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공학석사학위논문

A Segment-based Pricing Strategy  
for Reducing Congestion on Expressways

고속도로 혼잡 완화를 위한  
구간별 차등요금 부과전략

2015년 2월

서울대학교 대학원

건설환경공학부

이 은 호

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구간별 차등요금 부과전략

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이 논문을 공학석사 학위논문으로 제출함.

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## Abstract

# **A Segment-based Pricing Strategy for Reducing Congestion on Expressways**

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Congested zones that decrease the function of expressways have arisen due to population overcrowding in major cities. Specifically, congestion levels vary in different expressway sections despite being on the same expressway line. Demand management strategies are necessary to improve road operation efficiency by managing the congestion of each expressway section.

To this end, this study investigated a segment-based pricing strategy for expressways based on the second-best pricing method. We propose a bi-level problem, in which the upper level of the model is formulated to determine a toll level for each segment to minimize traffic congestion, and the lower level of the model is formulated as a variable demand assignment problem. We use a sensitivity

analysis-based algorithm to find the optimal solution for the upper level model.

Our case study application of the proposed model used the modified Sioux-Falls network. We composed scenarios of five different segmentation methods for evaluating the model: uniform pricing, line-based differentiated pricing, distance-based differentiated pricing, differentiated pricing for all expressway links, and differentiated pricing for all network links. The results show that the segment-based pricing strategy performs better than the initial pricing method with respect to the performance of the road network, due to the success in mitigating expressway congestion. Also, as the number of segments increases, the pricing results are closer to the results of marginal cost pricing. This study can be applied as a demand management method to improve the utility of excessively congested expressway segments.

**Keyword: Bi-level problem, Expressway toll, Segment-based pricing,  
Sensitivity analysis, Traffic congestion**

**Student Number: 2013-20935**

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# Chapter 1. Introduction

## 1.1 Background and Objective of Study

In contrast with general roads, expressways play a pivotal role in long distance inter-regional travel, and they are responsible for overall mobility rather than accessibility. However, the appearance of congested zones that diminish expressway mobility due to population overcrowding is a growing phenomenon in major cities. As urban areas create a high demand for travel, bottlenecks often occur at city limits. To illustrate this problem, Table 1 indicates that while most of the sections of the Gyeongbu expressway line in South Korea showed a C or D level of service in 2013, the Singal-Yangjae section shows an F level of service with the highest volume of traffic of all the expressway sections.

As we can see in the Gyeongbu line, the congestion level varies by section even though the sections are in the same line. Therefore appropriate demand management strategies are required to improve the operational efficiency of these roads. A number of techniques have been suggested to manage expressway demand, such as ramp metering, and road pricing. These techniques are effective measures for switching the social opportunity cost to productive benefits (Kwon, 2003). However, despite these suggestions, expressway congestion continues to be a pressing problem, and the introduction of demand management policies has yet to be considered.

<Table 1> Level of service of Gyeongbu line in sections, 2013

Section	Singal - Yangjae	Anseong - Singal	Cheonan - Anseong	Nami - Cheonan	Hoedeok - Nami	Biryong - Hoedeok
Level of service	F	E	E	D	D	D
Volume per lane (veh./day)	24,397	21,224	19,167	13,430	13,523	13,094
Section	Gimcheon - Biryong	Geumho - Gimcheon	Dongdaegu - Geumho	Eonyang - Dongdaegu	Yangsan - Eonyang	Busan - Yangsan
Level of service	B	C	E	C	C	C
Volume per lane (veh./day)	6,143	12,428	17,136	12,662	11,345	10,950

Source : Traffic Monitoring System (www.road.re.kr)

Road pricing is an effective and viable alternative for managing expressway demand, due to the advent of tolling systems using new technologies such as smart tolling—a multi-lane mainline tolling system. There have been numerous road pricing studies conducted from various perspectives, but most of them have been implemented on specific links or cordon lines, and few studies have been conducted on networks containing both toll and free roads.

Therefore, our study sought to develop a segment-based pricing strategy and to evaluate its effectiveness as part of a demand

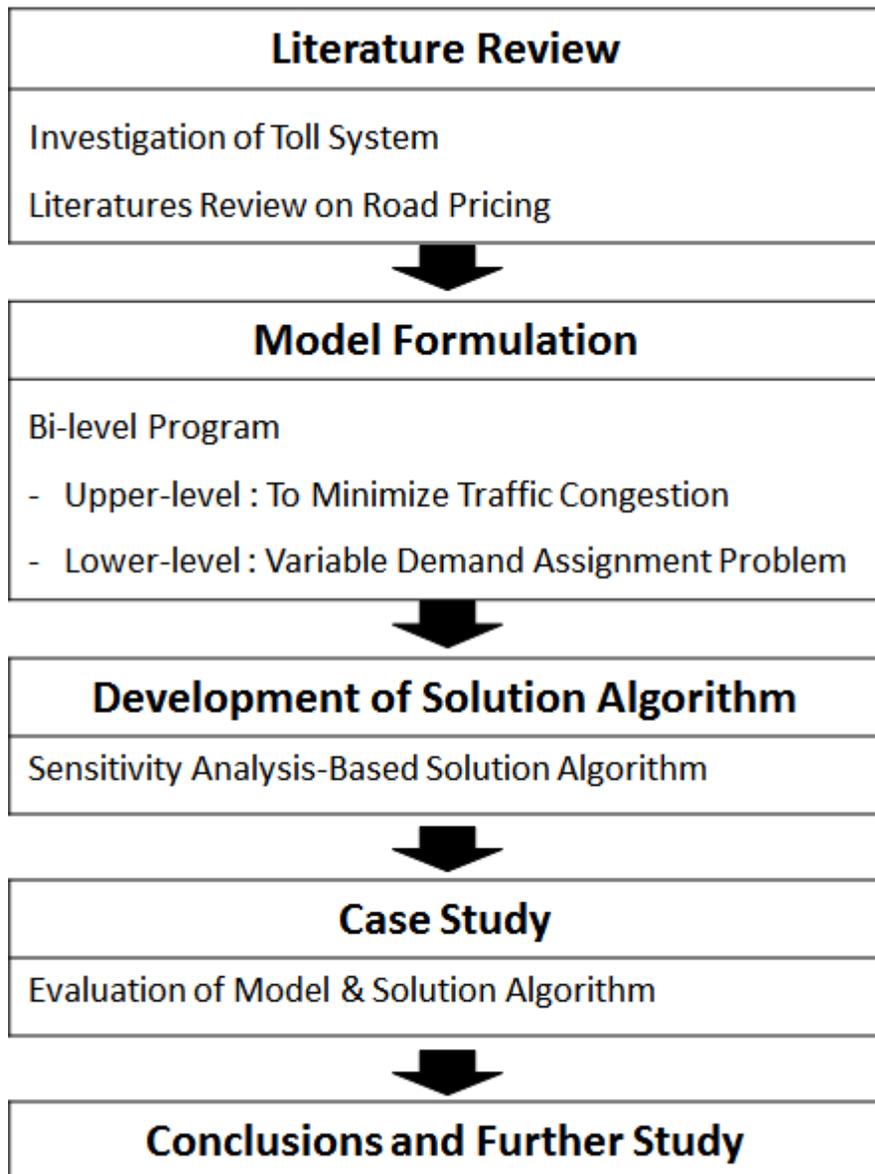
management scheme for expressways. The objective of segment-based pricing is to disperse the traffic of congested segments spatially, and to switch the demands in overcrowded regions to other modes and time periods. This pricing strategy will not only mitigate the congestion of the targeted expressway line, but also improve the efficiency of the entire network, to ultimately achieve an system optimum condition of road network.

## 1.2 Overview of Study

We implemented this study on a road network consisting of toll roads representing expressways and free roads representing national roads. Korean expressways use a two-part tariff which is composed of a fixed fee and a driving fee. The toll level is uniform across all lines. Lee et al. (2004) suggested that since a fixed fee is charged regardless of driving distance, it is reasonable to charge a congestion toll as part of the driving fee, to introduce the concept of social cost. This also accords with the concept of the benefit-pays principle. Therefore, we chose the driving fee as a design variable in this study.

In Chapter 2, we investigate toll systems briefly, and provide a thorough literature review on road pricing. Chapter 3 presents the bi-level problem, in which the upper level of the model is formulated to determine the toll level for each segment to minimize traffic congestion. The lower level of the model is formulated as a variable demand assignment problem. In addition, we develop a sensitivity analysis-based algorithm to solve the model. In Chapter 4, we evaluate the proposed model and the solution algorithm using a case study with several scenarios. Lastly, we present our study conclusions and suggest further areas of study in Chapter 5.

Figure 1 shows an outline of study in flow-chart format.



<Figure 1> Study flow chart

## **Chapter 2. Toll System and Literature Review**

### **2.1 Expressway Toll System in Korea**

Korean expressways may be classified as either public financial expressways operated by the Korea Expressway Corporation, and the privately financed expressways. Drivers on Korean expressways are currently tolled according to the 'Road Act' of 1963. According to 'Toll Road Act' Article 16, domestic expressways collect tolls for 30 years based on the rule for offsetting construction and maintenance costs. Also, an integrated self-supporting system has been adopted on Korean expressways, to adopt the user pay principle and to address the imbalances between deficit and surplus lines. Except for privately financed expressways, all expressway lines are charged the same toll. However, the expressway operator's debt has been increasing as toll revenues fall short of total construction and maintenance costs. Therefore, a new tolling system must be established to resolve the problem.

In 2004, a two-part tariff was adopted for expressway tolls. This tariff is composed of a fixed fee to offset construction investment costs, and a driving fee yearly maintenance costs. There are two types of expressway lines—the closed line and the open line. In the closed line, users take a toll road ticket at the expressway entrance toll plaza, and then pay the toll at the exit toll plaza. The closed type is applied to

sites that have heavy traffic and long gaps between toll plazas. Open-type expressway lines collect a toll at every toll plaza, and this type is adopted at sites where traffic levels are low and the gap between toll plazas is short. Table 2 shows the expressway toll calculation method used in 2014.

