공학석사학위논문

위상최적화를 이용한 굽음보 구조에서의 기계적 파동필터 설계

Topology optimization of mechanical wave filter in curved beam structures

2013 년 2 월

서울대학교 대학원

기계 항공 공학부

김 상 아

Abstract

We carried out a design of a mechanical wave filter to reduce wave transmission in a curved beam structure. Filtering phenomenon of the wave transmission in specific frequency range may be seen at infinitely periodic elastic structures composed of different materials. In a curved beam, however, implementation of the periodic structure is very limited. The dimension of the curved region which connects two waveguides is generally not long enough to cover broad target frequency. The periodic structure repeated only in a few times cannot guarantee the performance of wave filtering in the curved beam. To overcome the difficulty, we formulated the design problem of the curved beam as a topology optimization problem. A curved region between two straight waveguides was considered as a design domain to minimize the power transmission of waves at target frequencies. Compared to a periodic curved structure, the designed ones by topology optimization show the improvement of the performance in reducing the wave transmission at the target frequencies.

Keywords: Topology optimization, mechanical filter, curved beam

Student number: 2011-22884

Contents

List of Figures

Chapter1

Introductions

The vibration control has been a critical issue in the design of mechanical systems [1-3]. As well as passengers' comfortness in cars in everyday life, the performance and health of the systems in industry may be affected by vibrations. A typical strategy to the vibration control is to reduce the vibration amplitude at target regions in the systems by using additional masses, stiffeners, or damping materials directly. Although the direct treatments can reduce the vibrations of the systems, they must cause the increase of total mass and cost in production. More effective control of vibrations can be carried out by controlling waves which propagate in structures and transfer the power. More specifically, the reduction of vibrations can be accomplished by blocking the transmission of the waves to the target region. A specially designed connecting region can be considered between an excitation region and the target region. This connecting part is a mechanical wave filter.

A mechanical filter can allow or block the propagation of waves having specific ranges of frequencies, which are called pass band and stop band, respectively [4]. This phenomenon may be seen at periodic structures, composed of two different materials infinitely [5, 6]. For practical problems, however, more considerations in system size [7] and frequency ranges of interest should be taken account because as long as a sufficient number of unit cells should be needed (usually around three to four are required) [8].

The vibration of curved beams has been the subject of numerous studies due to their wide variety of potential applications, such as bridges and aircraft structures [9]. However, those structures with curved members are usually subjected to the corrosion, creep or fatigue under severe environmental or service condition [10]. So, vibration control of those structures is important for maintaining their status.

In the present work, we deal with topology optimization problems encountered in the study of wave propagation through a finite curved beam system shown in Fig. 1.1. Curved beams are frequently used in mechanical systems to joint structural parts smoothly, so that waves excited in one part are easily transferred to the other part. By using topology optimization in this work, the curved beam region which is connecting two perpendicular waveguides is designed to control transmission of waves through it at frequency ranges of interest. The two straight beams are considered to consist of the same elastic material with the identical cross-sectional area. If connecting curved beam has same cross-sectional area, impedance of random point of curved beam is determined by material property consisting of the curved beam. Then the design of this curved beam can be formulated as a topology design problem. The optimization results from the topology optimizations of curved beams are expected to be improved from periodic structure. Therefore, the results are discussed with the comparisons between the two models.

To represent the wave propagation in the curved beam, extensional, bending and torsional deformations should be considered. Among various mathematical approaches to calculated wave numbers, Flügge theory is employed because the longitudinal wave is mainly considered in this work [11, 12].

This thesis consists of eight chapters including this one. In the chapter 2, dispersion curve of curved beam structures is introduced. The structure and using theory is explained with some figures and equations. Next, in the chapter 3, we analyze the periodic structures. Principle of constructing periodic structure and dispersion cure is explained. Prior to formulate the topology design optimization in the chapter5, several equations for calculating power coefficient are induced. Then, the processes of the topology optimization formulation are briefly explained in the chapter 5. In the chapter 6, we show the optimization results obtained by the topology optimization and compare optimization result and periodic structure result.

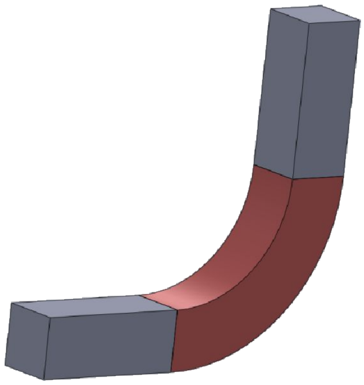


Fig. 1.1 Curved beam structure connecting two perpendicular waveguides.

Chapter2

Dispersion curve of curved beam structures

Let's consider a small segment of a curved beam with a subtended angle $d\theta$ at the center of curvature [11, 12] as shown in Fig. 2.1. The circumferential coordinate along the centerline of the segment is denoted by s and the radial coordinate normal to the centerline is z. The displacements of the centerline in the radial and tangential directions are denoted by w and u, respectively. The arc length of the segment is $ds = \mathrm{R}d\theta$ where R is radius of curvature of the centerline. Neglecting the effects of shear deformation and rotary inertia, the governing equations for free vibration in the radial and tangential directions are given by [12]

$$-EI(\frac{\partial^4 w}{\partial s^4} + \frac{2}{R}\frac{\partial^2 w}{\partial s^2} + \frac{W}{R^4}) - \frac{EA}{R}(\frac{w}{R} + \frac{\partial u}{\partial s}) = \rho A \frac{\partial^2 w}{\partial t^2}, \tag{2.1}$$

$$EA(\frac{\partial^2 u}{\partial s^2} + \frac{1}{R}\frac{\partial w}{\partial s}) = \rho A \frac{\partial^2 u}{\partial t^2}, \tag{2.2}$$

Where E is the Young's modulus, I the second moment of area, A the cross-sectional area, and ρ the density. The physical quantities such as the rotation φ of the cross-section and the normal force N, bending moment M, and shear force Q are given by:

$$\varphi = -\frac{u}{R} + \frac{\partial w}{\partial s}, \quad N = AE\left(\frac{w}{R} + \frac{\partial u}{\partial s}\right) + \frac{EI}{R}\left(\frac{w}{R^2} + \frac{\partial^2 w}{\partial s^2}\right), \tag{2.3-4}$$

$$M = EI\left(\frac{w}{R}+\frac{\partial^2 w}{\partial s^2}\right), \quad Q = -EI\left(\frac{1}{R^2}\frac{\partial w}{\partial s}+\frac{\partial^3 w}{\partial s^3}\right). \tag{2.5-6}$$

Equations describe the in-plane motion of thin, uniform, curved beams with constant curvature based on Flügge's theory. When the radius of curvature R tends to be infinity, the radial and tangential displacements are decoupled and the equations become those for a uniform, straight beam.

Using those equations, the dispersion relationship for each wave component in the curved beam can be obtained. The radial and tangential displacements are assumed to be time harmonic and of the form:

$$w(s,t) = C_w \mathrm{e}^{\mathrm{j}(\omega t - ks)}, \quad u(s,t) = C_u \mathrm{e}^{\mathrm{j}(\omega t - ks)}. \tag{2.7-8}$$

Substituting Eqs. (2.7, 2.8) into Eqs. (2.1, 2.2) gives

$$\begin{bmatrix} \frac{I}{AR^2}(k^2R^2-1)h^2+1-\frac{\rho}{E}R^2\omega^2 & -ikR \\ ikR & k^2R^2-\frac{\rho}{E}R^2\omega^2 \end{bmatrix}\begin{bmatrix} C_w \\ C_u \end{bmatrix}=0. \tag{2.9}$$

Setting the determinant of the matrix to zero gives the dispersion equation

$$k^6 - ({k_L}^2 + 2\kappa^2)k^4 + (\kappa^4 - {k_B}^4 + 2\kappa^2 {k_L}^2)k^2 - (\kappa^4 {k_L}^2 + \kappa^2 {k_B}^4 - {k_L}^2 {k_B}^4) = 0, \tag{2.10}$$

where $k_L = \sqrt{\rho\omega^2 / E}$ and are $k_B = \sqrt[4]{\rho A\omega^2 / EI}$ the longitudinal and bending wavenumbers for a straight beam, respectively and κ is the curvature. This characteristic equation in Eq. (2.10) can be plotted in a frequency-wave number plane, i.e., a dispersion curve. In a curved beam, the longitudinal wave is transmitted above cutoff frequency. So we can find which curve corresponds to the longitudinal wave in the dispersion curve of the curved beam where modes are mixed. In Fig. 2.2, blue line of dispersion curve corresponds to the longitudinal wave.

