공학석사학위논문

Mitigation Effect of Vortex-Induced Vibration of Suspension Bridge Hangers with Stockbridge Dampers

스톡브리지 댐퍼의
현수교 행어 와류진동 제진 효과

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서울대학교 대학원

건설환경공학부

방 민 규

ABSTRACT

PWS is polyethylene-coated hanger cable that has the advantages of maintenance and durability but because of its smooth surface, it has the weakness for wind resistant performance. This paper tries to supplement wind resistant performance by using Stockbridge damper known for its mitigating effect of vibration for a cable.

For the design of the Stockbridge damper, a suspension bridge in Korea was selected and one of the hanger cables of the bridge was set up as the target hanger cable. Through wind data obtained from anemometer on the bridge, the target frequency range was determined as 5~20Hz.

In order to evaluate the vibration of a hanger cable, Stockbridge damper was modeled as 2-DOF system. For verification of 2-DOF model of Stockbridge damper, excitation experiment of Stockbridge damper was carried out with SB8, SB12 and SB16. Dynamic properties obtained from 2-DOF model were compared with dynamic properties extracted from the experiment and similar results were obtained.

Amplitude of vortex-induced vibration was evaluated by Energy Balance Method with dynamic properties obtained from 2-DOF model and results showed that Stockbridge damper mitigates effectively the amplitude of the hanger cable in the vicinity of its natural frequencies. Based on this, asymmetric Stockbridge damper for the target hanger cable was designed and when designed Stockbridge damper was installed to the target hanger cable, the amplitude of the cable satisfied the allowable amplitude on target frequency range.

Keywords: Stockbridge damper, Suspension bridge hanger cable, Vortex-induced vibration, Energy balance method, Excitation experiment.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Hanger cables of a suspension bridge convey the load of girder system to main cable of the suspension bridge. There are two types of hanger cables, Center fit rope core(CFRC) and Parallel wire strand(PWS).

Figure 1.1 - Center fit rope cable (CFRC)

Figure 1.2 - Parallel wire strand (PWS)

PWS is polyethylene-coated hanger cable, so it has the advantage of maintenance and durability but because of its smooth surface, it has the weakness of wind resistant performance.[1]

In order to improve wind resistant performance of PWS, a surface treatment of the hanger cable and friction dampers are generally used. But, if vibration of the hanger cable occurred after construction, it is hard and expensive to reinforce by surface

treatment and friction dampers. Therefore, in this paper, the Stockbridge damper which is known for its mitigation effect of vibration for a cable is uesed.

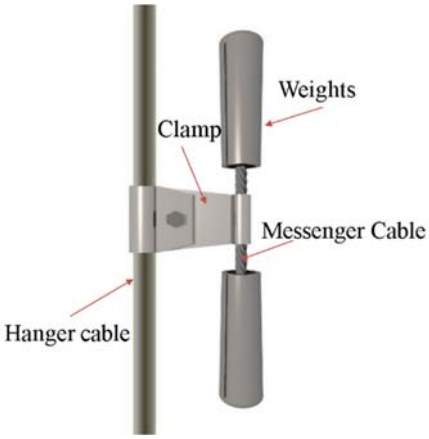

Figure 1.3 - Stockbridge damper on the hanger cable

Stockbridge damper was invented about 100 years ago and because of its mitigation effect, now it is widely used for transmission lines. Stockbridge damper consists of conductor, messenger cable and two weights. Conductor connects the hanger cable and Stockbridge damper, and messenger cable and weight shake to dissipate energy of vibration.[2]

Stockbridge dampers are already used in some suspension bridges and it is reported to be effective to mitigate hanger cable vibration. This paper offers proper methods to design Stockbridge damper for a hanger cable by showing the process for designing Stockbridge damper for an examined bridge.

CHAPTER 2

Target hanger cable

In this chapter, an examined suspension bridge is selected and assumes that vortex-induced vibration occurred on a hanger cable of the bridge. Through analysis of wind velocity and literature survey, target frequency range and allowable amplitude for mitigation are set.

2.1 Description of examined suspension bridge and a target hanger cable

For this paper, a suspension bridge in Korea is selected as an example. The overall length of the bridge is 1,800m, the length of main span is 1,150m and height of pylon is 203m. A hanger cable of the bridge is selected as a target hanger cable.