<Table 2> Calculation method of expressway toll in Korea, 2014

Types	Closed type	Open type
Fixed fee	900 won (450 won for two-lane)	720 won
Calculation of fee	Fixed fee + (driving distances × driving fee per km)	Fixed fee + (minimum distances of toll plazas × driving fee per km)

\* Unit price of driving fee per km :  
Type 1 : 41.4 won, Type 2 : 42.2 won, Type 3 : 43.9 won

Source : Korea Expressway Corporation (www.ex.co.kr)

## **2.2 Review of Road Pricing Studies**

### **2.2.1 Requirement for differentiated pricing on expressways**

The need for a differentiated pricing system on expressways has been recognized for the past 10 years. Lee et al. (2004) investigated ways to introduce a congestion toll on expressways. The authors suggested the consideration of various alternatives for differentiated congestion pricing, such as differentiated pricing by type of toll plaza, time period, expressway line, day of the week, and season.

Kim (2014) suggested the need for region-based differentiated pricing to improve road congestion by dispersing traffic and improving fiscal soundness by increasing the cost recovery rate. Multiple Pareto improvement solutions have been identified, which increase both revenue and benefits by region-based pricing. Region-based pricing results have confirmed the positive effects of the differentiated pricing scheme.

### **2.2.2 Studies of second-best pricing**

The study of road pricing began with a study of first-best pricing which derives a network toward a system optimum condition (Beckmann, 1965; Dafermos and Sparrow, 1971). In spite of its well-developed theoretical basis, the principle of first-best pricing is difficult to implement in current toll collection systems, and it is also unacceptable to road users. As such, second-best pricing schemes are considered to be a viable alternatives, where only a subset of links is

tolled.

Studies of second-best pricing have been conducted mainly with regard to determining toll levels on links or cordon lines. Yang and Huang (1998) investigated the marginal cost pricing principle with respect to congested road network with elastic demand and capacity constraints. In this paper, the authors made a theoretical investigation of how a classical economic principle might work in a generally congested road network.

Kim et al. (2004) considered congestion pricing as a network design problem. These authors provided a bi-level model to solve the congestion pricing problem, and calculated the congestion toll using the sensitivity analysis method.

Zhang and Yang (2004) studied the road pricing problem with respect to the selection of optimal toll levels and toll locations. They used a binary genetic algorithm and grid search method to determine optimal cordon locations and toll levels based on graph theory.

Zuo et al. (2010) developed a genetic algorithm-based bi-level programming solution to determine optimal toll levels and locations using second-best road pricing. As a case study, the model was applied to the Nagoya metropolitan area network.

Yu et al. (2009) investigated optimal toll locations and toll levels in areas, and cordon pricing, as opposed to first-best congestion pricing, to alleviate traffic congestion and improve social welfare. A comparison was implemented in a monocentric city.

## 2.3 Implications for Study

We reviewed the toll system in Korean expressways and previous studies on road pricing. Many researchers have been interested in the road pricing theory, and a significant number of studies on optimal toll pricing have been conducted. These have dealt mainly with finding optimal toll levels for predetermined links or toll locations, such as cordon line determination. In particular, various forms of second-best pricing schemes have been considered as a substitute for marginal cost pricing.

Even though there have been many studies on optimal toll pricing for individual links and cordon lines, differentiated pricing schemes are as yet very limited. Kim (2014) proposed an approach to determine a region-based pricing strategy, but the regions were simply set according to administrative district units. This study proposes a segment-based expressway pricing strategy that uses sensitivity analysis to maximize reductions in traffic congestion.

## Chapter 3. Model Formulation and Solution Algorithm

### 3.1 Segment-based Pricing Model

#### 3.1.1 Bi-level program

The road pricing problem targeted in this study is regarded as a network design problem (NDP). A network design problem is a mathematical program that determines optimal design variables to optimize a transportation system. In this study, the driving fee for each expressway line segment is set according to the design variables. Since the road toll is considered as a continuous design variable, methods for solving continuous network design problems may be applied to solve that proposed in this study.

Generally, a network design problem has the structure of a bi-level program, which optimizes a given objective function that reflects the behavior of network users. A bi-level program is a mathematical program composed of an upper level problem and a lower level problem. This program has been utilized in many transportation research areas such as road expansion, signal timing, and congestion toll pricing.

A bi-level program in a transportation system is a type of game theory. There are two kinds of frameworks for modelling the interaction of two game players. First, a Cournot-Nash game is a non-cooperative game in which each player tries to optimizing their own objective

without prior knowledge of the other player's function. In contrast, in the Stackelberg game, one player (the leader) knows how the other player (the follower) will respond to any decision he may make.

In the road pricing problem, it is reasonable to assume that the decision makers can anticipate the network users' behavior before establishing a policy. Therefore the model for this study can be represented as a Stackelberg game, where the upper level of the model is formulated to determine the toll level for each segment in order to minimize traffic congestion, whereas the lower level of the model is formulated as a variable-demand assignment problem.

### **3.1.2 Model Formulation**

The segment-based pricing model proposed in this study charges a different toll for each selected segment in an expressway line. With respect to the determination of a segment's toll level, trip assignment results are derived from the lower level problem, and then a new toll pattern can be calculated from the upper level problem using the trip assignment results. If the number of segments are identical to the number of links on a network, the network condition will reach a system optimum condition.

This model operates under some basic assumptions. The first assumption is that all road users will choose the minimum cost path, based on Wardrop's first principle. Also, all trips are assumed to be made only by passenger cars, and the value of time for all users is the

same. To consider the influence of a mode change and choosing alternative departure times, we applied the elastic demand function. We also assumed that an expressway toll is composed only of a driving fee. There is no fixed fee because a fixed fee does not affect the change in the total toll level, and the driving fee is the only parameter of interest in this study. Lastly, we assumed the demand pattern to be fixed in time.

We used the following notations:

Notation

$A$  : Set of links in the network,  $a \in A$

$R$  : Set of expressway lines,  $r \in R$

$S_r$  : Set of segments on line  $r$ ,  $s \in S_r$

$A_r^s$  : Set of links on line  $r$ , segments  $s$

$W$  : Set of O-D pairs,  $w \in W$

$K_w$  : Set of all routes between O-D pair  $w$ ,  $k \in K_w$

$f_k^w$  : Traffic flow on route  $k$ , O-D pair  $w$

$u^{\max}$  : Upper limit of driving fee

$u^{\min}$  : Lower limit of driving fee

$t_a(v_a)$  : Travel time function on link  $a$

$c_a(v_a, u)$  : Generalized travel cost on link  $a$

$$- c_a(v_a, u) = t_a(v_a) + (l_a u_r^s) / VOT, \quad a \in A_r^s$$

$$- c_a(v_a, u) = t_a(v_a), \quad a \in A - A_r^s$$

$l_a$  : Length of link  $a$

$VOT$  : Value of time

$d_w$  : Travel demand between O-D pair  $w$

$\mu_w$  : Travel cost between O-D pair  $w$

$D_w(\mu_w)$  : Demand between O-D pair  $w$  as a function of O-D travel cost  $\mu_w$

$D_w^{-1}(d_w)$  : Inverse of the demand function

$\delta_{ak}^w$  : 1 if route  $k$  between O-D pair  $w$  uses link  $a$ , and 0 otherwise

### Decision variables

$u_r^s$  : Driving fee on expressway line  $r$ , segment  $s$

$v_a$  : Flow on link  $a$

### Objective functions

$Z(u)$  : Objective function of upper level problem

$z(v)$  : Objective function of lower level problem

### (1) Upper level problem

The upper level problem determines the toll level for each segment in order to maximize the effect of reducing congestion, with given link flows obtained from the lower level problem. We note that it is desirable to minimize the total network travel cost for a fixed demand case. However, in the elastic demand case, the minimization of total travel cost as the objective function will lead the total travel demand to zero by imposing an extremely high toll level. Therefore, in the elastic demand model, the upper level problem can be formulated to consider both the demand and supply sides, as in the following mathematical program:

$$\min Z(u) = \sum_{a \in A} v_a t_a(v_a) - \sum_{w \in W} \int_0^{d_w} D_w^{-1}(x) dx \quad (3.1)$$

subject to

$$u^{min} \leq u_r^s \leq u^{max}, \quad \forall r \in R, \quad \forall s \in S_r \quad (3.2)$$

Equation (3.1) is the objective function of the upper level problem that road network performance can be maximized by minimizing the differences in the network total travel time and the total user benefit. We note that the inverse demand function  $D_w^{-1}(d_w)$  indicates the marginal benefit of users. The decision variables of the upper level problem are the toll level of each expressway line segment, and link

flows are given by the lower level equilibrium problem. Equation (3.2) calculates the upper and lower limits of the toll level.

## (2) Lower level problem

In the lower level problem, we implement the trip assignment with elastic demand according to the assumption that trip demand is variable with respect to travel cost. The lower level traffic network equilibrium problem can be found by solving the following convex network optimization problem (Sheffi, 1985).

$$\min z(v) = \sum_{a \in A} \int_0^{v_a} c_a(x, u) dx - \sum_{w \in W} \int_0^{d_w} D_w^{-1}(x) dx \quad (3.3)$$

subject to

$$\sum_{k \in K_w} f_k^w = d_w, \quad \forall w \in W \quad (3.4)$$

$$v_a = \sum_{w \in W} \sum_{k \in K_w} f_k^w \delta_{ak}^w, \quad \forall a \in A \quad (3.5)$$

$$f_k^w \geq 0, \quad \forall k \in K_w, \forall w \in W \quad (3.6)$$

$$d_w \geq 0, \quad \forall w \in W \quad (3.7)$$

## 3.2 Solution Algorithm

### 3.2.1 Sensitivity analysis method

A number of solution algorithms have been developed to achieve an optimal solution to the transportation network design problem. Among these, we chose to use the sensitivity analysis-based algorithm in this study. Since Tobin and Frieszs (1988) initiated efforts to solve the transportation network design model by using sensitivity information, it has been widely used in transportation network problems such as determining optimal ramp metering rates (Yang et al., 1994; Yang and Yagar, 1994), optimal signal timings (Wong and Yang, 1997, Yang and Yagar 1995), and congestion tolls (Yang and Lam, 1996; Yang and Bell, 1997). In a domestic study, Park (1999) conducted a sensitivity analysis for the transportation network design problem.

The advantage of sensitivity analysis is that the derivatives of demands, link flow, benefits, and other variables can be obtained according to the variations in the design parameters. This study adopts the sensitivity analysis method, designed for the network equilibrium problem, along with the elastic demand model proposed by Yang (1997). The procedure for the sensitivity analysis method presented here is an abridged version from Yang (1997).