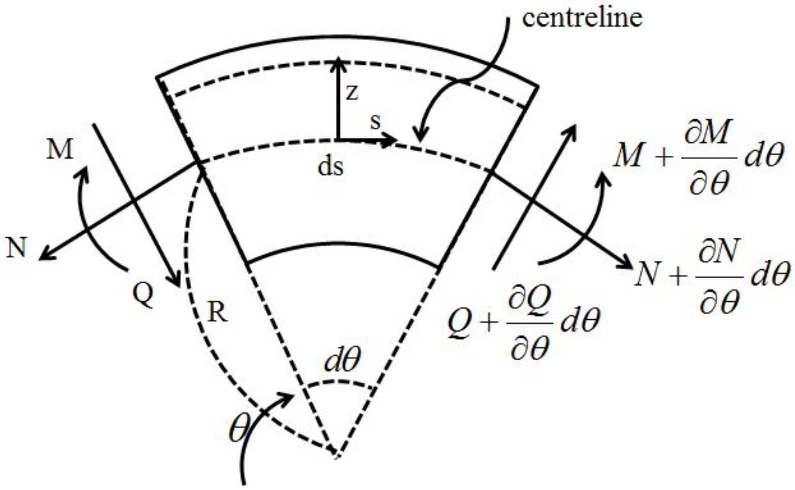
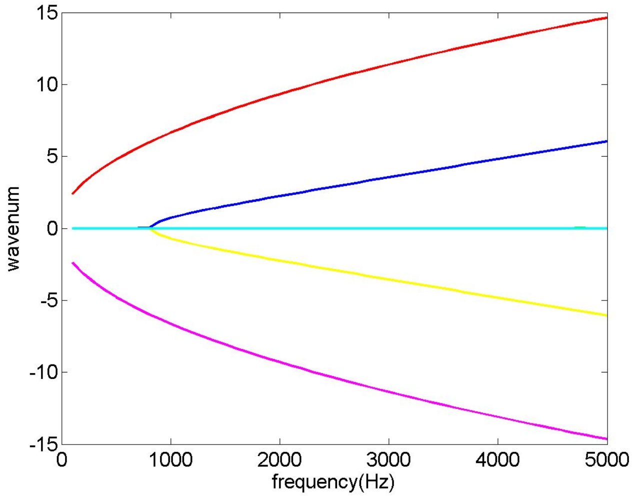
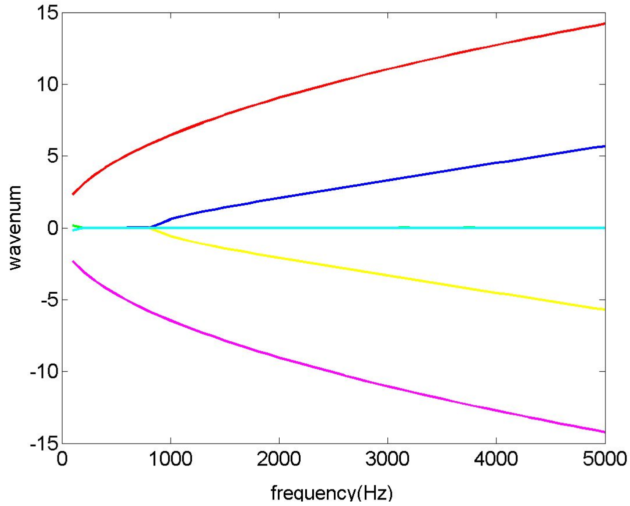


Fig. 2.1 Differential element of a thin, curved beam and sign convention of physical quantities.



(a)



(b)

Fig. 2.2 Real part of dispersion curve of curved beam structure (a) for steel (b) for aluminum.

Chapter3

Dispersion curve of periodic structures

Mechanical filter can be constructed by using wave propagation in periodic media such as multilayers structures. In these media, there are stop bands where the energy transmission coefficient appears to vanish or takes very low values, while in pass bands it oscillates with the frequency [13]. However, the number of unit cell is limited in curved beam structure due to curved beam's size as mentioned earlier. In this chapter, we construct proper periodic structures for some target frequency by using uniform curved beam's dispersion curve and Bragg condition [13, 14] to compare periodic structure result and optimized result. Periodic structure's performance is verified by considering dispersion curve of periodic structures which we constructed.

3.1 Bragg Condition

According to Bragg condition [13, 14], transmission energy is reduced effectively when periodic unit length is the same as $\lambda/2$, where λ is the wavelength of the propagating wave in the periodic structure. The idea is induced and a periodic structure consisting of two different materials is constructed. Each part's length matches with $\lambda/4$, a quarter of the wavelength. Using dispersion curve of uniform curved beam structures, proper periodic structure reducing power transmission at target frequency could be designed.

From Fig. 2.2, we could get wave number of each material matching some frequency (later, this become target frequency of optimization problem). Let's consider 2 kHz case. The wave number of 2 kHz is 2.246 for steel and 2.2087 for aluminum. For each case, we can calculate the wave lengths as follow as:

$$\lambda_{\text{steel}} = \frac{2\pi}{k} = \frac{2\pi}{2.246} = 2.7975m \tag{3.1}$$

$$\lambda_{\text{aluminum}} = \frac{2\pi}{k} = \frac{2\pi}{2.087} = 3.011m \tag{3.2}$$

From those values, a quarter of the wavelength for each material can be given

$$\frac{1}{4}\lambda_{\text{steel}} = 0.6994m \tag{3.3}$$

$$\frac{1}{4}\lambda_{\text{aluminum}} = 0.7528m \tag{3.4}$$

By using those values, we could predict periodic structure unit length which matches a quarter of the wave length and construct periodic structure in curved beam (Fig. 3.1). Because curved beam structure's height is so small in comparison with straight beam ($1/100$), we mainly consider centerline of curved. And here, two straight beams of Fig.1.1 consist of steel. So, we allocate aluminum part of unit cell first instead of steel to construct periodic structure.

3.2 Matrix Representation of Wave Motion

Let's consider dispersion equation (Eq. (2.10)) again. Here, we used longitudinal and bending wavenumbers for a straight beam. However, those values couldn't be calculated for periodic structure. So, we induce other method to calculate wave number of periodic structure which uses transfer matrix.

According to the dispersion relation in Eq. (2.8), six modes of wave propagation can be estimated. Three modes are positive–going waves and the remainders are negative-going ones in the curved beam.

$$w(s) = C_1 \mathrm{e}^{-\mathrm{j}k_1 s} + C_2 \mathrm{e}^{-\mathrm{j}k_2 s} + C_3 \mathrm{e}^{-\mathrm{j}k_3 s} + C_4 \mathrm{e}^{-\mathrm{j}k_4 s} + C_5 \mathrm{e}^{-\mathrm{j}k_5 s} + C_6 \mathrm{e}^{-\mathrm{j}k_6 s}, \tag{3.5}$$

$$\begin{aligned} u(s) = {} & \alpha_1 C_1 \mathrm{e}^{-\mathrm{j}k_1 s} + \alpha_2 C_2 \mathrm{e}^{-\mathrm{j}k_2 s} + \alpha_3 C_3 \mathrm{e}^{-\mathrm{j}k_3 s} + \alpha_4 C_4 \mathrm{e}^{-\mathrm{j}k_4 s} + \alpha_5 C_5 \mathrm{e}^{-\mathrm{j}k_5 s} \\ & + \alpha_6 C_6 \mathrm{e}^{-\mathrm{j}k_6 s}. \end{aligned} \tag{3.6}$$

Where α is the ratio of radial and tangential displacement of the curved beam. This is from Eq. (2.9) and given by

$$\alpha_i = \frac{i\kappa k_i}{k_L^{\ 2} - k_i^{\ 2}}, \quad i = 1, 2, \ldots 6 \tag{3.7}$$