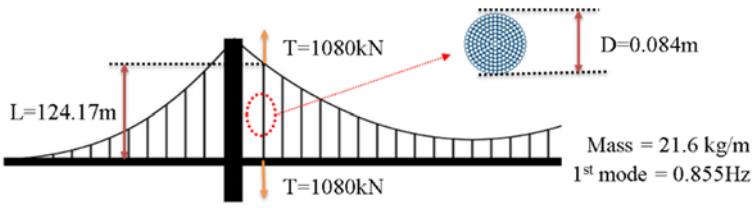

Figure 2.1 - The target hanger cable

The length of the target hanger cable is 124.17m, tension is 1,080kN, diameter is 0.084m, mass per length is 21.6kg/m and 1st natural frequency of the target cable is 0.855Hz and assumed that there is vortex-induced vibration whose amplitude is 56mm and frequency is 9.45Hz on the target hanger cable, when wind velocity is 6m/s.

2.2 Target frequency range and allowable amplitude

According to the following equation, the frequency of vortex-induced vibration is directly proportional to wind speed.

$$f_v = St\frac{V}{D} \tag{2.1}$$

where f_v is the frequency of vibration, V is velocity of the wind, D is the diameter of the cable and St is Strouhal number.

In order to set the target frequency range, wind data collected by an anemometer on a pylon of the examined bridge is used.

Figure 2.2 - Anemometer on the pylon

Because the wind data is measured on the top of the pylon, the data should be converted to the wind velocity on mid-height of the target hanger cable, as follows.[3]

$$V_{hanger} = \left(\frac{z_{hangrt}}{z_{pylon}} \right)^{\alpha_1} V_{pylon} \tag{2.2}$$

where V_{hanger} is the wind velocity on mid-height of the target hanger cable, V_{pylon} is the measured wind velocity on the top of the pylon, z_{hanger} (=129.1m) is altitude of hanger cable, z_{pylon} (=203m) is altitude of the pylon and α_1 (=0.12) is roughness coefficient. The results of analyzing wind data of September 2013 to July 2014 are as follows.

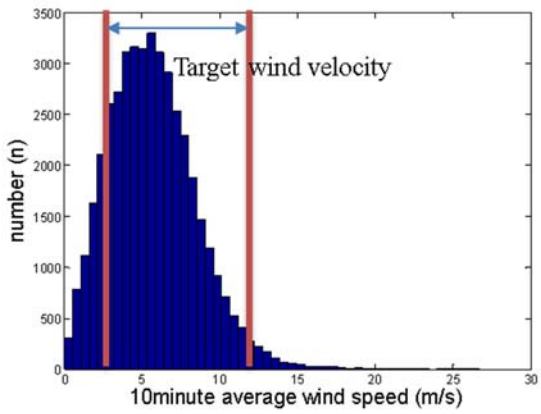

Figure 2.3 - 10 months wind data ('13.09~'14.07)

The upper limit of the target wind speed is determined to be 12m/s, because it covers 98% of wind data. Because of the lack of energy, if the wind velocity is lower than

3m/s, it is assumed that there isn't vortex-induced vibration on the target cable. So the lower limit of target wind speed is determined as 3m/s. Finally, through equation (2.1), the target frequency range is determined to 5~20Hz.

There is no specific design criteria for vibration of the hanger cable, we assume that the allowable amplitude of the hanger cable is D/10(8.4mm).

CHAPTER 3

Evaluation of mitigation effect of Stockbridge damper

This chapter explains how to evaluate amplitude of the hanger cable when Stockbridge damper is installed to the hanger cable.

3.1 Energy Balance Method

For evaluating vibration of the hanger cable, Energy Balance Method(EBM)[4] is generally used. Basic concept of this method is that sum of the energy dissipated by the hanger cable and Stockbridge damper is same to the energy brought by wind. It can be represented by,

$$P_{wind} = P_{cable} + P_{damper} \tag{3.1}$$

By solving this equation, it is possible to evaluate amplitude of the hanger cable.

3.1.1 Energy brought by wind

Through wind tunnel tests, there are various proposed formulas about energy brought by wind. In this paper, a formula among them is selected as,[2][5]

$$P_{wind} = Lf^3 D^4 \left(807.4\left(\frac{A}{D}\right)^{1.953} - 767.6\left(\frac{A}{D}\right)^2 - 3.2\left(\frac{A}{D}\right)^3 - 78.2\left(\frac{A}{D}\right)^4 \right) \quad (3.2)$$

where L is the length of a hanger cable, f is the frequency of vibration, D is the diameter of a hanger cable and A is amplitude of vibration.

3.1.2 Energy dissipated by the cable

Through vibration tests of cables, there are various proposed formulas about energy dissipated by cable. In this paper, a formula among them is selected as,[6]

$$P_{cable} = LK \frac{(\mathrm{A/\ D})^{2.43} f^{5.5}}{T^2} \quad (3.3)$$

Where L is the length of a hanger cable, K is proportionality factor, D is the diameter of a hanger cable A is amplitude of vibration, f is the frequency of vibration and T is the tension of a hanger cable.

