise modeling to get reliable and effectively performing feedback term. To solve this problem, we formulate optimal feedback control problem with these two task goals:(i) minimizing torque change and (ii) minimizing final state error. In this sense, minimizing torque change can be considered as a specific case of minimizing attention functional that makes optimization problem manageable. And to get the optimal feedback gains based on the minimum intervention principle, we solve the problem using iterative Linear Quadratic Gaussian method [23] with modeling of signal dependent noise in stochastic system. We also perform numerical experiments for robotic planar arm reaching motions and show that this control law shares many of the main features of human reaching motion, e.g., the transition from open-loop to closed-loop control during the movement, and improved robustness to low date rate and signal dependent noise. This approach is in fact applicable to much more general control settings, in which multi-tasking must be performed by the system under limited computation and communication resources.

Lastly, we try to analyze the experimental data on human arm movement. Related to the reported finding on the role of attention in directional bias for stroke production [1], we try to explain it using various well known optimization models, and conclude that both energy efficiency and control attention together explain some of the directional bias.

It is hoped that the results of this thesis will offer insight on how to develop control laws for robots that require less communication and computational resources. At the same time, some of the control laws, and more generally the methodologies developed for quantitatively measuring the robustness of control laws to spatiotemporal quantization, and the amount of control attention required, can serve as a useful tool for comparatively examining competing theories on human motor control.

Bibliography

[1] Natalia Dounskaia and Jacob A Goble. The role of vision, speed, and attention in overcoming directional biases during arm movements. *Experimental brain research*, 209(2):299–309, 2011.

[2] Anatol G Feldman. Functional tuning of nervous system with control of movement or maintenance of a steady posture. 2. controllable parameters of muscles. *BIOPHYSICS-USSR*, 11(3):565, 1966.

[3] Anatol G Feldman. Once more on the equilibrium-point hypothesis (λ model) for motor control. *Journal of motor behavior*, 18(1):17–54, 1986.

[4] Hiroaki Gomi and Mitsuo Kawato. Equilibrium-point control hypothesis examined by measured arm stiffness during multijoint movement. *Science*, 272(5258):117–120, 1996.

[5] Tamar Flash and Neville Hogan. The coordination of arm movements: an experimentally confirmed mathematical model. *The journal of Neuroscience*, 5(7):1688–1703, 1985.

[6] Yoji Uno, Mitsuo Kawato, and Rika Suzuki. Formation and control of optimal trajectory in human multijoint arm movement. *Biological cybernetics*, 61(2):89–101, 1989.

[7] Christopher M Harris and Daniel M Wolpert. Signal-dependent noise determines motor planning. *Nature*, 394(6695):780–784, 1998.

[8] Emanuel Todorov and Michael I Jordan. Optimal feedback control as a theory of motor coordination. *Nature neuroscience*, 5(11):1226–1235, 2002.

[9] Emanuel Todorov. Optimality principles in sensorimotor control. *Nature neuroscience*, 7(9):907–915, 2004.

[10] Jacob A Goble, Yanxin Zhang, Yury Shimansky, Siddharth Sharma, and Natalia V Dounskaia. Directional biases reveal utilization of arm's biomechanical properties for optimization of motor behavior. *Journal of neurophysiology*, 98(3):1240–1252, 2007.

[11] Daniel M Wolpert, Zoubin Ghahramani, and Michael I Jordan. Perceptual distortion contributes to the curvature of human reaching movements. *Experimental Brain Research*, 98(1):153–156, 1994.

[12] Daniel M Wolpert, Zoubin Ghahramani, and Michael I Jordan. Are arm trajectories planned in kinematic or dynamic coordinates an adaptation study. *Experimental brain research*, 103(3):460–470, 1995.

[13] Girish N Nair, Fabio Fagnani, Sandro Zampieri, and Robin J Evans. Feedback control under data rate constraints: An overview. *Proceedings of the IEEE*, 95(1):108–137, 2007.