At high frequencies, displacement ratio α_3 and α_6 tend to infinity. So the radial and tangential displacements of the curved beam are given, respectively, by

$$\begin{aligned} w(s) = {} & C_1 \mathrm{e}^{-\mathrm{j}k_1 s} + C_2 \mathrm{e}^{-\mathrm{j}k_2 s} + (\alpha_3)^{-1} C_3 \mathrm{e}^{-\mathrm{j}k_3 s} + C_4 \mathrm{e}^{-\mathrm{j}k_4 s} + C_5 \mathrm{e}^{-\mathrm{j}k_5 s} \\ & + (\alpha_6)^{-1} C_6 \mathrm{e}^{-\mathrm{j}k_6 s}, \end{aligned} \tag{3.8}$$

$$u(s)=\alpha_1 C_1 \mathrm{e}^{-\mathrm{j}k_1 s}+\alpha_2 C_2 \mathrm{e}^{-\mathrm{j}k_2 s}+C_3 \mathrm{e}^{-\mathrm{j}k_3 s}+\alpha_4 C_4 \mathrm{e}^{-\mathrm{j}k_4 s}+\alpha_5 C_5 \mathrm{e}^{-\mathrm{j}k_5 s} \\ +C_6 \mathrm{e}^{-\mathrm{j}k_6 s}. \tag{3.9}$$

The generalized displacements and corresponding internal forces can be grouped in vector form as:

$$w=\begin{Bmatrix} w \\ \varphi \\ u \end{Bmatrix} \quad \begin{array}{l} w: \text{ radial displacement} \\ u: \text{ tangential displacement} \\ \varphi: \text{ rotation of cross-section} \end{array} \tag{3.10}$$

$$f=\begin{Bmatrix} Q \\ M \\ N \end{Bmatrix} \quad \begin{array}{l} Q: \text{ shear force} \\ M: \text{ bending moment} \\ N: \text{ normal force} \end{array} \tag{3.11}$$

Note that the rotation φ and internal forces Q, M and N are obtained from Eqs. (2.3-6). The wave vectors consisting of the amplitudes of the waves can be defined by

$$a^+(s)=\begin{Bmatrix} C_1 \mathrm{e}^{-\mathrm{j}k_1 s} \\ C_2 \mathrm{e}^{-\mathrm{j}k_2 s} \\ C_3 \mathrm{e}^{-\mathrm{j}k_3 s} \end{Bmatrix},\ a^-(s)=\begin{Bmatrix} C_4 \mathrm{e}^{-\mathrm{j}k_4 s} \\ C_5 \mathrm{e}^{-\mathrm{j}k_5 s} \\ C_6 \mathrm{e}^{-\mathrm{j}k_6 s} \end{Bmatrix}. \tag{3.12}$$

Here, the matrices Ψ and Φ define the transformation from the wave domain to the physical domain given by

$$\begin{array}{l} \Psi^+=[\psi_1 \quad \psi_2 \quad \psi_3],\quad \Psi^-=[\psi_4 \quad \psi_5 \quad \psi_6], \\ \Phi^+=[\phi_1 \quad \phi_2 \quad \phi_3],\quad \Phi^-=[\phi_4 \quad \phi_5 \quad \phi_6]. \end{array} \tag{3.13-16}$$

where the column vectors ψ_i and ϕ_i for $i = 1,2,4,5$ are

$$\psi_i = \begin{Bmatrix} 1 \\ -(\kappa\alpha_i + ik_i) \\ \alpha_i \end{Bmatrix} \tag{3.17}$$

$$\phi_i = \begin{Bmatrix} iEk_i(\kappa^2 - k_i^2) \\ EI(\kappa^2 - k_i^2) \\ EA(\kappa - ik_i\alpha_i) + EI\kappa(\kappa^2 - k_i^2) \end{Bmatrix} \tag{3.18}$$

and ψ_i and ϕ_i for $i = 3,6$ are

$$\psi_i = \frac{1}{\alpha_i}\begin{Bmatrix} 1 \\ -(\kappa\alpha_i + ik_i) \\ \alpha_i \end{Bmatrix}, \tag{3.19}$$

$$\phi_i = \frac{1}{\alpha_i}\begin{Bmatrix} iEk_i(\kappa^2 - k_i^2) \\ EI(\kappa^2 - k_i^2) \\ EA(\kappa - ik_i\alpha_i) + EI\kappa(\kappa^2 - k_i^2) \end{Bmatrix}. \tag{3.20}$$

3.3 Transfer Matrix of Periodic Structure

We use transfer matrix to calculate wave number of periodic structure [15]. The transfer matrix method was introduced in the study of wave propagation in periodic composites. Through introducing a new definition for the dispersion relation using transfer matrix, the pass bands and stop bands are calculated for finite system. Here, we used displacement and internal force vectors to construct transfer matrix.

Using the displacement and internal force vectors, initial vector and other vector in material 1 are given by:

$$\begin{Bmatrix} w_0 \\ f_0 \end{Bmatrix} = \begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix} \begin{Bmatrix} a_0{}^+ \\ a_0{}^- \end{Bmatrix}$$
$$= \begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix} \begin{bmatrix} e^{-jk_{11}s(s=0)} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & e^{-jk_{16}s(s=0)} \end{bmatrix} \begin{Bmatrix} C_1 \\ ... \\ C_6 \end{Bmatrix} \quad (3.21)$$
$$= \begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix} \begin{Bmatrix} C_1 \\ ... \\ C_6 \end{Bmatrix}.$$

$$\begin{Bmatrix} w_1 \\ f_1 \end{Bmatrix} = \begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix} \begin{bmatrix} e^{-jk_{11}s_1} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & e^{-jk_{16}s_1} \end{bmatrix} \begin{Bmatrix} C_1 \\ ... \\ C_6 \end{Bmatrix}. \quad (3.22)$$

Where the wave numbers k_{1i} $(i = 1, 2, ...6)$ relate to material1. w_0 f_0, w_1 and f_1 are displacement and force vector of initial point and point 1, respectively.

Using them, transfer matrix of material 1 could be calculated by using amplitude of each components are identical,

$$\begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix}^{-1} \begin{Bmatrix} w_0 \\ f_0 \end{Bmatrix} = \left(\begin{bmatrix} \Psi^+ & \Psi^- \\ \Phi^+ & \Phi^- \end{bmatrix} \begin{bmatrix} e^{-jk_{11}s_1} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & e^{-jk_{16}s_1} \end{bmatrix} \right)^{-1} \begin{Bmatrix} w_1 \\ f_1 \end{Bmatrix} \quad (3.23)$$

$$\mathrm{T}_{0(1)} d_0 = \mathrm{T}_{1(1)} d_1 \quad (3.24)$$

$$d_1 = TM_1 d_0 \quad (3.25)$$

In a similar way,

$$\left(\begin{bmatrix}\Psi^{+} & \Psi^{-}\\ \Phi^{+} & \Phi^{-}\end{bmatrix}\begin{bmatrix}e^{-jk_{21}s_1} & 0 & 0\\ 0 & \dots & 0\\ 0 & 0 & e^{-jk_{26}s_1}\end{bmatrix}\right)^{-1}\begin{Bmatrix}w_1\\ f_1\end{Bmatrix} = \left(\begin{bmatrix}\Psi^{+} & \Psi^{-}\\ \Phi^{+} & \Phi^{-}\end{bmatrix}\begin{bmatrix}e^{-jk_{21}s_2} & 0 & 0\\ 0 & \dots & 0\\ 0 & 0 & e^{-jk_{26}s_2}\end{bmatrix}\right)^{-1}\begin{Bmatrix}w_2\\ f_2\end{Bmatrix} \tag{3.26}$$

$$\mathrm{T}_{1(2)}d_1 = \mathrm{T}_{2(2)}d_2 \tag{3.27}$$

$$d_2 = TM_2 d_1 \tag{3.28}$$

Where the wave numbers k_{2i} $(i = 1, 2, \dots 6)$ relate to material2.