Perturbed equilibrium can be expressed as shown below, with the perturbed variational inequality as the perturbation parameters  $\mathbf{u}$  existing in the link performance functions  $t(\mathbf{v}, \mathbf{u})$  and the O-D demand

functions  $D(\boldsymbol{\mu}, \mathbf{u})$ .

$$t(\mathbf{v}^*, \mathbf{u})^T \cdot (\mathbf{v} - \mathbf{v}^*) - D^{-1}(\mathbf{d}^*, \mathbf{u})^T \cdot (\mathbf{d} - \mathbf{d}^*) \geq 0 \quad (3.8)$$

It is assumed that the link performance functions and the O-D demand functions are differentiable once, with respect to  $\mathbf{u}$ .

The initial assignment results  $\mathbf{d}^*(0)$  and  $\mathbf{v}^*(0)$  are given, and  $t(\mathbf{v}, \mathbf{u})$  and  $D(\boldsymbol{\mu}, \mathbf{u})$  are assumed to be strongly monotonic in  $\mathbf{v}$  and  $\boldsymbol{\mu}$ , respectively. The problem in the network equilibrium is that the path flows are not unique. To overcome this problem, Tobin and Friezs (1998) proposed a restriction approach whereby a non-degenerate extreme point is selected in the feasible region of the equilibrium path flow. The non-degenerate extreme point can be obtained by the Frank-Wolfe algorithm. An equilibrium path flow pattern and a link/path incidence matrix are provided by the O-D minimum path set generated from each iteration of the Frank-Wolfe algorithm. Then, a non-degenerate extreme point can be selected from the following equilibrium path flow set.

$$\Omega^* = \{(\mathbf{f}, \mathbf{d}^* | \mathbf{v}^* = \Delta \mathbf{f}, \mathbf{d}^* = \Lambda \mathbf{f}, \mathbf{f} \geq 0\} \quad (3.9)$$

Let the selected  $\mathbf{f}^*$  be a non-degenerate extreme point in the region of the equilibrium path flow set  $\Omega^*$ . Then the necessary condition of

Equation (3.8) is that there exists a solution satisfying the following equations at  $\mathbf{u} = 0$ .

$$\hat{\mathbf{t}}(\mathbf{f}^*, 0) - \boldsymbol{\pi} - \Lambda^T \boldsymbol{\mu} = 0 \quad (3.10)$$

$$\boldsymbol{\pi}^T \mathbf{f}^* = 0 \quad (3.11)$$

$$\Lambda \mathbf{f}^* - D(\boldsymbol{\mu}, 0) = 0 \quad (3.12)$$

$$\boldsymbol{\pi} > 0, \mathbf{f}^* \geq 0 \quad (3.13)$$

where  $\hat{\mathbf{t}}$  is a vector of the path cost functions, and  $\boldsymbol{\pi}$  is a vector of the multipliers associated with the non-negativity condition of the path flow.

If link flows and O-D demands are assumed to be positive, the non-binding non-negative condition in Equation (3.13) can be eliminated because they will not be bound near  $\mathbf{u} = 0$  either. The Lagrange multipliers associated with the non-binding constraints become zero and at values near  $\mathbf{u} = 0$ . Considering the non-degenerate extreme point of the positive path flow and eliminating the nonbinding constraints, the above equations from (3.10) to (3.13) can be reduced to the following:

$$\hat{\mathbf{t}}^0(\mathbf{f}^*, 0) - \boldsymbol{\pi} - \Lambda^{0T} \boldsymbol{\mu} = 0 \quad (3.14)$$

$$\Lambda^0 \mathbf{f}^{0*} - D(\boldsymbol{\mu}, 0) = 0 \quad (3.15)$$

where  $^0$  indicates reduced vectors and matrices.

Equation (3.16) is obtained after differentiating both sides of Equations (3.14) and (3.15) with respect to the perturbation parameter  $\mathbf{u}$ .

$$\begin{bmatrix} \nabla_{\mathbf{u}} \mathbf{f}^0 \\ \nabla_{\mathbf{u}} \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0) & -\Lambda^{0T} \\ \Lambda^0 & -\nabla_{\mathbf{u}} \mathcal{D}(\boldsymbol{\mu}, 0) \end{bmatrix} \begin{bmatrix} -\nabla_{\mathbf{u}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0) \\ \nabla_{\mathbf{u}} \mathcal{D}(\boldsymbol{\mu}, 0) \end{bmatrix} \quad (3.16)$$

Let  $\mathbf{J}_{\mathbf{f}^0, \boldsymbol{\mu}}(0) = \begin{bmatrix} \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0) & -\Lambda^{0T} \\ \Lambda^0 & -\nabla_{\mathbf{u}} \mathcal{D}(\boldsymbol{\mu}, 0) \end{bmatrix}$ . This Jacobian matrix  $\mathbf{J}_{\mathbf{f}^0, \boldsymbol{\mu}}(0)$

is non-singular with respect to  $(\mathbf{f}^0, \boldsymbol{\mu})$ , coming from Equations (3.14) and

(3.15). Therefore the matrix elements  $[\mathbf{J}_{\mathbf{f}^0, \boldsymbol{\mu}}]^{-1} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$  can be

calculated as below.

$$\mathbf{B}_{22} = [-\nabla_{\mathbf{u}} \mathcal{D}(\boldsymbol{\mu}, 0) + \Lambda^0 \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0)^{-1} \Lambda^{0T}]^{-1} \quad (3.17)$$

$$\mathbf{B}_{12} = \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0)^{-1} \Lambda^{0T} \mathbf{B}_{22} \quad (3.18)$$

$$\mathbf{B}_{21} = -\mathbf{B}_{22} \Lambda^0 \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0)^{-1} \quad (3.19)$$

$$\mathbf{B}_{11} = \nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0)^{-1} [\mathbf{E} + \Lambda^{0T} \mathbf{B}_{21}] \quad (3.20)$$

where  $\mathbf{E}$  is an unitary matrix of appropriate dimension, and

$$\nabla_{\mathbf{f}} \hat{\mathbf{t}}^0(\mathbf{f}^*, 0) = \Delta^{0T} \nabla_{\mathbf{v}} \mathbf{t}(\mathbf{v}^*, 0) \Delta^0 \quad (3.21)$$

Therefore, the derivative of each parameter with respect to the perturbation parameter  $u$  is written as:

$$\nabla_u f^0 = -B_{11} \nabla_u \hat{t}^0(f^*, 0) + B_{12} \nabla_u D(\mu, 0) \quad (3.22)$$

$$\nabla_u v = -\Delta^0 B_{11} \Delta^{0T} \nabla_u t(v^*, 0) + \Delta^0 B_{12} \nabla_u D(\mu, 0) \quad (3.23)$$

$$\nabla_u \mu = -B_{21} \Delta^{0T} \nabla_u t(v^*, 0) + B_{22} \nabla_u D(\mu, 0) \quad (3.24)$$

$$\nabla_u d = \nabla_u D(\mu, 0) + \nabla_u D(\mu, 0) \nabla_u \mu \quad (3.25)$$

The above sensitivity analysis results are independent of the choice of an extreme point in the set of equilibrium path flows, and always yield the same results.

### 3.2.2 Sensitivity analysis-based solution algorithm

Based on the sensitivity analysis method described above, we developed the sensitivity analysis-based solution algorithm. The process of applying the algorithm is outlined below.

[Step 0] Solve the lower level equilibrium problem, charging an initial toll pattern  $(u^{(0)})$  for all expressway lines

[Step 1] With the initial trip assignment result, obtain the objective function derivative of the upper level problem with respect to each link, and assign the segments for differentiated pricing

[Step 2] Solve the lower level equilibrium problem for a given  $u^{(n)}$ , and obtain the O-D demands  $d(u^{(n)})$  and link flows  $v(u^{(n)})$

[Step 3] Calculate  $\nabla_u d$  and  $\nabla_u v$  using the sensitivity analysis method

[Step 4] Formulate first-order Taylor linear approximations for the upper level problem, and solve the linear problem such that it yields an auxiliary solution  $y$

[Step 5] Compute  $u^{(n+1)} = u^{(n)} + \alpha_n (y - u^{(n)})$

( $\alpha_n$  is step size)

[Step 6] Apply convergence criterion

If  $|u^{(n+1)} - u^{(n)}| \leq \epsilon$ , then stop. Otherwise, let  $n = n + 1$  and go to [Step 2]

To solve the bi-level program, solve the lower level problem (trip assignment) before (Step 2). Once the O-D demands and link flows are generated from the lower level problem, the upper level problem can be solved, to yield a new toll pattern (Step 3, Step 4). The algorithm adopted to calculate the lower level problem is a Frank-Wolfe convex combination algorithm.

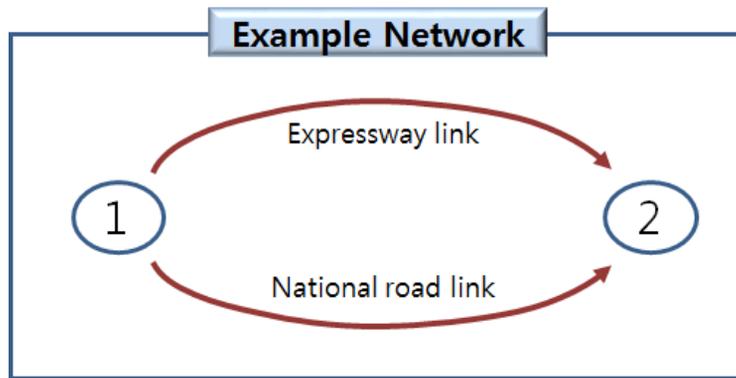
At [Step 5], the step size  $\alpha_n$  is predetermined to be monotonically decreasing. For convergence, the following two conditions should be satisfied (Powell and Sheffi, 1982).

$$\sum_{n=1}^{\infty} \alpha_n = \infty, \quad \sum_{n=1}^{\infty} \alpha_n^2 < \infty$$

In this study,  $\alpha_n = 1/(n+1)$  is adopted as a simple way of satisfying the above conditions.

### 3.2.2 Validation of algorithm

Before implementing the case study for different scenarios, we validated the sensitivity analysis-based algorithm to ensure its reliability in finding the solution. We validated the algorithm on the simple network shown in Figure 2, which is composed of an expressway link and a national road link. Table 3 presents the network characteristics.