3.1.3 Energy dissipated by Stockbridge damper

It is possible to calculate energy dissipated by Stockbridge damper with following equation,

$$P_{damp} = \frac{1}{2}\left[Z\dot{y}_0^2 \cos\alpha \right] \quad (3.4)$$

where Z is impedance which means the ratio between force and velocity of Stockbridge damper, $\dot{y}_0$ is maximum velocity of Stockbridge damper and α is phase lag between force and velocity of Stockbridge damper. Especially, energy dissipated by Stockbridge damper on a hanger cable is calculated following equation, [7]

$$P_D = \frac{1}{4} T C_w k^2 \frac{1-\left(h^2+g^2\right)}{1+h^2+g^2} D^2 \left(\frac{A}{D}\right)^2 \tag{3.5}$$

where

$$h = -\frac{\sin^2 kl_1 \left(\sin 2kl_1 + 2\gamma \sin\alpha\right)}{\sin^2 kl_1 + \gamma^2 + 2\gamma \sin kl_1 \sin\left(kl_1+\alpha\right)},$$

$g = \dfrac{\sin^2 kl_1 \cos 2kl_1 + \gamma^2 + \gamma \sin 2kl_1 \sin\alpha}{\sin^2 kl_1 + \gamma^2 + 2\gamma \sin kl_1 \sin\left(kl_1+\alpha\right)}$, $k = 2\pi f \sqrt{m_L / \mathrm{T}}$ is wave number, $c_w = \sqrt{T / m_L}$ is wave velocity, $\gamma = T / Zc_w$ and l_1 is the length between Stockbridge damper and hinge of a hanger cable. Dynamic properties of Stockbridge damper, impedance(Z) and phase lag(α), are determined by 2-DOF model of Stockbridge damper.

3.2 Dynamic properties of Stockbridge damper

3.2.1 2-DOF model of Stockbridge damper

In order to evaluate the vibration of a hanger cable by Energy Balance Method, it is necessary to find out dynamic properties, Z (impedance) and α (phase lag), of Stockbridge damper. For determining these properties, Stockbridge damper is modeled as 2-DOF system[8], as shown in Figure 3.1:

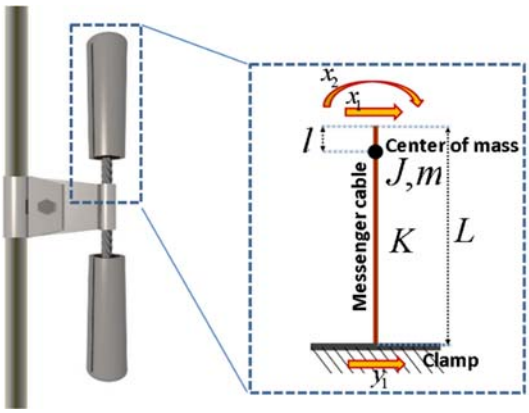

Figure 3.1 - 2-DOF model of Stockbridge damper

where y_1 is vibration of the hanger cable, x_1 and x_2 are movements of messenger cable end, L is length of messenger cable and l is distance between the end of messenger cable and center of mass. Assuming the messenger cable is a massless cantilever beam, K (stiffness matrix) is obtained. Weight is assumed that a concentration of the mass located in the center of mass. Based on these, the

motional equation of Stockbridge damper is established as,

$$M\ddot{x} + C\dot{x} + Kx = C\dot{y} + Ky \tag{3.6}$$

where

$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} m & ml \\ ml & J \end{bmatrix}$, m is mass of weight, J is moment of inertia of weight, $K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 4k & 2kL \\ 2kL & 4kL^2 / 3 \end{bmatrix}$, $k = 3EI / L^3$ and $C = a_0 M + a_1 K$ is assumed Rayleigh damping 10% with reference to experimental results.

Through this equation of motion, the natural frequencies of the first and second modes of the Stockbridge damper can be obtained as,

$$\omega_{1,2} = \sqrt{u \pm v} \tag{3.7}$$

where

$$u = \frac{\left(k_{11} + k_{22}m_{11} - k_{12}m_{12}\right)}{2\left(m_{11}m_{22} - m_{12}^{\ 2}\right)},$$

$$v = \frac{\sqrt{\left(2k_{12}m_{12} - k_{11} - k_{22}m_{11}\right)^2 - 4\left(m_{11}m_{22} - m_{12}^{\ 2}\right)\left(k_{11}k_{22} - k_{12}^{\ 2}\right)}}{2\left(m_{11}m_{22} - m_{12}^{\ 2}\right)}.$$