Using two equations, a transfer matrix of a periodic structure is given by

$$\therefore d_2 = TM_2 d_1 = TM_2 TM_1 d_0 = TMd_o \tag{3.29}$$

$$\begin{aligned}\mathrm{TM} &= TM_2\mathrm{TM}_1\\ &= \mathrm{T}_{2(2)}{}^{-1}\mathrm{T}_{1(2)}\mathrm{T}_{1(1)}{}^{-1}\mathrm{T}_{0(1)} \quad (\mathrm{TM}:\ \textit{transfer matrix})\end{aligned} \tag{3.30}$$

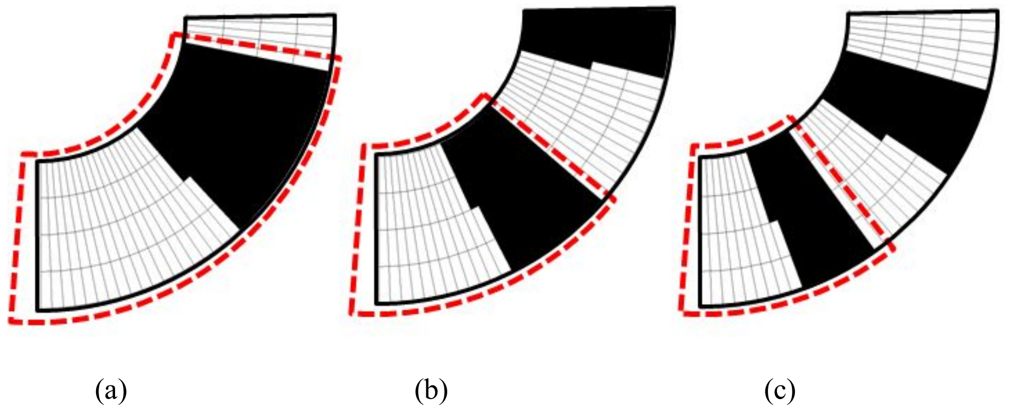
The wave number of periodic structure's unit cell is calculated by using transfer matrix (eigenvalue problem).

$$d_2 = \mathrm{TM}d_0 = (e^{-ik(d_2-d_0)} \times I)d_0 \tag{3.31}$$

3.4 Dispersion Curve of Periodic Structure

Periodic structure for reducing power transmission at target frequency is predicted by using the dispersion curve of the uniform curved beam. From periodic structure, we can calculate wave number of unit cell of periodic structure by using transfer matrix and eigenvalue problem.

Fig. 3.3 is dispersion curve of periodic structure (Fig. 3.2). As mentioned earlier, longitudinal wave is transmitted above cutoff frequency. So, green line of dispersion curve corresponds to the longitudinal wave number. Note at 3 kHz because it is target frequency of the periodic structure. At this frequency, imaginary part of the wave number exists. It means this frequency is in stop band. Because wave couldn't be transmitted at stop band, we can conclude suggested periodic structure is considered as proper structure for reducing power transmission coefficient at the target frequency (3 kHz). However, we can't control wave propagation sufficiently since we couldn't construct enough unit cell at given curved beam structure. Power transmission value of periodic structure at target frequency is calculated in chapter6. Fig. 3.4 and Fig. 3.5 are dispersion curves of other periodic structures.



(a) (b) (c)

Fig. 3.1 Predicted periodic structure of curved beam for target frequency (a) 2 kHz, (b) 3 kHz, (c) 4 kHz (red dash line means unit cell).

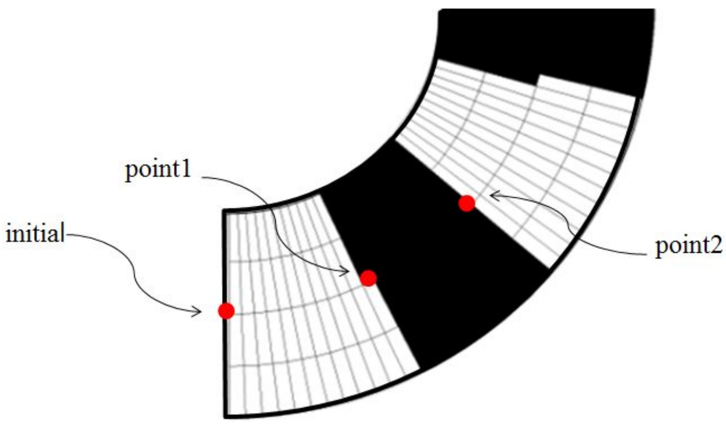
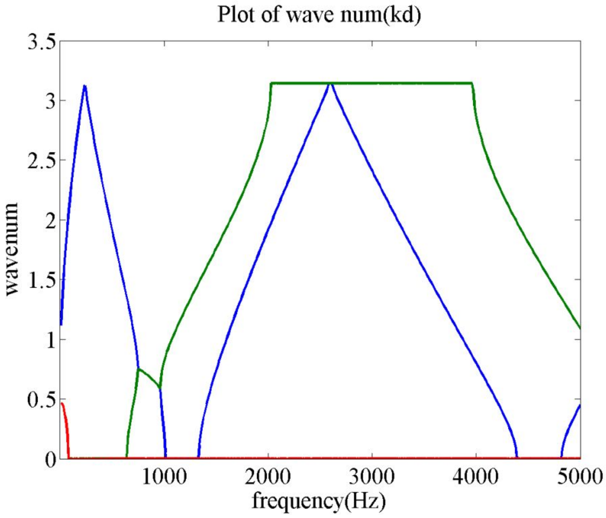
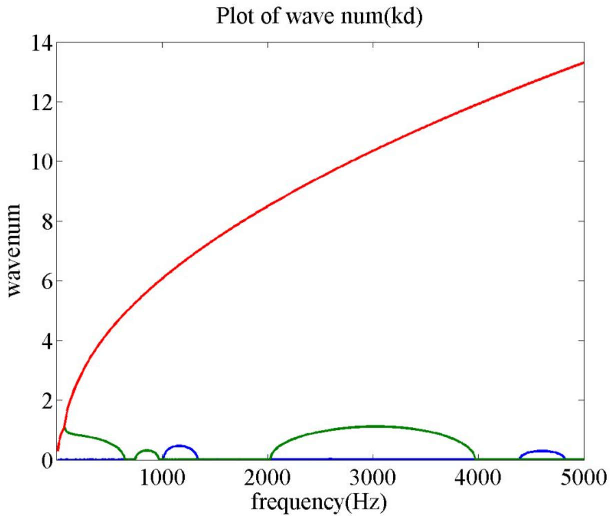


Fig. 3.2 Predicted periodic structure of curved beam for target frequency 3 kHz. White means mateirl1 and black means material2.

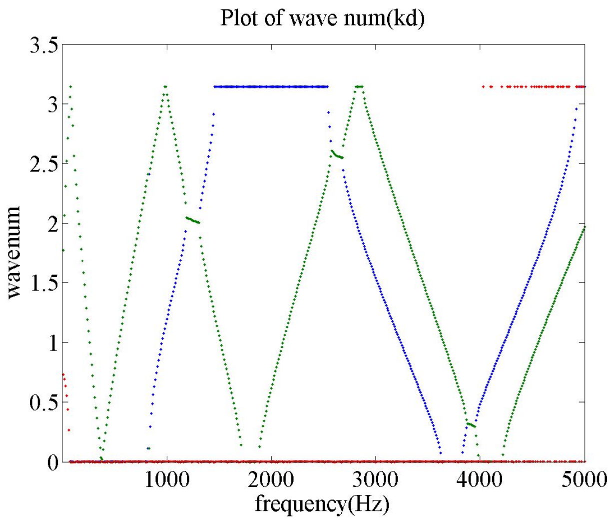


(a)

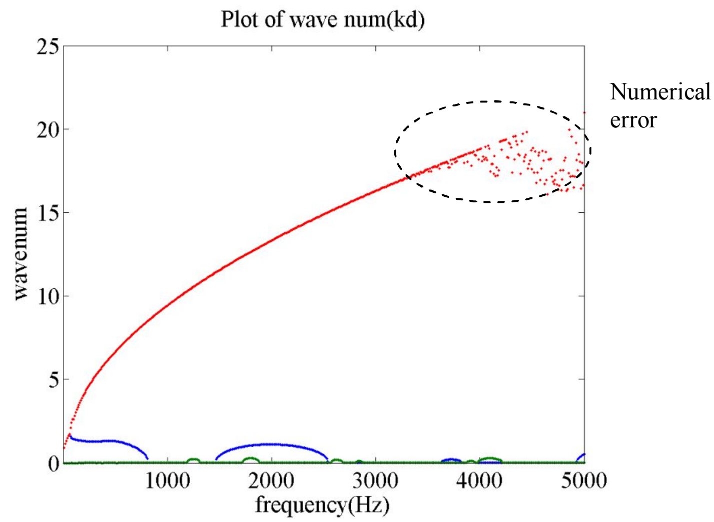


(b)

Fig. 3.3 dispersion curve of periodic structure for target frequency 3kHz (a) dispersion curve(real) (b) dispersion curve(imaginary), (green line is dispersion curve of longitudinal wave and red and blue lines are dispersion curve of transverse wave), curved beam structure (black is steel, white is aluminum).