[14] Dimitrios Hristu. *Optimal control with limited communication.* PhD thesis, Harvard University, 1999.

[15] Daniel Liberzon. Hybrid feedback stabilization of systems with quantized signals. *Automatica*, 39(9):1543–1554, 2003.

[16] K Li and J Baillieul. Robust and efficient quantization and coding for control of multidimensional linear systems under data rate constraints. *International Journal of Robust and Nonlinear Control*, 17(10-11):898–920, 2007.

[17] Roger W Brockett. Minimum attention control. In *Decision and Control, 1997., Proceedings of the 36th IEEE Conference on*, volume 3, pages 2628–2632. IEEE, 1997.

[18] RW Brockett. Minimizing attention in a motion control context. In *Decision and Control, 2003. Proceedings. 42nd IEEE Conference on*, volume 4, pages 3349–3352. IEEE, 2003.

[19] Donghyun Kim, Cheongjae Jang, and Frank C Park. Kinematic feedback control laws for generating natural arm movements. *Bioinspiration & Biomimetics*, 9(1):016002, 2014.

[20] Sekhar Tatikonda and Sanjoy Mitter. Control under communication constraints. *Automatic Control, IEEE Transactions on*, 49(7):1056–1068, 2004.

[21] Sohee Lee and Frank Chongwoo Park. Mobile robot motion primitives that take into account the cost of control. In *Latest Advances in Robot Kinematics*, pages 429–436. Springer, 2012.

[22] Eri Nakano, Hiroshi Imamizu, Rieko Osu, Yoji Uno, Hiroaki Gomi, Toshinori Yoshioka, and Mitsuo Kawato. Quantitative examinations of internal representations for arm trajectory planning: minimum commanded torque change model. *Journal of Neurophysiology*, 81(5):2140–2155, 1999.

[23] Emanuel Todorov and Weiwei Li. A generalized iterative lqg method for locally-optimal feedback control of constrained nonlinear stochastic systems. In *American Control Conference, 2005. Proceedings of the 2005*, pages 300–306. IEEE, 2005.

국문초록

로봇 산업이 발전함에 따라 로봇은 점점 더 복잡한 일을 수행하게 되었지만, 지금 쓰이고 있는 제어 및 동작 계획 방법들은 가용 메모리 용량 및 계산 처리 속도 내에서 원하는 일을 수행하기에 어려움이 많다. 이에 따라 이 논문에서는 좀 더 세밀하고 어려운 일들을 동시에 처리할 수 있는 제어기를 소개하고 이를 평가 하기 위한 방법을 제안 하였다. 사람이 동시에 여러 가지 일을 하기 위해 각각의 일에 집중을 시간에 따라 옮겨 가며 처리하는 것으로 부터 영감을 받아, 양자화 관점에서 문제를 접근하였다.

우선 결정적 동적 시스템의 2 자유도 팔의 방사형 도달 움직임에 대한 시뮬레이션을 시행하였으며 최소 집중 제어기가 다른 제어기에 비해 제한된 환경에서도 잘 작동하는 것을 확인하였다. 확률론적 동적 시스템의 경우 문제를 간단히 하기 위해 집중 지수 대신에 토크 체인지를 고려하였으며 최소 간섭 원리에 따라 최적화된 피드백 항을 구하였다. 첫번째 시뮬레이션과 같은 방식으로 2 자유도 팔의 방사형 도달 움직임에 대한 시뮬레이션을 시행하였으며 낮은 정보 처리 속도 내에서 노이즈가 모델링 되었을때 잘 작동함을 확인함 뿐만 아니라, 사람의 움직임과도 비교하여 분석하였다. 또한 팔 동작 생성의 방향 편향성이 사람의 집중과 관련 있다는 실험 결과를 이용하여 이 제어 기법이 사람의 집중과 연관성이 있는지에 대하여 논의하였다.

주요어: 집중, 사람과 유사한 팔 동작, 최적 피드백 제어

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