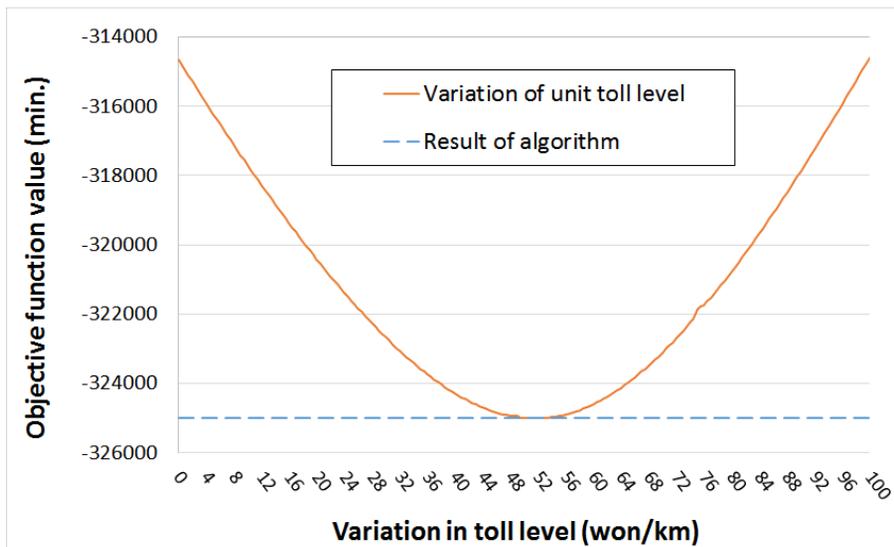
<Figure 2> Example network

<Table 3> Characteristics of example network

Potential demand ( $\bar{d}_w$ )		4,000 trip/hr
Sensitivity ( $\theta$ )		0.01
Expressway link	Capacity	2,200 veh./hr
	Free flow speed	100 km/hr
	Length	14 km
National road link	Capacity	1,800 veh./hr
	Free flow speed	70 km/hr
	Length	14 km

We validated the algorithm in two steps. In the first step, the expressway link toll charged was increased 0.5 won/km in the range from 0 won/km to 100 won/km, and trip assignment was implemented for each toll level to find the toll that minimized the value of the objective function. Then, we compared the optimal toll level to the result obtained by implementing this study's algorithm, whether or not the algorithm found the optimal value.

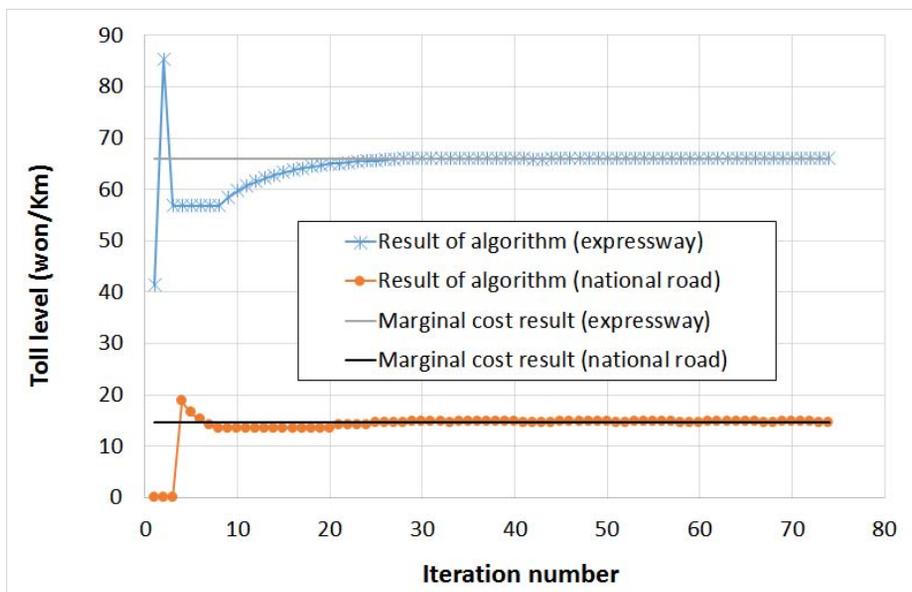
The objective function value is at a minimum at a toll level of 51.57 won/km. We note that the 10 times additional implementation of trip assignments were conducted for 0.01 won/km increase near the minimum objective value. The algorithm found the exact same toll level of 51.57 won/km after 40 iterations. Figure 3 shows the variation in the objective function values with respect to the variation in toll levels.



<Figure 3> Variation in objective function values

The second validation step is to check whether or not a system optimum can be achieved. In this step, both the expressway and national road links were to be charged. We implemented trip assignment with marginal cost pricing to find the system optimum condition, and then compared the optimal toll level with the results from the sensitivity analysis-based algorithm.

We found that the algorithm yielded approximately the same values as the marginal cost pricing results. The converged toll values were 66.05 won/km for the expressway, and 14.65 won/km for the national road. These values are the same within two decimal places of the marginal cost pricing results. Figure 4 shows the iterative variation in the toll levels.



<Figure 4> Variation in toll levels

## Chapter 4. Case Study

This chapter describes the application process of the proposed model and algorithm. We conducted a case study is for the numerical experiments using the example network with several scenarios. We constructed the scenarios to highlight the effects of various segmentation methods. We implemented the solution algorithm using the GAMS/CONOPT solver.

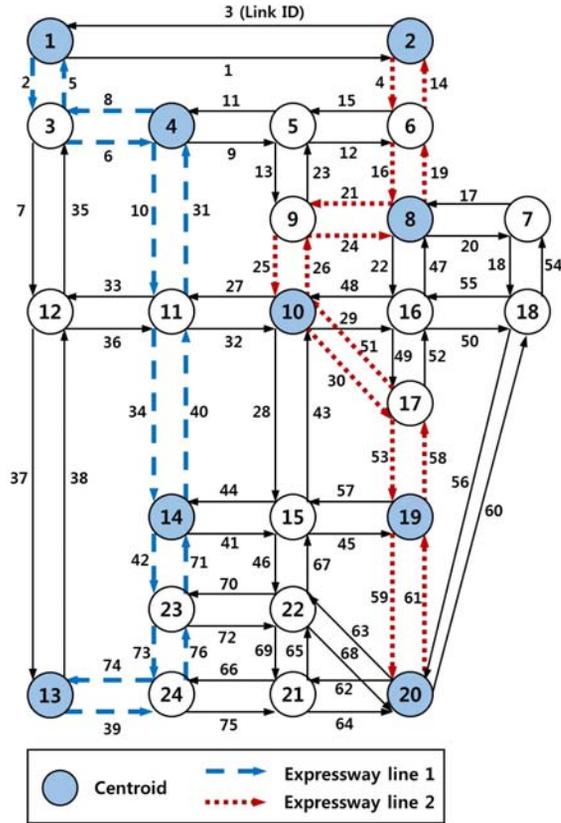
### 4.1 Experimental Design

#### 4.1.1 Network characteristics

We used the Sioux-Falls network to verify the model and algorithm used in this study. The Sioux-Falls network has been widely used in previous studies addressing continuous transportation design problems. We modified the network for the purposes of the study by selecting expressway links and national road links, and Table 6 shows the link characteristics set. The network, as shown in Figure 5, contains 24 nodes and 76 links. We set two expressway lines, and consider 9 centroid nodes as travel demand zones. These two expressway lines are set to pass near a node where demand is concentrated, representing sites such as the central business district.

<Table 4> Link characteristics of Sioux-Falls network

Link	Length (km)	Cap. (veh./hr)									
1	81	1,800	20	30	1,800	39	24	2,200	58	21	2,200
2	21	2,200	21	30	2,200	40	39	2,200	59	42	2,200
3	81	1,800	22	18	1,800	41	27	1,800	60	86	1,800
4	21	2,200	23	18	1,800	42	18	2,200	61	42	2,200
5	21	2,200	24	30	2,200	43	39	1,800	62	30	1,800
6	24	2,200	25	18	2,200	44	27	1,800	63	38	1,800
7	36	1,800	26	18	2,200	45	30	1,800	64	30	1,800
8	24	2,200	27	27	1,800	46	18	1,800	65	24	1,800
9	27	1,800	28	39	1,800	47	18	1,800	66	27	1,800
10	36	2,200	29	30	1,800	48	30	1,800	67	18	1,800
11	27	1,800	30	35	2,200	49	18	1,800	68	38	1,800
12	30	1,800	31	36	2,200	50	30	1,800	69	24	1,800
13	18	1,800	32	27	1,800	51	35	2,200	70	27	1,800
14	21	2,200	33	24	1,800	52	18	1,800	71	18	2,200
15	30	1,800	34	39	2,200	53	21	2,200	72	27	1,800
16	18	2,200	35	36	1,800	54	18	1,800	73	24	2,200
17	30	1,800	36	24	1,800	55	30	1,800	74	24	2,200
18	18	1,800	37	81	1,800	56	86	1,800	75	27	1,800
19	18	2,200	38	81	1,800	57	30	1,800	76	24	2,200



<Figure 5> Modified Sioux-Falls Networks

We assume that the O-D demands vary with respect to travel costs. We applied the negative exponential demand function as in Equation (4.1), where  $\bar{d}_w$  is the potential demand of O-D pair  $w$ , and  $\theta$  is the sensitivity of demand with respect to travel cost. The value of  $\theta$  is set to be 0.01 for all O-D pairs.

$$D_w(\mu_w) = \bar{d}_w \exp(-\theta \mu_w) \quad (4.1)$$

A total of eight O-D pairs' potential demands are given in Table 5. The destination for all trips is set to be node 10. This trip pattern is intended to represent the situation in which demands are concentrated in the central business district (e.g. urban city) in a specific time period (e.g. rush hour) when congested zones appear along the expressway.

<Table 5> Potential demand for O-D pairs

Origin	Destination	$\bar{d}_w$ (trip/hour)
1	10	2,000
2	10	2,000
4	10	1,500
8	10	2,000
13	10	3,000
14	10	2,000
19	10	1,500
20	10	2,000

The BPR (Bureau of Public Roads) function is used as the link travel time function.

$$t_a(v_a) = t_{0a} \left[ 1 + \alpha \left( \frac{v_a}{C_a} \right)^\beta \right] \quad (4.2)$$

where,

$t_{0a}$  : Free flow travel time of link  $a$

$C_a$  : Capacity of link  $a$

$\alpha, \beta$  : Parameters of BPR function

The free flow travel time of the link is defined as the link length divided by the free flow speeds of the expressways and national roads. The free flow speeds are set to 100 km/h for expressways and 70 km/h for national roads. The BPR function parameter values,  $\alpha$  and  $\beta$ , are taken to be 0.15 and 4, respectively.