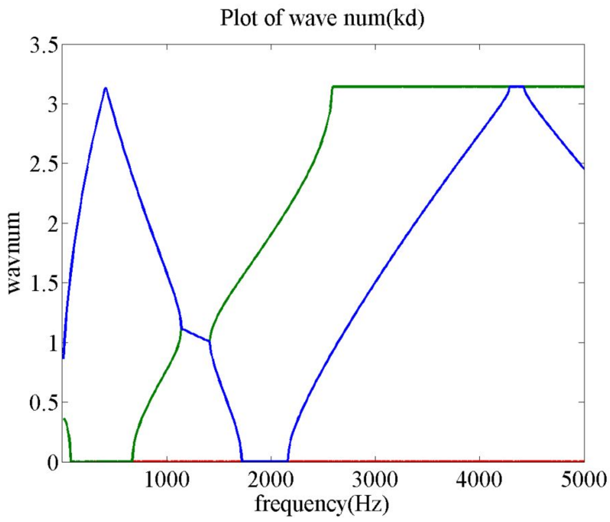


(a)

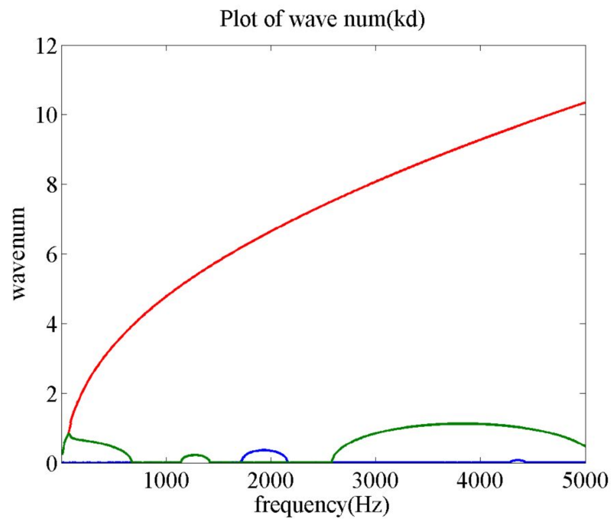


(b)

Fig. 3.4 A dispersion curve of periodic structure for target frequency 2kHz (a) dispersion curve(real) (b) dispersion curve(imaginary), (green line is dispersion curve of longitudinal wave and red and blue lines are dispersion curve of transverse wave), curved beam structure (black is steel, white is aluminum).



(a)



(b)

Fig. 3.5 dispersion curve of periodic structure for target frequency 4kHz (a) dispersion curve(real) (b) dispersion curve(imaginary), (green line is dispersion curve of longitudinal wave and red and blue lines are dispersion curve of transverse wave), curved beam structure (black is steel, white is aluminum).

Chapter4

Finite element analysis of curved beam structures

For topology optimization design of curved beam structures, the objective function is defined as the power transmission coefficient. Thus, the coefficients of incident and transmitted wave component should be calculated. In this research, Two point method [16] is employed to induce component coefficients of each wave guides.

4.1 Calculation of Power Coefficient

At first, we can calculate of displacements of each point of whole system (including straight beam and curved beam structures) as follow:

$$(K-\omega^2 M)u = F \tag{4.1}$$

$$u = -(K-\omega^2 M)^{-1} F \tag{4.2}$$

To calculate each component coefficients, we used Tow point method [16]. u_1 and u_2 are x-axial displacements of incident domain. Those can be expressed as follow. We neglect reflection component by using PML (perfectly matched layer) [17] at end of each straight beams:

$$u_1 = I_{il}\mathrm{e}^{\mathrm{j}(\omega t - k_i x_1)} + R_{rl}\mathrm{e}^{\mathrm{j}(\omega t + k_i x_1)} \tag{4.3}$$

$$u_2 = I_{il}\mathrm{e}^{\mathrm{j}(\omega t - k_i x_2)} + R_{rl}\mathrm{e}^{\mathrm{j}(\omega t + k_i x_2)} \tag{4.4}$$

Where k_i is wave number of incident plane, I_{il} and R_{il} incident and reflect coefficient of longitudinal wave respectively. Next, construct matrix and calculate each coefficients.

$$\begin{bmatrix} \mathrm{e}^{-\mathrm{j}k_i x_1} & \mathrm{e}^{\mathrm{j}k_i x_1} \\ \mathrm{e}^{-\mathrm{j}k_i x_2} & \mathrm{e}^{\mathrm{j}k_i x_2} \end{bmatrix} \begin{bmatrix} I_{il} \\ R_{il} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4.5}$$

$$\therefore \begin{bmatrix} I_{il} \\ R_{il} \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{-\mathrm{j}k_i x_1} & \mathrm{e}^{\mathrm{j}k_i x_1} \\ \mathrm{e}^{-\mathrm{j}k_i x_2} & \mathrm{e}^{\mathrm{j}k_i x_2} \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4.6}$$

And transmission coefficient of longitudinal wave T_{tl} can be induced as follow: u_3 is x-axis displacement of transmission plane.

$$u_3 = T_{tl}\mathrm{e}^{\mathrm{j}(\omega t - k_t x_3)} \tag{4.7}$$

Next, time-averaged power of incidence and transmission can be expressed as

$$P_{il} = \frac{1}{2}\rho_i C_{bi} I_{il} I_{il}^{*}, \quad P_{tl} = \frac{1}{2}\rho_t C_{bt} T_{tl} T_{tl}^{*} \tag{4.8-9}$$

Where ρ_i and ρ_t are the densities of incidence and transmission plane and C_{bi} and C_{bt} are group velocity of incidence and transmission plane respectively. For sensitivity of object function, differential value of those coefficients could be induced similar process.

$$(K'-\omega^2 M')u+(K-\omega^2 M)u'=0 \tag{4.10}$$

$$u'=-(K-\omega^2 M)^{-1}(K'-\omega^2 M')u \tag{4.11}$$

$$u_1'=I_{il}'\mathrm{e}^{\mathrm{j}(\omega t-k_i x_1)}+R_{rl}'\mathrm{e}^{\mathrm{j}(\omega t+k_i x_1)} \tag{4.12}$$

$$u_2'=I_{il}'\mathrm{e}^{\mathrm{j}(\omega t-k_i x_2)}+R_{rl}'\mathrm{e}^{\mathrm{j}(\omega t+k_i x_2)} \tag{4.13}$$

$$\begin{bmatrix} I_{il}' \\ R_{il}' \end{bmatrix}=\begin{bmatrix} \mathrm{e}^{-\mathrm{j}k_i x_1} & \mathrm{e}^{\mathrm{j}k_i x_1} \\ \mathrm{e}^{-\mathrm{j}k_i x_2} & \mathrm{e}^{\mathrm{j}k_i x_2} \end{bmatrix}^{-1}\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} \tag{4.14}$$

$$u_3'=T_{tl}'\mathrm{e}^{\mathrm{j}(\omega t-k_t x_3)} \tag{4.14}$$

4.2 PML

A perfectly matched layer (PML) [17] is an absorbing layer model for linear wave equations that absorbs, almost perfectly, propagating waves of all non-tangential angles-of-incidence and of all non-zero frequencies. It is defined as one governed by a modification of the equations for the elastic medium, with the modification motivated by a continuous, complex-valued, uncoupled coordinate stretching. In this research, we neglect reflection of longitudinal waveguides from each straight beam's end (see Fig.1.1). So, we adjust one-dimensional PML system.