Assuming vibration of the hanger cable to the sinusoidal oscillation, solutions of motional equation are

$$x_1 e^{i\alpha^*} = y_1 \frac{\left(C - \frac{FB}{E}\right)}{A - \frac{BD}{E}} \quad \text{and} \quad x_2 e^{i\beta^*} = \frac{-Dx_1 e^{i\alpha^*} + Fy_1}{E} \tag{3.8}$$

where

$$A = \left(-\left(1 + \frac{a_0}{i\omega}\right)\omega^2 m_{11} + \left(1 + i\omega a_1\right) k_{11}\right), \quad B = \left(-\left(1 + \frac{a_0}{i\omega}\right)\omega^2 m_{12} + \left(1 + i\omega a_1\right) k_{12}\right),$$

$$C = \left(-\frac{a_0}{i\omega}\omega^2 m_{11} + \left(1 + i\omega a_1 k_{11}\right)\right), \quad D = \left(-\left(1 + \frac{a_0}{i\omega}\right)\omega^2 m_{21} + \left(1 + i\omega a_1\right) k_{21}\right),$$

$$E = \left(-\left(1 + \frac{a_0}{i\omega}\right)\omega^2 m_{22} + \left(1 + i\omega a_1\right) k_{22}\right), \quad F = \left(-\frac{a_0}{i\omega}\omega^2 m_{21} + \left(1 + i\omega a_1 k_{21}\right)\right),$$

ω is frequency of vibration and α^*, β^* are the respective phase angles. Finally, the dynamic properties of Stockbridge damper can be obtained as,

$$F\left(t\right) = -2m\left(\ddot{x}_1 + l\ddot{x}_2\right) = Z\dot{y}_0 \sin\left(\omega t + \alpha\right). \tag{3.9}$$

3.2.2 Excitation experiment of Stockbridge damper

For verification of 2-DOF model of Stockbridge damper, excitation experiment of Stockbridge damper was carried out.[9] Three symmetric types of pre-made Stockbridge dampers were tested and in particular, SB16 is the largest commercial Stockbridge damper.

Figure 3.2 - Three types of Stockbridge dampers (from above SB8, SB12 and SB16)

Table 3.1 Material properties of each Stockbridge dampers

Symbol		SB8	SB12	SB16
m	Mass of weight, kg	1.815	2.75	3.65
J	Moment of inertia of weight, kg.m^2	0.01156	0.033959	0.04071
L	Length of messenger cable, m	0.1885	0.227	0.2645
l	Distance between messenger cable end and center of mass, m	0.0067	0.0157	0.0193
K	Stiffness of messenger cable, N/m	7907.65	6480.07	4096.19

For the excitation experiment, load cell and accelerometer were placed on the shaking table and Stockbridge damper was installed on them.

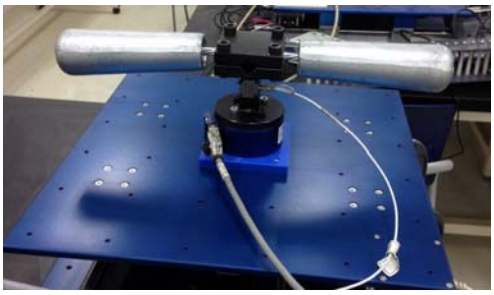

Figure 3.3 - Setting of the excitation experiment

Operating the shaking table with amplitude 2.5mm, data of force and velocity was

measured for 20 seconds with 200 per second of sampling frequency between 5Hz and 20Hz in the frequency interval of 0.5 Hz. With the measured data, the dynamic properties(Z and α) of Stockbridge damper can be extracted using the following formula,

$$\dot{y}(t) = \dot{y}_0 \sin \omega t \quad \text{and} \quad F(t) = \hat{F} \sin(\omega t + \alpha) = Z\dot{y}_0 \sin(\omega t + \alpha) \qquad (3.10)$$

where $\dot{y}$ is velocity of Stockbridge damper, F is force of Stockbridge damper.

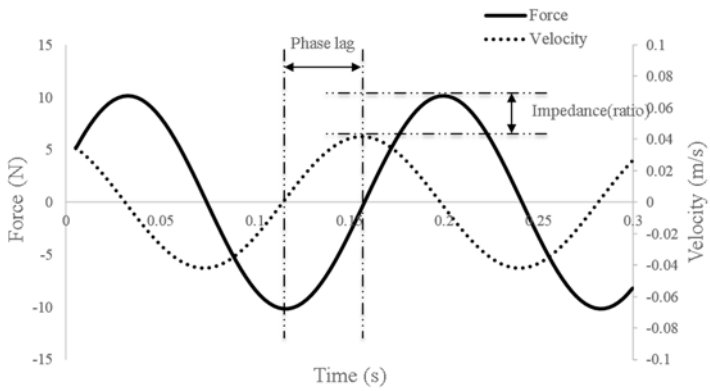

Figure 3.4 - Example of measured data (SB8, 6Hz)

3.2.3 Comparison of dynamic properties

Dynamic properties obtained from 2-DOF model were compared with dynamic properties extracted from the experiment. The results are as follows.