We set the initial driving fee for the expressway to be based on the driving fee for a passenger car on September 2014, which is 41.4 won/km. The upper and lower limits of the driving fee are set to 200 won/km and 0 won/km, respectively. These limits are set such that they do not converge to unrealistic values and to search for solutions as quickly as possible. For practical use, a study of user willingness to pay is necessary in order to set realistic toll limits. We set the value of time, 249.8 won/min, to be identical for all users, with reference to the Korea Transport Database (KTDB).

#### **4.1.2 Analysis scenarios**

To evaluate the proposed pricing strategy, we considered various scenarios using different segmentation methods. In constructing the scenarios, we focused on ways to assign the expressway line segments. Since a segment is a subset of an expressway link, determining which links to put together is a crucial aspect of the segment-based pricing strategy. Each segmentation method has its own practical outcomes. Thus, the effect of segment-based pricing must be evaluated according to the methods used to assign the segments.

The scenarios were composed using five different segmentation methods. The first scenario used uniform pricing, which is the same as the current pricing method for Korean expressways. In this scenario, all the expressway links are considered to be one segment to find an optimal toll level. The objective of using this scenarios is to confirm that the use of the road network concept will improve outcomes, by a simple change in the unit toll level. Also, the uniform pricing method is the best match for the concept of the integrated self-supporting system, which is the current concept adopted in Korea's expressway toll system.

The second scenario employs differentiated pricing by lines. With this pricing method, each expressway line is set as one segment, and all lines are to be charged a different toll. This scenario can produce an optimal toll level for each expressway line, with consideration of its unique traffic conditions, such as the average level of service for each line. This pricing method corresponds well with the concept of line-specific self-supporting systems, and is being used in some toll road systems, such as the privately financed expressways in Korea, and national toll roads in Japan.

The third scenario uses differentiated pricing by distance from the centroid node. In this scenario, we consider two alternatives. In the first, only the links near the centroid node are assigned to a segment, and in the second, the links outer the centroid node are also assigned to a segment as well as the links near the centroid node. This pricing method represents the congestion toll pricing concept for expressway

segments near city limits, or the concept of discounts for long distance travel.

In the fourth scenario, every single link in the expressway is assigned to a segment, and all expressway links are charged separately. Traditionally, the collection of different toll amounts for each link has been considered to be too difficult in practice. However, newly developed smart-tolling systems that use multi-lane mainline tolling systems are now available, so the collection of tolls for every link is likely to be possible in the near future.

In the last scenario, all network links are charged individually. This scenario is intended for comparison with marginal cost pricing results. Also, to compare with the results of proposed scenarios, we determined the trip assignment results for the initial uniform pricing method and marginal cost pricing. The performance indices are the upper level objective function values that indicate the amount of reduced traffic congestion, the total travel demand and travel time, and the volume-capacity ratio ( $v/c$ ) of the expressway lines and the congested links.

## 4.2 Results

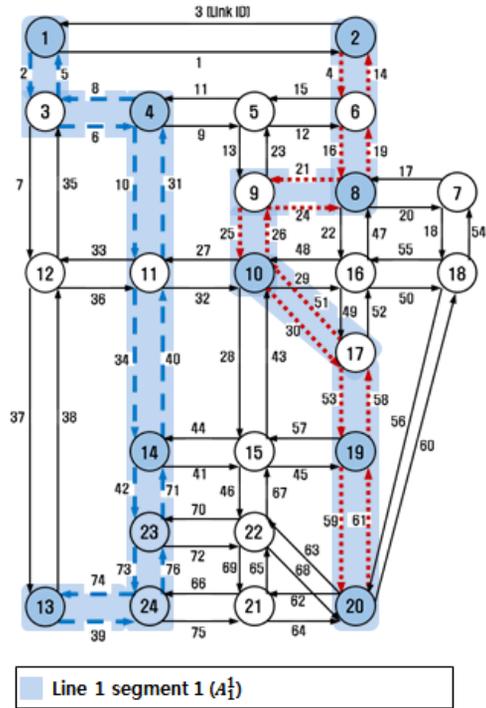
### 4.2.1 Results for each scenario

For the each of the above segmentation scenarios, we used the sensitivity analysis-based algorithm described in Chapter 3 to determine the optimal toll level for each segment. We applied these scenarios to an arbitrarily composed network and demand pattern, which is intended to represent the phenomenon of travel demand concentration in city's central business district.

#### (1) Uniform pricing

With the first scenario, we considered uniform pricing, whereby all expressway links are charged the same toll level. We set these links as  $A_1^1$  (Line 1 segment 1), as shown in Figure 6. Table 6 gives the optimal toll results for this first scenario. The results show that the final value for  $A_1^1$  toll (49.99 won/km) is higher than the initial toll level (41.40 won/km).

Table 7 gives the values from the upper-level objective function for scenario 1 with total travel time and total benefits, and compares these values with the initial toll pricing and marginal cost pricing results. We found that uniform pricing effectively improved the performance of the road network by modifying the unit toll level.



<Figure 6> Segmentation for scenario 1

<Table 6> Toll results for scenario 1

(unit : won/km)

Section	Line 1 segment 1
Initial toll	41.40
Final toll	49.99

<Table 7> Values from upper-level objective function for scenario 1

(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
<b>Scenario 1</b>	<b>450,232.80</b>	<b>1,326,165.77</b>	<b>-875,932.97</b>	<b>-1,561.2</b>
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3

## (2) Line-based differentiated pricing

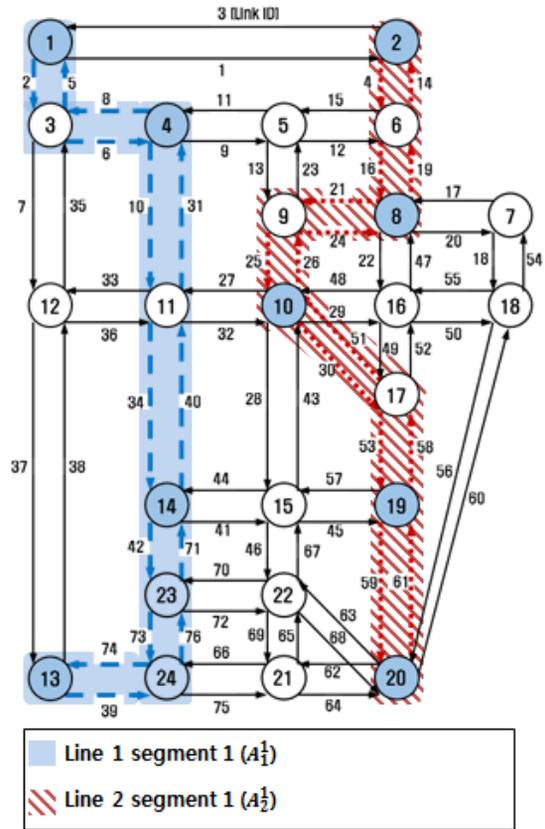
In second scenario, each expressway line is assigned to a segment. The modified Sioux-Falls network contains two expressway lines, so we set all the links on expressway line 1 to  $A_1^1$  (Line 1 segment 1), and all the links on expressway line 2 to  $A_2^1$  (Line 2 segment 1), as shown in Figure 7. The optimal toll results for this scenario, as listed in Table 8, show that both  $A_1^1$  and  $A_2^1$  charged higher tolls than the initial toll level.

A comparison of the upper-level objective function values for scenario 2 with the results of scenario 1, initial toll pricing, and marginal cost pricing show that the objective function value for scenario 2 is slightly higher than that of scenario 1, but there isn't much difference. With scenario 2, while there is an improvement in the total travel time, the total benefits decreased in comparison with scenario 1.

<Table 8> Toll results for scenario 2

(unit : won/km)

Section	Line 1 segment 1	Line 2 segment 1
Initial toll	41.40	41.40
Final toll	51.62	49.35



<Figure 7> Segmentation for scenario 2

<Table 9> Values from upper-level objective function for scenario 2  
(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
Scenario 1	450,232.80	1,326,165.77	-875,932.97	-1,561.2
<b>Scenario 2</b>	<b>449,755.67</b>	<b>1,325,728.83</b>	<b>-875,973.16</b>	<b>-1,601.4</b>
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3

### (3) Distance-based differentiated pricing

The third scenario used differentiated pricing by distance from the centroid node, and in this study's network, node 10 was the centroid node in which demands were concentrated. Scenario 3 considered two alternatives—scenario 3-1 and scenario 3-2. In scenario 3-1, only the links located near the centroid (node 10) were assigned to a segment. In scenario 3-2, other links located outer the node 10 were assigned to a segment in addition to the links located near the centroid.

In scenario 3-1, the links located between nodes 4 and 14 were assigned to  $A_1^1$  (Line 1 segment 1), and the links located between nodes 8 and 19 were assigned to  $A_2^1$  (Line 2 segment 1), as shown in Figure 8-(a). The optimal toll results for scenario 3-1 are given in Table 10. Both segments charged higher tolls than in the initial toll pricing scheme.