In perfectly matched medium, the coordinate x replaced by a stretched coordinate $\tilde{x}$, defined as [18]

$$\tilde{x}:=\int_0^x \lambda(s)ds \tag{4.15}$$

where λ is a nowhere-zero, continuous, complex-valued coordinate stretching function. The stretching function is expressed in terms of an attenuation function, which controls the reflection due to PML. By the continuity of λ ,

$$\frac{d\tilde{x}}{dx} = \lambda(x) \tag{4.16}$$

Which formally implies

$$\frac{d}{d\tilde{x}} = \frac{1}{\lambda(x)}\frac{d}{dx} \tag{4.17}$$

If the function λ is defined in terms of real-valued, continuous attenuation functions f as

$$\lambda(x) := 1 - i\frac{f(x)}{k_s} \tag{4.18}$$

Where $f(x)$ is attenuation function. To facilitate a meaningful discussion of the effects of the parameter, the attenuation function is chosen to be of the form

$$f(x) := f_0(\frac{x}{L_p})^m \qquad (m=1) \tag{4.19}$$

In this research problem, we consider x-axis displacement. So continuation attenuation function of x-axis and y-axis are

$$\lambda_1 := 1 - i\frac{f(x)}{k_s}, \quad \lambda_2 = 1 \tag{4.20-21}$$

As a result, stiffness matrix K_p and mass matrix M_p are expressed as

$$K_p = \int B_p{}^T D_p B_p dV = B_p{}^T D_p B_p (\frac{t_p b_p h_p}{\lambda_1}) \tag{4.22}$$

$$M_p = \int \rho_p H_p{}^T H_p dV = H_p{}^T H_p (\frac{t_p b_p h_p}{\lambda_1}) \tag{4.23}$$

Where B_p is the strain-displacement matrix, D_p is elasticity matrix, ρ_p is density and H_p is displacement interpolation matrix, t_p is thickness, b_p is width and h_p is height of PML elements. In here, strain-displacement matrix can be expressed in terms of λ_1. But displacement interpolation matrix is expressed like general case.

$$B_p = \frac{1}{4b_p h_p}\begin{bmatrix} \frac{-(h_p-y)}{\lambda_1} & 0 & \frac{(h_p-y)}{\lambda_1} & 0 & \frac{-(h_p+y)}{\lambda_1} & 0 & \frac{(h_p+y)}{\lambda_1} & 0 \\ 0 & \frac{-(b_p-x)}{\lambda_2} & 0 & \frac{-(b_p+x)}{\lambda_2} & 0 & \frac{(b_p+x)}{\lambda_2} & 0 & \frac{(b_p-x)}{\lambda_2} \\ \frac{-(b_p-x)}{\lambda_2} & \frac{-(h_p-y)}{\lambda_1} & \frac{-(b_p+x)}{\lambda_2} & \frac{(h_p-y)}{\lambda_1} & \frac{(b_p+x)}{\lambda_2} & \frac{-(h_p+y)}{\lambda_1} & \frac{(b_p-x)}{\lambda_2} & \frac{(h_p+y)}{\lambda_1} \end{bmatrix} \tag{4.24}$$

Chapter5

Topology Optimization Formulation

In this chapter, the topology optimization design problems for curved beam structures are formulated. To obtain optimal configurations of the curved beam structures, the design domain is defined and discretized by finite elements. For the penalization fi material properties, the SIMP model is employed [19]. For sensitivity analysis, the direct method [20] is utilized and the process of the analysis is briefly stated.

5.1 Problem Definition

As shown in Fig. 5.1, two dimensional straight beams are arranged whose cross-sectional areas are identical each other and a curved beam connecting them is selected as a design domain. Fig. 9 shows the design domains of the curved beam structures. The design domain is discretized by finite elements. Material property of each elements can be expressed as a function of design variables ($0 \le \chi_e \le 1$) which are allocated to every element in the design domain. Value 1 means steel and value 0 means aluminum.

The problem of design optimization with the objective of minimizing power transmission of longitudinal wave for curved beam can be formulated as follows:

$$\min\{T_{power}\} \tag{5.1}$$

$$\text{subject to } 0 \le \chi_e \le 1. \tag{5.2}$$

In Eq. (5.1), the symbol T_{power} means transmitted power coefficient which is defined as the ratio of incident power of the longitudinal wave to the transmitted power of the longitudinal wave in the discretized curved beam.

$$T_{power} = \frac{|P_{tl}|}{|P_{il}|} = \left(\frac{T_{tl} T_{tl}^{*} \rho_t C_{bt}}{I_{il} I_{il}^{*} \rho_i C_{bi}} \right) = |T|^2 \left(\frac{\rho_t C_{bt}}{\rho_i C_{bi}} \right) \tag{5.3}$$

5.2 Optimization Formulation

For the optimization formulation, the Young's Modulus E and density are penalized using the SIMP model as follows:

$$E = x_e^{3} E_1 + (1 - x_e)^3 E_2 \tag{5.4}$$

$$\rho = x_e \rho_1 + (1 - x_e) \rho_2 \tag{5.5}$$

Where E_1, E_2, ρ_1 and ρ_2 represent the properties of steel and aluminum respectively.

For the analysis of the sensitivities of the objective function with respect to the design variables, the direct method [20] is employed. Also incidence plane and transmission plane are same material, transmitted power coefficient is summarized the function of transmission coefficients, by using the direct method, we have the gradient with respect to the design variable

$$\begin{aligned}\frac{\mathrm{d}T_{power}}{\mathrm{d}x_e} &= \frac{(T_{tl}{}'T_{tl}^{*} + T_{tl}T_{tl}^{*}{}')I_{il}I_{il}^{*} - (I_{il}{}'I_{il}^{*} + I_{il}I_{il}^{*}{}')T_{tl}T_{tl}^{*}}{(I_{il}I_{il}^{*})^2} \\ &= \frac{2\,\mathrm{Re}(T_{tl}{}'T_{tl}^{*})I_{il}I_{il}^{*} - 2\,\mathrm{Re}(I_{il}{}'I_{il}^{*})T_{tl}T_{tl}^{*}}{(I_{il}I_{il}^{*})^2}.\end{aligned} \tag{5.6}$$

Finally, the method of moving asymptotes (MMA) [21] is used as an optimizer in order to update the design variables for each iteration.

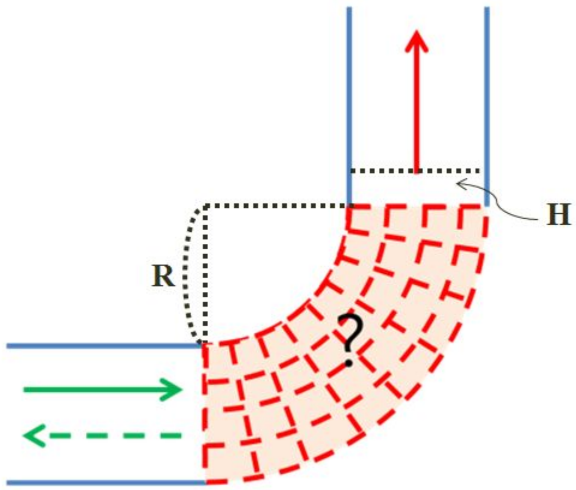


Fig. 5.1 Design domain for optimal curved beam structure connecting two perpendicular steel waveguides is discretized into elements.

Chapter6

Numerical Example

As numerical case studies, the topology optimization of curved beam structures was constructed for two types of materials: steel and aluminum. In all cases considered, power transmission of longitudinal wave for curved beam is calculated at specific element which is located in the center line of straight beam because straight beam's height is very small in comparison with its length($1/100$).

6.1 Optimization for curved beam structure

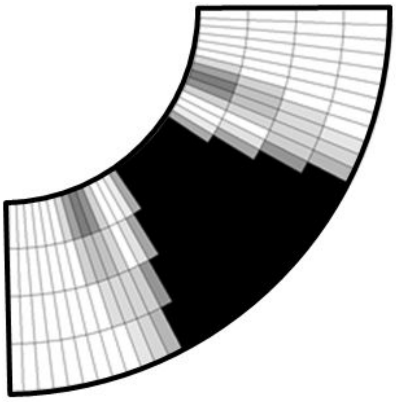
The design domain of curved beam has the radius of $R = 1\ m$, height of $H = 0.1\ m$ and thickness of $t = 1\ m$ (see Fig. 5.1). The design domain is discretized into 40×4 finite elements and initial condition is established intermediate material ($\chi_e = 0.6$).