Table 3. 2 Natural frequencies of each Stockbridge damper

Method		SB8	SB12	SB16
Experiment	Natural frequency, Hz	9.6	6.2	4.2
2-DOF model	Natural frequency, Hz	9.1	6.5	4.9

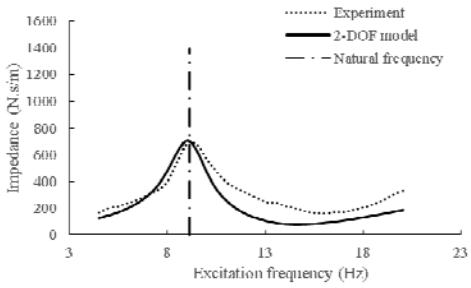

Figure 3.5 - Impedance of SB8

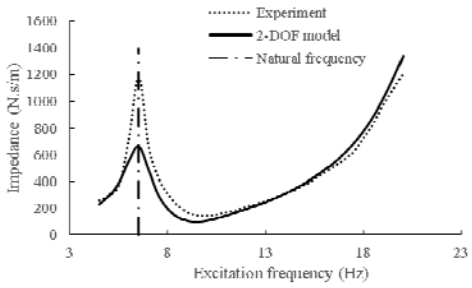

Figure 3.6 - Impedance of SB12

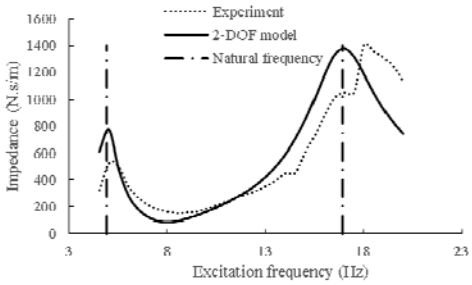

Figure 3.7 - Impedance of SB16

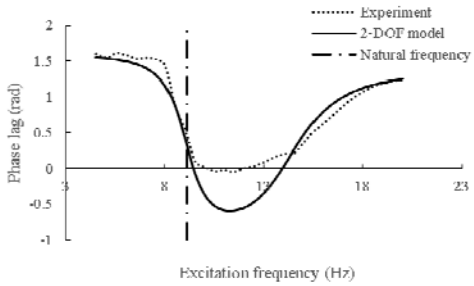

Figure 3.8 - Phase lag of SB8

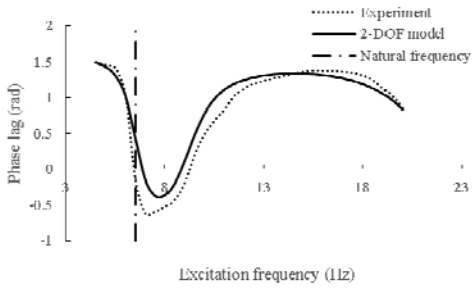

Figure 3.9 - Phase lag of SB12

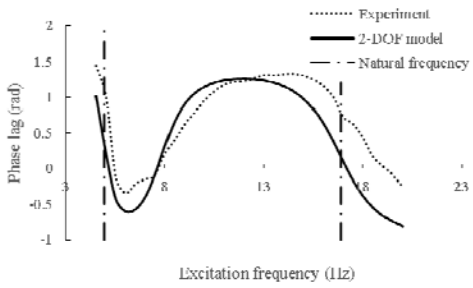

Figure 3.10 - Phase lag of SB16

Natural frequencies of first mode of SB-8, SB-12 and SB-16 dampers are 9.1Hz, 6.5Hz and 4.9Hz respectively. For all Stockbridge dampers, both graphs of impedance obtained from 2-DOF model and excitation experiment have the maximum value on the natural frequency of each damper. For SB16, there is another maximum value in both graphs obtained from 2-DOF model and excitation experiment at the natural frequency of second mode, 16.9 Hz. In the case of phase lag, both the data obtained from 2-DOF model and the data from excitation experiment show similar trend on the target frequency range for all three types of Stockbridge dampers. As a result, these six graphs show that 2-DOF model of Stockbridge damper has rationality.