<Table 10> Toll results for scenario 3-1

(unit : won/km)		
Section	Line 1 segment 1	Line 2 segment 1
Initial toll	41.40	41.40
Final toll	58.90	48.66

In scenario 3-2, we divided the expressway lines into four different segments.  $A_1^1$  (Line 1 segment 1) and  $A_2^1$  (Line 2 segment 1) are the same as in scenario 3-1. The links located outside nodes 4 and 14 on

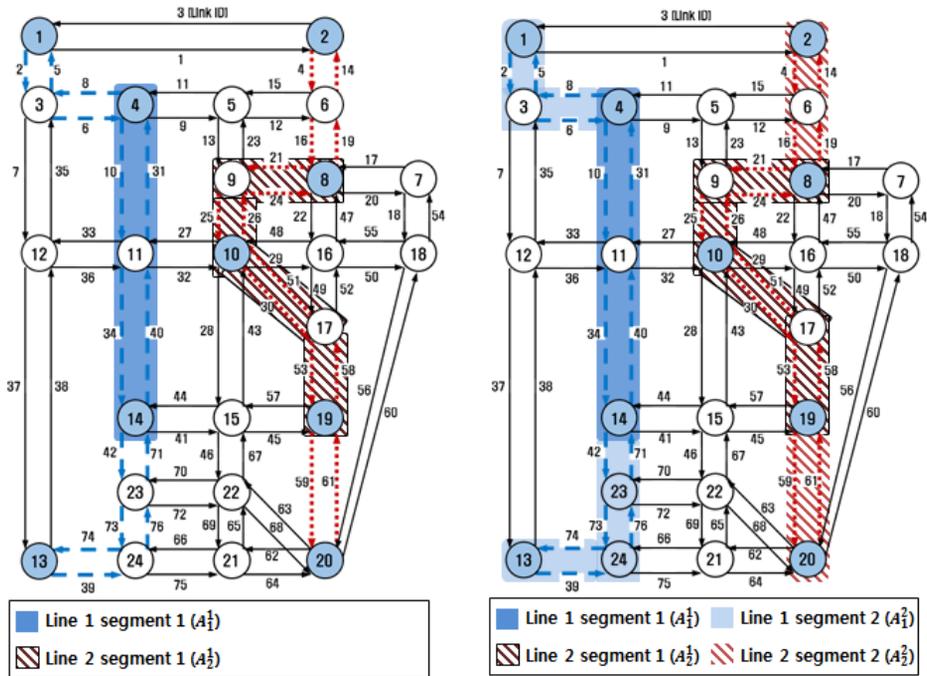
line 1 were set to  $A_1^2$  (Line 1 segment 2), and the links located outside nodes 8 and 19 on line 2 were set to  $A_2^2$  (Line 2 segment 2), as shown in Figure 8-(b). The optimal toll results for scenario 3-2, as given in Table 10, show that segments  $A_1^1$  and  $A_1^2$ , located near the centroid node, converged at a higher level than the initial toll level, whereas the converged toll level of segments far from the centroid node was lower than the initial toll level. This indicates that expressway sections near the central area where demands are concentrated need to be charged higher tolls than expressway sections further away from the central business district.

<Table 11> Toll results for scenario 3-2

(unit : won/km)

Section	Line 1 segment 1	Line 2 segment 1	Line 2 segment 1	Line 2 segment 2
Initial toll	41.40	41.40	41.40	41.40
Final toll	55.52	30.04	54.92	10.08

The upper-level objective function values for scenario 3, as shown in Table 12, indicate that the road network performance significantly improved in both cases, as compared to the initial toll pricing result. The objective function value for scenario 3-2, in particular, is significantly higher than the value in scenario 3-1.



(a) Scenario 3-1

(b) Scenario 3-2

<Figure 8> Segmentation for scenario 3-1, 3-2

<Table 12> Values from upper-level objective function for scenario 3  
(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
<b>Scenario 3-1</b>	<b>455,468.67</b>	<b>1,331,548.19</b>	<b>-876,079.53</b>	<b>-1707.8</b>
<b>Scenario 3-2</b>	<b>465,351.55</b>	<b>1,341,814.18</b>	<b>-876,462.63</b>	<b>-2,090.9</b>
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3

#### (4) Differentiated pricing for all expressway links

In scenario 4, every expressway link was assigned to a segment. Links located at the same point but with opposite directions were considered to be the same link, because there is no case in which both links in opposite directions were used simultaneously with this demand pattern. Accordingly, each line was designated as being composed of seven segments.

The final toll results for scenario 4, as shown in Table 13, reveal a tendency for the links nearer to the centroid node to converge at a higher toll level. Table 14 shows the upper-level objective function value for scenario 4, which is much closer to the marginal cost pricing result than that of previous scenarios.

<Table 13> Toll results for scenario 4

(unit : won/km)

Line 1 (link #)	Final toll	Line 2 (link #)	Final toll
Segment 1 (2, 5)	38.52	Segment 1 (4, 14)	13.25
Segment 2 (8, 6)	38.52	Segment 2 (16, 19)	13.25
Segment 3 (10, 31)	49.48	Segment 3 (21, 24)	52.98
Segment 4 (34, 40)	57.40	Segment 4 (25, 26)	48.45
Segment 5 (42, 71)	27.76	Segment 5 (30, 51)	43.95
Segment 6 (73, 76)	27.76	Segment 6 (53, 58)	43.95
Segment 7 (39, 74)	27.76	Segment 7 (59, 61)	16.97

<Table 14> Values from upper-level objective function for scenario 4  
(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
<b>Scenario 4</b>	<b>465,461.24</b>	<b>1,342,135.90</b>	<b>-876,674.66</b>	<b>-2,302.9</b>
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3

### (5) Differentiated pricing for all network links

Scenario 5 is not strictly an expressway pricing scheme because national road links are also charged. We implemented this scenario in order to compare its results with those of marginal cost pricing. Every network link was subject to being charged a toll.

Table 15 shows the final toll results for scenario 5 along with the marginal cost pricing results, which show only those links that experienced traffic volume. As we can see from these results, some of the final link tolls are significantly different from the marginal cost pricing tolls, while the others show a similar toll level.

The marginal cost pricing values are determined independently from the performance function and link flow of each link. In contrast, the model in this study calculates the toll level for each link to minimize objective functions with iterations, and the changes in links tolls have a influence on other link toll levels. Therefore, the calculations of the links toll levels by this study's model are not mutually independent. In particular, it is difficult to determine the optimal link toll when the surrounding links have high traffic flow, because the link toll

adjustment is more sensitive in such cases. This feature is demonstrated in scenario 5 results. The links that show the greatest difference from the marginal cost pricing results are located near the centroid node, where demand is concentrated.

The upper-level objective function values for scenario 5, as shown in Table 16, do not reach the marginal cost pricing levels, but show the highest values among all five scenarios.

<Table 15> Toll results for scenario 5

(unit : won/km)

Line 1	Final toll	Marginal cost	Line 2	Final toll	Marginal cost	National road	Final toll	Marginal cost
Link # 2	1.68	1.77	Link # 4	1.68	4.27	Link # 9	46.90	5.95
Link # 6	1.68	1.77	Link # 16	1.68	4.27	Link # 13	46.90	5.95
Link # 10	65.62	1.58	Link # 21	80.61	20.37	Link # 22	18.50	5.45
Link # 40	60.71	3.4	Link # 25	51.90	117.64	Link # 48	18.50	5.45
Link # 71	1.68	4.73	Link # 51	49.11	49.41	Link # 32	59.59	120.49
Link # 76	1.68	4.73	Link # 58	49.11	49.41	Link # 43	29.03	20.93
Link # 39	1.68	4.73	Link # 61	4.74	3.27	Link # 41	29.03	20.93

<Table 16> Values from upper-level objective function for scenario 5

(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
<b>Scenario 5</b>	<b>451,527.54</b>	<b>1,328,501.03</b>	<b>-876,973.49</b>	<b>-2,601.7</b>
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3

### **4.2.1 Summary of results**

We have excluded results from scenario 5, since it is not exclusively an expressway pricing scheme. We have summarized the results with respect to the upper-level objective function values that represent the effects of road network improvements, changes in travel demand and travel time, and the volume-capacity ratio ( $v/c$ ) of the congested links and of all the expressway lines. In an appendix, we also include the variations in toll levels and the objective function values with iterations for each scenario.

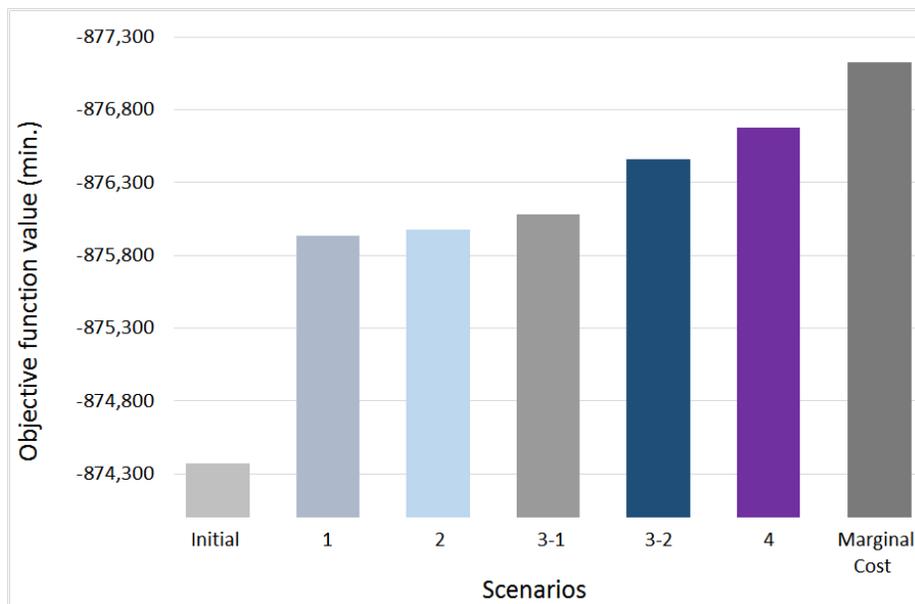
#### **(1) Comparison of upper-level objective function values**

The upper-level objective function value indicates the performance of the road network. This value is calculated by subtracting the net benefits from the total travel time. The corresponding values and results for all scenarios are presented in Table 17 and Figure 9. There was a 2,754.3 minute difference in the objective function value between the initial pricing and marginal cost pricing. A significant result was that in scenario 1, which used the uniform pricing method, simply by changing the unit toll level a substantial improvement of the road network performance was made. Also objective function values were understandably increased as the number of segments increased, while the increment amounts decreased with respect to increasing the number of segments.

<Table 17> Results for upper-level objective function values

(unit : min.)