6.2 Result for different target frequency

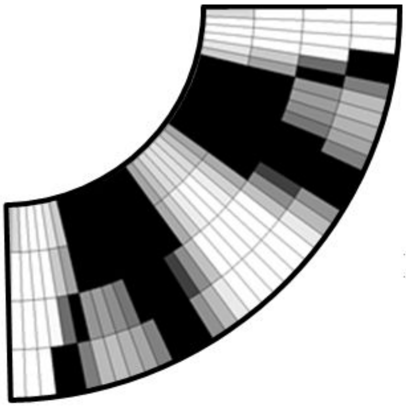
The result are different each other at different target frequency (Fig. 6.1). The target frequencies are $2\ \mathrm{kHz}$ and $3\ \mathrm{kHz}$. As you can see, the topology optimized shapes of the curved beam are dependent upon target frequencies. And, as shown in Fig. 6.2-3, the transmission coefficient of curved beam is greatly reduced after topology optimization in comparison with periodic structure result whether target frequency is different. At steel curved beam, transmission coefficient of longitudinal wave is $T_{power} = 1$. However, after topology optimization, the transmission coefficients of curved beam are $T_{power} = 0.0964$ at target frequency 2000 Hz and $T_{power} = 0.0081$ at target frequency 3000 Hz. Those are greatly reduced in comparison with uniform beam.

6.3 Comparison between periodic and optimized curved beam

Comparing the power transmission coefficient result of predicted periodic structure and optimized curved beam, later design reduces the power transmission coefficient more at each target frequency. Let's analyze cause of this result. In the dispersion curve of periodic structure which we constructed, target frequency is located in the stop band. In the stop bands, all incident waves are effectively attenuated. However, periodic structure unit number isn't enough to reduce power transmission coefficient in curved beam. It couldn't reduce transmission coefficient of curved beam effectively. On the other hands, topology optimization doesn't have limits like domain size. So, by using of topology optimization, we got structure reducing transmission coefficient more than periodic structure.



(a)



(b)

Fig. 6.1 Topology optimized curved beam for different target frequencies ((a) 2 kHz (b) 3 kHz). (Black is steel and white is aluminum).

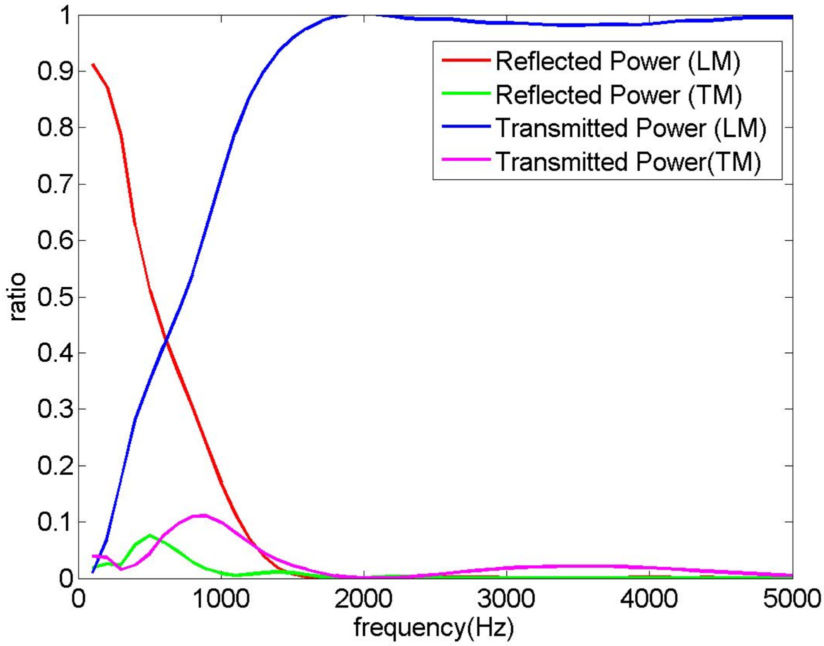
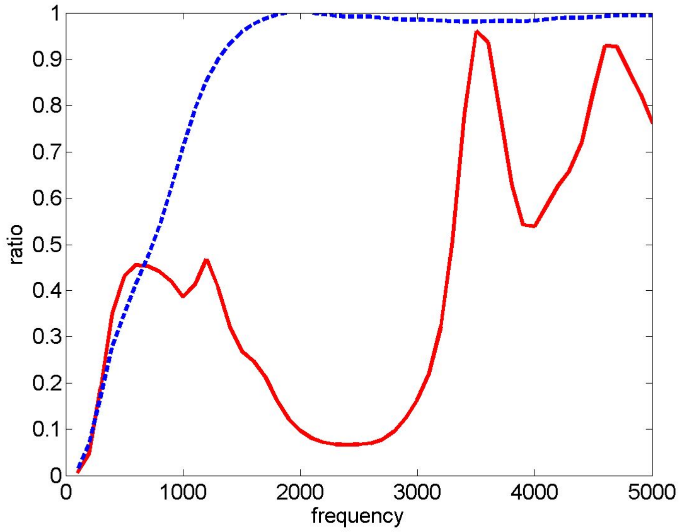
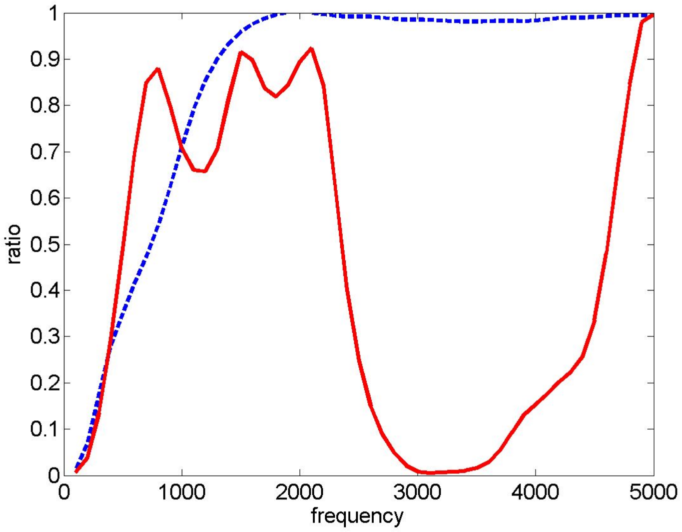


Fig. 6.2 Power transmission coefficient of optimized curved beam.

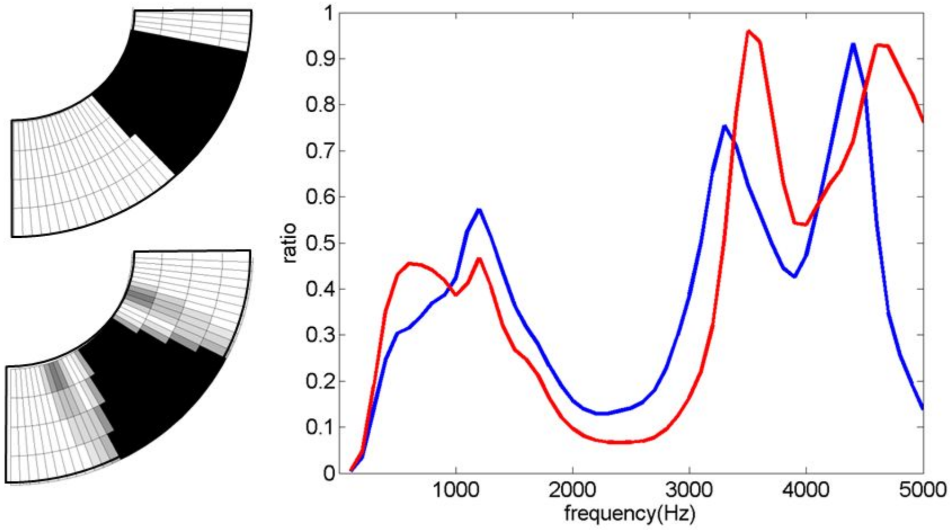


(a)