3.3 Evaluation of amplitude for a target hanger cable

When three types of Stockbridge dampers(SB8, SB12, and SB16) are installed on the target hanger cable, amplitude of vortex-induced vibration is evaluated by Energy Balance Method with dynamic properties obtained from 2-DOF model. At this time, assume that Stockbridge damper is installed 2.8m away from the hinge of the hanger cable because Stockbridge damper mitigates effectively the amplitude of the target hanger cable for target frequency range in that case.

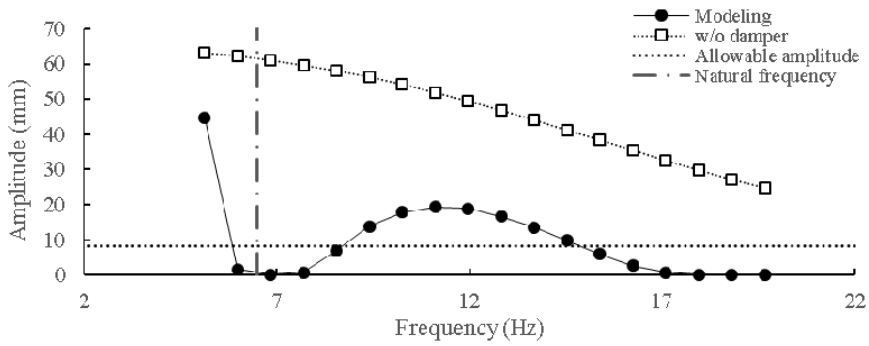

Figure 3.11 - Amplitude of the hanger cable with SB8

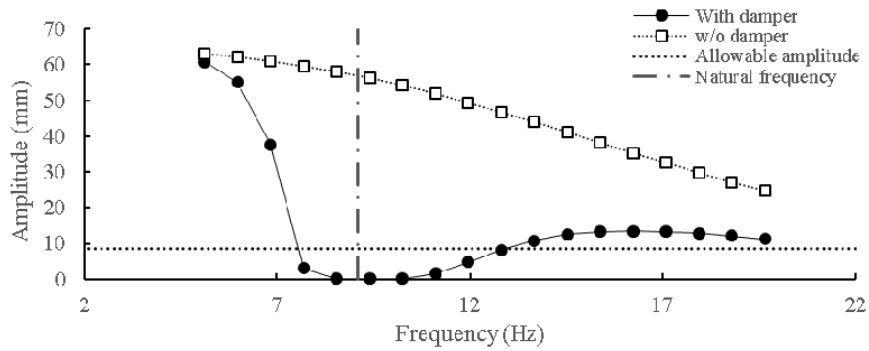

Figure 3.12 - Amplitude of the hanger cable with SB12

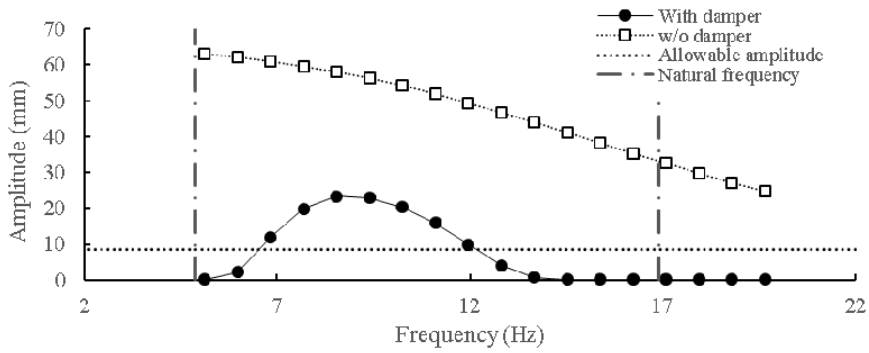

Figure 3.13 - Amplitude of the hanger cable with SB16

When SB8 is installed on the target hanger cable, in the vicinity of 1st mode natural frequency of damper, amplitude of the hanger cable satisfies the allowable amplitude.

Otherwise, the mitigation effect of SB8 is relatively reduced. Similarly, when SB12 is installed, damper can reduce the amplitude of vibration for the target hanger cable, especially near its natural frequency. In the case of SB16, there are 1st and 2nd mode natural frequencies in the target frequency range and near its natural frequencies, SB16 damper has high mitigation effect.

The results of evaluation for the target hanger cable with Stockbridge damper are summarized below.

1. Stockbridge damper mitigates effectively the amplitude of the hanger cable in the vicinity of its natural frequencies.

2. Pre-made Stockbridge damper can't satisfy the allowable amplitude for all target frequency range, so it is necessary to design a new Stockbridge damper which can widely works.