Pricing method	Total travel time	Total benefits	Obj. func. value (F)	$\Delta F$
Initial toll	459,527.78	1,333,899.54	-874,371.76	-
Scenario 1	450,232.80	1,326,165.77	-875,932.97	-1,561.2
Scenario 2	449,755.67	1,325,728.83	-875,973.16	-1,601.4
Scenario 3-1	455,468.67	1,331,548.19	-876,079.53	-1707.8
Scenario 3-2	465,351.55	1,341,814.18	-876,462.63	-2,090.9
Scenario 4	465,461.24	1,342,135.90	-876,674.66	-2,302.9
Marginal pricing	459,891.80	1,337,017.85	-877,126.05	-2,754.3



<Figure 9> Results for upper-level objective function values

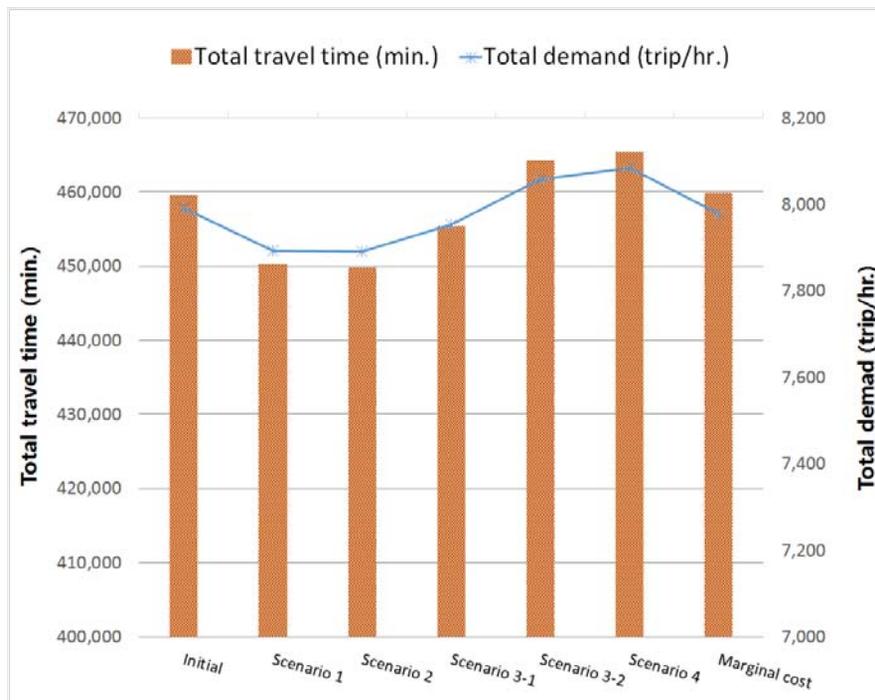
## **(2) Comparison of travel demand and travel time**

We also considered elastic the demand function in which demand varies with respect to the path cost. The total travel demand represents the benefit to network users, so high demand will bring high benefits. However, high demand also leads to increased total travel time. Therefore, there is a trade-off between total travel demand and total travel time. The solution algorithm determines the trade-off point that minimizes the objective function.

Table 18 shows the total demand and travel time on expressways and national roads. Total demand has ranged from 7,890 to 8,085 trip/hour, and total travel time has ranged from 449,756 to 465,461 minutes. We found that the proportion of travel time on expressways decreased in all scenarios, compared to that with initial toll pricing. The reason is that the overall toll level was increased rather than the initial toll level. This indicates that expressway congestion was mitigated by the segment-based pricing method. In Figure 10, the trade-off between total travel demand and total travel time is clearly visible.

<Table 18> Results for total demand and travel time

Pricing method	Total demand (trip/hr.)	Travel time (min.)				
		Expressway	%	National road	%	Total travel time
Initial toll	7,991	291,774	63.5	167,754	36.5	459,528
Scenario 1	7,892	273,500	60.7	176,733	39.3	450,233
Scenario 2	7,890	273,599	60.8	176,156	39.2	449,756
Scenario 3-1	7,953	275,429	60.5	180,040	39.5	455,469
Scenario 3-2	8,058	275,489	59.3	188,811	40.7	464,300
Scenario 4	8,085	280,518	60.3	184,943	39.7	465,461
Marginal pricing	7,979	280,599	61	179,295	39	459,892



<Figure 10> Results for total demand and total travel time

### (3) Comparison of volume-capacity ratio

In order to confirm the effect of a segment-based pricing strategy in reducing expressway congestion, we calculated the volume-capacity ratio ( $v/c$ ) before/after implementing the strategy. The volume-capacity ratios of two expressway lines (line 1, line 2) and the congested links (link 40, link 21, and link 25) for the four scenarios are shown in Table 19. While the volume-capacity ratio for all the lines did not greatly decrease, the ratio values of the congested links decreased more than 0.1 in some cases. These results indicate that the segment-based pricing strategy is effective in managing expressway congestion by segments.

<Table 19> Results for volume-capacity ratio

Pricing method	Line 1	Line 2	Link 40	Link 21	Link 25
Initial toll	0.43	0.72	0.50	0.81	1.19
Scenario 1	0.42	0.68	0.48	0.69	1.09
Scenario 2	0.41	0.68	0.47	0.69	1.10
Scenario 3-1	0.41	0.69	0.48	0.68	1.14
Scenario 3-2	0.42	0.68	0.45	0.64	1.07
Scenario 4	0.43	0.69	0.47	0.68	1.08
Marginal pricing	0.43	0.69	0.44	0.69	1.07

## Chapter 5. Conclusions and Further Study

### 5.1 Conclusions

In this study, we suggest a segment-based differentiated pricing strategy to alleviate traffic congestion in expressways and to increase the efficiency of road networks by dispersing traffic in congested segments. We formulated a bi-level program to determine the optimal toll level for each expressway line segment, and developed a sensitivity analysis-based algorithm to calculate the model solution. The sensitivity analysis method provides the derivatives of demands, link flow, benefits, and other variables with respect to the variation of design parameters, and we utilized these results to solve for the objective function and assign segments.

For a case study, we applied the model to the Sioux-Falls network. In order to compare the effectiveness of the proposed model when using various segmentation methods, we introduced five scenarios in the case study: uniform pricing, line-based differentiated pricing, distance-based differentiated pricing, differentiated pricing for all expressway links, and differentiated pricing for all network links. The numerical results show that the proposed bi-level model improved the efficiency of the road network, and reduced expressway congestion.

The tolling strategy we proposed in this study can be applied as a demand management method to improve the utility of excessively

congested expressway segments. The study results also proved that the segment-based pricing scheme, which has never been applied to actual road networks, improves expressway congestion. Thus, we anticipate that this study will be used to provide guidance for the development of future expressway tolling systems.

## 5.2 Further Study

In this study, we applied the elastic demand function to determine the effect of a volume switch to the other modes. To confirm the tangible effects of the mode switch, a modal split model should be introduced. In addition, differentiated pricing may introduce equity issues to the groups who have different time of values or live in different regions. Thus an index that represents equity could be added to the model, to ensure the feasibility of implementing the differentiated pricing scheme.

We also note that the demand pattern applied in this study was set to represent the concentration of demand in the central area. However, to confirm the effect of the segment-based pricing strategy in a more general case, more diverse network conditions and demand patterns, including those in real networks, must be considered.

This study proposed an alternative to the current pricing system, but prior to its application or any change in pricing strategy, a revision to the 'Toll Road Act' would be necessary.

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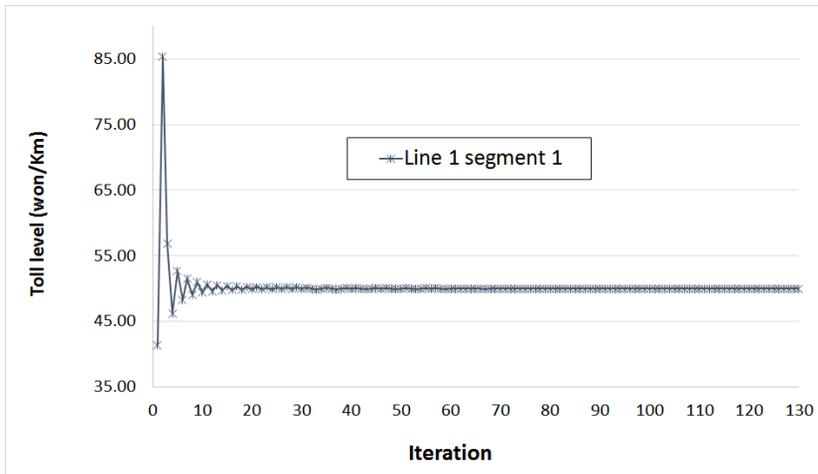
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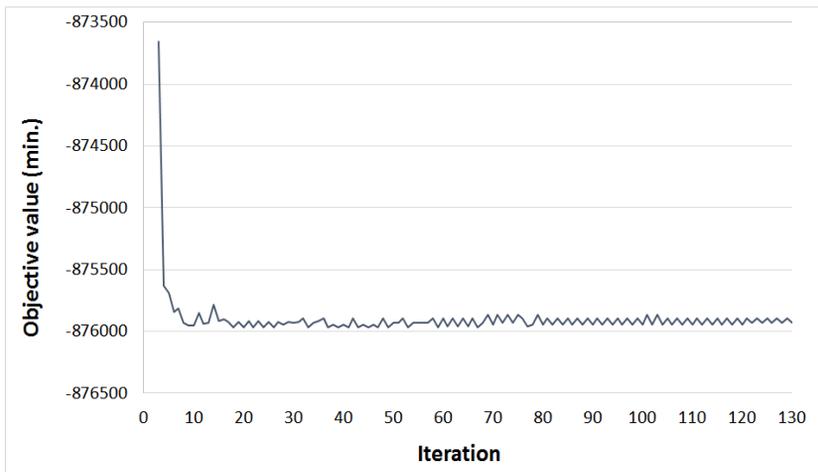
# Appendix

## 1. Variation in toll level and objective function value with iterations for each scenario

### (1) Uniform pricing (130 iterations)



(a) Variation in toll level

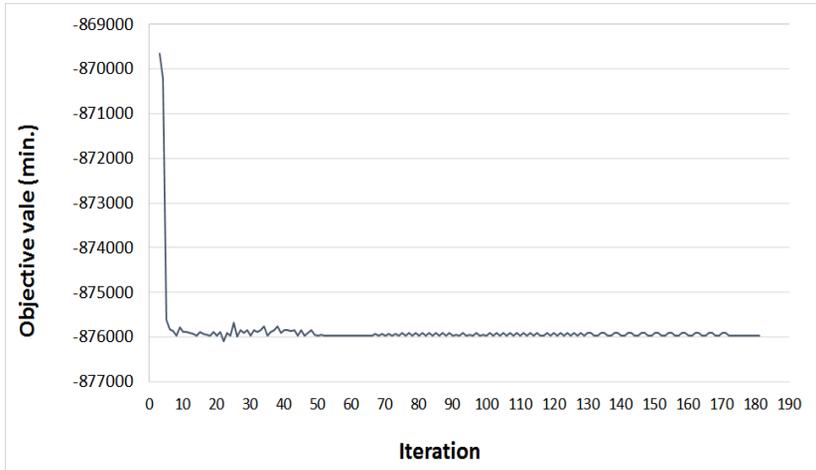


(b) Variation in objective function value

(2) Line based differentiated pricing (181 iterations)