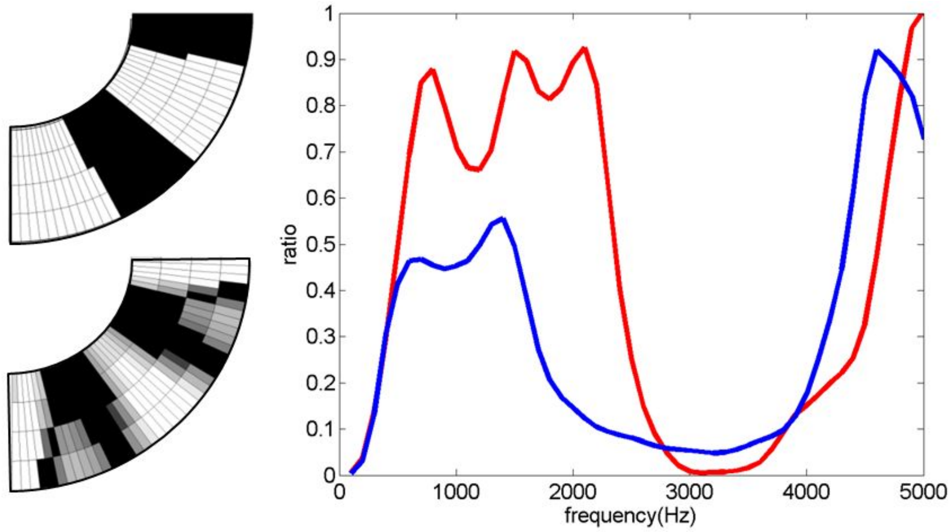


(b)

Fig. 6.3 Power transmission coefficient of optimized curved beam for (a) 2 kHz and (b) 3 kHz (blue dash line for steel curved beam and red line for optimized curved beam.



(a)



(b)

Fig. 6.4 Power transmission coefficient of periodic structure and optimized curved beam for (a)2 kHz and (b)3 kHz (blue line for periodic structure and red line for optimized curved beam(black is steel and white is aluminum).

Chapter7
Conclusion

In the present paper, we designed two dimensional connecting curved beams for reducing transmission coefficient by using topology optimization. When the amount of transmission coefficient is calculated, reflected wave from end of the system wasn't considered. Through perfectly matched layers, reflected wave components from each straight beam's end were neglected. The curved design domain was discretized into many elements and the material property of each element was interpolated between those of two different materials. The optimized results demonstrated the effectiveness of the design approach in this work. The topology optimized shapes of the curved beam are dependent upon target frequencies. Also, the performances the curved beam in reducing the power transmission are improved in comparison with those of the periodic structure. Those periodic structures used comparative model was constructed by using Bragg condition and Flügge theory.

Acknowledgement

This work was supported by the National Research Foundation of Korea (NRF) grant (No: 20120005693) funded by the Korean Ministry of Education, Science and Technology (MEST) contracted through IAMD at Seoul National University and the WCU program (No. R31-2008-000-10083-0) through NRF funded by MEST.

References

[1] J. S. Jensen, “Topology optimization problems for reflection and dissipation of elastic waves,” Journal of Sound and Vibration, vol. 301, pp. 319-340, 2007.

[2] B. Kang, C. H. Riedel and C. A. Tan, “Free vibration analysis of planar curved beams by wave propagation,” Journal of Sound and Vibration, vol. 260, pp. 19-44, 2003.

[3] J. L. Horner and R. G. White, “Prediction of vibrational power transmission through bends and joints in beam－ like structures,” Journal of Sound and Vibration, vol. 147(1), pp. 87-103, 1991.

[4] W. B. Bickford and B. T. Strom, “Vibration of plane curved beams,” Journal of Sound and Vibration, vol. 39(2), pp. 135-146, 1975.

[5] W. J. Zhou and M. N. Ichchou, “Wave propagation in mechanical waveguide with curved members using wave finite element solution,” Computer Methods in Applied Mechanics and Engineering, vol. 199, pp. 2099-2109, 2010.

[6] M. I. Hussein, G. M. Hulbert and R. A. Scott, “Dispersive elastodynamics of 1D banded materials and structures: analysis,” Journal of Sound and Vibration, vol. 289, pp. 779-806, 2006.

[7] M. I. Hussein, K. Hamza, G. M. Hulbert, et al. , “Multiobjective evolutionary optimization of periodic layered materials for desired wave dispersion characteristics,” Structural and Multidisciplinary Optimization, vol. 31, pp. 60-75, 2006.

[8] N. Olhoff, B. Niu and G. Cheng, “Optimum design of band-gap beam structures,” International Journal of Solids and Structures, vol. 49, pp. 3158-3169, 2012.

[9] A. Søe-Knudsen, “Design of stop-band filter by use of curved pipe segments and shape optimization,” Structural and Multidisciplinary Optimization, vol. 44, pp. 863-874, 2001.

[10] J. Dahl, J. S. Jensen and O. Sigmund, “Topology optimization for transient wave propagation problems in one dimension, ” Structural and Multidisciplinary Optimization, vol. 36, pp. 585-595, 2008.

[11] S. J. Walsh and R. G. White, “Vibrational power transmission in curved beams,” Journal of Sound and vibration, vol. 233(3), pp. 455-488, 2000.

[12] S. K. Lee, B. R. Mace and M. J. Brennan, “Wave propagation, reflection and transmission in curved beams,” Journal of Sound and Vibration, vol. 306, pp.636-656, 2007.

[13] F. Kobayashi, S. Biwa and N. Ohno, “Wave transmission characteristics in periodic media of finite length: multilayers and fiber arrays,” International Journal of solids and Structures, vol. 41 pp. 7361-7375, 2004.

[14] J. M. Bendickson and J. P. Diwling, “Analytic expressions for the electromagnetic mode density in finite, one-dimensional, photonic band-gap structures,” Physical review E, vol.53, pp. 4107-4121, 1996.

[15] W. Cao and W. Qi, “Plane wave propagation in finite 2-2 composites,” Journal of Applied Physics, vol. 78, pp.4627-4632, 1995.

[16] ASTM E1050-98(2006) Standard Test Method for Impedance and Absorption of Acoustical Materials Using a Tube, Two Microphones and Digital Frequency Analysis System.

[17] U. Basu and A. K. Chopra, “Perfectly matched layers for time-harmonic elastodynamics of unbounded domains: theory and finite-element implementation,” Computer Methods in Applied Mechanics and Engineering, vol. 192, pp. 1337-1375, 2003.

[18] W.C Chew, J.M. Jin and E. Michielssen, "Complex coordinate stretching as a generalized absorbing boundary condition," Microwave and Optical Technology Letters 15(6), pp.363-369, 1997.

[19] M.P. Bendsøe and O. Sigmund*, Topology optimization: Theory, Methods, and Applications* (Springer, Germany, 2003).

[20] K. K. Choi, and N. Kim, "Structural Sensitivity Analysis and Optimization 1: Linear Systems," New York: Springer, 2004, pp.122-123.

[21] K. Svanberg, "The Method of Moving Asymptotes: A New model for Structural Optimization," International Journal for Numerical methods in Engineering, vol.24, pp. 359-373, 1987.

초록

본 연구의 목적은 굽음보 구조에서 위상최적화를 이용하여 기계적 필터를 설계하는 것이다. 일반적으로 특정 주파수에서 파동을 선별하여 투과시키는 필터를 만들기 위하여 다른 물질로 구성된 무한개의 주기적인 구조체가 사용되어 왔다. 그러나 굽음보 구조에서는 이러한 주기적인 구조를 만드는데 한계가 있는데 그 이유는 두 개의 도파관을 연결하는 굽음보의 공간은 다양한 목적주파수를 다루기에 충분히 길지 않기 때문이다. 반복되는 횟수가 많지 않은 주기적인 구조의 경우 파동필터로서의 기능을 보장하기 힘들다. 이러한 문제점을 해결하기 위하여 본 연구에서 제안하는 해석 기법은 위상최적화의 물성치 보간 기법이다. 각 목적주파수에서의 파동의 투과를 최소화하기 위한 설계영역으로는 두 개의 도파관을 연결하는 굽음보 영역을 고려하였다. 예제로서 목적주파수에 따라 최적화 형상이 다름을 확인할 수 있었으며, 목적주파수에 적합한 예상되는 주기적인 구조를 설계하고 그 결과값과 비교해 봤을 때 위상최적화를 이용하여 설계한 결과값이 더 개선됨을 확인할 수 있었다.

주요어 : 위상최적화, 기계적 필터, 굽음보

Student Number : 2011-22884