CHAPTER 4

Design of Stockbridge damper for the hanger cable

Because of narrow effective range, mitigation effect of pre-made Stockbridge can't satisfy the allowable amplitude. Asymmetrical damper is known to be effective for wider range due to different natural frequencies of top and bottom.[3] This chapter suggest how to design asymmetrical Stockbridge damper for the target hanger cable.

4.1 Design process

In the design of asymmetrical Stockbridge damper, the following method were adopted.

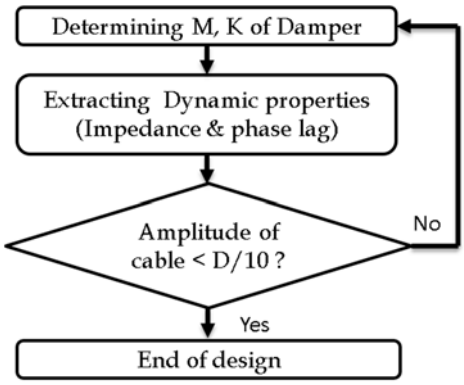

Figure 4.1 - Design process of Stockbridge damper

Step1. Reflecting the target frequency range, determine material properties of Stockbridge damper. For example, set first and second natural frequencies of heavy

side in the target frequency range and set first natural frequency of light in the middle of the target frequency range.

Step2. Using material properties from step1, calculate the dynamic properties of Stockbridge damper from 2-DOF model.

Step3. Evaluate amplitude of the target hanger cable by Energy Balance Method and check whether the result satisfies the allowable amplitude or not. If mitigation effect of Stockbridge damper is not enough, increase material properties.

Step4. If mitigation effect is enough, finish the design process.

4.2 Dsigned Stockbridge damper

By proposed design process, Stockbridge damper for the target hanger cable is as follows.

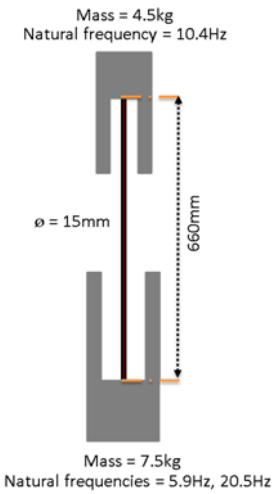

Figure 4.2 - Designed Stockbridge damper for the target hanger cable

Length of steel messenger cable is 660mm, diameter is 15mm, mass of heavy side is 7.5kg, 1st natural frequency is 5.9Hz, 2nd natural frequency is 20.5Hz, mass of light side is 4.5kg and 1st natural frequency is 10.4Hz. At this time, natural frequencies of heavy side belong to target frequency range and 1st natural frequency of light side is in the middle of natural frequencies of heavy side.

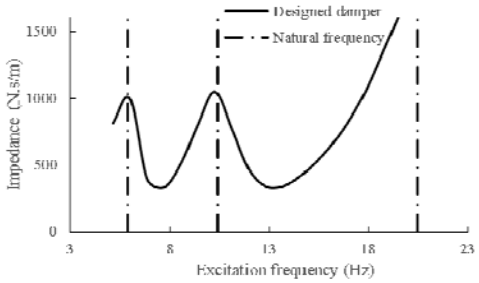

Figure 4.3 - Impedance of designed Stockbridge damper

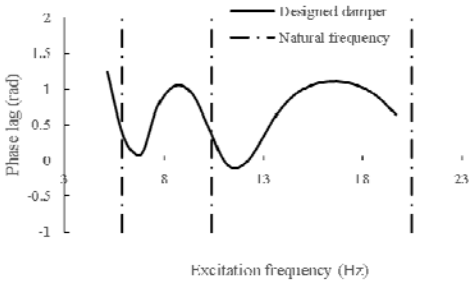

Figure 4.4 - Phase lag of designed Stockbridge damper

Dynamic properties of designed Stockbridge damper is calculated from 2-DOF model. Figure 4.3 shows that the impedance of designed Stockbridge damper has three maximum values on natural frequencies of Stockbridge damper. Using these dynamic properties, amplitude of the target hanger cable was evaluated by Energy Balance Method.

Figure 4.5 shows that if the designed Stockbridge damper is installed on the target hanger cable, amplitude of the hanger cable satisfies the allowable amplitude on target frequency range.