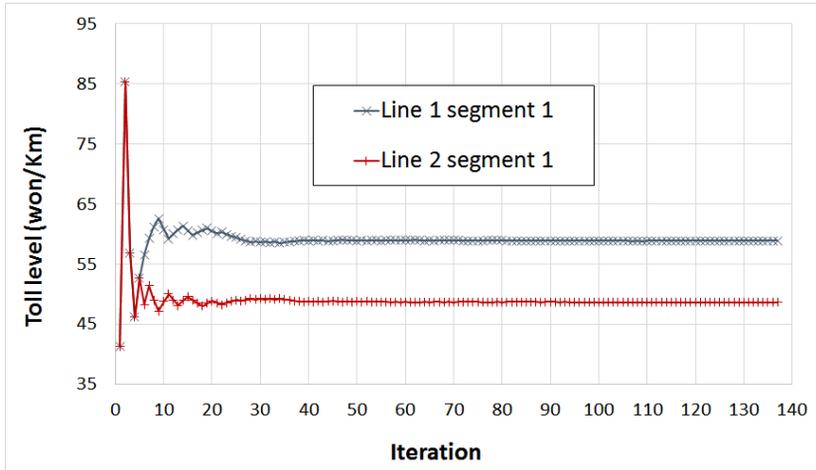
(a) Variation in toll level



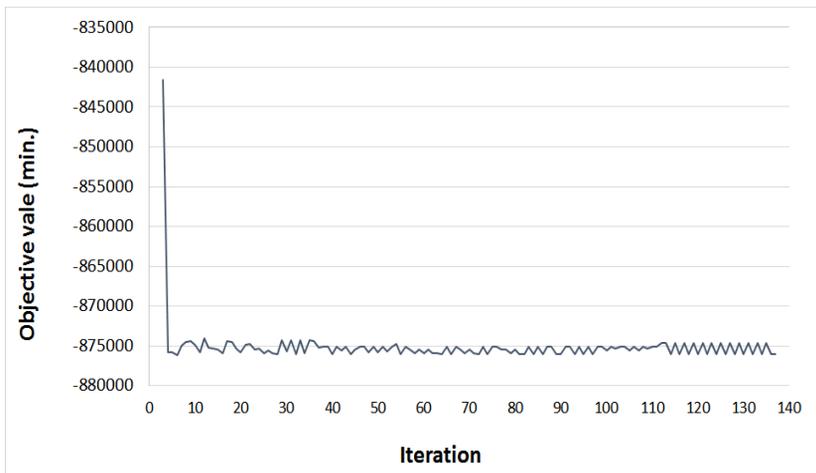
(b) Variation in objective function value

### (3) Distance based differentiated pricing

<Scenario 3-1> (137 iterations)

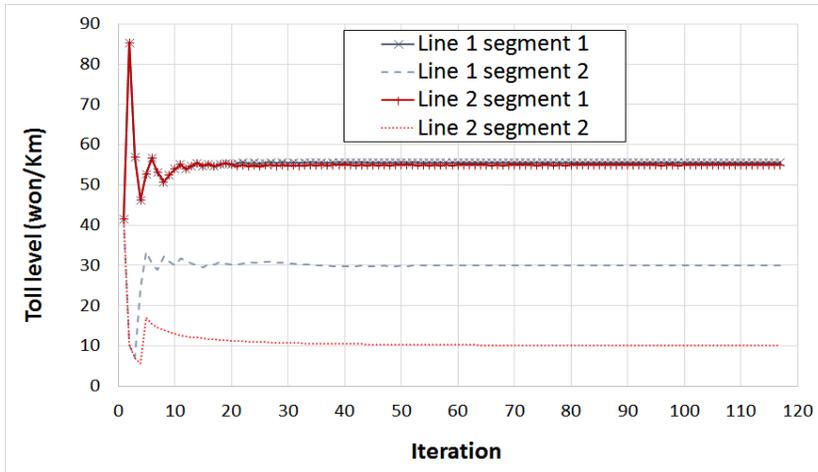


(a) Variation in toll level

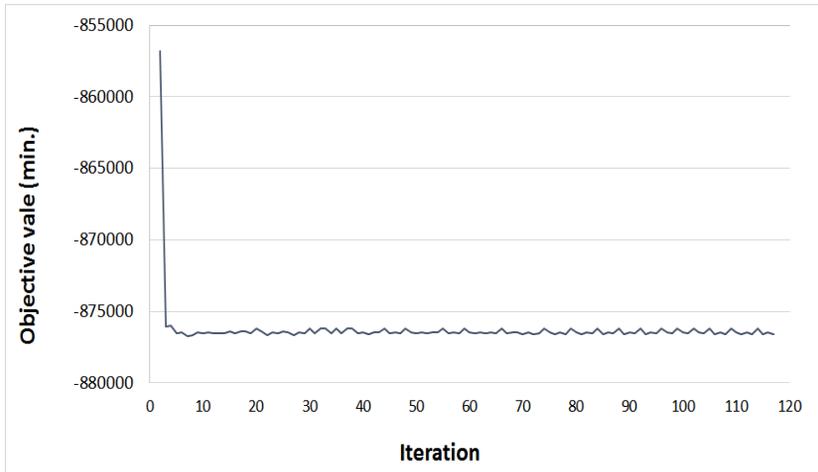


(b) Variation in objective function value

<Scenario 3-2> (117 iterations)

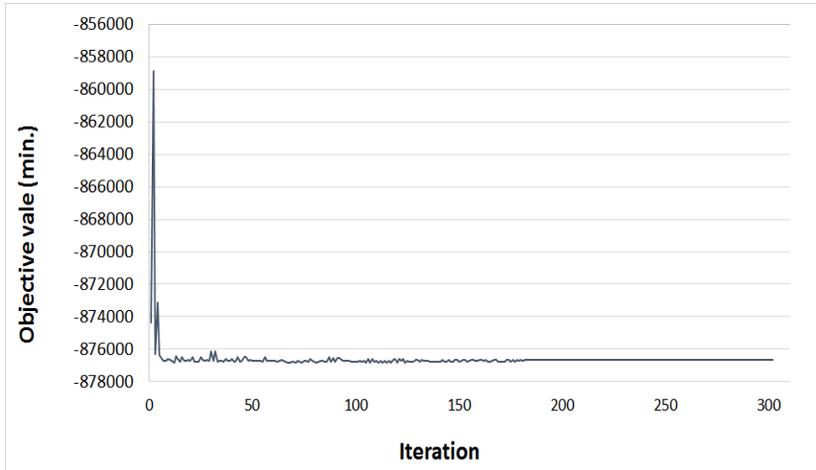


(a) Variation in toll level



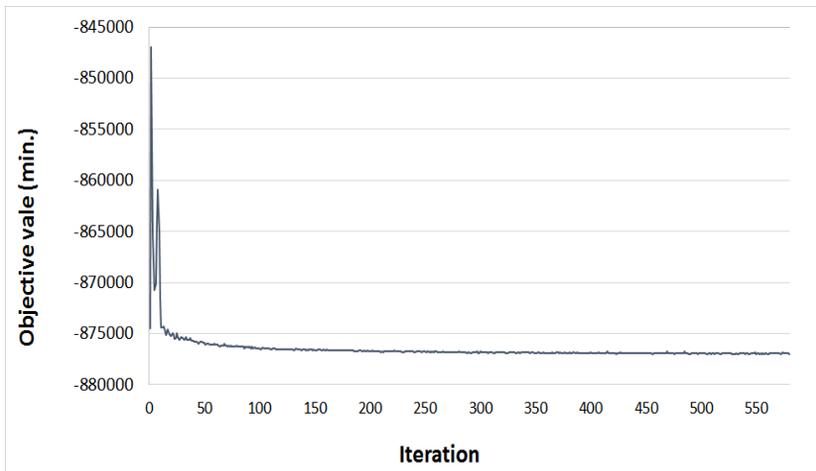
(b) Variation in objective function value

**(4) Differentiated pricing for all expressway links (302 iterations)**



(a) Variation in objective function value

**(5) Differentiated pricing for all network links (580 iterations)**



(a) Variation in objective function value

## 국문초록

대도시권의 인구 과밀화 현상이 지속되면서 고속도로의 기능을 저하시키는 혼잡한 구간이 발생하였으며 특히 같은 노선 내에서도 구간별로 상이한 서비스 수준을 나타내고 있다. 따라서 고속도로 구간별로 혼잡을 관리하여 도로 운영 효율을 높일 수 있는 고속도로 수요관리 방안이 필요한 시점이다.

이에 본 연구에서는 차선통행료 방식에 근거한 고속도로 구간별 차등요금 부과전략을 개발하였다. 상위단계의 교통혼잡 최소화를 위한 각 구간의 주행요금을 산정하는 문제와 하위단계의 가변수요 통행배정문제로 구성된 바이레벨 형태의 모형을 제시하였다. 상위단계 문제의 최적 해를 찾기 위해 민감도 분석 기반의 풀이 알고리즘을 이용하였다.

제안된 모형의 검증을 위해 수정된 Sioux-Falls 네트워크에 적용하였다. 분석 시나리오는 구간 설정 방식에 따라 균일요금, 노선별, 수요지 거리별, 고속도로 모든 링크별, 그리고 네트워크 모든 링크별 부과 등 총 5개로 구성하였다. 적용 결과 모든 방식이 초기 요금을 부과한 경우보다 도로 네트워크의 성능을 향상시켰으며, 이는 고속도로 구간의 혼잡 완화로 인한 것임을 확인할 수 있었다. 또한 구간의 개수가 많아질수록 한계비용 부과방식의 결과에 근접함을 알 수 있었다.본 연구는 고속도로 특정 구간의 과도한 혼잡으로 인해 발생하는 비효율을 절감하기 위한 수요관리 방안으로 적용될 수 있을 것으로 판단된다.

**주요어:** 바이레벨 문제, 고속도로 통행요금, 구간별 차등요금,  
민감도 분석, 교통혼잡

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