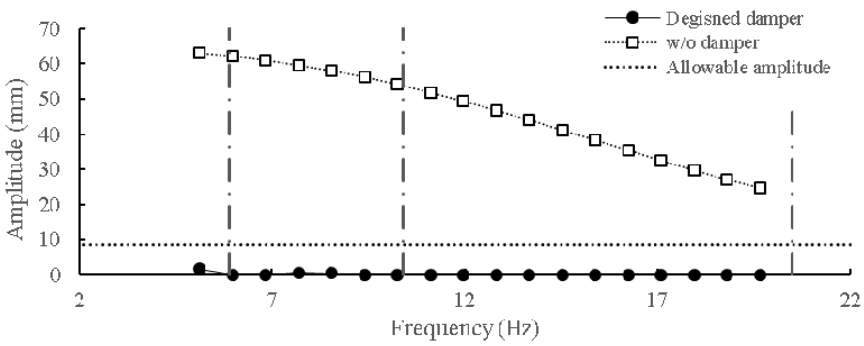

Figure 4.5 - Amplitude of the hanger cable with designed Stockbridge damper

CHAPTER 5

CONCLUSIONS

The objective of this study is to propose a design method of Stockbridge damper for a hanger cable. The following conclusions can be made based on the obtained results.

1. The target hanger cable of the examined suspension bridge was selected. Through the wind velocity measured by anemometer on the pylon of the bridge, the frequency of vortex-induced vibration for the target hanger cable was calculated to be 5~20Hz and it was chosen as the target frequency range for Stockbridge damper to cover.
2. Through a literature survey, based on Energy Balance method, an equation to evaluate the amplitude of vibration for a hanger cable was proposed.
3. Stockbridge damper was modeled as 2-DOF model and the equation of motion for Stockbridge damper was set up. For verification of 2-DOF model, the excitation experiment was performed and the properties from 2-DOF model and from the experiment were compared. As a results, it was confirmed that 2-DOF model of Stockbridge damper had rationality.
4. Using 2-DOF model of Stockbridge damper and Energy Balance Method, the amplitude of the target hanger cable was evaluated. As the results, it was confirmed that Stockbridge damper mitigates effectively the amplitude of

the hanger cable in the vicinity of its natural frequencies.

5. Design process of asymmetric Stockbridge damper for the target hanger cable was set up. If Stockbridge damper designed from this process is installed to the target hanger cable, the amplitude of the cable satisfies the allowable amplitude on target frequency range.

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국문 초록

현수교 행어케이블은 Parallel wire strand (PWS)와 Center fir rope core(CFRC) 두 종류가 있다. 이중 PWS는 케이블의 표면을 폴리에틸렌으로 도금한 것으로, 내구성 및 유지관리에는 장점이 있지만, 매끈한 표면 때문에 내풍성능이 떨어진다는 단점이 있다. 본 논문에서는 PWS에 송전선 등 케이블 진동 제진에 사용되는 댐퍼인 스톡브리지 댐퍼를 이용하여, PWS의 부족한 내풍성능을 보완하고자 하였다.

현수교 행어케이블에 적용할 수 있는 스톡브리지 댐퍼를 설계하기 위해 우선 예시 교량을 설정하고, 그 교량의 행어케이블 중 한 케이블을 대상 행어케이블로 설정하였다. 교량에 설치된 풍속계 데이터를 통해 대상 행어케이블의 와류진동의 진동수 범위를 산정하고, 이를 바탕으로 5~20Hz를 제진 할 목표 진동수 범위로 선정하였다.

스톡브리지 댐퍼의 성능평가를 위해 스톡브리지 댐퍼를 2자유도 모델로 모델링 하였고, 이를 검증하기 위해 SB8, SB12 그리고 SB16, 3종류의 스톡브리지 댐퍼에 대해서 가진 실험을 수행하였다.

2자유도 모델을 통해 구한 댐퍼의 동적 특성과 실험에서 추출한 동적 특성을 서로 비교해 본 결과, 서로 일치하는 경향을 나타내는 것을 확인할 수 있었다.

계산된 스톡브리지 댐퍼의 동적 특성을 이용하여, 스톡브리지 댐퍼가 설치되었을 때 대상 행어케이블의 진폭을 에너지 평형법을 이용해 평가해 보았고, 스톡브리지 댐퍼는 댐퍼의 고유진동수 부근에서 높은 제진효과를 보임을 알 수 있었다. 이 결과를 바탕으로, 대상 행어케이블에 적용할 스톡브리지 댐퍼를 설계해 보았고, 설계된 댐퍼가 대상 행어케이블에 설치되었을 때 진폭을 에너지 평형법으로 평가한 결과 설정한 목표 진동수 범위에서 허용 진폭을 만족함을 확인 할 수 있었다.

주요어: 스톡브리지 댐퍼, 현수교 행어 케이블, 와류진동, 에너지 평형법, 가진 실험.